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Neutrino Mass es and Dark Matter

David McGraw Jr. PhD

McNeese State University

(Lake Charles, Louisiana, United States)

E-mail: dmcgraw1@mcneese.edu

Abstract

Neutrinos have provided us with new physics beyond the Standard Model of Particle Physics. This paper will mathematically calculate the mass of new species of Neutrinos. The Standard Model fails to explain many of the parameters that are the foundations of Particle Physics. Many of the Problems can be explained by adding mass to Neutrinos, and by finding new species of neutrinos.

Keywords Neutrinos masses, Heavy Neutrinos, and Dark Matter.

1. Introduction

Even though the Standard Model of Particle Physics has been successful at explaining physics at the subatomic level. There are still areas where the theory fails, an example is spacetimes singularities. Also, the Standard Model fails when it comes to Neutrino masses. According to Particle Physics, neutrinos are massless particles. However, experiments have confirmed that neutrinos do have some mass. Physicists have tried to add this new situation to the Standard Model. It has led to new theoretical issues within the Standard Model. It looks like the mass terms need to be small. Also, it is not clear how the masses came about. In Particle Physics neutrinos have no mass because it contains only left-handed neutrinos. There are no right right-handed in Particle physics, so it is not possible to add a mass term to the Standard Model. It is clear that neutrinos change flavors. In that situation, then it would seem that neutrinos have mass. We need to go beyond the Standard Model to explain how Neutrinos got their mass. Also, we will need to explain and calculate new species of neutrinos. Attempts have been made to add mass to the Neutrinos by the seesaw mechanism. The mass terms mix neutrinos of different generations. Unlike what we see with Quark mixing, neutrino mixing seems to be maximal. This has led to the explanation of symmetries between generations that explain mixing patterns. The mixing matrix used in this paper contains several phases that break CP invariance. These phases led to a surplus of leptons in the early universes. This asymmetry also converts in later stages to an excess of baryons over ant-baryons. This explains the matter-antimatter asymmetry in many universes. If neutrinos have mass then we are looking at new physics. Neutrino masses and mixing go beyond the Standard Model of Particle Physics. This paper will calcite new species of Neutrinos. The light neutrinos can help explain some of the cold dark matter in many of the universes. The heavier neutrinos are possible candidates for dark matter. [1][2][3][4].

2. Theory

A two-component Weyl spinor field can describe a neutrino. Neutrinos have chirality, L or R, and they can be defined as

$$\text{ML}=\frac{1-\gamma_5}{2} \qquad \text{MR}=\frac{1+\gamma_5}{2} \tag{1}$$

They have the following properties:

$$M_L^2=\text{M}_L\,,\quad M_R^2=M_R\,,\quad \text{MLMR}=\text{MRML}=0 \qquad \text{ML}+\text{MR}=1. \tag{2}$$

For relativistic particles, chirality coincides with helicity. Then the projection operators are

$$\text{M}_{\pm}=\frac{1}{2}\left(1 \pm \frac{\sigma\rho}{|\rho|}\right) \tag{3}$$

They satisfy relations similar to equation (2). For a free fermion, helicity is conserved. However, generally, chirality is not conserved. Chirality is only conserved in the limit m = 0 because, in this case, chirality coincides with helicity [5].

For the particle-antiparticle conjugation operator $\hat{B}$, its action is defined as

$$\hat{B}:\ \psi \rightarrow \psi^{B} = B\overline{\psi}^{S}, \qquad B = i\gamma_2\gamma_0 \tag{4}$$

Some of its properties are

$$B^{\kappa} = B^{S} = B^{-1} = -B \qquad B\gamma_{\mu}B^{-1} = -\gamma_{\mu}^{S}$$

$$(\psi^{B})^{B} = \psi,\quad \overline{\psi}^{B} = \psi^{S}B,\qquad \overline{\psi}_1\psi_2^{B} = \overline{\psi}_2^{B}\psi_1,\quad \overline{\psi}_1 D\psi_2 = \overline{\psi}_2^{B}(BD^{S}B^{-1})\psi_1^{B} \tag{5}$$

where $\psi, \psi_1, and \psi_2$ fermion fields and D are an arbitrary 4x4 Matrix. Using the commutative properties of the Dirac γ matrices acting on a chiral field $\hat{B}$ flips its chirality:

$$\hat{B}:\psi_L \rightarrow (\psi_L)^{B} = (\psi_L)^{B} = (\psi^{B})_R \quad \psi_R \rightarrow (\psi_R)^{B} = (\psi^{B})_L . \tag{6}$$

