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INFLUENCE OF SAMPLE SIZE, ESTIMATION METHOD AND NORMALITY ON FIT INDICES IN CONFIRMATORY FACTOR ANALYSIS

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ABSTRACT

In this study, Monte Carlo simulation is used to evaluate the characteristics of CFA fit indices under different conditions (such as sample size, estimation method and distributional conditions). The simulation study was performed using seven different samples where sample has a different sample size such as 50, 100, 200, 400, 800, 1600, 4000, four different estimation methods (Maximum Likelihood, Generalized Least Square, Least Square and Weighted Least Square) and three distribution conditions (normal, slightly non-normal and moderately non-normal). A simulation study was conducted with EQS software to examine the effect of these conditions on the most common eleven fit indices that are studied in CFA and SEM. As a result of this study, all of the factors studied are shown to have an influence on the fit indices.

Indexing terms/Keywords

Confirmatory Factor Analysis, Monte Carlo Simulation, EQS, Structural Equation Modelling, Fit Indices

Academic Discipline And Sub-Disciplines

Statistic; Research; Statistical Method

SUBJECT CLASSIFICATION

E.g., Mathematics Subject Classification; Library of Congress Classification

TYPE (METHOD/APPROACH)

Provide examples of relevant research types, methods, and approaches for this field: E.g., Historical Inquiry; Quasi-Experimental; Literary Analysis; Survey/Interview

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Provide examples of relevant academic disciplines for this journal: E.g., History; Education; Sociology; Psychology; Cultural Studies;

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INTRODUCTION

Structural Equation Modelling (SEM) is an effective method for model testing and development which allows testing of theoretical models as a whole. SEM enables researcher to determine direct and indirect effect between variables. SEM is a multivariate statistical approach which models by including interactions between theoretical structures, measurement errors, and relations between errors in a model [1-4]. SEM is also defined as a comprehensive statistical technique used for testing the causal relationships between observed (manifest) and latent (unobserved) variables. In recent years, it has become a very popular tool for researchers in psychology and educational, social and behavioural sciences. SEM is also a significant statistical approach used in such fields to test the consistency of experimental and non-experimental data with theory [5].

In the present study, for non-normally distributed variables, the effect of estimation methods on the consistency measures was compared with the Monte Carlo simulation. For this purpose, first multivariate normal and abnormal data sets were generated in different sample volumes and later parameters and consistency criteria were computed with four parameter estimation methods for the generated data sets. At the end of the study, the effects of estimation methods used on the consistency criteria were discussed.

MATERIAL AND METHOD

Confirmatory Factor Analysis

Confirmatory factor analysis (CFA) is a part of SEM dealing with measurement models of the relationships between latent and observed variables. The purpose of exploratory factor analysis (EFA) is to discover the structure consisting of a number of unknown factors underlying a set of variables. The presupposition is that any variable depends on any factor. The purpose of CFA is to test statistically the significance of a structure (model) consisting of a known number of factors. In other words, CFA is used to check whether the data of a sample verifies a proposed model [6].

In CFA, three elements must be specified to carry out required analysis. The number of factors, the loading of each observed variable on each factor, and the correlation between each factor pair are pre-specified [7]. CFA model [1,7-9] can be shown as in Equation 1 and the variance-covariance matrix of y is given in Equation 2 as observed variables are defined as, latent factors as and unique variances as.

$$y = \Lambda_y \eta + \varepsilon \quad (1)$$

$$\Sigma = \Lambda_y \Psi \Lambda_y^T + \Theta_\varepsilon \quad (2)$$

∑ in Equation 2 is the pxp symmetric variance-covariance matrix of p number of observed variables. In the CFA model, is a matrix of size pxp of factor loadings. Ψ is a symmetric matrix of size mxm of factor correlations and the diagonal elements of matrix pxp are a vector size of p.

Methods of Estimation Used In Confirmatory Factor Analysis

Maximum Likelihood

Maximum likelihood estimation (MLE) is the most preferred method of estimation [10]. The fitting function of maximum likelihood estimator is as shown in Equation 3.

$$F_{ML} = \log\left|\Sigma(\theta)\right| - \log\left|S\right| + tr\left[S\Sigma(\theta)^{-1}\right] - p \quad (3)$$

In Equation 3, F_ML is the discrepancy function computed for estimation, p is the number of observed variables, and tr is the trace of the matrix [3]. $(N-1)F_{ML}$ also has the distribution χ^2 of the degree of freedom $\frac{1}{2(p(p+1))} - k$ where is the number of unknown parameters [1].

Several studies in the literature have examined the performance of ML estimators in terms of improper solutions, non-convergence, bias of estimators, the size of the sample volume, and the occurrence of normal or non-normal distribution of variables [1, 10, 12-15].

Maximum likelihood estimation requires multivariate normality assumption. A feature of ML estimator is that the information on the first (mean) and second (variance) order moments of observed variables is enough to compute the fitting function. Thus, the third (skewness) and fourth (kurtosis) order moments are not need to compute the fitting function [2].

Some simulation studies have shown that for non-normally distributed variables, the ML estimator was consistent but not efficient enough [16, 17].

