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## Measuring one way speed of light

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#### Abstract

In this paper, we present a method for measuring the one-way speed of light. In order to achieve this, it is necessary to solve two problems. The first problem is how to synchronize the clocks at the source and the detector, and the second problem is to prove that the slow movement of one of the clocks does not significantly affect the already established synchronization between the two clocks.


Keywords: one-way speed of light, clock synchronisation

## 1. Introduction

In his famous paper [1], Einstein formulated a definition of two synchronized clocks. The author begins with a descriptive definition of simultaneous events :

If, for instance, I say, "That train arrives here at 7 o'clock," I mean something like this: "The POINTING OF THE SMALL HAND OF MY WATCH TO 7 and the arrival of the train are simultaneous events,

If the observer is at the railway station, then he could confirm that the train arrived at 7 h , but if the observer is sitting in a city park, the fact that his watch shows 7 h does not mean that the train really arrived at 7 h .

The author further assumes that there are two clocks at points $A$ and $B$, which we will denote by $C_{A}$ and $C_{B}$, respectively.

If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events. If there is at the point B of space another clock in all respects resembling the one at A.

What does it mean that the watch $C_{B}$ in all respects resembles the watch $C_{A}$ ? We can understand that watches $C_{A}$ and $C_{B}$ are physically identical, but does that mean that they also measure identical time? In other words, how can an observer at a train station and an observer sitting in the city park know that their clocks measure the identical time? And finally, assuming that the speed of light is constant, the author formulates a definition of two synchronized clocks.
...We establish by definition that the "time" required by light to travel from A to B equals the "Time" it Requires to travel from B to A. Let a ray of light start at the "A time $t_{A}$ " from A
towards B, Let it at the "B time" $t_{B}$ Be Reflected at B in the direction of A, and arrive again at A at the "A time" $t_{A}^{\prime}$. In accordance with definition the two clocks synchronize if

$$
\begin{equation*}
t_{B}-t_{A}=t_{A}^{\prime}-t_{B} \tag{1}
\end{equation*}
$$

We ASSume that this definition of synchronism is free from contradictions.

From Equation (1) it follows that:

$$
\begin{equation*}
t_{B}=\frac{t_{A}+t_{A}^{\prime}}{2} \tag{2}
\end{equation*}
$$

According to the definition, the clocks $C_{A}$ and $C_{B}$ are synchronized if Equation(2) holds. If the initial times on the clocks $C_{A}$ and $C_{B}$ have been set independently of each other, then Equation (2) can only be true by chance.

It is obvious that this definition has two major drawbacks. Firstly, it remains unclear how the initial times were set on the clocks and secondly, on the basis of which experiment the author concluded that the speed of light is constant.

Therefore, we cannot accept the assumption that 'THE DEFINITION OF SYNCHRONISM IS FREE FROM CONTRADICTIONS' so it is necessary to reformulate the definition of two synchronized clocks.

## 2. Synchronization of clocks without the assumption of a constant speed of light

Suppose that at the same altitude we have three collinear points that we will mark with $A, S$ and $B$. At each point there is a clock, which we will mark with $C_{A}, C_{S}$ and $C_{B}$. All three clocks measure time in the same time units and the times measured by clocks $C_{A}, C_{S}$ and $C_{B}$ will be denoted by $t_{A}, t_{S}$ and $t_{B}$ respectively.

Let us define a one-dimensional coordinate system whose origin is determined by the point $O, \operatorname{Fig}(1)$. Suppose we have chosen four points $A, S, B$ and $B^{\prime}$ so that $S A=S B$.


Figure 1: At points $A, S$ and $B$ there are three clocks marked with $C_{A}, C_{S}$ and $C_{B}$ respectively

From point $S$ we will simultaneously emit two signals in the direction of points $A$ and $B$. The time on clock $C_{S}$ at that moment is equal to zero $\left(t_{S}=0\right)$. When the signal arrives at point $A$, the time on clock $C_{A}$ will be reset $\left(t_{A}=0\right)$. The same applies to the $C_{B}$ clock. If with $c_{B}$ and $c_{A}$ denote the speed of light in the direction $S B$ and $S A$ respectively, then we have the following equations:

$$
\begin{array}{r}
c_{B}=c+v \\
c_{A}=c-v \\
l=S A=S B \\
\Delta t_{A}=\frac{l}{c_{A}} \\
\Delta t_{B}=\frac{l}{c_{B}} \tag{7}
\end{array}
$$