So then, the antiparticle of a left-handed fermion is right-handed. One should not confuse particle-antiparticle operator B with charge conjugation operator B. For massive fermion, the Lagrangian has the form

$$-\Im_n = n\overline{\psi}\ \psi = \overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L . \tag{7}$$

A massive field must have both components, so th

$$\psi = \psi_L + \psi_R . \tag{8}$$

If we have a Dirac field, then the right-handed component of a massive field can be completely independent of the left-handed one. For a Majorana field, the right-handed field can just be $\hat{B}$, then

$$\psi = \psi_L + \eta(\psi_B)_R = \psi_L + \eta(\psi_L)^{B} \tag{9}$$

where the phase factor is equal to $\eta = \varepsilon^{i\theta}$. From the last equation, it immediately follows that the $\hat{B}$ - conjugate field coincides with itself up to a phase factor, then

$$\psi^{B} = \eta^{*}\psi . \tag{10}$$

This means that Majorana fields are neutral. Majorana particles are fermionic analogs of photons and mesons. To construct a massive Dirac field, one needs two independent two compounds Weyl fields. Nevertheless, this is not the case with a Majorana fermion; they have only two degrees of freedom. For Majorana neutrinos, particle-antiparticle conjugation and charge conjugation leave the field unchanged because they do not have charges [5][6]. For n fermion species, the Majorana mass term can be written like

$$-\Im_m = \frac{1}{2}[\overline{\psi}_L^{B}M\psi_L + \overline{\psi}_L M'(\psi_L)^{B}] \tag{11}$$

$$-\Im_m = \frac{1}{2}[\psi_L^{G}BM\psi_L + \overline{\psi}_L B M'\overline{\psi}_L^{G}] \tag{12}$$

$$-\Im_m = \frac{1}{2}[\psi_L^{G}BM\psi_L + h.c.] \tag{13}$$

where $\psi = (\psi_1 \ldots \psi_n)^{G}$ is a vector in the flavor space, and M is an n x n matrix. The equation shows an essential difference between Dirac and Majorana mass terms. The Dirac mass terms $\overline{\psi}\psi$ are invariant, and they conserve the charge. The Majorana mass terms break all the charges that the field ψ has two units. The process means that no charged particle has Majorana mass, and so only neutrinos can be Majorana particles [6][7][8]. If neutrinos have Majorana masses, the total lepton number is not conserved. In many areas of the Multiverse, quarks and charged fermions get their mass through the Yukawa couplings with the Higgs field H = ($H^{t} H^{\circ})^{G}$:

$$-\Im_{y} = h^{u}_{ik}\overline{Q}_{Lj}u_{Rj}\widetilde{H} + h^{d}_{jk}Q_{Lj}d_{Rk}H + f^{e}_{jk}\bar{l}_{Lj}e_{Rk}H + h.c. \tag{14}$$

where Q_{Lj} *and* l_{Lj} are left-handed quark and lepton doublets, u_{Rj} , d_{Rk} , and e_{Rj} are right-handed singlet fields of up-type quarks, down-type quarks, and charged leptons, and j, k is the generation indices? In the Multiverse, neutrinos are not always massless. The neutrinos in the Multiverse can have heavy masses. In addition, the neutrinos in the Multiverse can have Majorana masses. The Majorana mass has the form $\nu^{G}_{L}B\nu_{L}$. In the Multiverse, there is an isotriplet Higgs field Δ ≈ (3, 2):

$$-\Im^{\Delta}_{YUK} = f_{\Delta}(l^{G}Bi\tau_{2}\tau l)\Delta + \text{h.c.} \tag{15}$$

