



Checking presence of excess volatility in forecasting volatility of a set of market indexes through an empirical comparison of three GARCH model

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ABSTRACT

Classical financial theory is based on Efficient Market Hypothesis (EMH). Several researchers like 'Schiller (1981) (1990), Le Roy and Porter (1980) have extensively argued for the invalidity of EMH.

Volatility excess has been detected and highlighted by many researchers; however it has not been explained very well by EMH.

For this reason, we conducted an empirical study to identify the variable characteristics of volatility by comparing three GARCH models (GARCH, E-GARCH and GRJ-GARCH) over five different market indexes to examine prediction of returns volatility.

This comparison led us to detect several volatility characteristics like volatility clustering and leverage effect. This change in volatility regime is an irrefutable proof of the presence of volatility excess.

Given the inability of classical financial theory in explaining volatility excess, researchers started to focus on behavioural finance (Barret and Saphister (1996)).

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INTRODUCTION

Since the 1987 stock market crash, modeling and forecasting the financial markets volatility has received a significant attention from academics and practitioners because of its central position in many financial applications: assets valuation, allocation of wealth and hedging against risk.

In addition, the financial world has witnessed the bankruptcy or pre-bankruptcy of several institutions that have incurred great losses because of their exposure to impromptu market movements for over a decade. These financial disasters highlighted more the significance of predicting volatility in risk management. Given these facts, the quest for accurate forecasts still appear in progress.

While it is admitted that volatility is often marked by a number of stylized facts, such as persistence, volatility clustering, time-varying volatility and leptokurtic data behaviour, the introduction of the GARCH model developed by Engle (1982) and Bollerslev (1986), has created a new approach useful to recommend these time dependencies to financial econometricians. It has become a popular tool for modeling and forecasting volatility.

However, despite success of the GARCH model, it has been marked by a failure to capture asymmetric volatility¹. This limitation has been overcome by introducing more flexible treatment of volatility through recommending asymmetric responses of volatility to positive and negative shocks.

This new class of asymmetric GARCH models include exponential GARCH (EGARCH) of Nelson (1991) and threshold GARCH² (GJR-GARCH) of Glosten, Jagannathan and Runkle (1993). In this section, we compare the predictive power of volatility for these three models over 5 markets.

1 - PRESENTATION OF THE THREE GARCH MODELS:

Let $r_t = 100 \times (\ln P_t - \ln P_{t-1})$ which denotes the continuous composite returns rate in period $t - 1$ to t , where P_t is the level of asset prices at date t . Ω_{t-1} denotes the set of all information of observed returns up to $t - 1$.

1-1 – GARCH-type Predictive Models of Volatility:

A symmetric GARCH (1,1) model with a basic mean can be formulated as follows:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t | \Omega_{t-1} \sim F(0,1) \text{ et i. i. d} \quad (1)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (2)$$

Where μ and μ and σ_t^2 denote respectively conditional mean and variance of returns. ε_t is the innovation process, while $F(0,1)$ is the density function with a zero mean and variance equal to unity. In addition, ω, α and β are non-negative parameters with an $\alpha + \beta < 1$ restriction to ensure positivity of conditional variance and stationarity.

¹ For assets prices, negative shocks (bad news) have generally a higher impact on volatility than positive shocks (good news). This phenomenon is also known as leverage effect.

² Threshold GARCH.



Two simple classes of models, which may cope with asymmetric volatility in response to asymmetric shocks, are the GJR-GARCH model proposed by Glosten et al. (1993) and the exponential GARCH (EGARCH) model advocated by Nelson (1991). The GJR-GARCH model differs from the (2) model as follows:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \delta I_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

Where the I_{t-1} dummy function takes the value of unity if $\varepsilon_t < 0$ and zero otherwise. The dummy variable distinguishes between positive and negative shocks, in a way that asymmetric effects in data will be captured by δ .

Thus, in GJR-GARCH, positive information has an impact α and negative information has an impact $\alpha + \delta$, with the fact that negative (positive) information has a greater effect on volatility if $\delta > 0$ ($\delta < 0$). In addition, $\omega, \alpha, \delta, \beta$ are non-negative parameters with a $\alpha + \beta + 0,5 \delta < 1$ restriction, therefore estimating the sum $\alpha + 0,5 \delta$ should still be positive (Ling and McAleer (2002)).

