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Doppler effect and one-way speed of light

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Abstract:

The Doppler effect is one of the most important and most elementary phenomenon in nature. If we accept that fact, then there is no need to use any other theories to understand its essence. There is only one Doppler effect, regardless of whether the signal between the sender and the receiver travels directly or through a medium. Therefore, there is only one formula that is valid in all cases. The goal of this paper is to determine that formula. Using the obtained results, we will show that the one-way speed of light can be measured by physical experiment.

Keywords: *Doppler effect, one-way speed of light, absolute velocity*

1. Introduction

In physics, there are two formulas for the Doppler effect. Simply speaking, one formula applies to sound and the other to electromagnetic waves. In the first case sound waves propagate in a medium. The velocity of the observer and of the source are relative to the medium in which the waves are transmitted. The Doppler effect therefore results from motion of the source or motion of the observer [1].
For electromagnetic waves the Doppler effect is defined as the apparent shift in frequency of a wave in relation to an observer who is moving relative to the wave source. This means that only the relative velocity between the observer and the source needs to be considered [1].

2. Doppler effect equation

Let us define a one-dimensional coordinate system whose origin is determined by the point O and OA defines the positive direction of the coordinate axis.

$$\mathbf{e} = \frac{\mathbf{OA}}{OA} = [1] \tag{1}$$

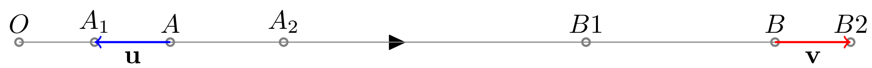

Figure 1: At time interval t_A two photons were emitted from a point A towards point B

Suppose that at some point the first photon has been emitted from the point A, Fig(1). The direction in which the photon was emitted will be denoted by the unit vector $\mathbf{a}$. The point A moves at a uniform velocity $\mathbf{u}$ with respect to the fixed point O, and its position after time t_A is denoted by A_1. The second photon has been emitted from the point A_1. The position of the first photon in that moment, is noted by point A_2. Let c denotes the speed with which the photons move from point A to point B.

$$\mathbf{a} \in \{\mathbf{e}, -\mathbf{e}\} \tag{2}$$

$$u_r = \mathbf{u} \cdot \mathbf{a} \tag{3}$$

$$AA_1 = u_r * t_A \tag{4}$$

$$AA_2 = c * t_A \tag{5}$$

$$A_1A_2 = (c - u_r) * t_A \tag{6}$$

After some time, the first photon will arrive at the point noted with B, and the second photon will arrive at the point noted with B_1. The point B moves at a uniform velocity $\mathbf{v}$ with respect to the fixed point O. Let us denote by t_B the time required for the second photon to arrive at point B, whose position at that moment is denoted by B_2 , Fig(1).

$$v_r = \mathbf{v} \cdot \mathbf{a} \tag{7}$$

$$B_1B = A_1A_2 = (c - u_r) * t_A \tag{8}$$

$$BB_2 = t_B * v_r \tag{9}$$

$$B_1B_2 = t_B * c \tag{10}$$

$$B_1B_2 = B_1B + BB_2 \tag{11}$$

$$t_B * c = (c - u_r) * t_A + t_B * v_r \tag{12}$$

$$(c - v_r) * t_B = (c - u_r) * t_A \tag{13}$$

$$t_B = \frac{c - u_r}{c - v_r} t_A \tag{14}$$

$$f_B = \frac{1}{t_B} \tag{15}$$

$$f_A = \frac{1}{t_A} \tag{16}$$

$$f_B = \frac{c - v_r}{c - u_r} f_A \tag{17}$$

$$f_B = \frac{1 - \frac{v_r}{c}}{1 - \frac{u_r}{c}} f_A \tag{18}$$

$$f = f_A \tag{19}$$

$$f' = f_B \tag{20}$$

Equation (17) can be written in the following ways

$$f' = \frac{c - v_r}{c - u_r} f \tag{21}$$

$$f' = \frac{c - v_r}{c - u_r} f = \frac{c - v_r + u_r - u_r}{c - u_r} f = \frac{c - u_r - (v_r - u_r)}{c - u_r} f \tag{22}$$

$$\Delta v_r = v_r - u_r \tag{23}$$

$$f' = \left(1 - \frac{\Delta v_r}{c - u_r}\right) f \tag{24}$$

$$\Delta f' = -\frac{\Delta v_r}{c - u_r} f \tag{25}$$

If instead of two photons we considered the emission of two successive crests of a electromagnetic wave, then we would get identical formulas. This means that the formulas we have derived do not depend on what the nature of light is.