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Weighted Least Squares

If variables have continuous but non-normal distribution, then the method of weighted least-squares (WLS) is used for parameter estimation [1, 11]. A number of previous studies have suggested use of WLS estimation for non-normally distributed variables although MLE (or robust MLE) has been shown to have better performance [11]. Contrary to MLE, WLS estimation requires raw data for data analysis.

This estimation method is also referred to as asymptotically distribution free (ADF) estimation in the related literature. ADF estimation is referred to as Weighted Least Squares (WLS) in LISREL and as Arbitrary Distribution Generalized Least Squares (AGLS) in EQS [11].

The minimized fitting function of AGLS is as shown in Equation 4.

$$F_{WLS} = [S - \Sigma(\theta)]' W^{-1} [S - \Sigma(\theta)] \quad (4)$$

In Equation 4, θ represents the vector of parameters; S is the variance-covariance matrix of the sample,$\Sigma(\theta)$ is the reproduced variance-covariance matrix; W^{-1} is the kxk positive definite weight matrix$(k = p(p+1)/2,\ p =$ number of observed variables) which is the inverse of the weight matrix [11].

The main advantage of the AGLS method is that it includes the least assumptions about the distribution of the observed variables. Studies conducted with non-normal variables have observed that the AGLS method is relatively not affected by the characteristics of the distribution [2, 11, 14, 17].

In addition to its advantages, the AGLS method has also disadvantages. An increase in the number of observed variables increases the (k) number, which leads to in turn the rapid growth of the weight matrix. Thus, it would be harder to solve the estimating equations due to the growing weight matrix [1].

The fitting functions of the Generalized Least Squares (GLS) and Least Squares (LS) estimations which are special cases of AGLS are obtained by function customization as presented in Equation 5.

$$F = \frac{1}{2} tr\left[\left(S - \Sigma(\theta)\right) W^{-1}\right]^2 \quad (5)$$

GLS estimator is obtained by replacing the matrix W^{-1} with the variance-covariance matrix of the sample (S) and LS estimator is obtained by replacing the matrix W^{-1} with the unit matrix (I).

Least Squares

In the LS method, the fitting function which is used to evaluate the fitting of the model and for that purpose minimized is as follows in Equation 6.

$$F_{LS} = \frac{1}{2} tr\left[\left(S - \Sigma(\theta)\right)\right]^2 \quad (6)$$

In Equation 6, F_{LS} is the discrepancy function computed for estimation and tr is the trace of the matrix [1, 11, 18, 19].

Generalized Least Squares

In the GLS method, the fitting function which is used to evaluate the fitting of the model and for that purpose minimized is as follows in Equation 7.

$$F_{GLS} = \frac{1}{2} tr\left[\left(S - \Sigma(\theta)\right) S^{-1}\right]^2 \quad (7)$$

Equation 7, F_{GLS} is the discrepancy function computed for estimation; tr is the trace of the matrix; S^{-1}; is the (pxp) weight matrix of the errors [1,11,18,19].

When the normality assumption cannot be met, the AGLS estimation method may be preferred. However, it is necessary to remember that the AGLS estimation method requires larger sample sizes [2, 11, 20].

Model Fit

The model fit determines the fit of the variance-covariance matrix to the structural equation model (SEM). Chi-square test and GFI (Goodness of Fit Index), AGFI (Adjusted Goodness of Fit Index) and RMR (Root Mean Square Residual) indices are widely used for the model fit. The measures basically use the differences between the variance-covariance matrix of the sample (S) and the reproduced variance-covariance matrix ($\sum$). They are known as Chi-square, RMSEA (Root Mean Square Error of Approximation), GFI (Goodness-of-fit Index), AGFI (Adjusted Goodness-of-fit Index), MFI (McDonald's Fit Index), RMR (Root Mean Square Residuals), SRMR (Standardized Root Mean Square Residual).

Model Comparison

Comparative fit indices compute the fit by comparing the proposed model to the null model which is more restrictive. An independent model is what generally assumes that there is no relationship between indicators. They are called IFI (Incremental Fit Index), NFI (Normed Fit Index), TLI (Turker-Lewis Index) - NNFI (Non-normed Fit Index), CFI (Comperative Fit Index).

Monte Carlo Simulation

A Monte Carlo (MC) simulation method can determine the characteristics of the variable distributions using randomly generated numbers [21]. Asymptotic properties of an estimator are usually knows; however its finite-sample properties are not known. MC simulation enables researchers to determine the performances of estimators in finite samples. Knowledge of the sample distribution is the most significant assumption of statistical knowledge of behaviour[22].

For Structural Equation Modelling (SEM), MC simulation has become a fairly common method in evaluating statistical estimations. It is a superior method for examination of the estimations and the goodness-of-fit statistics under several conditions such as large sample size, non-normal distribution, model complexity and misspecification of the model [22].

In this study, the research question was established as the comparison of MLE, LS, GLS and AGLS estimation methods by the fitting criteria used in the EQS in different sample sizes (50, 100, 200, 400, 800, 1600, and 4000) in conditions where the variables meet or fail to meet the assumption of multivariate normality (slight non-normal – moderate non-normal) in the CFA model.