Where $c$ and $v$ are unknowns that have yet to be determined. After the signals have arrived at points $A$ and $B$, the following equations apply:

$$
\begin{array}{r}
t_{S}=t_{A}+\Delta t_{A} \\
t_{S}=t_{B}+\Delta t_{B} \\
t_{A}+\Delta t_{A}=t_{B}+\Delta t_{B} \\
t_{B}=t_{A}+\Delta t_{A}-\Delta t_{B} \\
t_{A}=t_{B}+\Delta t_{B}-\Delta t_{A} \tag{12}
\end{array}
$$

We will say that the clocks $C_{A}$ and $C_{B}$ are synchronized if there exists a function $f()$ and its inverse function $f^{-1}()$ such that $t_{B}=f\left(t_{A}\right)$ and $t_{A}=f^{-1}\left(t_{B}\right)$, where $t_{A}$ and $t_{B}$ denote the times that were simultaneously measured by the clocks $C_{A}$ and $C_{B}$ respectively. We will say that the times $t_{A}$ and $t_{B}$ are equivalent and write this in the following way $t_{A} \sim t_{B}$. It is obvious that the relation $\sim$ is reflexive, symmetric, and transitive.

We will define the functions $f_{A}$ and $f_{B}$ in the following way

$$
\begin{array}{r}
t_{A}=f_{A}\left(t_{S}\right)=t_{S}-\Delta t_{A} \\
t_{S}=f_{A}^{-1}\left(t_{A}\right)=t_{A}+\Delta t_{A} \\
t_{B}=f_{B}\left(t_{S}\right)=t_{S}-\Delta t_{B} \\
t_{S}=f_{B}^{-1}\left(t_{B}\right)=t_{B}+\Delta t_{B} \\
\left(t_{A} \sim t_{S}\right) \wedge\left(t_{S} \sim t_{B}\right) \Rightarrow\left(t_{A} \sim t_{B}\right) \\
t_{B}=f_{B}\left(t_{S}\right)=f_{B}\left(f_{A}^{-1}\left(t_{A}\right)\right)=f_{B} \circ f_{A}^{-1}\left(t_{A}\right)=t_{A}+\Delta t_{A}-\Delta t_{B} \\
t_{A}=f_{A}\left(t_{S}\right)=f_{A}\left(f_{B}^{-1}\left(t_{B}\right)\right)=f_{A} \circ f_{B}^{-1}\left(t_{B}\right)=t_{B}+\Delta t_{B}-\Delta t_{A} \tag{19}
\end{array}
$$

Now we will estimate the error in measuring the speed of light using two clocks in the case that one clock moves with a uniform velocity relative to the point $O, \operatorname{Fig}(1)$. Suppose clock $C_{B}$ has been moved with velocity u from point $B$ to point $B^{\prime}$. Denote by $t_{B B^{\prime}}$ the time measured by clock $C_{A}$ that is required for clock $C_{B}$ to be moved from point $B$ to $B^{\prime}$.

$$
\begin{gather*}
L=B B^{\prime}  \tag{20}\\
t_{B B^{\prime}}=\frac{L}{u} \tag{21}
\end{gather*}
$$

According to $S T R$ the clock $C_{B}$ starts to measure time in the different time units compared to clock $C_{A}$. Denote by $\tau_{B B^{\prime}}$ the time measured by clock $C_{B}$ that is required for clock $C_{B}$ to be moved from point $B$ to $B^{\prime}$. Then, the following relation holds between times $t_{B B^{\prime}}$ and $\tau_{B B^{\prime}}[1]$.

$$
\begin{array}{r}
\tau_{B B^{\prime}}=t_{B B^{\prime}} \sqrt{1-\frac{u^{2}}{c^{2}}} \\
\epsilon=t_{B B^{\prime}}-\tau_{B B^{\prime}}=t_{B B^{\prime}}-t_{B B^{\prime}} \sqrt{1-\frac{u^{2}}{c^{2}}}=|u \ll c| \approx t_{B B^{\prime}} \frac{u^{2}}{2 c^{2}}=\frac{L}{u} \frac{u^{2}}{2 c^{2}}=\frac{L u}{2 c^{2}} \\
\epsilon \approx \frac{L u}{2 c^{2}} \tag{24}
\end{array}
$$

