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Photon Characteristics and Behavior under Refractive Index

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Abstract;

The effective mass, energy and momentum of photon in medium is described. The photon has both kinetic and potential energy and displays an effective mass when travelling through a medium. The photon is a type of elementary particle that serves as the quantum of the electromagnetic field and is explained through equations to have an elastic constant, volume, momentum, force, power and zero rest mass. The theory that photons a massless only applies to rest mass. A photon undergoes different dynamics such as strain and changes in effective mass under the influence of the refractive index that gives rise to a refractive force. This study synthesizes some properties of energy such as elastic constant, weight, and volume in order to explain the nature of photons and photon dynamics under refractive index. In this study, a helical spring model has been used to describe the behavior of a photon in the medium. The model connects together the exhibited properties such as energy, effective mass, spin, elastic constant and volume. Further explanation is given to the effect of gravitational force on wavelength and refractive index of air, and how gravitational force causes red and blue shifts in photon propagation. The quantization of energy has been extended to the quantization of space, time and matter. This study is consistent and consolidates the existing classical and quantum theories. There are further potential applications in optical communication, quantum cryptography, quantum optics, and medicine, especially in the use of two-photon excitation microscopy and photodynamic therapy.

Keywords: photon, mass, refractive index, wavelength, energy

Introduction

It is understood that all matter in the form of particles or system of particles contains and/or is supported by one form of energy or another [1-2]. The state of the universe, for example, is determined by its state of energy. The interaction of forces in nature simply leads to the exchange of energy between particles and systems [3]. It is therefore clear that energy is very fundamental to nature. Photons are the smallest packets (quantum) of the different forms of energy. Photons are regarded as the building block of energy, being the smallest particle in the energy ensemble [4]. Further, a photon can be described as a self-assembling particle once it is created by its source. This study synthesizes some properties of energy and provides explanation for some of the expected properties of photons in different media.

Further, a photon is explained through equations to have an effective mass, elastic constant, volume, momentum, force and power. A photon undergoes different dynamics such as strain and change of mass under the influence of refractive index, that gives rise to a refractive force. This theory is consistent and consolidates the existing theories in classical and quantum mechanics.

A helical spring model has been used to describe the shape and behavior of a photon. The model connects together the exhibited properties such as spin, elastic constant, volume and surface area.

A discussion is made on the effect of gravitational force on wavelength and refractive index and how gravitational force causes red and blue shift in photon propagations.

This study has several applications in thermodynamics and explains, in a more precise manner, what leads to the behavior photons when they interact with air, fluids or solids. There are further potential applications in optical communication, quantum cryptography, quantum optics, and medicine, especially in the use of two-photon excitation microscopy and photodynamic therapy.

Photons are considered the smallest packets of electromagnetic (EM) waves. EM waves are a spectrum of waves that carry energy in different forms such as radio signals, heat, light and gamma particles among others. The energy of the photons can be described in terms of energy density. The volume of a photon is related to the energy density, also called the energy - volume ratio [5, 6]. The energy density is given as

$$< U > = \varepsilon_0 E^2 \quad (1)$$

Where $< U >$ is the energy density, E is the electric field, and ε_0 is the permittivity of free space.

Photons have been described as possessing energy and momentum, with momentum given by

$$p = \frac{h}{\lambda} \quad (2)$$

Where h is plank constant and λ is the wavelength.

There has been a long-standing debate on the relationship between energy and mass of the photon when it is under the medium. It is however generally agreed by many scholars [7] that a photon has zero rest mass. We equally agree on the concept of zero rest mass. However, the concept of an effective mass of a photon is still under debate and there is scanty information on this subject.

Furthermore, there is limited information on photon dynamics as light passes through a refractive medium. Most information available is centered on angles of reflection and refraction. There is also little information that describes photon geometry and on the changes to wavelength, energy, amplitude, mass and momentum through different media. This study therefore endeavors to fill the information gap on these aspects.