The confirmatory factor analysis (CFA) developed by Raykov and Marcoulides (2006) was used as the research model. The CFA model employed in the present study is given in Figure 1.

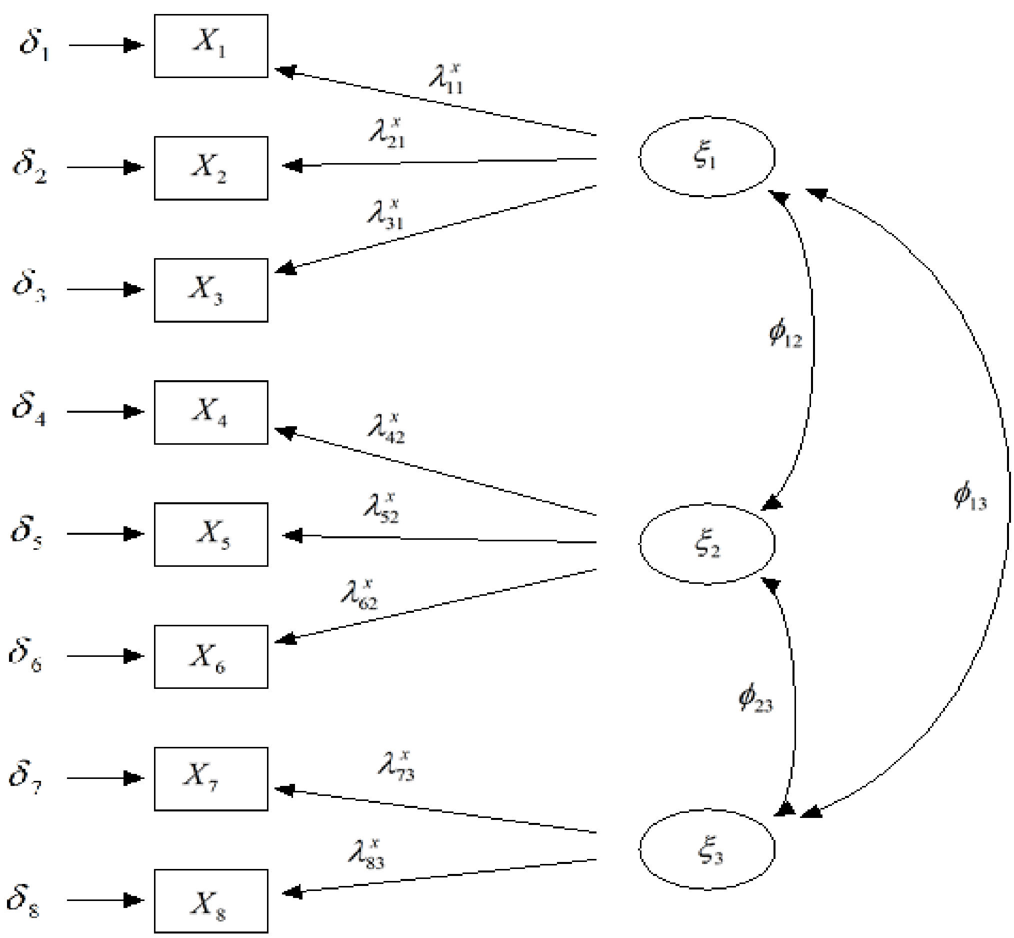

Figure 1. CFA Model[23]

The parameters used in the present study are given in Equations 8, 9 and 10.

$$\Phi = \begin{bmatrix} 1 & & \\ .636 & 1 & \\ .276 & .681 & 1 \end{bmatrix} \quad (8)$$

$$\Lambda_x^T = \begin{bmatrix} .519 & .615 & .521 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .511 & .594 & .595 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .614 & .635 \end{bmatrix} \quad (9)$$

$$\theta_\delta(diag) = \begin{bmatrix} .181 & .182 & .178 & .288 & .308 & .256 & .203 & .217 \end{bmatrix} \quad (10)$$

A comprehensive literature search must be performed to decide which software program to use. Different strengths and weakness of the programs depend on the research question. Thus, the software chosen must demonstrate the most appropriate fit with the research question. In this study, it was decided to use EQS (Version 6.2, Build 104) (Bentler (2013)) for the MC simulation. Successful results of the EQS program in the previous simulation studies [11, 22, 24] as well as its success in generating data from non-normal distribution are considered as the ability to generate data appropriate to the model in a single step.

For the 84 conditions defined in the study, 16800 different data sets will be obtained, with each condition being repeated 200 times. The number of iterations here was determined according to the literature [24, 25] and thought to be sufficient.