When the electrically neutral component Δ develops a VEV, a Majorana neutrino mass can be generated. In this way, Majorana neutrino species exist in the Multiverse. The operator $H^{G}i\tau_{2}\tau H \approx (3,2)$ has the correct quantum numbers. The term $(l^{G}Bi\tau_{2}\ \tau l)$ ($H^{G}i\tau_{2}\tau H)$ has the dimension d= 5, so then,

$$\frac{f}{M}(l^{G}Bi\tau_{2}\tau l)(H^{G}i\tau_{2}\tau H). \tag{16}$$

This produces a Majorana mass term for neutrinos mL = f ν^{2}/M , when the Higgs field develops a nonvanishing VEV, with M being the characteristic mass scale of the particles in the loop. The total lepton number is not conserved in this situation. Flavors of Majorana states cannot be transformed according to an arbitrary representation of their symmetry groups. The dual model is related by the reflection of roots concerning the origins of coordinates and by a change in infinitesimal operators. For equivalent dual representation, we have

$$h^{t}t^{B}h = -t^{BT} \tag{17}$$

This matrix raises and lowers indices on ψ . Complex conjugate representations are equivalent to symmetric terms. They include regular expressions of all groups. The Sp(n/2) has an antisymmetric matrix h:

$$h^{T} = -h, \qquad h^{t} = -h. \tag{18}$$

The analogous identity holds for ψ_{L} . It follows from equation (18) that nonzero Majorana masses may exist upon the breakdown of flavor symmetry to the emergence of a mass matrix without residual symmetries. This matrix would ensure the appearance of non-identical mass states in the flavor spectrum. For h = hT both a Majorana and a Dirac mass may exist without symmetry breaking. A breakdown of flavor symmetry leads to a vacuum expectation value $\overline{\psi}^{C}(x)\overline{\psi}_{d}(x)$ for S and J operators. So then,

$$(M_{RR})^{f}_{e} = \langle \overline{\psi}^{f}_{R}(x)\psi_{\mathrm{Re}}(x) \rangle \tag{19}$$

$$(M_{LL})^{f}_{e} = \langle \overline{\psi}^{f}_{L}(x)\psi_{Le}(x) \rangle \tag{20}$$

The Majorana State has the form

$$\psi_R(x) = \gamma_5 hB\overline{\psi}_R^T(x) \quad (21)$$

$$\psi_L(x) = -\gamma_5 hB\overline{\psi}_L^T(x) \quad (22)$$

The invariant Dirac mass can then be written:

$$\overline{\psi}_R^e(x)\psi_{Le}(x) = \overline{\psi}_R^e(x)\psi_{Le}(x) - \overline{\psi}_L^e(x)\psi_{\mathrm{Re}}(x) \neq 0 \quad (23)$$

This form in equation (23) corresponds to an imaginary Dirac mass. The condition in equation (22) corresponds to the phase; if we combine it with equation (21), these equations together render the Dirac mass in equation (23) as real-valued. Equations (21) and (22) and the anticommutation properties of ψ lead to the following relation

$$M_e^f = -h_{ee'} M_{e'}^{f'} h^{ff}. \quad (24)$$

$$h_{ef} = -h^{ef} = (-1)^e \delta_{n+1-e,f}, \qquad e,f = 1,2,3,\ldots\ldots n. \quad (25)$$

One can see that Matrix M has zero trace, which is evident from equation (24). The condition in equation (24) entails the vanishing of all odd-order principal minors of the matrices in equations (19) and (20). The Dirac form of equation (23) admits the existence of the invariant mass μ_{RL} :

$$(\mu_{RL})_e^f = \frac{1}{n}\langle\psi_R^G(x)\psi_{LG}(x)\ \delta_e^f. \quad (26)$$

From the properties in equations (21) and (22) for the operator product $\psi_L^e\psi_{Lf}$, we have

$$\overline{\psi}_R^e(x)\psi_{Lf}(x) = h_{ff'}\overline{\psi}_L^{f'}(x)\ \psi_{\mathrm{Re}'}(x)h^{e'e}. \quad (27)$$

Because the expectation values of these quantities are fundamental, then the elements $(\mu_{RL})_e^f$ are related by the following equation

$$(\mu_{RL})_e^f = h_{ff'}(\mu_{RL})_{e'}^{f'} h^{e'e}, \quad (28)$$

then,

$$(\mu_{RL})_e^f = (\mu_{RL})_f^e. \quad (29)$$

For both MRR and for MLL, n2/4 relations arise from equation (24). Because of this, then

$$\mathrm{M} = \left(M_{RR}\ \mu_{RL}\ \mu_{LR}\ M_{LL}\right) \quad (30)$$

.