Model Description

Einstein's photon model theorized that energy and radiation are made up of quanta. Earlier Max Planck already proposed that energy is made up of individual units or packets of energy called quanta. The quantum field theory incorporates the electromagnetic radiation of all electrically charged matter. The relativity work of Einstein laid the foundation for the creation of the quantum field theory. With exception of the *standard model* of particle physics, which requires extension of the quantum theory in order to explain the behavior of sub-nuclear processes, all behaviors of matter are unified and can be explained by quantum field theory. The quantum field theory provides the best way to described typical behaviors of a photon. Photons displays the wave-particle duality as described by Werner Heisenberg. Photons add an invariant mass to any system they encounter as best predicated by the special theory of relativity.

In this study, a helical spring model has been used to describe the behavior of a photon in a medium. The model connects together the exhibited properties such as spin, elastic constant, volume and surface area. A discussion is made on the effect of gravitational force on wavelength and refractive index and how gravitational force causes red and blue shift in photon propagations.

Photon In Perspective

A photon is a combination of a pair of oscillating electric and magnetic fields (Fig. 1) described as an electromagnetic wave. Photons are discrete ensembles possessing both wave and particle properties [8]. The electric and magnetic fields are not separate, but have a common origin [9]. They originate from charges. An electric field originates from a stationary or moving charge whilst a magnetic field originates from a moving charge. When a charge such as an electron oscillates between two points, it emits photons. Once generated, a photon becomes a self-assembling particle of continuous changing electric and magnetic fields oscillating perpendicular to each other and propagating into space as a transverse wave. A photon has a frequency at which it self-assembles and the velocity at which it propagates. A photon is capable of self-assembling because it has internal energy and force within it that comes from the mutual oscillations of the two fields. A photon consists of a line of in-phase but diagonally oscillating electric and magnetic fields as shown in Fig. 1.

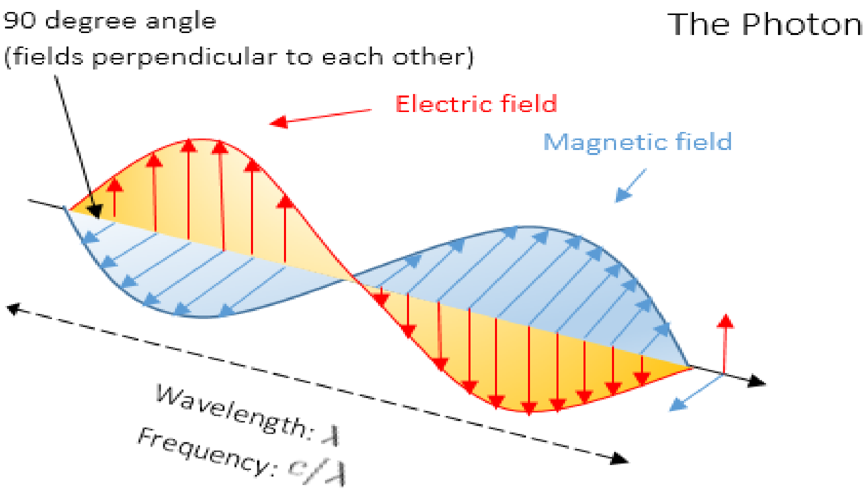


Figure 1: Illustration of the wavelength, electric and magnetic fields and propagation vector (k) of a photon (courtesy of illusterati.com)

The energy, *e*, of a photon is given by the equation

$$e = hf \tag{3}$$

Where h is plank constant and *f* is the linear frequency. The linear frequency is given by the equation

$$f = \frac{c}{\lambda} \tag{4}$$

Where *c* is the velocity of photon in vacuum and λ is the wavelength of the photon. Equation 1 becomes

$$e = \frac{hc}{\lambda} \tag{5}$$

Using the concepts of frequency and circular motion we can visualize a photon as having an amplitude to which it oscillates. The relationship between wavelength and amplitude can be derived, Fig. 2.