Once the mass parameter values and the mass design of the target model are established, the researcher creates the mass variance-covariance matrix. Determining the covariance matrix, whether the assumption of multivariate normality is met or not must be considered. The multivariate normality of the variance-covariance matrix and its level of non-normality are determined by the skewness and kurtosis. In accordance with the research problem, three different variance-covariance matrices are necessary. The characteristics of these matrices are defined as follows. The first matrix was taken from the study of Raykov and Marcoulides (2006) and has a skewness coefficient of 0 and a kurtosis coefficient of 0. For the covariance matrices to be obtained by the two other conditions, that is, by the occurrence of non-normality, the first of two conditions required to be satisfied is the level of non-normality and the second is the multivariate use of this level. Fleishman's method is used to satisfy these conditions. To do this, first it is necessary to generate 8 one-dimensional variables having the required coefficients of skewness and kurtosis using the Fleishman coefficients and then to obtain the required variance-covariance matrix using these variables [16, 25, 26]. For the slight non-normal variance-covariance matrix, the Fleishman coefficients having a skewness value of 0,75 and a kurtosis value of 2 were used. For the moderate non-normal variance-covariance matrix, the Fleishman coefficients having a skewness value of 2 and a kurtosis value of 7 were used [26]. Generating data in this way, previous studies have obtained highly successful results [16, 25-28].

The iteration number was determined to be 25. ESQ provides non-convergent solutions and incorrect solutions in fit output files. It is an advantage of using ESQ.

Results

The present study, in which the simulation study was conducted with the assumption of multivariate normality and slight non-normal and moderate non-normal variables, examined the results of , GFI, AGFI, IFI, MFI, NFI, NNFI, CFI, RMR, SRMR and RMSEA in accordance with the outputs obtained by using different sample sizes and estimation methods. 1758 errors arising from the simulation study was excluded from the study and the result of 15042 solutions was examined.

In the simulation results, NOR represents that the assumption of multivariate normality is met, SNN is the slight non-normal distribution, and MNN is the moderate non-normal distribution. LS represents least squares method, GLS represents generalized least squares method, ML represents maximum likelihood method, and AGLS represents weighted least squares method. Lastly, the fit indices were referred to with their English abbreviations as defined in the literature.

In the simulation study, in accordance with the characteristics of the fit measures, the values of , RMSEA, RMR and SRMR are expected to be close to 0 while the values of NNFI and CFI are expected to be close to 1 when a true model is studied. This being given, IFI and NNFI were exempted from the evaluation because they yielded inconsistent results as shown in Tables 1, 2 and 3.

Table 1. Simulation Result (The mean fit indices)

		Chi-square				GFI				AGFI				IFI			
		LS	GLS	ML	AGLS	LS	GLS	ML	AGLS	LS	GLS	ML	AGLS	LS	GLS	ML	AGLS
NOR	50	22.277	18.250	22.323	33.763	0.987	0.907	0.907	0.832	0.972	0.803	0.803	0.645	0.969	0.985	0.968	0.896
	100	26.037	23.571	26.115	31.414	0.992	0.940	0.942	0.923	0.984	0.874	0.877	0.836	0.973	0.927	0.972	0.908
	200	34.308	33.212	34.340	38.740	0.995	0.958	0.961	0.952	0.990	0.912	0.918	0.899	0.973	0.908	0.973	0.904
	400	50.405	51.426	50.464	55.265	0.997	0.968	0.971	0.966	0.993	0.932	0.939	0.927	0.974	0.902	0.974	0.902
	800	83.094	87.277	83.161	90.859	0.998	0.973	0.976	0.972	0.995	0.942	0.950	0.940	0.974	0.899	0.974	0.900
	1600	150.188	160.642	150.230	164.439	0.998	0.975	0.979	0.974	0.996	0.947	0.955	0.946	0.974	0.897	0.974	0.898
	4000	343.136	370.741	343.214	374.657	0.998	0.977	0.981	0.977	0.996	0.951	0.959	0.951	0.975	0.898	0.975	0.899
SNN	50	14.968	13.532	15.916	23.652	0.947	0.931	0.931	0.882	0.888	0.854	0.853	0.751	1.593	5.269	1.637	0.937
	100	13.718	14.555	15.305	18.497	0.971	0.963	0.964	0.954	0.938	0.922	0.924	0.902	1.495	2.509	1.344	1.072
	200	14.319	14.993	15.480	16.608	0.983	0.981	0.981	0.979	0.964	0.960	0.960	0.956	1.597	0.683	1.598	1.218
	400	15.523	16.050	16.528	17.155	0.990	0.990	0.990	0.989	0.979	0.979	0.978	0.977	1.402	1.362	1.276	1.146
	800	17.000	18.291	18.896	19.156	0.994	0.994	0.994	0.994	0.988	0.988	0.988	0.987	1.109	1.088	1.009	1.016
	1600	20.774	23.241	23.672	24.135	0.996	0.996	0.996	0.996	0.993	0.992	0.992	0.992	0.958	0.879	0.857	0.907
	4000	31.270	35.979	36.208	37.147	0.998	0.998	0.998	0.998	0.995	0.995	0.995	0.995	0.786	0.715	0.703	0.698
MNN	50	14.596	13.232	15.728	23.181	0.949	0.932	0.931	0.884	0.892	0.857	0.855	0.754	1.393	0.948	1.266	0.955
	100	13.606	13.823	15.118	18.062	0.971	0.965	0.964	0.955	0.940	0.926	0.925	0.905	1.348	3.149	1.372	1.043
	200	14.458	14.786	15.056	16.256	0.983	0.981	0.982	0.979	0.964	0.961	0.961	0.956	1.321	1.209	1.396	1.438
	400	14.312	15.555	15.866	16.265	0.991	0.990	0.990	0.990	0.981	0.979	0.979	0.979	3.206	0.448	2.879	0.797
	800	15.940	17.269	17.494	18.181	0.995	0.995	0.995	0.994	0.989	0.989	0.989	0.988	1.288	1.194	1.181	1.080
	1600	19.936	21.998	22.478	22.471	0.997	0.997	0.997	0.996	0.993	0.993	0.993	0.993	0.980	0.915	0.890	0.891
	4000	33.654	36.129	36.893	35.993	0.998	0.998	0.998	0.998	0.995	0.995	0.995	0.995	0.717	0.676	0.668	0.675