The EGARCH model of Nelson (1991) provides an alternative asymmetric model in the following lines:

$$\log(\sigma_t^2) = \omega + \alpha \left[\gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{2/\pi} \right] + \beta \log(\sigma_{t-1}^2) \tag{4}$$

Where the coefficient γ captures the asymmetric impact of information with the fact that negative shocks have a greater impact than similar positive shocks if $\gamma < 0$. The effect of clustering is captured by a significant α coefficient. Finally, the use of the logarithmic form allows parameters to be negative without conditional variance becoming so.

1-2 - Distribution Hypotheses:

Another finding in the GARCH literature is the abnormal characteristics of distribution of financial returns. To model such a phenomenon when estimating a GARCH model, Theodossiou (1998) forewarned a SGT³ distribution that constitutes a flexible tool for modeling distribution of financial data which exhibits fat tails, leptokurtic shapes and asymmetry (skewness).

Probability density function of SGT (pdf⁴) for standardized residuals (z_t) of returns can be defined as follows:

$$F(z_t; N, k, \lambda) = C \left(1 + \frac{|z_t + \delta|^k}{\binom{N+1}{k} (1 + \text{sign}(z_t + \delta) \lambda)^k O^k} \right)^{-(N+1)/k} \tag{5}$$

Where :

$$C = 0,5 k \times \binom{N+1}{k}^{-1/k} \times B\left(\frac{N}{k}, \frac{1}{k}\right)^{-1} \times O^{-1} \tag{6}$$

$$O = (g - \rho^2)^{-1/2} \tag{7}$$

$$\rho = 2\lambda \times B\left(\frac{N}{k}, \frac{1}{k}\right)^{-1} \times \binom{N+1}{k}^{-1/k} \times B\left(\frac{N-1}{k}, \frac{2}{k}\right) \tag{8}$$

$$g = (1 + 3\lambda^2) \times B\left(\frac{N}{k}, \frac{1}{k}\right)^{-1} \times \binom{N+1}{k}^{-1/k} \times B\left(\frac{N-2}{k}, \frac{3}{k}\right) \tag{9}$$

$$\delta = \rho \times O \tag{10}$$

Where z_t is standardized residual with a zero mean and a variance equal to unity, N is the tail thickness parameter⁵ with a $N > 2$ constraint, k is a leptokurtic parameter with $k > 0$; λ governs density skewness under

³ Skewed Generalized t.: biased generalized t statistic.

⁴ Probability density function



the $|\lambda| < 1$ constraint, $sign$ is the sign function and $B(\cdot)$ denotes the beta function. (.) In particular, the SGT distribution characterizes several well-known distributions. More specifically, for $k = 2$ et $\lambda = 0$ it gives a t-student distribution and for $N = \infty, k = 2$ et $\lambda = 0$ it gives the normal distribution.

However, several recent papers have reported more new findings for heavy-tailed distributions (HT)⁶, like Politis (2004) and Hung et al. (2008). With a HT distribution, probability density function of innovations becomes:

$$F(z_t, a_0, 1) = \frac{(1+a_0 z_t^2)^{-1.5} \exp\left(-\frac{z_t^2}{2(1+a_0 z_t^2)}\right)}{\sqrt{2\pi}(\phi(a_0^{-0.5}) - \phi(-a_0^{-0.5}))} \quad (11)$$

Where 1 denotes the standard deviation of z_t and ϕ denotes the cumulative probability density function of the standard normal distribution. The Sharpe parameter, a_0 , reflects the degree of heavy tails with the $0 < a_0 < 1$. constraint. When $a_0 \rightarrow 0$, HT will be reduced to a standard normal distribution, while the distribution have denser tails than the normal distribution when $a_0 \rightarrow 1$ (or Politis (2004)).

In our study, we make recourse to three model specifications to model GARCH, GJR-GARCH and EGARCH volatility.

2 - HYPOTHESES AND DATA:

Our empirical specification will aim at testing the following hypotheses:

- Hypothesis 1: Volatility of markets index returns has different regimes; therefore there is volatility excess;
- Hypothesis 2: Volatility of markets index returns follows a single regime; therefore there is no volatility excess.

To test our hypotheses, we will compare the predictive power of the three asymmetric volatility models of GARCH, GJR-GARCH and EGARCH. This procedure is done on 5 markets, notably BVSP (Brazil), MERV (Argentina), MXX (Mexico), HSI (China) and GDAXI (Germany).