The circumference gives a full cycle which is a full wavelength.

$$\lambda = 2\pi r \quad \text{and} \quad r = \frac{\lambda}{2\pi} \tag{6}$$

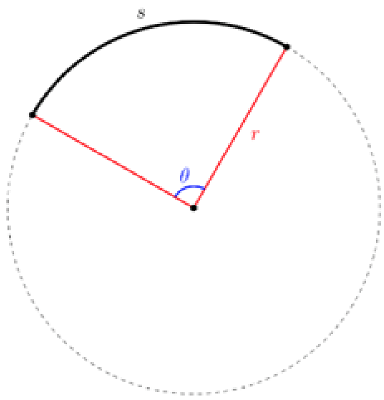


Figure 2: Depiction of circular motion. *r* is the radius of the circle representing the amplitude of motion. θ is the angle of displacement with arc length, S.

Results and Discussion

Effective Mass of a Photon

We define the effective mass of photon as non-rest mass. According to photon wave-particle duality principle, when treated as a wave, a photon behaves and is regarded as massless. However, when treated as a particle, it exhibits attributes of mass. This implies the mass of a photon can be illusive; exhibiting some real and imaginary aspects to itself, depending on how it is viewed. Some earlier prediction has stated that a photon has zero mass whether at rest or in motion. This prediction is based on the assumption that since all electromagnetic waves have same velocity in vacuum, it follows therefore that these electromagnetic particles are massless [10]. To the contrary our theory attributes the constant velocity of light in vacuum to quantization of space and time. Recent research has revealed that the rest mass of a photon is complex when it comes into contact with matter, and that the mass depends upon the scalar curvature of the surface of matter and wavelength of the photon [7, 11].

Our theoretical view, with regard to the photon's interaction with refractive index of a transparent media, is that the rest mass (m_{rest}) of a photon is zero ($m_{rest} = 0$). The reason for this will be given later in the subsections. However, the effective mass (m) of a photon is not zero. The effective mass of a photon is its mass whilst in motion, which is closely related to the definition of relative mass [12].

Other sources have suggested that the mass of a photon is entirely zero whether at rest or in motion. This cannot be entirely true because then a photon will have no momentum eqn. (2), and the energy-mass equation as we know it will be rendered null and void. Indeed, the effective mass of the photon has also been described by Salih, A. 2013[13].

We can further prove that a photon has an effective mass associated with it by using dimensional analysis on momentum of a photon of eqn.2. The units of Plank constant are Joules Seconds and the unit of wavelength is the meter (or length). Therefore,

$$p = \frac{h}{\lambda} \equiv \frac{[Joules][T]}{[L]} \tag{7}$$

$$Joules \equiv \frac{[Kg][L]^2}{[T]^2} \tag{8}$$

$$\frac{h}{\lambda} \equiv \frac{[Kg][L]^2}{[T]^2} \cdot \frac{[T]}{[L]} \tag{9}$$

$$\frac{h}{\lambda} \equiv \frac{[Kg][L]}{[T]} \tag{10}$$

Where [Kg] is mass, [L] is length, [T] is time.

From eqn. 8 and 10 we can see that energy and momentum cannot exist without mass; as their fundamental units includes mass (Kg). It is also important to note that energy and momentum are derived quantities as opposed to being regarded as primary quantities. Therefore, it is a self-contradiction to state that a photon has energy and momentum but zero mass. However, we do agree that the photon rest mass is zero. The effective mass can be derived from the mass-energy equation

$$e = mc^2 \tag{11}$$

Equation 11 is the energy-mass equation derived by Albert Einstein. This equation was later modified by Paul Dirac to include the particle's energy both at rest and in motion for the conservation of energy [10]. The modified equation is

$$e = \sqrt{p^2c^2 + (m_{rest}c^2)^2} \tag{12}$$

Can also be written as

$$e = \sqrt{(e_{motion})^2 + (e_{rest})^2} \tag{13}$$

Where

$$p = mc; \tag{14}$$

e is the total energy, e_{motion} is energy in motion, e_{rest} is energy at rest, m_{rest} is the rest mass, m is the mass whilst in motion and p is the momentum. But, for a photon,

$$e_{rest} = 0 \tag{15}$$

This is because a photon never rests (never stops) [14] as shall be discussed in details in subchapters. In a nutshell, rest energy does not apply to a photon. For the same reason its rest mass is zero,