After arriving at point $B^{\prime}$, the clock $C_{B}$ starts again to measure time in the same time units as the stationary clock $C_{A}$. This claim should be experimentally confirmed or rejected. This means that the following equality must hold:

$$
\begin{equation*}
t_{A}^{\prime}-t_{A}=\tau_{B^{\prime}}^{\prime}-\tau_{B^{\prime}} \tag{25}
\end{equation*}
$$

Where $t_{A}$ and $t_{A}^{\prime}$ are the times measured by the clock $C_{A}$ and $\tau_{B^{\prime}}$ and $\tau_{B^{\prime}}$ are the corresponding times in relation to $t_{A}$ and $t_{A}^{\prime}$, measured by the clock $C_{B}$. We measure time $t_{A}$ and send a signal in the direction of point $B^{\prime}$. When the signal arrives at point $B^{\prime}$ with $\tau_{B^{\prime}}$ we will mark the corresponding time on the clock $C_{B} \ldots$

In addition to the relativistic interpretation of the reasons why a moving clock measures different time than a stationary clock, we will offer an alternative explanation. If we prove that the speed of light is not constant, then the $S T R$ is not correct either. Therefore, the movement of the clock $C_{B}$ in relation to the clock $C_{A}$ does not affect time, which is absolute, but the instrument which we use to measure time. This means that the actual "time dilation" should be determined empirically.

## 3. One way speed of light

In the previous section, we defined the method of synchronization between two clocks. In this section we will prove that the slow movement of one of the clocks does not significantly affect the already established synchronization between them.

Suppose that a signal is sent from point $A$ in the direction of point $B^{\prime}$. Let's denote by $T_{A}$ the time shown by the clock $C_{A}$ at that moment. Let's denote by $\tau_{B^{\prime}}$ the time, measured by the clock $C_{B}$, when the signal arrived at point $B^{\prime}$ and was then reflected in the direction of point $A$. And finally, mark with $T_{A}^{\prime \prime}$ the time, measured by the clock $C_{A}$, when the signal arrived at point $A$.

We will denote with $T_{A}^{\prime}$ the time measured by the clock $C_{A}$ so that $T_{A}^{\prime} \sim \tau_{B^{\prime}}$

From the Equation(12) it follows:

$$
\begin{equation*}
T_{A}^{\prime}=\tau_{B^{\prime}}+\Delta t_{B}-\Delta t_{A} \tag{26}
\end{equation*}
$$

We will define $\Delta T^{\prime}$ and $\Delta T^{\prime \prime}$ as it follows:

$$
\begin{array}{r}
\Delta T^{\prime}=T_{A}^{\prime}-T_{A} \\
\Delta T^{\prime}=\tau_{B^{\prime}}-T_{A}+\Delta t_{B}-\Delta t_{A} \\
\Delta T^{\prime \prime}=T_{A}^{\prime \prime}-T_{A}^{\prime} \\
\Delta T^{\prime \prime}=T_{A}^{\prime \prime}-\tau_{B^{\prime}}-\Delta t_{B}+\Delta t_{A} \tag{30}
\end{array}
$$

Our task is to measure the time it takes for light to travel from point $A$ to point $B^{\prime}$ (and vice versa). We will denote that time with $\Delta T_{A B^{\prime}}$ and compare it with the $\Delta T^{\prime}$. Due to the "time dilation" caused by the movement of the clock $C_{B}$ it follows that:

$$
\begin{align*}
& \Delta T_{A B^{\prime}}=\Delta T^{\prime}+\epsilon  \tag{31}\\
& \Delta T^{\prime}=\Delta T_{A B^{\prime}}-\epsilon \tag{32}
\end{align*}
$$

$$
\begin{array}{r}
c+v=\frac{2 l+L}{\Delta T_{A B^{\prime}}} \\
\Delta e=\frac{\epsilon}{\Delta T_{A B^{\prime}}}=\frac{L u}{2 c^{2}} \frac{c+v}{2 l+L}=\frac{L}{2 l+L} \frac{c+v}{c} \frac{u}{2 c} \\
\epsilon=\Delta T_{A B^{\prime}} \Delta e \tag{35}
\end{array}
$$

Unknown $c$ nor $v$ have not yet been determined, but we assume that $(v \ll c)$.