The matrix MRR and MLL are full matrices involving off-diagonal elements. The set of gap equations then contains n(2n + 1) equations for the elements of the matrix M, but there are n2 constraints in equations (24) and (26) [8][9]. In the problem of Majorana masses, a local gauge interaction with a vector meson $F_\mu^c(x)$ is a preferable answer for us to consider. The currents that determine the vector interaction of F_μ^c with chiral fermions $\Psi_{(R,L)}^e$,

$$k_{(R,L)\mu}^c(x) = \Psi_{(R,L)}^e(x)\gamma_\mu t_e^{cf}\Psi_{(R,L)f}(x), \quad (31)$$

this equation can be written in terms of the Majorana operators $\psi_{(R,L)}(x)$. By considering the anticommutation $\Psi_{(R,L)}(x)$, we have

$$K^{c}_{(R,L)\mu}(x) = \overline{\psi}^{e}_{(R,L)}(x)\gamma_{\mu}t^{cf}_{e}\psi_{(R,L)f}(x) \tag{32}$$

then,

$$K^{c}_{(R,L)\mu}(x) \equiv {}_{2}\, k^{c}_{(R,L)\mu}(x). \tag{33}$$

If we consider the pseudovector current of the Majorana states for matrices, then

$$K^{(5)c}_{\mu}(x) = \overline{\psi}(x)v_{\mu}\gamma_{5}t^{c}\psi(x) = 0 \tag{34}$$

The situation is the opposite for currents with matrices symmetric with the identity matrix. The vector currents for Majorana operators are zero; however, pseudovector currents are equal to chiral vector currents. The symmetric properties of the interaction between Majorana particles are crucial for us to understand. These particles help find the solutions to equations for the parameters of spontaneous symmetry breaking. There are two critical issues to address, the mechanism of mass generation for vector particles F^{c}_{μ} and the fermion mass generation. The effective potential can be determined in the functional integral for the amplitudes. Then we can perform integration concerning the fields $F^{c}_{\mu}(x)$. The local and nonlocal combination would depend on this type,

$$\overline{\psi}^{c}Z_{1}\psi_{c}\, ,\overline{\psi}^{c}Z_{2}\ \psi_{d},\ \overline{\psi}^{d}Z_{3}\ \psi_{c}\, ,\ldots\ldots \tag{35}$$

for R and L operators in the role of ψ and $\overline{\psi}$. The quantities of the type $h^{cd}\psi^{T}_{c}\ldots\psi_{d}$ transform into $\overline{\psi}^{c}\ldots\ldots\psi_{c}$ equations (22) and (23). This outcome leads to a distinction between the terms of chiral operators and the terms of Majorana operators[9]. For this situation to occur, we consider Veff in the second order. One can separate the mechanism of mass generation F^{c}_{μ} and introduce an auxiliary scalar field with nonzero vacuum expectation values. If the mass MF is large, then

$$V_{eff} = \frac{-g^{2}_{F}}{2M^{2}_{F}}k^{c}_{\mu}(x)k^{c\mu}(x)$$

$$V_{eff} = \frac{-g^{2}_{F}}{8M^{2}_{F}}K^{c}_{\mu}(x)K^{c\mu}(x). \tag{36}$$

This equation can be transformed into the following relations:

$$\sum_{C} = t^{Cd}_{c}t^{Ce}_{c} = \frac{1}{4}(\sigma^{d}_{c}\sigma^{e}_{c} - h_{cd}h^{de}) \tag{37}$$

If we use equation (36) and the Fierz transformation, we have

$$V_{RR} = \frac{g^{2}_{F}}{4M^{2}_{F}}\left(\overline{\psi}^{c}_{R}\frac{1-\gamma_{5}}{2}\psi_{Rd}\right) \mathrm{x} \left(\overline{\psi}^{d}_{R}\frac{1+\gamma_{5}}{2}\psi_{Rc}\right) \tag{38}$$

This relation can be simplified to,

$$V_{RR} = \frac{g_F^2}{16M_F^2} \times \left[(\overline{\psi}_R^c \psi_R^d)(\overline{\psi}_R^d \psi_{Rc}) - (\overline{\psi}_R^c \gamma_5 \psi_{Rd})(\overline{\psi}_R^d \gamma_5 \psi_{Rc}) \right] \tag{39}$$