We will use daily prices of these indexes over the period stretching from 18th January 2002 to 31st December 2012, a total of 2709 days. The prices were taken from the Yahoo Finance database.

3- THE RESULTS AND THEIR INTERPRETATIONS:

3-1- Descriptive Statistics of Time-series

Table 1 reports the characteristics of the returns of the chosen five market indexes:

⁵ Tail-thickness parameter

⁶ Heavy-tailed distribution

**Table 1. Descriptive statistics of the five markets returns**

| Statistics | BVSP | GDAXI | HSI | MERV | MXX |
|---------------------------------|-----------|-----------|------------|------------|-----------|
| Mean | 0,0225205 | 0,000371 | 0,004473 | 0,0297005 | 0,0267008 |
| Maximum | 5,939677 | 4,68928 | 5,822503 | 6,999315 | 4,534343 |
| Minimum | -5,253248 | -3,228312 | -5,898598 | -5,624822 | -3,590464 |
| Skewness | -0,08614 | 0,0648492 | -0,0454581 | -0,0771208 | 0,0450354 |
| Kurtosis | 6,551229 | 7,213832 | 10,76151 | 7,851749 | 7,124008 |
| Median | 0,0524867 | 0,031519 | 0,139656 | 0,0475758 | 0,0528936 |
| Stand. Dev | 0,8575317 | 0,7091557 | 0,7221079 | 0,9557759 | 0,6507112 |
| Normality test ⁷ (W) | 0,96829 | 0,94828 | 0,92495 | 0,93922 | 0,94959 |
| ADF Test | -51,753 | -54,079 | -53,418 | -49,700 | -47,119 |
| KPSS Test ⁸ | 0,229 | 0,107 | 0,0727 | 0,114 | 0,116 |
| LM ⁹ Test | 0,00615 | 0,40295 | 0,89698 | 0 | 0,00001 |

This table shows us that the returns of the five markets are differently distributed. We notice divergent means. They vary between 0,000371 (GDAXI) and 0,0297005 (MERV). However, maximum values range between 4,534343 (MXX) and 6,999315 (MERV). The minimum values are all negative and vary between -5,898598 (HSI) and -3,228312 (GDAXI).

We found two types of distributions; GDAXI and MXX have tails skewed to the right and distributions skewed to the left. The other indexes have distributions skewed to the right and tails skewed to the left. Kurtosis coefficients are superior to 3, therefore the 5 markets returns distributions are leptokurtic.

Shapiro-Wilk normality test reported null p-values and consequently, daily returns of the five markets are not normally distributed. Dickey-Fuller Augmented test reported negative values inferior to the critical values corresponding to the number of observations, then the null hypothesis which assumes presence of a unit root is rejected.

However, the KPSS test indicates that the null hypothesis of the presence of stationarity is rejected for all markets except for BVSP for which stationarity hypothesis is accepted even at the 1% level. Finally, Lagrange-Multiplier test indicates that only GDAXI and HSI present errors autocorrelations.

⁷ Shapiro-Wilk normality test, if p-value < 0, then the null hypothesis is rejected. Therefore, data is not normally distributed.

⁸ Stationarity test of Kwiatkowski-Phillips-Schmidt-Shin.

⁹ Lagrange-Multiplier test of auto-correlation of returns.



Ces analyses préliminaires des données encouragent l'adoption, d'une part, des distributions sophistiquées ayant des queues épaisses, et d'autre part, des modèles conditionnels qui permettent des volatilités qui varient dans le temps.

3-2- The Results and Their Interpretations:

To compare the three GARCH models, first we should look into their overall significance. Table 2 reports the results of the Wald Chi² test:

Table 2. the overall significance of the three models through the Wald Chi² test

| Indexes | GARCH | | EGARCH | | GJR-GARCH | |
|---------|-----------|---------|-----------|---------|-----------|---------|
| | Statistic | P-value | Statistic | P-value | Statistic | P-value |
| BVSP | 10,63 | 0,0011 | 10,24 | 0,0014 | 10,46 | 0,0012 |
| GDAXI | 4,46 | 0,0347 | 8,51 | 0,0035 | 5,55 | 0,0184 |
| HSI | 22,36 | 0 | 17,90 | 0 | 27,42 | 0 |
| MERV | 18,85 | 0 | 18,38 | 0 | 22,59 | 0 |
| MXX | 15,15 | 0,0001 | 16,05 | 0,0001 | 18,13 | 0 |

The Wald Chi² test indicates that the three models are overall significant at the 5% level. Nevertheless, accepting the overall significance hypothesis may be done at the 1% level for the three models and only for four markets (BVSP, HSI, MERV and MXX), and only for the EGARCH model for GDAXI. Indeed, the p-values are inferior to 1% for 16 of the 18 conducted regressions.

Overall significance may be better tested by the log-likelihood ratio test. This latter is articulated around one hypothesis, i.e. the distribution takes a Chi² distribution. The Wald Chi² test has an additional hypothesis, which knowing in advance standard error. Table 3 reports the results of the log-likelihood test.

Table 3. overall significance through the log-likelihood ratio test

| Indexes | GARCH | EGARCH | GJR-GARCH |
|---------|-----------|-----------|-----------|
| BVSP | -3430,314 | -3427,400 | -3428,453 |
| GDAXI | -2819,362 | -2845,994 | -2818,297 |
| HSI | -2806,589 | -2842,037 | -2801,311 |
| MERV | -3591,131 | -3621,846 | -3589,307 |
| MXX | -2551,024 | -2572,759 | -2549,473 |

The log-likelihood statistics are negative and low, and therefore we may conclude that the three models are overall significant for the five markets.

As a second step, we will examine individual significance of each model's parameters for each market. Table 4



reports the results:

Table 4. Individual significance of regressions parameters

| Model | Parameters | BVSP | | GDAX | | HSI | | MERV | | MXX | |
|-----------|---------------------------|-----------|---------|-----------|---------|-----------|---------|-----------|---------|-----------|---------|
| | | Statistic | P-value | Statistic | P-value | Statistic | P-value | Statistic | P-value | Statistic | P-value |
| GARCH | MC ¹⁰ | 3,26 | 0,001 | 2,11 | 0,035 | 4,73 | 0,000 | 4,34 | 0,000 | 3,89 | 0,000 |
| | L1 ¹¹ ARCH | -0,68 | 0,497 | 12,23 | 0,000 | 14,67 | 0,000 | 12,27 | 0,000 | 12,68 | 0,000 |
| | Cons ¹² ARCH | 59,75 | 0,000 | 55,06 | 0,000 | 50,40 | 0,000 | 48,36 | 0,000 | 50,20 | 0,000 |
| EGARCH | MC | 3,20 | 0,001 | 2,92 | 0,004 | 4,23 | 0,000 | 4,29 | 0,000 | 4,01 | 0,000 |
| | L1 EARCH ¹³ | 1,41 | 0,159 | 2,10 | 0,036 | 4,40 | 0,000 | 1,45 | 0,146 | 0,89 | 0,373 |
| | L1 EARCH-A ¹⁴ | -3,45 | 0,001 | 10,01 | 0,000 | 13,77 | 0,000 | 15,29 | 0,000 | 13,81 | 0,000 |
| | Con EARCH-A ¹⁵ | -19,27 | 0,000 | -37,94 | 0,000 | -44,27 | 0,000 | -7,02 | 0,000 | -47,04 | 0,000 |
| GJR-GARCH | MC | 3,23 | 0,001 | 2,36 | 0,018 | 5,24 | 0,000 | 4,75 | 0,000 | 4,26 | 0,000 |
| | L1 ARCH | -2,19 | 0,028 | 7,12 | 0,000 | 11,00 | 0,000 | 9,85 | 0,000 | 7,69 | 0,000 |
| | L1 TARCH ¹⁶ | 1,79 | 0,074 | 2,10 | 0,036 | 5,42 | 0,000 | 2,87 | 0,004 | 2,61 | 0,009 |
| | Con TARCH ¹⁷ | 58,43 | 0,000 | 55,36 | 0,000 | 45,67 | 0,000 | 47,88 | 0,000 | 49,53 | 0,000 |

¹⁰ Conditional mean

¹¹ First lag of GARCH.

¹² Constant of GARCH.

¹³ First lag of EARCH.

¹⁴ First lag of EARCH-A.

¹⁵ Constant of EARCH-A.

¹⁶ First lag of TARCH.

¹⁷ Constant of TARCH.