Table 2. Simulation Result (The mean fit indices)

		MFI				NFI				NNFI				CFI			
		LS	GLS	ML	AGLS	LS	GLS	ML	AGLS	LS	GLS	ML	AGLS	LS	GLS	ML	AGLS
NOR	50	0.951	0.989	0.951	0.852	0.878	0.681	0.878	0.804	0.944	0.983	0.944	0.815	0.960	0.918	0.961	0.887
	100	0.957	0.968	0.956	0.932	0.924	0.768	0.924	0.812	0.953	0.864	0.953	0.836	0.971	0.909	0.970	0.899
	200	0.958	0.961	0.958	0.948	0.948	0.825	0.948	0.839	0.955	0.838	0.955	0.834	0.972	0.901	0.972	0.899
	400	0.959	0.958	0.959	0.953	0.961	0.859	0.961	0.864	0.957	0.833	0.957	0.835	0.974	0.898	0.974	0.900
	800	0.960	0.957	0.960	0.955	0.968	0.878	0.968	0.880	0.958	0.831	0.958	0.833	0.974	0.897	0.974	0.899
	1600	0.959	0.956	0.959	0.955	0.971	0.886	0.971	0.887	0.957	0.829	0.957	0.830	0.974	0.896	0.974	0.897
	4000	0.960	0.957	0.960	0.956	0.973	0.894	0.973	0.894	0.958	0.832	0.958	0.832	0.975	0.898	0.975	0.898
SNN	50	1.022	1.037	1.013	0.940	0.521	0.497	0.482	0.714	84.652	-2.109	-0.425	0.885	0.465	0.376	0.412	0.881
	100	1.017	1.013	1.009	0.993	0.551	0.494	0.501	0.600	1.294	1.109	0.993	0.807	0.564	0.508	0.519	0.828
	200	1.007	1.005	1.004	1.001	0.542	0.505	0.504	0.559	0.392	1.347	-0.243	0.168	0.564	0.520	0.553	0.724
	400	1.002	1.001	1.001	1.000	0.538	0.517	0.506	0.532	4.760	-1.221	0.768	3.124	0.560	0.535	0.533	0.634
	800	1.000	0.999	0.999	0.999	0.561	0.525	0.508	0.527	0.982	0.904	0.697	0.548	0.741	0.698	0.663	0.679
	1600	0.999	0.998	0.998	0.998	0.582	0.532	0.521	0.534	0.160	0.408	-0.072	0.635	0.786	0.700	0.682	0.700
	4000	0.998	0.998	0.998	0.997	0.607	0.554	0.542	0.542	0.570	0.425	0.398	0.392	0.735	0.651	0.635	0.630
MNN	50	1.026	1.040	1.015	0.944	0.526	0.500	0.484	0.718	0.274	-0.709	-1.688	0.452	0.501	0.334	0.469	0.885
	100	1.017	1.016	1.010	0.995	0.546	0.514	0.498	0.602	-0.123	1.218	-0.039	12.016	0.563	0.520	0.499	0.824
	200	1.006	1.006	1.005	1.002	0.534	0.501	0.508	0.558	-2.569	11.247	-3.254	0.079	0.581	0.558	0.559	0.706
	400	1.003	1.002	1.001	1.001	0.556	0.513	0.515	0.542	6.154	0.762	6.446	0.295	0.584	0.531	0.551	0.697
	800	1.001	1.000	1.000	0.999	0.565	0.520	0.516	0.526	-0.089	0.995	-0.838	0.547	0.732	0.667	0.665	0.666
	1600	0.999	0.998	0.998	0.998	0.568	0.524	0.518	0.521	0.732	1.105	0.345	0.516	0.760	0.675	0.648	0.668
	4000	0.998	0.998	0.998	0.998	0.538	0.509	0.504	0.509	0.415	0.328	0.315	0.327	0.642	0.589	0.582	0.589

Table 3. Simulation Result (The mean fit indices)

		RMR				SRMR				RMSEA			
		LS	GLS	ML	AGLS	LS	GLS	ML	AGLS	LS	GLS	ML	AGLS
NOR	50	0.031	0.057	0.034	0.099	0.059	0.103	0.063	0.176	0.068	0.040	0.068	0.131
	100	0.024	0.035	0.025	0.052	0.043	0.063	0.046	0.092	0.065	0.055	0.065	0.085
	200	0.019	0.025	0.020	0.032	0.034	0.045	0.036	0.058	0.068	0.065	0.068	0.076
	400	0.015	0.020	0.016	0.024	0.028	0.035	0.029	0.042	0.069	0.070	0.069	0.074
	800	0.014	0.016	0.014	0.018	0.024	0.029	0.025	0.032	0.069	0.071	0.069	0.073
	1600	0.013	0.015	0.013	0.016	0.022	0.026	0.023	0.028	0.070	0.072	0.070	0.073
	4000	0.012	0.014	0.012	0.014	0.021	0.024	0.021	0.025	0.069	0.072	0.069	0.072
SNN	50	0.083	0.119	0.087	0.140	0.084	0.116	0.087	0.139	0.023	0.014	0.029	0.074
	100	0.059	0.076	0.063	0.083	0.060	0.076	0.062	0.082	0.008	0.011	0.016	0.028
	200	0.044	0.051	0.045	0.053	0.044	0.050	0.045	0.053	0.007	0.009	0.010	0.014
	400	0.033	0.035	0.033	0.037	0.033	0.035	0.033	0.036	0.009	0.009	0.011	0.012
	800	0.025	0.026	0.025	0.026	0.025	0.026	0.025	0.026	0.008	0.010	0.011	0.012
	1600	0.020	0.020	0.020	0.021	0.020	0.020	0.020	0.020	0.010	0.013	0.013	0.014
	4000	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.013	0.016	0.016	0.016
MNN	50	0.081	0.116	0.086	0.138	0.083	0.115	0.086	0.138	0.022	0.012	0.028	0.072
	100	0.058	0.073	0.061	0.081	0.059	0.073	0.061	0.081	0.008	0.008	0.015	0.028
	200	0.044	0.050	0.044	0.052	0.045	0.050	0.044	0.052	0.008	0.007	0.009	0.013
	400	0.031	0.034	0.032	0.035	0.032	0.034	0.032	0.035	0.006	0.008	0.009	0.009
	800	0.024	0.025	0.024	0.025	0.024	0.025	0.024	0.025	0.006	0.008	0.009	0.010
	1600	0.019	0.020	0.019	0.020	0.019	0.020	0.019	0.020	0.009	0.012	0.013	0.013
	4000	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.015	0.016	0.016	0.016

When the chi-square goodness-of-fit criterion is evaluated according to Table 1, LS and ML estimation methods yielded similar results in all the sample sizes and under all the distributional conditions. The chi-square test gave great values for NOR in the sample sizes of 400, 8000 and 1600 in all the estimation methods while it rejected the model fit in the sample size of 4000. According to all the estimation methods and in all the sample sizes, the chi-square test yielded better fit results for SNN and MNN.

Under all the distributional conditions and all the sample sizes, GFI yielded better results in the LS estimation method. Under all the distributional conditions, as the sample size increased, so did the fit level. Under all the distributional conditions, AGLS did not yield appropriate result for the sample size of 50 units.

While AGFI yielded better results for LS, the model turned to be unfit for the sample sizes of 200 and smaller under the normality assumption. GLS and ML estimation methods resulted in poor fit for the sample sizes of 50 and 100 units. As for SNN and MNN, it resulted in poor fit only for the sample size of 50.

For only AGLS estimation method and under the normality assumption, MFI indicated misfit in the sample size of 50 units while it showed high fit in all other conditions. It was seen that MFI could take values greater than 1 in the true model.

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NFI revealed the misfit of the model under SNN and MM distributional conditions in all the sample sizes. For NOR, in the ML and LS estimation methods, as the sample size increased, so did the model fit; and the fit was poor for the sample size of 50 units. For NOR, in the GLS and AGLS estimation methods, the model was found misfit in all the sample sizes.

CFI revealed the misfit of the model under SNN and MM distributional conditions in all the sample sizes. For NOR, in the GLS and AGLS estimation methods, the model was found misfit in all the sample sizes.

Considering all the conditions, RMR fit index yielded similar results. Furthermore, under all the distributional conditions and in all the sample sizes, as the sample size increased, the RMR value got closer to zero. It indicates the increase in the model fit.

For SNN and MNN, SRMR fit index yielded very similar results to RMR. As for NOT, it yielded higher results than RMR.

According to RMSEA fit index, LS and ML estimation methods had similar results. For SNN and MNN, the model fit is much better while the best results were obtained in the sample sizes of 200 and 400 units.

A factorial ANOVA was conducted to evaluate the significance of the differences in the simulation results. Table 4 shows p probability (significance) values derived from the analysis.

Chi-square, MFI, NFI, CFI, RMR SRMR and RMSEA fit criteria in Table 4 were significant under all conditions involving main effects and interaction. Considering the distributional conditions and sample size interaction, there was no significant difference in GFI and AGFI.

Table 4. P Values

	Chi-square	GFI	AGFI	IFI	MFI	NFI	NNFI	CFI	RMR	SRMR	RMSEA
DC	<0.001	<0.001	<0.001	0.020	<0.001	<0.001	0.486	<0.001	<0.001	0.002	<0.001
S	<0.001	<0.001	<0.001	0.013	<0.001	<0.001	0.576	<0.001	<0.001	<0.001	<0.001
PT	<0.001	<0.001	<0.001	0.183	<0.001	<0.001	0.395	<0.001	<0.001	<0.001	<0.001
DC*S	<0.001	0.977	0.976	0.075	<0.001	<0.001	0.434	<0.001	<0.001	<0.001	<0.001
DC*PT	<0.001	<0.001	<0.001	0.448	<0.001	<0.001	0.343	<0.001	<0.001	<0.001	<0.001
S*PT	<0.001	<0.001	<0.001	0.173	<0.001	<0.001	0.313	<0.001	<0.001	<0.001	<0.001
DC*S*PT	<0.001	<0.001	<0.001	0.409	<0.001	<0.001	0.420	<0.001	<0.001	<0.001	<0.001

Table 5 shows the result of the pairwise comparison of the distributional conditions. According to Table 5, the comparisons of the NOR-SNN and NOR-MNN pairs were significant in all fit indices. The comparisons of the SNN-MNN were not significant in Chi-square, GFI, AGFI, MFI and CFI at a significance level of %5. However, they were significant in NFI, RMR, SRMR and RMSEA.

Table 5. Pairwise Comparisons of Distributional Conditions

	Chi-square	GFI	AGFI	MFI	NFI	CFI	RMR	SRMR	RMSEA
NOR-SNN	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
NOR-MNN	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
SNN-MNN	0.067	0.062	0.063	0.091	0.001	0.145	<0.001	0.031	0.013

Table 6 shows the pairwise comparison of the sample sizes. The Chi-square statistic was significant only for the comparison between the sample size of 50 units and the sample size of 100 units. For GFI, AGFI, RMR and SRMR, the differences between all the pairwise comparisons were significant. For MFI, the differences between the pairwise comparison of the sample sizes of 200-400, 400-800, 800-1600, 800-4000 and 1600-4000 were not significant. For CFI, the differences between the sample sizes of 100-200, 200-400 and 800-1600 were not significant. For RMSEA, the differences between the sample sizes of100-1600, 100-4000, 200-400, 200-800 and 400-800 were not significant.

Table 6. Pairwise comparisons of Size

		Chi-square	GFI	AGFI	MFI	NFI	CFI	RMR	SRMR	RMSEA
50	100	.743	<0.001	<0.001	<0.001	.103	<0.001	<0.001	<0.001	<0.001
	200	<0.001	<0.001	<0.001	<0.001	.006	<0.001	<0.001	<0.001	<0.001
	400	<0.001	<0.001	<0.001	<0.001	<0.001	.001	<0.001	<0.001	<0.001
	800	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	1600	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	4000	<0.001	<0.001	<0.001	.002	<0.001	<0.001	<0.001	<0.001	<0.001
100	50	.743	<0.001	<0.001	<0.001	.103	<0.001	<0.001	<0.001	<0.001
	200	<0.001	<0.001	<0.001	<0.001	.248	.520	<0.001	<0.001	<0.001
	400	<0.001	<0.001	<0.001	<0.001	<0.001	.044	<0.001	<0.001	<0.001
	800	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	1600	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.245
	4000	<0.001	<0.001	<0.001	<0.001	<0.001	.028	<0.001	<0.001	.100
200	50	<0.001	<0.001	<0.001	<0.001	.006	<0.001	<0.001	<0.001	<0.001
	100	<0.001	<0.001	<0.001	.040	.248	.520	<0.001	<0.001	<0.001
	400	<0.001	<0.001	<0.001	.095	.002	.175	<0.001	<0.001	.943
	800	<0.001	<0.001	<0.001	.004	<0.001	<0.001	<0.001	<0.001	.699
	1600	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.002
	4000	<0.001	<0.001	<0.001	<0.001	<0.001	.005	<0.001	<0.001	<0.001
400	50	<0.001	<0.001	<0.001	<0.001	<0.001	.001	<0.001	<0.001	<0.001
	100	<0.001	<0.001	<0.001	<0.001	<0.001	.044	<0.001	<0.001	<0.001
	200	<0.001	<0.001	<0.001	.095	.002	.175	<0.001	<0.001	.943
	800	<0.001	<0.001	<0.001	.229	.014	<0.001	<0.001	<0.001	.753
	1600	<0.001	<0.001	<0.001	.032	<0.001	<0.001	<0.001	<0.001	.002
	4000	<0.001	<0.001	<0.001	.023	<0.001	<0.001	<0.001	<0.001	<0.001
800	50	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	100	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	200	<0.001	<0.001	<0.001	.004	<0.001	<0.001	<0.001	<0.001	.699
	400	<0.001	<0.001	<0.001	.229	.014	<0.001	<0.001	<0.001	.753
	1600	<0.001	<0.001	<0.001	.337	.019	.311	<0.001	<0.001	.005
	4000	<0.001	<0.001	<0.001	.275	.002	<0.001	<0.001	<0.001	<0.001
1600	50	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	100	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.245
	200	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.002
	400	<0.001	<0.001	<0.001	.032	<0.001	<0.001	<0.001	<0.001	.002
	800	<0.001	<0.001	<0.001	.337	.019	.311	<0.001	<0.001	.005
	4000	<0.001	.002	.002	.895	.451	<0.001	<0.001	<0.001	.006
4000	50	<0.001	<0.001	<0.001	.002	<0.001	<0.001	<0.001	<0.001	<0.001
	100	<0.001	<0.001	<0.001	<0.001	<0.001	.028	<0.001	<0.001	.100
	200	<0.001	<0.001	<0.001	<0.001	<0.001	.005	<0.001	<0.001	<0.001
	400	<0.001	<0.001	<0.001	.023	<0.001	<0.001	<0.001	<0.001	<0.001
	800	<0.001	<0.001	<0.001	.275	.002	<0.001	<0.001	<0.001	<0.001
	1600	<0.001	.002	.002	.895	.451	<0.001	<0.001	<0.001	<0.001