For VLL, one must consider equations (19), (33), and ψR →ψL. We have also to consider R x L products, then

$$V_{RL} = -2\frac{g_F^2}{4M_F^2}\left[(\overline{\psi}_R^c \frac{1-\gamma_5}{2}\psi_{Lc}) \times (\overline{\psi}_L^d \frac{1+\gamma_5}{2}\psi_{Rd}) - (\overline{\psi}_R^c \frac{1-\gamma_5}{2}\psi_{Ld}) \times (\overline{\psi}_R^d \frac{1+\gamma_5}{2}\psi_{Lc}) \right]. \tag{40}$$

Our Majorana conditions make it possible to prove symmetry and interchange R→L. For the critical parameters, the set of gap equations contains independent elements of the orthogonal matrix that diagonalizes the Matrix M in equation (31). The eigenvalues of this matrix, which are

the masses of Majorana particles, are real transformations. Equations (25) and (31) impose on the matrix elements the requirement that ψR and ψL be Majorana states. In all, there are n(2n + 1) equations and n(2n + 1) unknown parameters. Spontaneous symmetry breaking creates an invariant form of the matrix μRL. The system takes the path of the smallest symmetry breaking under the Majorana conditions, and then the matrix is symmetric. For this case, the n2/2 conditions in equations (29) and (30) replicate some of the n(n + 1)/2 equations for the elements of the symmetric matrix that has identical diagonal elements. Only these n(n + 1)/2 equations must be considered. We can now write the following relations:

$$\frac{n(n+1)}{2} + \frac{n^2}{4} + \{n(n+1)\} + \frac{n(n+1)}{2} - n(2n+1) = \frac{n(n-1)}{2}, \tag{41}$$

If MRR ≠ 0 and MLL = 0, we have a solution of n = 6. The equality MLL = 0 is a necessary condition..Also, we need to remember that the Majorana matrix MRR has pairs of identical eigenvalues[9]. We can then conclude that if the mass scale M for MRR is much larger than the Dirac scale μ, then n = 6 means the presence of three new Dirac neutrinos having a small mass and three new neutrinos having a large mass. With n = 6 and MLL = 0, and if we assume that Majorana mass scale is much larger than the Dirac scale, we have

$$M >> \mu \tag{42}$$

If we employ the transformation

$$\psi'_R = U\psi_R \tag{43}$$

We can diagonalize MRR by using the orthogonal matrix U. Simultaneously; we can go over to,

$$\psi'_L = U\psi_L. \tag{44}$$

The process does not change either the diagonal matrix or MLL = 0. We then have a 12 x 12 matrix of the form

$$\text{UTMU} = \left[M_{R1}\ 0\ \mu\ 0\ 0\ M_{R6}\ 0\ \mu\ \mu\ 0\ 0\ 0\ 0\ \mu\ 0\ 0 \right] \tag{45}$$

This matrix factorizes into the two-dimensional matrices discussed before. So then,

$$m^D = \left[M_D\ \mu\ \mu\ 0 \right] \qquad D = 1,2,3 \ldots 6. \tag{46}$$

If μ << MD, and two eigenvalues of mD differ by μ2/MD2, then

$$\lambda_{1,2}^D = \frac{M_D}{2} \pm \sqrt{\frac{M_D^2}{4} \oplus \mu^2} \tag{47}$$

$$\lambda_{1,2}^D = M^D + \frac{\mu^2}{M_D} - \frac{\mu^2}{M_D} \tag{48}$$

Equations (47) and (48) are valid for any MD sign. The rotation of unit vectors determines the eigenfunctions of the matrix in equation (46) for this matrix utilizing the transformation

$$V_D = [\alpha_D \beta_D \; - \beta_D \alpha_D] \tag{49}$$

where the quantities αD and βD are given by

$$\alpha_D = \frac{1}{\sqrt{1+(\mu)/M_D)^2}}, \tag{50}$$

$$\beta_D = \frac{\mu/M_D}{\sqrt{1+(\mu/M_D)^2}} \quad , \tag{51}$$

$$\alpha^2 + \beta^2 = 1 \tag{52}$$

For the matrix in equation (46), we change the masses MRi so that MR6 = - MR1, MR5 = - MR2, and MR4 = -MR3. We will then have three pairs of large masses and three pairs of small masses. The eigenfunctions of the diagonalized states are

$$\psi_{\pm D} = U^C_{\pm D}(\alpha_{\pm D}\psi_{Rc} + \beta_{\pm D}\psi_{Lc}), \tag{53}$$

$$\Psi_{\pm D} = U^c_{\pm D}(-\beta_{\pm D}\psi_{Rc} + \alpha_{\pm D}\psi_{Lc}). \tag{54}$$

Because of a choice of signs in equations (50), (51), and (52), we have

$$\alpha_D = \alpha_{-D} \qquad \beta_D = -\beta_{-D} \quad . \tag{55}$$

The wave functions in equations (53) and (54) have a property similar to the Majorana relations. So then,

$$\gamma_5 hE\overline{\psi}^{TF} = \psi_{-D} \quad , \quad \gamma_5 hE\overline{\Psi}^{TF} = -\Psi_{-D} \quad . \tag{56}$$

The arrangement of the masses was chosen so that hD'D = h-D'D. If one needs to prove the relation in equation (56), we can write

$$\gamma_5 hE\psi^{TD} = hU^{+T}h^+(\alpha_D\gamma_5 hE\psi^T_R + \beta_D\gamma_5 hE\psi^T_L). \tag{57}$$

For the matrices U diagonalizing MRR, there is the following relationship between the elements:

$$h_{D'D}U^{+TD}_d h^{cd} = \pm U^c_{-D} \qquad D' = D; \quad c,d = 1,2,3,\ldots 6. \tag{58}$$

Equation (56) makes it possible to construct Dirac states of positive masses. If we create Majorana states for each diagonal, D = 1,2,3. We obtain,

$$\chi_1^{(M_D)} = \frac{\psi_D + E\overline{\psi}^{TD}}{\sqrt{2}} = \frac{\psi_D + \gamma_5 h\psi_{-D}}{\sqrt{2}} \tag{59}$$

And,

$$\chi_2^{(M_D)} = \frac{\psi_D - E\overline{\psi}^{TD}}{\sqrt{2}} = \frac{\psi_D - \gamma_5 h^+\psi_{-D}}{\sqrt{2}}. \tag{60}$$

Similar procedures can be applied to the small mass states. Finally, the massive states of positive mass can be written in the form,

$$\psi = \frac{\chi_1 + i\chi_2}{\sqrt{2}}, \qquad \psi = \frac{\overline{\chi}_1^T - i\overline{\chi}_2^T}{\sqrt{2}}E^{+T} \tag{61}$$

for any MD > 0 and mD > 0. Thus, twelve Majorana states form 3 new heavy and 3 new light Dirac particles. The process led to twelve independent elements whose values are determined by the gap equations. We can then estimate values arising from the matrix form. The characteristic equations for the three new light neutrinos are,

$$2.7x^3-5.6x^2-3.3x + 1.77 = 0. \tag{62}$$

After finding the roots and going through the conversions, we have, ms1 = .75 MeV/c2, ms2 = .36 MeV/c2 , and finally m3s = 2.4 MeV/c2. The three heavy neutrinos masses are also found the same way by using the characteristic equations, so then we have

$$0.88x^3 -4.05x^2 - 12.55x + 13.67 = 0 \tag{63}$$

After finding the roots and going through the conversions, we have the three heavy neutrinos masses mh1 = 2.7 GeV/c2 mh2 = 0.88 GeV/c2 , and finally mh3 = 6.4 GeV/c2[9] [10].

3. Conclusion

In conclusion, new discussions about neutrino masses are playing an important part in physics beyond the Standard Model. A matrix model that includes neutrino mixing and the seesaw mechanism for neutrino masses has been used to try and calculate new masses for new species of Neutrinos. In the Standard Model neutrinos have zero mass. One must look beyond the Standard Model to find new species of Neutrinos. Many Physicists are starting to believe that neutrinos play an important part in dark matter. By using the seesaw mechanism physicists can explain the connection between neutrino masses and dark matter. These new models bring about the need for heavier neutrino masses. This paper has tried to calculate new masses for these heavier neutrinos. The seesaw mechanism can also help us explain why there is more matter than antimatter in the different universes. [10] [11] [12] [13] [14].

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