$$
\begin{array}{r}
(v \ll c) \Rightarrow\left(\frac{c+v}{c} \rightarrow 1\right) \\
(u \rightarrow 0) \\
(36) \wedge(37) \Rightarrow(\Delta e \rightarrow 0) \\
\frac{2 l+L}{\Delta T^{\prime}}=\frac{2 l+L}{\Delta T_{A B^{\prime}}-\epsilon}=\frac{2 l+L}{\Delta T_{A B^{\prime}}-\Delta T_{A B^{\prime}} \Delta e}=\frac{2 l+L}{\Delta T_{A B^{\prime}}} \frac{1}{1-\Delta e} \approx \frac{2 l+L}{\Delta T_{A B^{\prime}}}(1+\Delta e) \\
\frac{2 l+L}{\Delta T^{\prime}}=(c+v)+\frac{L}{2 l+L}\left(\frac{c+v}{c}\right)^{2} \frac{u}{2} \approx(c+v) \tag{40}
\end{array}
$$

Making an error $\epsilon$, arbitrarily small compared to $\Delta T_{A B^{\prime}}$, it follows that:

$$
\begin{array}{r}
\Delta T^{\prime}=\frac{2 l+L}{c+v} \\
\frac{2 l+L}{c+v}=\tau_{B^{\prime}}-T_{A}+\Delta t_{B}-\Delta t_{A} \\
\frac{2 l+L}{c+v}-\Delta t_{B}+\Delta t_{A}=\tau_{B^{\prime}}-T_{A} \\
\frac{2 l+L}{c+v}-\frac{l}{c+v}+\frac{l}{c-v}=\tau_{B^{\prime}}-T_{A} \\
\frac{l+L}{c+v}+\frac{l}{c-v}=\tau_{B^{\prime}}-T_{A} \tag{45}
\end{array}
$$

In a similar way, we will determine the speed of light in the direction $B A$.

$$
\begin{array}{r}
\Delta T^{\prime \prime}=\frac{2 l+L}{c-v} \\
\frac{2 l+L}{c-v}=T_{A}^{\prime \prime}-\tau_{B^{\prime}}-\Delta t_{B}+\Delta t_{A} \\
\frac{2 l+L}{c-v}+\Delta t_{B}-\Delta t_{A}=T_{A}^{\prime \prime}-\tau_{B^{\prime}} \\
\frac{2 l+L}{c-v}+\frac{l}{c+v}-\frac{l}{c-v}=-T_{A}^{\prime \prime}-\tau_{B^{\prime}} \\
\frac{l}{c+v}+\frac{l+L}{c-v}=T_{A}^{\prime \prime}-\tau_{B^{\prime}} \tag{50}
\end{array}
$$

Thus, from Equations (45) and (50), we obtained a system of two linear equations with two unknowns $\frac{1}{c+v}$ and $\frac{1}{c-v}$.

$$
\begin{gather*}
(l+L) \frac{1}{c+v}+l \frac{1}{c-v}=\left(\tau_{B^{\prime}}-T_{A}\right)  \tag{51}\\
l \frac{1}{c+v}+(l+L) \frac{1}{c-v}=\left(T_{A}^{\prime \prime}-\tau_{B^{\prime}}\right)  \tag{52}\\
\Delta=\left|\begin{array}{cc}
l+L & l \\
l & l+L
\end{array}\right|=L^{2}+2 L l=L(L+2 l)
\end{gather*}
$$

In order to have non-trivial solutions, it is necessary that $L>0$.

$$
\begin{gather*}
\Delta_{+}=\left|\begin{array}{cc}
\left(\tau_{B^{\prime}}-T_{A}\right) & l \\
\left(T_{A}^{\prime \prime}-\tau_{B^{\prime}}\right) & l+L
\end{array}\right| \\
\Delta_{-}=\left|\begin{array}{cc}
l+L & \left(\tau_{B^{\prime}}-T_{A}\right) \\
l & \left(T_{A}^{\prime \prime}-\tau_{B^{\prime}}\right)
\end{array}\right| \\
c+v=\frac{\Delta}{\Delta_{+}}  \tag{53}\\
c-v=\frac{\Delta}{\Delta_{-}} \tag{54}
\end{gather*}
$$

It is obvious that from Equations (53) and (54) we can easily determine $c$ and $v$. In this way, we can claim that we have fully established the synchronization between the clocks $C_{A}$ and $C_{B}$ according to the Definition 2.

Now we can make measurements in different directions taking into account that the clocks are at the same altitude, and compare the obtained values for $c$ and $v$.

If there is at least one direction such that $(v>0)$ then the assumption of a constant speed of light should be rejected.

## References

[1] A. Einstein
(1905) On The Electrodynamics of moving bodies
https://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf

