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Monetary Growth Models: An Evaluation on Tobin Model

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ABSTRACT

Growth models have an important place in the economics. These models try to represent the behavior of an economy so that we can understand and analyze the growth mechanism in economies. Growth theory first focusing on the production function tried to explain the growth in the physical production. However, in the real world, if the saving part of that production is not channeled into physical capital, the scenario of the early growth models cannot reflect the real world. In this sense, Tobin model has made the first step including money option for the individual saving portfolio and turned the growth model into the monetary growth model. In this study, we revisited important discussions on Tobin model and by conducting a critical analysis on the critically important features we are hoping to attract attention to the uncompleted arguments. In this manner we evaluate that the stableness of the equilibrium can be translated differently so that point needs to be cleared in described classification. The critical first step of articulation of saving portfolio can put the model in a strategic reference point situation as we would like to underline.

Indexing terms/Keywords

Tobin model; monetary growth model; growth; monetary growth; saving portfolio

Academic Discipline And Sub-Disciplines

Economics; Growth theory

SUBJECT CLASSIFICATION

JEL Codes: O41; O42; E10; B22

TYPE (METHOD/APPROACH)

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INTRODUCTION

Monetary growth models can be seen as a part of the evaluation of the growth theory. In this development stages, first we can see the development of the Harrod- Domar (Harrod, 1939; Domar, 1946) and Neo- Clasical real growth models. In Harrod- Domar model and Neo-classical real growth model[1], the effects of real variables are taken into account, yet the effects of the monetary variables are not considered in the models. The existence of the money and its effects on the economic variables and parameters are neglected. In 'non-monetary' models there is no place for the portfolio selection; and the wealth owners can only save capital goods as assets. But of course, the existence of the various monetary assets, in addition to real assets, can generate different results in the analysis.

In this development path, after the real growth models, it is needed to form monetary models in order to analyze the effects of the monetary factors on the capital intensity. Combining the growth theory with the monetary theory, Tobin (1965) was first to take the money as a variable into account in a growth model. Such an inclusion can analyze the existence of the long- term relationship of money supply with the real variables such as the equilibrium capital intensity, real wage and the per capita consumption. For the case of existence of such determined relationship, it can be searched for the optimum level of money supply in order to maximize the long- term growth rate. In this development path, Tobin (1965) model represents the first step for its inclusion of money in the frame of a neo- classical real growth model.

In this paper, summarizing the properties of the Tobin model, the first example of the neoclassical monetary models, and comparing their distinctive features with real growth models, we aim to analyze the distinctive parts of the logic of the monetary growth models. Moreover, evaluating their place in the growth models spectrum we aim to contribute the discussion on the subject. Second part of the paper investigates the main properties of a neoclassical monetary growth models by reproducing Tobin model. Third part raises a discussion through comparing models. Fourth part concludes the paper.

2. A NEO- CLASICAL MONETARY GROWTH MODEL: THE TOBIN MODEL

The neo- classical monetary growth models take the Solow model as basic and integrate the monetary variables to the model. A discussion on the effects of the volume of monetary assets on the steady- state equilibrium conditions became possible first after the neo- classical monetary growth models bringing money into the neo- classical growth frame.

Several basic assumptions of the neo- classical monetary growth models can be counted. Investment (or capital accumulation rate) is always equal to the intended savings. The markets are on equilibrium being independent from the price variations. Lastly, the wealth of the wealthy people consists of real and monetary assets.

2.1. Tobin Monetary Growth Model

The monetary variables and the price changes were first included in the economic growth models by the paper of Tobin (1965) in the literature. Money is accepted as a value storing device as the capital goods do. For the very same reason, while a part, but not all, of the savings transform into the physical capital, the other part of the savings are hold as money in the model.