Therefore, we can write the Doppler effect equation, regardless of whether we observe sound or electromagnetic waves, in the following way:

$$f' = \frac{c - \mathbf{a} \cdot \mathbf{v}}{c - \mathbf{a} \cdot \mathbf{u}} f \tag{26}$$

We apply formula (26) in the case when the sender and receiver of the signal move along the $x - axis$. The general solution for the Doppler effect is given in [2].

It follows from Equation (21) that:

$$c = \frac{f' u_r - f v_r}{f' - f} \tag{27}$$

$$c = \frac{f' u_r - f v_r}{f' - f} = \frac{f' u_r - f v_r + f u_r - f u_r}{f' - f} = \frac{(f' - f) u_r - f(v_r - u_r)}{f' - f} \tag{28}$$

$$c = u_r - \frac{f \Delta v_r}{f' - f} \tag{29}$$

$$c = u_r - \frac{\Delta v_r}{\Delta f'} f \tag{30}$$

In the special case if $(u_r = v_r)$ then it follows that:

$$\Delta v_r = 0 \tag{31}$$

$$f' = f \tag{32}$$

$$c = \frac{f v_r - f v_r}{f - f} = \frac{0}{0} \tag{33}$$

This means that in this case the speed of light c is undetermined.

If the signal is emitted from point B towards point A, we have the following equations:

$$\mathbf{a} = -\mathbf{e} \tag{34}$$

$$u_r = \mathbf{v} \cdot \mathbf{e} = -v_r \tag{35}$$

$$v_r = \mathbf{u} \cdot \mathbf{e} = -u_r \tag{36}$$

$$f' = \frac{c + u_r}{c + v_r} f \tag{37}$$

This formula was already derived in a different way in [2].

We will apply the Equation (27) to calculate the one way speed of light in relation to the Earth-centered inertial frame [3].

3. One way speed of light regarding the Earth-centered inertial frame

We will consider the case when the center of the coordinate system coincides with the center of the Earth.

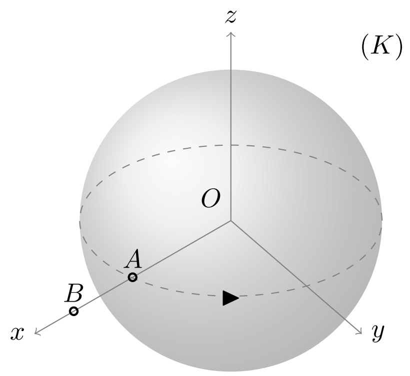

Figure 2: Earth-Centered Inertial Coordinate System

The Earth-Centered Inertial Coordinate System Fig(2), which we will denote by (K) is defined as follows:

1. The origin is at the centre of the Earth.
2. The fundamental plane (x, y) lies in the plane of the Earth's equator.
3. A right-handed convention.
4. The $x - axis$ is fixed relative to distant stars.

Let's assume that we made measurements along AB which has the same direction as the positive $x - axis$, Fig(2). The sender is marked by point A and moves with a velocity $\mathbf{u}$ with respect to the coordinate system (K). The receiver is marked by point B and moves with respect to the coordinate system (K) with a velocity $\mathbf{v}$. The signal frequencies at point A and B will be denoted by f and f^+ respectively.

In relation to the coordinate system (K) we have the following equations:

$$\mathbf{u} = [u_x, u_y, u_z] \tag{38}$$

$$\mathbf{v} = [v_x, v_y, v_z] \tag{39}$$

$$\mathbf{e} = [1, 0, 0] \tag{40}$$

$$\mathbf{a} = \mathbf{e} \tag{41}$$

$$\mathbf{u} \cdot \mathbf{a} = u_x \tag{42}$$

$$\mathbf{v} \cdot \mathbf{a} = v_x \tag{43}$$

$$u_r = u_x \tag{44}$$

$$v_r = v_x \tag{45}$$

$$\Delta v_x = v_x - u_x \tag{46}$$

$$\Delta v_r = v_r - u_r = v_x - u_x = \Delta v_x \tag{47}$$

Equation (26) can be written in the following way:

$$f^+ = \left(\frac{c_x^+ - v_x}{c_x^+ - u_x} \right) f \tag{48}$$

Where the speed of the light in positive $x - axis$ is noted by c_x^+.

From Equation (48) it follows that:

$$c_x^+ = u_x - \frac{f}{f^+ - f} \Delta v_x \tag{49}$$

$$f^+ = \left(1 - \frac{\Delta v_x}{c_x^+ - u_x} \right) f \tag{50}$$

$$\Delta f^+ = f^+ - f \tag{51}$$

$$\Delta f^+ = -\frac{\Delta v_x}{c_x^+ - u_x} f \tag{52}$$

We will analyze three possibilities:

1) c_x^+ is equal to some constant value

From Equation (52) it follows that the change in frequency Δf^+ directly depends on u_x, which is not true. Therefore, this possibility should be rejected. An alternative possibility is to introduce some new arithmetical rules for the classical equations of motion to explain this "anomaly".

2) $(c_x^+ - u_x)$ is equal to some constant value

We will define W_x^+ as follows:

$$W_x^+ = c_x^+ - u_x = -\frac{f}{f^+ - f}\Delta v_x = -\frac{\Delta v_x}{\Delta f^+}f \tag{53}$$

$$c_x^+ = u_x + W_x^+ \tag{54}$$

$$\Delta f^+ = -\frac{\Delta v_x}{W_x^+}f \tag{55}$$

$$\lambda^+ = -\frac{\Delta v_x}{\Delta f^+} \tag{56}$$

$$W_x^+ = \lambda^+ f \tag{57}$$

If we vary f,$\mathbf{u}$ and $\mathbf{v}$ then W_x^+ is equal to some constant value, which should be proven by experiment.

In the direction of negative $x-axis$ we have the following equations:

$$\mathbf{a} = -\mathbf{e} \tag{58}$$

$$u_r = \mathbf{u}\cdot\mathbf{a} = -u_x \tag{59}$$

$$v_r = \mathbf{v}\cdot\mathbf{a} = -v_x \tag{60}$$

$$\Delta v_r = v_r - u_r = -\Delta v_x \tag{61}$$

The speed of the light in that direction is noted by c_x^-.

Equation (26) can be written in the following way:

$$f^- = \left(\frac{c_x^- + v_x}{c_x^- + u_x}\right)f \tag{62}$$

From Equation (62) it follows that:

$$c_x^- = -u_x + \frac{f}{f^- - f}\Delta v_x \tag{63}$$

We will define W_x^- as follows:

$$W_x^- = c_x^- + u_x = \frac{f}{f^- - f}\Delta v_x = \frac{\Delta v_x}{\Delta f^-}f \tag{64}$$

$$c_x^- = -u_x + W_x^- \tag{65}$$

$$\Delta f^- = \frac{\Delta v_x}{W_x^-}f \tag{66}$$

$$\lambda^- = \frac{\Delta v_x}{\Delta f^-} \tag{67}$$

$$W_x^- = \lambda^- f \tag{68}$$

If we vary f,$\mathbf{u}$ and $\mathbf{v}$ then W_x^- is equal to some constant value, which should be proven by experiment.

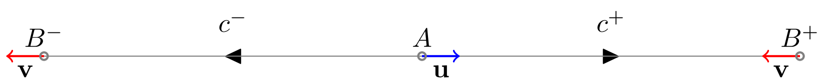

Figure 3: From point A, the signals were simultaneously sent in the direction of points B^+ and B^-, respectively

For the sake of simplicity, we assumed that the velocity **u** is the same in both measurements. The same applies to the velocity **v**, Fig(3).

Let define c_x, λ_x, w_x and c_H as it follows:

$$c_x = \frac{c_x^+ + c_x^-}{2} = \frac{W_x^+ + W_x^-}{2} \tag{69}$$

$$2c_x = -\frac{f}{f^+ - f}\Delta v_x + \frac{f}{f^- - f}\Delta v_x = \frac{f^+ - f^-}{(f^+ - f)(f^- - f)} f\Delta v_x \tag{70}$$

$$c_x = \frac{\Delta f^+ - \Delta f^-}{2\Delta f^+ \Delta f^-}\Delta v_x f \tag{71}$$

$$\lambda_x = \frac{\lambda^+ + \lambda^-}{2} \tag{72}$$

$$\lambda_x = \frac{\Delta f^+ - \Delta f^-}{2\Delta f^+ \Delta f^-}\Delta v_x \tag{73}$$

$$c_x = \lambda_x f \tag{74}$$

$$w_x = W_x^+ - c_x = -W_x^- + c_x \tag{75}$$

$$c_H = \frac{2c_x^+ c_x^-}{c_x^+ + c_x^-} \tag{76}$$

By definition c_x is equal to the arithmetic mean between the c_x^+ and c_x^- while the speed of light in two directions noted by c_H is equal to the harmonic mean between the c_x^+ and c_x^-. The inequality $c_x \geq c_H$ is always valid.

