

DOI: <https://doi.org/10.24297/jap.v23i.9773>**Laser Cooling Conditions Dependent on the Photon Density and Frequency Using Quantum Model based on Fluid Laws and Treating Atoms as Strings**Habib. A. H<sup>1</sup>, Mubarak Dirar AbdAlla<sup>1</sup>, Najwa Idris A. Ahamed<sup>2</sup>, Einas M.A. Widaa<sup>3</sup>, Elharam A. E. Mohammed<sup>4</sup><sup>1</sup>Department of Physics, Faculty of Science, Sudan University of Science and Technology, Khartoum, Sudan Email: hhabib857@gmail.com, mubarakdirar@gmail.com<sup>2</sup>Department of Physics, College of Science, Qassim University, Buraydah 51452, Saudi Arabia n.ahamed@qu.edu.sa<sup>3</sup>Physical Science Department, Faculty of Science, Taif University, P.O. Box. 11099, Turaba 21945, Kingdom of Saudi Arabia emwidaa@tu.edu.sa<sup>4</sup>Department of Physical Sciences, Physics Division, College of Science, Jazan University, P.O. Box 114, Jazan 45142, Saudi Arabia alharamm@jazanu.edu.sa**Abstract**

Using the laws of fluid mechanics for gases and treating atoms as vibrating strings subjected to gas and photon pressure, a useful expressions for cooling conditions was found. These conditions can be applied to nano and quantum systems as far as the model recognises the particle dual nature, which is the cornerstone of quantum physics. These expressions indicated that the atoms density and the degree of cooling, as well the photon density and frequency, affected the cooling process. Dense and high cooling degree requires increasing the applied photon density or the frequency or both.

**Keywords:** cooling, atoms, photon, density, frequency, quantum, nano**Introduction**

Laser is one of the most importance applications in our day life. It is a single wave length, very narrow beam, powerful intensive light [1]. Laser can be generated due to the so called stimulated emission of radiation. When a photon having energy equal to the energy difference between the ground and excited states is incident in an atom, it forces the electron in the excited state to return back to the ground state emitting a photon having the same frequency, direction and phase of the incident photon [2]. The two photons when incident on two excited atoms force them to emit two additional coherent photons. This amplification process continues till larger number of coherent photons emerge in the form of a highly very narrow powerful beam known as laser [3]. Laser is used in modern technology in a wide variety of applications such as computer optical fibers in industry and medicine [4].

Recently laser cooling represents none one of the importance industrial applications for superconductors that can be used in improving the performance of magnetic resonance imaging devices. Cooling can also be utilized to generate powerful magnetic field which in turn can generate powerful electric energy [5,6,7]. The cooling process requires slowing the atoms and molecules motion, and decreasing their velocities, using photon pressure. According to the kinetic theory of gases the decrease of velocity decreases the temperature [8,9,10].

Ordinary laser cooling is based on the mechanical effect of single-photon transitions between ground states and electronically excited states.

One can also use multiple laser wavelengths and multiple atomic transitions for laser cooling and atoms trapping by quantum optics tool. Traditional laser cooling relies largely on mechanical forces due to light scattering from a single frequency laser. Multiple wavelengths and transitions can lead to usual "single-photon" cooling: access to substantially different effective photon momenta, access to different atomic line widths and saturation intensities, the possibility of coherence and EIT effects, and the possibility to easily separate atom fluorescence from laser excitation by the three-laser excitation. Such technique is based on mechanical forces arising from excited state to excited state transitions. For example, replacing the single laser excitation by the three-laser excitation can lead to larger light scattering forces, and the fluorescence would be at a very different wavelength from the excitation lasers, which could be easily filtered away. Many attempts were made for laser cooling conditions. The laser cooling of a cryogenic buffer-gas beam of calcium monohydride (CaH) molecules was investigated. The vibrational branching ratios for laser cooling transitions for both excited electronic states *A* and *B* were also determined. The repeated photon scattering via the  $A \rightarrow X$  transition is found to be achievable at a rate of  $\sim 1.6 \times 10^6$  photons  $s^{-1}$ . A sub-Doppler cooling technique, namely the magnetically assisted Sisyphus effect, and use it to cool the transverse temperature of a molecular beam of CaH were also determined. Using a standing wave of light, the transverse temperature was lowered from 12.2(1.2) mK to 5.7(1.1) mK. This gives a clear pathway for creating a magneto-optical trap (MOT) of CaH molecules. Such a MOT could serve as a starting point for production of ultracold hydrogen gas via dissociation of a trapped CaH cloud [10] to an equal split of the cooling light into two fibers (1) and (2). This work study theoretically laser cooling feasibility of the molecule LuF, in the fine structure level of approximation. An ab-initio complete active space self-consistent field (CASSCF)/ MRCI with



Davidson correction calculation has been done in the  $\Lambda^{(\pm)}$  and  $\Omega^{(\pm)}$  representations. The corresponding adiabatic potential energy curves and spectroscopic parameters have been investigated for the low-lying electronic states. The results of the internuclear distances of the  $X^3\Sigma_{0+}$  and  $(1)^3\Pi_{0+}$  states show the candidacy of the molecule LuF for direct laser cooling. Franck–Condon factors, the radiative lifetimes, the total branching ratio, the slowing distance, and the laser cooling scheme study prove that the molecule LuF is a good candidate for Doppler laser cooling [11]. Isotopes using gray molasses operating on their respective  $D_1$  atomic transitions was investigated. For  $^7\text{Li}$  the results show that the sub-Doppler cooling can be achieved with two distinct  $\Lambda$ -type transitions, where the upper level can be either of the two  $2^2P_{1/2}$  hyperfine states. A temperature of 85  $\mu\text{K}$  was obtained with atom numbers of  $10^8$ , and phase-space densities in the range of  $10^{-6}$ – $10^{-5}$  for both isotopes. These conditions provide a good starting point for loading the mixture into an optical dipole trap and performing evaporative cooling to quantum degeneracy. This provides a valuable simplification for the preparation of ultracold  $^6\text{Li}$ – $^7\text{Li}$  mixtures, which were proven to be a successful system for the study of impurity physics and Bose-Fermi superfluids [12]. Different other attempts were also done concerning laser cooling [13,14,15].

(4.3) Force for fluid and cooling conditions According to Newton's second law the force  $F$  can be defined in terms of the mass  $M$  and the velocity  $v$  to be

$$F = \frac{dMv}{dt} \quad (1)$$

The pressure is defined by

$$P = \frac{F}{A} \quad (2)$$

For uniform velocity the force takes the form

$$F = v \frac{dM}{dt} \quad (3)$$

But the fluid total mass  $M$  existing in a cylinder having area  $A$  and length  $x$  is given by

$$M = mN = mnAx \quad (4)$$

Thus the force is given by

$$F = v \frac{dM}{dt} = v \frac{dnAxm}{dt} \quad (5)$$

According to the definition of the velocity, one gets

$$F = v n A m \frac{dx}{dt} = v^2 n A m \quad (6)$$

The pressure  $P$  is thus given for a photon of speed  $c$  and number density  $n$  by

$$P = \frac{F}{A} = v^2 n m = n m c^2 \quad (7)$$

But for the photon having frequency  $f$  Planck hypothesis and special relativity gives the energy to be in the form

$$mc^2 = hf \quad (8)$$

This means that the pressure exerted by the photon is given by

$$p_p = P = nhf \quad (9)$$

Consider a stream of atoms moving in the positive  $x$ -direction with speed  $v$  and pressure  $p$  such that stream of photons moves in an opposite direction to exerted an inward pressure  $P_p$  on the  $a$  atoms. Therefore the fluid equation of motion for the particles with speed  $v$  is given by

$$\frac{dMv}{dt} = (\Delta p - \Delta P_p) \quad (10)$$

In most cases the medium atoms are in the form of vibrating strings having variable non uniform velocity. Thus when no forces are exerted on them the atoms become at rest

$$0 = (\Delta p - \Delta P_p) \quad (11)$$

Thus the fluid pressure is equal to the photon pressure

$$P = p_p \quad (12)$$

If the atoms have kinetic energies corresponding to thermal energy  $\frac{1}{2}n_a kT$ . thus

$$P = \frac{1}{2}n_a m v^2 = \frac{1}{2}n_a kT \quad (13)$$

Thus to stop a toms to cool them by laser to change its temperature from  $T$  to zero, the photon frequency and intensity  $n$  must satisfy equation (12) inserting (9) and (13) in (12) gives

$$nhf = \frac{1}{2} nakT_1 \quad (14)$$

To cool the same a toms density to from  $T_1$  and to 0 and from  $T_2$  to 0 requires either changing the photons frequency or their density  $n$  or both. In general, cooling to 0 for  $T_1$  and  $T_2$  needs.

$$N_1 hf_1 = \frac{1}{2} nakT_1 \quad (15)$$

$$N_2 hf_2 = \frac{1}{2} nakT_2 \quad (15)$$

For photons of the same frequency equations (14) and (15) gives

$$\frac{1}{2} nak (T_2 - T_1) = (n_2 - n_1) hf = nhf$$

$$T_2 - T_1 = \frac{2nhf}{nak} \quad (16)$$

Thus to cool a toms from  $T_2$  to  $T_1$  one needs photons density and frequency to satisfy equations (15).

For photons having the same density but different frequencies (14) and (15) give

$$\frac{1}{2} n_a k (T_2 - T_1) = nh (f_2 - f_1)$$

Thus

$$T_2 - T_1 = \frac{2nh}{n_a k} (f_2 - f_1) \quad (17)$$

The requirement for cooling can be expressed in terms of laser intensity  $I$  and power. For laser having frequency  $f$  and density  $n$ , the power passing through area  $A$  is given by  $Pr$  = energy passing through area  $A$  per unit time ( $t = 1$ )

(Total number of photone inside the cylinder emerging in one second)  $\times$  energy in one photon

= photon density  $\times$  cylinder volume  $\times hf$

$$= n \times l \times Ahf = n \times c \times A \times hf$$

$$Pr = nhf c A \quad (18)$$

The intensity  $I$  is the power emerged per unit area

$$I = \frac{Pr}{A} = nhf c \quad (19)$$

According to equations (10), (18) and (17) one can express the cooling conditions in terms of the laser intensity and power to get

$$(T_2 - T_1) = \frac{2I}{h_a k c} \quad (20)$$

Since the intensity and the power are related in terms of the area  $A$  by

$$I = Pr/A$$

Thus

$$(T_2 - T_1) = \frac{2Pr}{h_a k A} \quad (21)$$

However the situation is different for particles affected by a field having potential  $V$ . In this case equation (10) can be rewritten to be

Type equation here.

$$\frac{d(Mv)}{dt} = \Delta p - \Delta P_p - \Delta V$$

(22)

When the a toms becomes at rest the left hand side vanishes to get

$$0 = (\Delta P - \Delta P_p - \Delta V) \quad (23)$$

This requires the zero colling condition to be

$$P_p = P - V \quad (24)$$

In view of equations (4. 3. 9), (4. 3. 13) . one gets

$$nhf = \frac{1}{2}n_a kT - V \quad (25)$$

To cool atoms from  $T_2$  to zero and from  $T_1$  zero , one gets

$$n_2 hf_2 = \frac{1}{2}n_a kT_2 - V \quad (26)$$

$$n_1 hf_1 = \frac{1}{2}n_a kT_1 - V \quad (27)$$

For laser having the some frequency one gets

$$n_2 hf = \frac{1}{2}n_a kT_2 - V$$

$$n_1 hf = \frac{1}{2}n_a kT_1 - V \quad (28)$$

Substracting the two relations gives

$$\frac{1}{2}n_a k(T_2 - T_1) = (n_2 - n_1)hf \quad (29)$$

$$T_2 - T_1 = \frac{2nhf}{n_a k} \quad (30)$$

Fortunately the cooling condition depends only on the properties of the laser only and dose not depend on the field permeated the medium. Assume now that atoms moving in the frictional medium, having frictional coefficient .  $\propto$  In this case equation (10) gives

$$\frac{dMv}{dt} = (\Delta p - \Delta p_p)A - \alpha v \quad (31)$$

For the atoms to be at rest, again the left hand side vanishes to get

$$0 = \Delta(p - p_p)A - \alpha v \quad (32)$$

In are dimension

$$(P - P_p)A = \alpha \int v dx = p_f A \quad (33)$$

Where  $P_f$  stands for friction pressure

Therefore

$$p_f = P - P_p \quad (34)$$

Hence

$$\frac{1}{2}n_a kT = nhf + p_f \quad (35)$$

Again the cooling condition requires

$$\frac{1}{2}n_a kT_2 - \frac{1}{2}n_a kT_1 = (n_2 - n_1)hf - p_f + p_f$$

$$T_2 - T_1 = \frac{2nhf}{n_a k} \quad (36)$$

One can also assumes that when the temperature beam was applied thus the new temperature

Satisfies

$$\frac{1}{2}n_a kT_2 - n_2 hf = \frac{1}{2}n_a kT_1 \quad (37)$$

Again the cooling conditions depend on the laser properties only and are not affected by the medium friction

### 3. Discussion

The equation for a fluid depending on Newton second law was used to describe the behavior of atoms in the form of vibrating strings when laser photons having pressure  $P_p$  was applied on two opposite sides to oppose the atoms pressure  $P$  which was treated as a gas. The model was put completely under the umbrella of quantum laws by recognizing the particle wave dual nature as shown in equation (1). Newton second law takes care of the particle nature, while string solution secures the wave nature. The atoms can be cooled to rest according to this approach when the photon pressure which depends on the photon density  $n$  and frequency  $f$  According to equation (9) equals the atoms pressure as shown in equation (12). The atoms pressure is temperature dependent according to equation (13) which is based on the kinetic theory of gases. Any gas can be cooled to zero from  $T_1$  by applying appropriate photon density and frequency. High temperature  $T_1$  and high density atoms  $n_a$  requires increasing either the photon density  $n$  or the frequency  $f$  or both. This agrees with experimental work and common sense. Equation (16) indicated that lowering temperature from  $T_2$  to  $T_1$  requires applying photons with density  $n$  frequency  $f$  or increasing the applied photon density by an amount  $n$ . The same results can be ground according to equation (17) but here the frequency needs to be increased. Equations (20) and (21) showed that increasing intensity and power of the photon causes more lowering to the gas temperature. Applying uniform potentials as shown in equation (22) and taking into account the effect of the friction as shown in equation (31) was shown in equations (29,30,36,37) gives the same results.

### Conclusion

Based on a new quantum model recognising the wave particle dual nature,

useful expressions for cooling conditions was found for nano and quantum systems. These expressions showed that the atoms density and the degree of cooling beside the photon density and frequency affected the cooling process. Dense and high cooling degree requires increasing the applied photon density or the frequency or both.

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