To understand the dynamics of the model it is better to reproduce the Tobin model. Production level (Y) depends on the labor force in the economy (L) and the existing capital stock (K).

$$Y = Y(K, L) \qquad (1)$$

The production function in Eq. (1) is a first order homogenous function; and the output of the production function can be assumed as a homogenous commodity. Defining y=Y/L and k=K/L, these assumptions can be summarized and the production function of Eq. (1) can be expressed as a function of capital intensity as given in Eq. (2).

$$y = y(k) \qquad \text{where} \quad y' > 0, \quad y' < 0 \qquad (2)$$

As generally used in formulation of the Solow model, small letters here represent the per capita values. Similarly, k=K/L term represents the capital intensity, in other words the capital formation per capita. The input levels to be used in production process depend on their productivity rates, $w = Y_L(K, L)$ and $r = Y_K(K, L)$; and when we express the production function as a function of capital intensity as in Eq. (2), productivity rate of per capita capital, *k*, will be as in Eq. (3) below.

$$rk = y_k = d\,y(k)/\,dk \qquad (3)$$

$$k = K/.L \quad \text{and}$$

[1] It is also known as Solow model atributing to Robert Solow`s (1946) work.

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$$L = L_0 e^{n t} \quad (4)$$

Taking derivative (D = d/dt) of Eq. (4) we reach first dynamic equation of the model in Eq. (5):

$$Dk = (DK)/ L - n K/ L = (DK)/ L - n k \quad (5)$$

So far in building the model there is no difference with the Solow model. The differentiation starts with the expression of the term of the capital change, DK in Equation (5), which gives the dynamic structure to the model. The change in capital in Solow model is directly equal to savings; we need to find out the new equation for savings and therefore for the investment. We will look in how the inclusion of money makes change in savings.

As stated before, inclusion of money in the model constitutes the most important feature of Tobin model. In the model, money affects the economic system through disposable income (Y_D) as in Eq. (6). This inclusion of money appears as two important behavioral modifications feature in Tobin model which differentiate it from its Solowian base. First of all, changing disposable income it affects the consumption and savings; then according to portfolio preference it affects the investments which constitute the dynamic structure of the growth models. As given in Equation (6) the change rate of real money balances adds up to yield of production to form the disposable income.

$$Y_D = Y + D(M/ P) = Y + (M/ P)(\mu - \pi) \quad (6)$$ [2]

In Equation (6) D represents derivation according to time (D = d/dt); μ represents the change in money supply (DM/ M); π represents the change in general price level (DP/ P). Equation (6) shows us a change in the change rate of real money balances changes the disposable income[3].

Savings (S) is a function of disposable income. Individuals save a proportion (s) of their disposable income. Therefore multiplying Eq. (6) we will have Eq. (7).

$$S = s\, Y_d = s\,[Y + (M/ P)(\mu - \pi)] \quad (7)$$

The second behavioral modification that the inclusion of money in Tobin model differentiates it from the Solow model can be introduced at this point. Money stocks are defined as outside money here; and the outside money represents a debt of an entity other than private sector[4]. Wealth (W) is composed of physical capital (K) and real money balances (M/ P). In Tobin model, different from the Solow model, wealth can be hold in terms of money or capital goods. This feature, to underline again, constitutes one of the important differences of Tobin model from Solow model.

$$W = K + M/ P \quad (8)$$

Net increase in wealth is equal to savings.

$$DW = S = D(K + M/ P)$$

$$DW = S = DK + D(M/ P) = DK + (\mu - \pi)\,(M/ P) \quad (9)$$

Investments (I) is equal to net change in capital stock.

$$I = DK \quad (10)$$

Using (9) and (10) investment can be arrange as in Eq. (11) below.

$$I = DK = s\,Y - (I - s)\,(\mu - \pi)\,(M/ P) \quad (11)$$

Wealth owners make their portfolio arrangements according to the return rates of these assets. The return rate, or marginal productivity, of the physical capital (r) is equal to marginal productivity of capital; and the return rate (or real yield) of real money balances is equal to decrease in the general price level (-π).

An argument can be raised at this point, on the price stability concept. Stability of the portfolio preference can be related with the achievement of the price stability. In another words, prices stay unchanged if portfolio preference are in equilibrium. Otherwise, if the wealth owners prefer to hold more physical capital goods in their wealth than real money balances, it will increase the demand of individuals' demand for goods and cause a rise in the general price level (dp/ dt >

[2] $D(M/ P) = DM\,.\,P - DP\,.\,M/ P^2 = (DM/ M)\,.\,M/ P - (DP/ P)\,.\,M/ P = M/ P\,.\,(\mu - \pi)$

[3] Moreover, according to the Eq. (6) we can note that a one-time change in the general price level or nominal money balances can change the real money balances: and consequently, this shift can cause a variation in the disposable income and then the consumption and saving levels.

[4] This outside money is supplied by government by the means of transfer payments. The implicit assumption here is that the government does not make spending on the commodities produced by the private sector.

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0). It can be concluded that the necessary and sufficient condition to achieve the portfolio equilibrium in an economy is the achievement of the price stability (dp/ dt = 0).

Individuals hold a proportion (β) of their incomes as money balances. The opportunity cost of the holding money (i) is equal to capital return rate (r) minus the rate of return of money balances (-π). The opportunity cost of holding money (i) is given in Eq. (12) and the money balance proportion of income (β) is given as a function of the opportunity cost in Eq. (13).

$$i = r - (-\pi)] = r + \pi \quad (12)$$

$$\beta = \beta\,(r + \pi) \quad (13)$$

The last important behavioral feature of Tobin model is that how the money balance preference will be realized in the proportion of wealth. This arrangement mechanism can be reflected into model by alternative methods. Here we use β as the proportion of income to be directed into real balances in order to achieve the aimed wealth proportion of money balances. The equation of money demand:

$$M / P = \beta\, Y \quad (14)$$

Equation (14) can be also related with the quantity equation. Real money balances to be demanded should be backed by the income comes from the production of goods and services. According to Eq. (14) the individuals decide what proportion of their income they will allocate for money balances; and they decide that proportion of β according to the opportunity cost given in Eq. (12) and (13).

Using the equality in (14) we can rewrite the investment in Eq. (11) as depending on income (Y) instead of real money balances as in Eq. (15).

$$I = DK = s\, Y - (1 - s)\,(\mu - \pi)\,\beta\, Y \quad (15)$$

Putting (15) in the place of the DK term in the dynamic equation of Eq. (5) we have Eq. (16):

$$Dk = s\, Y/L - (1 - s)\,(\mu - \pi)\,\beta\, Y/L - n\, k$$

$$Dk = s\, y - (1 - s)\,(\mu - \pi)\,\beta\, y - n\, k \quad (16)$$

In steady state case, change in capital to labor ratio equals zero[5] (Dk = 0), labor force grows by the n rate (DL/ L_0 = n) and the real money balance does not change (and $\mu - \pi = n$)[6]. Therefore, the equilibrium in the steady state will imply the Eq. (17) and the steady state value of the capital intensity is shaped as in Eq. (18).

$$n\, k = [s - (1 - s)\,\lambda\, n]\, y \quad (17)$$

$$k_1 = [s\, y(k)/ n] - (1 - s)\,\lambda\, y(k) \quad (18)$$

To compare that value with the one from Solow model, we can recall the steady- state level of capital intensity was equal to, $k_0 = [s\, y(k)]/ n]$. This comparison can be seen more clearly on the figure. The points A and B in Figure 1 shows the steady state values of capital intensity in Tobin and Solow growth models respectively.

In Figure 1, we can see the dynamics of the Equation (16) which brings the system to the equilibrium values in Eq. (18). In the dynamic equation of Tobin monetary growth model (Eq. 16), the leakages part, $n{\cdot}k$, remains same as in the Solow model; and given as a line in Figure 1. The savings part is different in Tobin model and it is subtracted by the second term of $(1 - s){\cdot}(\mu - \pi){\cdot}\beta{\cdot}y$ which represent the allocation of money balances from total savings to be added to the wealth. Because this second term is positive the savings curve of Tobin model places below that of Solow model. The savings channeled into investment minus the leakage of population growth ($n{\cdot}k$) will be equal to the derivative change in capital intensity (k) given in vertical axis; hence, it will be added to the capital intensity value in next term. Consequently, the intersection point will refer the steady state capital intensity value in which the derivative change of capital intensity, given in Eq. 16, becomes zero. This will refer that point A and point B in Figure 1 will be the steady state values of Solow and Tobin models respectively.