The table above first indicates that the components of GARCH are significant at the 1% level for all markets except for BVSP. The L1 parameter is non-significant for Brazil ($p\text{-value} > 5\%$). Conditional mean is significant at the 1% level for the four markets: BVSP, HSI, MERV and MXX and at the 5% level for GDAX.

Concerning EGARCH, we found that conditional mean is significant at the 1% level for the five markets. The same is true for L1 parameter and the constant Con EARCH-A. Nevertheless, these three markets do not present errors autocorrelation. Therefore, EGARCH explains very well volatility of the markets with auto-correlated errors, notably GDAXI and HSI.

GJR-GARCH parameters are all significant at the % level for HSI, MERV and MXX, whereas for GDAX, MC and L1 TARCH are significant at the 5% level. As for BVSP, L1 TARCH is not significant and it is only significant at the 5% level. BVSP is marked by stationarity of returns.

In summary, GARCH models explained very well volatility of HSI, MERV, MXX and GDAX. The simplest GARCH family model could not function for VSP because the KPSS test showed stationarity of its returns. We may conclude that volatility of returns of HSI, MERV, MXX and GDAX presents symmetry of responses to positive and negative market information.

The EGARCH model explained very well volatility of returns only for GDAX. We may deduce that this model is not adequate. This model differs from the GARCH model by the fact that it efficiently captures volatility clustering and leverage effects. Since GDAX has been very well explained by GARCH, therefore volatility does not present a leverage effect of negative information against positive information. EGARCH highlighted clustering of volatility of returns for this index.

Volatility clustering is defined by Mandelbrot (1963) as "substantial changes followed by other substantial changes of whatever sign and small changes are followed by small changes". A quantitative illustration of this fact is that when returns are not auto-correlated, absolute returns ($|r_t|$) show a significant auto-correlation function and start slowly to disappear.

The GARCH model is marked, on the one hand, by uncorrelated serial error processes (u_t) with a zero mean, and on the other hand by a symmetric response of current volatility to positive or negative lagged errors (u_{t-1}). However, since error u_t is not correlated to its past, it may be considered as a measure of new information entering the financial market at moment t . Consequently, negative information will have a typically higher impact on future volatility of returns than positive information.

Such asymmetry of responses to market information is called leverage effect. It cannot be captured by a basic GARCH model. The EGARCH model is an exponential extension of the GARCH model. Volatility is measured by conditional variance. It is considered as a multiplying function of lagged innovations by opposition to the GARCH model which is an additional function of lagged error terms. Consequently, variance becomes very sensitive to the model's parameters.

The GJR-GARCH allows for having different impacts of positive and negative lagged innovations. It allows for detecting the leverage effect. The GJR-GARCH model showed performance similar to that of the GARCH model. Absence of a unit root, non-normality of returns and their leptokurtic distributions has contributed to such a result. The GJR-GARCH is more sophisticated than the EGARCH model. It allows the leverage effect for positive and negative variations. It shows therefore more acceptable performance in explaining volatility of the five markets. Then, we accept the first hypothesis which assumes that volatility of returns has different regimes, thus there is volatility excess. This excess is illustrated by changing volatility regimes captured by these three models: volatility clustering and leverage effect. This result corroborates that of Blitz, Pang, Van Vliet (2012) and Conrad and Loch (2012).

Comparing performance of the GARCH-type models in explaining volatility of returns of the five markets points to the insufficiency of the EGARCH model compared to GARCH and GJR-GARCH models because of errors terms characteristics. However, we were able to study the performance of other models belonging to the same family, mainly



GARCH-N, GARCH-t, GARCH-HT, etc. Detecting volatility characteristics (leverage and clustering effects) leads us to check for presence of volatility excess, thus checking our first hypothesis.

Moreover, we could use the SPA¹⁸ test, proposed by Hansen (2005), which is a robust and a more accurate test. It examines the relative performance of GARCH-type models aimed at studying symmetric or asymmetric distributions. This test requires inter-daily data (5, 15 and 60 minutes) unavailable in this study.

Volatility is a fundamental variable in theory and practice. Its central role is to evaluate assets and manage risk. Since true volatility cannot be observed, we chose variance as a proxy. It is computed through sums of squared inter-daily returns. Nevertheless, the precision of this estimator is affected by market microstructure noise, especially when data is high-frequency data.

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¹⁸ Superior Predictive Ability test



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