Table 7 shows the pairwise comparisons of the estimation methods (LS-GLS, LS-ML, LS-AGLS, GLS-ML, GLS-AGLS, ML-AGLS) according to all the fit criteria when the normality condition and sample size are not taken into account. The comparisons yielded significant differences.

Table 7. Pairwise Comparisons of Prediction Technique

		Chi-square	GFI	AGFI	MFI	NFI	CFI	RMR	SRMR	RMSEA
LS	GLS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.010
	ML	<0.001	<0.001	<0.001	.001	<0.001	<0.001	<0.001	.001	<0.001
	AGLS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
GLS	LS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.010
	ML	<0.001	.009	.008	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	AGLS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
ML	LS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	.001	<0.001
	GLS	<0.001	.009	.008	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	AGLS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
AGLS	LS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	GLS	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	ML	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

When Table 8 is examined according to the results of the factorial ANOVA, the most affected criterion by the distributional conditions was chi-square, which was followed by NFI and RMSEA. Chi-square was the most affected by the sample size and as the second most affected indices, RMR and SRMR were equally affected. When the distributional condition and

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the sample size were evaluated together, the most affected was chi-square, which was followed by RMR. The other measures were little affected. When being evaluated with the estimation method, the distributional condition affected GFI and AGFI. When the sample size was evaluated together with the estimation method, it affected GFI, AGFI, RMR and SRMR. When the distributional condition, the sample size and the estimation method were evaluated together, the effect on all the measures was weak.

Table 8. Partial Eta Square

	Chi-square	GFI	AGFI	MFI	NFI	CFI	RMR	SRMR	RMSEA
DC	.905	.196	.196	.368	.807	.235	.405	.001	.566
S	.902	.751	.751	.007	.016	.017	.811	.804	.066
PT	.032	.294	.294	.070	.070	.021	.211	.239	.062
DC*S	.934	<0.001	<0.001	.024	.078	.010	.299	.033	.017
DC*PT	.018	.216	.216	.004	.112	.023	.009	.063	.004
S*PT	.022	.371	.371	.183	.085	.035	.324	.348	.130
DC*S*PT	.027	.072	.072	.007	.035	.020	.010	.072	.006

In the next section, the results are compared with the literature and the significant findings are interpreted.

Discussion

The present study concludes that the use of arbitrary distribution generalized least squares (AGLS) method is inappropriate when the sample size is smaller than 200. Boomsma and Hoogland (2001) have observed similar results in their study. In their simulation study, Orson et al. (2002) have suggested that AGLS method is preferable to maximum likelihood estimation (MLE) and generalized least squares (GLS) estimation methods for the sample sizes of 1000 or more and for different values of kurtosis. In the present study, the four estimation techniques yielded close results for sample sizes of 400 units or more and for the deviations from multivariate normality.

Under the assumption of multivariate normality, the chi-square fit measure yielded unrealistic results by taking greater values as the sample size increased. However, the chi-square fit was not very much affected by the increase in the sample size when the assumption of multivariate normality was not considered. As IFI and NNFI yielded inconsistent results for the slight non-normal and moderate non-normal distributions where the normality assumption was not met, it may be suggested not to use IFI and NNFI. NFI and CFI yield good results only for ML and LS estimation methods and only when the normality assumption is met; thus, it may be suggested to use NFI and CFI only under such conditions.

The present study examined two conditions where the normality assumption was violated. In order to obtain more detailed results, further studies may increase the study scope by dealing with different non-normal distributional conditions and increasing the degree of deviation from normality. Since it is thought that the sample sizes larger than 4000 units would yield better results in cases when the normality assumption is not met, larger sample sizes (5000, 7500, 10000, etc.) may be examined. In conclusion, although the literatures have observed that EQS program produce fairly good results, similar cases may be examined using different simulation software programs and the results may be compared.

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