[5] In other words, the capital and labour will grow with same rate in the steady state.
[6] If D (M/ P L) equals zero, it implies D (log M – log P – log L) = 0, or equally $\mu - \pi - n = 0$.

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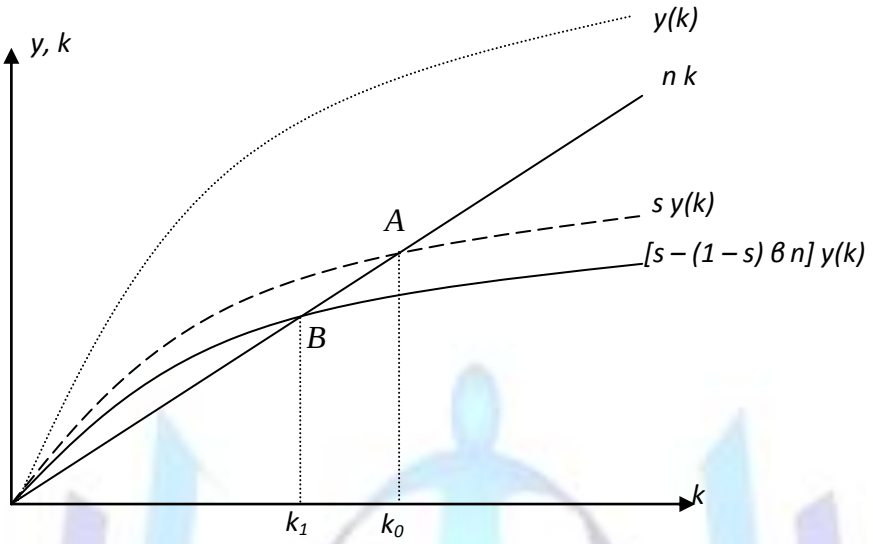

Fig 1: The equilibrium capital intensity levels of Tobin and Solow models

As in Solow model, the point B in Figure 1 refers to a stable equilibrium, although there are some objection to that point in the literature (for the discussion on the issue see, Nagatani, 1970; Johnson, 1967; Stein, 1966) this point is discussed in the next subsection).

2.2. Inferences of Tobin Model and A Discussion

First of all, we can see the steady state capital intensity determined by the Equations (16) to (18) is less than the steady state capital intensity value of Solow model, If we take account of that the marginal saving propensity "s" is less than 1 and positive. In addition to this, per capita consumption and production levels are determined at lower levels (Figure 1). That would indicate that the barter economy is more efficient than the monetary economy. This point has been criticized and constituted a discussion in the literature (see, Patinkin and Levhari, 1968; Patinkin, 1972). The critiques raise the point that if it had been correct the evolution into monetary economy would not have occurred in the history; consequently it contradicts with the historical perspective. As in Patinkin and Levhari model, this point constitutes one of the motivations of the endeavors of developing the preceding models.

Comparing the feature of the model with its predecessors, it is noted that the disequilibrium which occurs when the necessary growth rate differs from the natural growth rate in Harrod-Domar can be solved by the monetary policies in Tobin model. In Harrod-Domar growth model, the necessary growth rate (s/v) is determined by the propensity to saving rate (s) and the capital-yield coefficient (v); natural growth rate, rather, is determined by the population growth rate (n). However there is no automatic mechanism to equate these 2 growth rates. According to Tobin, those 2 rates can be equated by the means of the monetary policy. In such a way that the changing the rate of money supply change (μ) may affect the real variables. For example, in case of the necessary growth rate is greater than natural growth rate the state can increase the return of the real money balances by the means of money policy. In this case first the portfolio equilibrium breaks and then to maintain that equilibrium again some portion of savings is transform from real balance to real money balances. Therefore the capital intensity in the economy decreases and consequently the necessary growth rate decreases and equated to natural growth rate.

These two different discussions from the literature seem also contradicting each other. Inefficiency of a monetary model which denies the historical evolution of today's economies into the modern economic equilibrium state implies an unstable equilibrium for Tobin model. Second discussion, in contrast, arguing that Tobin model actually solves of the "knife-edge" feature of the Harrod- Domar model, draws a totally different picture. An analytical analysis of Nagatami (1970) can be added to this controversy. In Nagatani (1970) Tobin model defined through a production function of per capital (1/ K) terms instead of the per capita terms (1/ L) and they point an unstable equilibrium and imply to a diverging economic structure (Figure 2). According to Figure 2, Tobin model behaves unstable rather than to be a solution to the instability of growth process. We can conclude here that this discussion in the literature is inconclusive needs more attention (see for wider

discussions, Dimand, 2014; Dimand and Durlauf, 2009; Orphanides and Solow, 1990). We evaluate also this first step of including money into the growth models has important to understand the behavior in today economies.

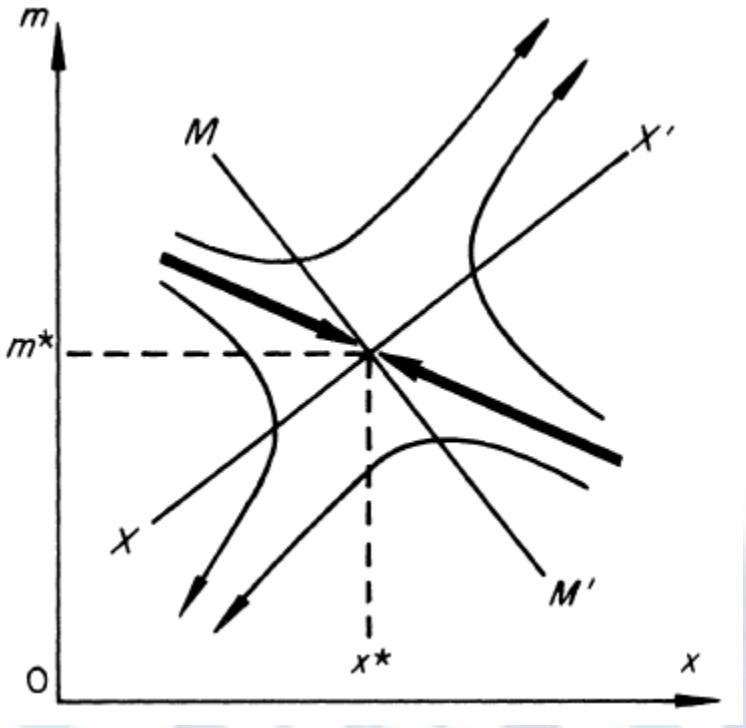

Fig 2: The phase diagram of Tobin model (Source: Nagatani, 1970, p. 173)

As discussed above, changes in money supply affects the capital intensity and growth rate in Tobin model, and it is one of the most important features of the model. Other than the change rate of real money balances, it can be seen from Eq. (6) that a one-time change in real money balances will affect the disposable income changing real money balances base in Eq. (6). It can be thought as that a change of the real money balances will change the calculation of change rate by altering the reference point.

In the analysis of Tobin model, another point is on the price stability concept. Stability of the portfolio preference can be related with the achievement of the price stability. If portfolio preference reaches to a stable equilibrium state, then prices can stay unchanged. Otherwise, in case the wealth owners prefer to hold more physical capital goods in their wealth than real money balances they will increase the demand for goods; and consequently the general price level will rise ($dp/dt > 0$). It means that the portfolio equilibrium in an economy is a condition to achieve a price stability ($dp/dt = 0$) in the economy.

3. CONCLUSION

Tobin model, having included the money preference as a saving option in the growth model, represents an important step in the development of growth models. The model also constitutes the first monetary growth model and precedes to its successors. Although the present day portfolio options are much more articulated as an alternative to the physical capital investment, the model can give us important clues of behavior of the modern economies too. In this concept, this paper tries to raise a discussion by revisiting the model dynamics. Some of the old discussions have been stayed inconclusive on the development track but we evaluate that the Tobin model has a potential to serve as a reference point to the possible new discussions in the growth literature.

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