Now we have that:

$$W_x^+ = c_x + w_x \tag{77}$$

$$W_x^- = c_x - w_x \tag{78}$$

$$c_x^+ = (u_x + w_x) + c_x \tag{79}$$

$$c_x^- = -(u_x + w_x) + c_x \tag{80}$$

In the case if (**v=0**) it follows that:

$$\Delta v_x = v_x - u_x = -u_x \tag{81}$$

$$c_x^+ = -\Delta v_x - \frac{f}{f^+ - f}\Delta v_x = -\Delta v_x \frac{f^+}{\Delta f^+} \tag{82}$$

$$c_x^+ = \frac{f^+}{f} W_x^+ \tag{83}$$

Equation (83) has already been proved in [4].

Equation (71) can be written in the following way:

$$c_x = \frac{\Delta f^- - \Delta f^+}{2\Delta f^+ \Delta f^-} u_x f \tag{84}$$

In a similar way, we can perform measurements in the same directions as the positive and negative $y-axis$. We will define the c_y^+, c_y^-, w_y and c_y in an analogous way as the c_x^+, c_x^-, w_x and c_x were defined.

And finally we will perform measurements in the same directions as the positive and negative $z-axis$. We will define the c_z^+, c_z^-, w_z, and c_z in an analogous way as the c_x^+, c_x^-, w_z and c_x were defined.

If the following equations

$$c_x = c_y = c_z \tag{85}$$

are valid, then we can define constant speed of light noted by c as follows:

$$c = c_x = c_y = c_z \tag{86}$$

In that case, we can also define the velocity $\mathbf{w}$ as it follows:

$$\mathbf{w} = [w_x, w_y, w_z] \tag{87}$$

Let the unit vector $\mathbf{a}$ denotes the direction in which light moves, and the velocities $\mathbf{u}$ and $\mathbf{v}$ are given as follows:

$$\mathbf{u} = u\mathbf{a} \tag{88}$$

$$\mathbf{v} = v\mathbf{a} \tag{89}$$

$$\Delta v = \Delta\mathbf{v} \cdot \mathbf{a} \tag{90}$$

$$w_a = \mathbf{a} \cdot \mathbf{w} \tag{91}$$

The formulas for the change in frequency Δf_a and speed of light c_a along any direction can be written in the following way:

$$\Delta f_a = -\frac{\Delta\mathbf{v} \cdot \mathbf{a}}{c + \mathbf{w} \cdot \mathbf{a}} f = -\frac{\Delta v}{c + w_a} f \tag{92}$$

$$c_a = (u + w_a) + c \tag{93}$$

We will consider two possibilities:

2.1) ($||\mathbf{w}|| >> 0$)

Then we could say that we detected and calculated the absolute velocity noted by $\mathbf{w}$. But, because w_a changes its value, it follows from the Equation (92) that Δf_a depends on the direction in which the light propagates. We are not aware of any experiment that could confirm this. Therefore, we can formally reject this possibility as well.

2.2) ($||\mathbf{w}|| \to 0$)

We have following equations:

$$w_a = 0 \tag{94}$$

$$c = -\frac{\Delta v}{\Delta f} f \tag{95}$$

$$c_a = u + c \tag{96}$$

If our analyzes are correct, then the expression on the right side of Equation (95) remains unchanged when when we vary Δv, f and the direction in which the light propagates.
The speed c_a depends on the velocity $\mathbf{u}$ at which the transmitter moves. We do not see any contradiction here because the speed of light in one direction has not been experimentally measured so far.
remains unchanged when
3) $(c_x^+ - u_x)$ is not equal to some constant value.

This is an unexpected result. Maybe the proposed formula is not correct or there are errors in the measurements.

4. Conclusion

Instead of The Earth-Centered Inertial Coordinate System, we could assume that the measurements refer to some other coordinate system, for example Heliocentric-ecliptic Coordinate System.
It is easy to prove that the speed of light c as defined by Equation (95) does not change its value, while the speed of light c_a as defined by Equation (96) changes its value in accordance to the velocity with which the signal transmitter moves.

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