$m_{rest} = 0$ (16)

Therefore eqn. 12 for the photon energy reduces to

$e^2 = p^2c^2$ or $e = pc$ (17)

Which is further written in the same form expressed by Albert Einstein as

$e = mc^2$ (18)

and we have

$m = \frac{h}{c\lambda}$ (19)

In this case the mass of photon varies with velocity and wavelength. Further interpretation of eqn. 19 is that *energy has mass*. Radiation with shorter wavelengths is more massive compared to those with longer wavelengths. Because energy has mass, it follows that *energy has weight*. An object that gains energy increases in weight, and conversely losses weight when it emits energy**.** Space with shorter wavelength is heavier compared to one containing longer wavelengths. It can further be deduced that blue stars are heavier than red stars if they occupied the same size of space.

Momentum of a Photon

The linear momentum (*p*) of a photon in vacuum is given in terms of plank constant and wavelength.

$p = mc$ or $p = \frac{h}{c\lambda} * c$ or $p = \frac{h}{\lambda}$ (20)

The momentum in the medium ($p^{'}$) is given by

$p^{'} = \frac{h}{\lambda^{'}}$ or $p^{'} = \frac{nh}{\lambda}$ or $p^{'} = np$ (21)

From eqn. 21 we see that the momentum of a photon increases by the factor of the refractive index as it travels from vacuum into media. The increase in momentum can also be attributed to the increase in mass of the photon. The corresponding change in momentum is given below in terms of the net refractive index.

$\Delta p = p^{'} - p$ or $\Delta p = np - p$ or $\Delta p = N^{'}p$ (22)

Whereas the linear momentum of a photon increases at the interface, its kinetic energy remains constant.

The angular momentum (*L*) of a photon in vacuum is equal to half the reduced plank constant.

$L = I\omega = \frac{mr^2\omega}{2}$ or $L = \frac{h}{c\lambda} * \frac{\lambda^2}{2(2\pi)^2} * \frac{2\pi c}{\lambda}$ or $L = \frac{h}{4\pi}$ or $L = \frac{\hbar}{2}$ (23)

The moment of inertia has been taken to be that of a cylinder spinning about the central axis as explained in details in the section on volume of a photon.

Force of A Photon

The force of a photon is defined by the energy required to produce the displacement or amplitude in the self-assembling process. A second definition is that this force is a centripetal force as depicted (Fig. 2). In both cases the formula is derived. By using the first definition,

$e = F.R$ or $F = \frac{e}{R}$ (24)

Where *F* is the force of a photon, *e* is the energy and *R* is the amplitude of oscillation.

By using eqn. 5 and 6, the above equation gives force of a photon as

$F = \frac{2\pi hc}{\lambda^2}$ (25)

On the other hand, if we used the second definition of the force, we have

$F = ma_c$ (27)

Where *m* is the mass of photon and a_c is the centripetal acceleration of the photon given by

$a_c = \frac{c^2}{R}$ or $a_c = \frac{2\pi c^2}{\lambda}$ (28)

Substituting eqn. 19 and 28 into eqn. 27 we get

$F = \frac{h}{c\lambda} * \frac{2\pi c^2}{\lambda}$ or $F = \frac{2\pi hc}{\lambda^2}$ (29)

Equations 25 and 29 are agreeable to the definitions of the force of a photon.

This force can also be expressed in terms of the amplitude, R as

$F = \frac{2\pi hc}{(2\pi R)^2}$ or $F = \frac{\hbar c}{R^2}$ (30)

Which clearly obeys the inverse square law. From the equation above, the energy of a photon becomes

$e = F.R$ or $e = \frac{\hbar c}{R}$ (31)

Which shows that the force and energy of a photon are dependent on amplitude of oscillation and not the amount of electric charge producing them. Here we should note that photons with shorter wavelengths implies smaller amplitudes of oscillation and more energy.

Further, it is important to note that a photon undergoes two types of accelerations, visa-vis centripetal and linear accelerations. The centripetal acceleration is given by equation 28. The linear acceleration of a photon occurs on three scenarios; at photon creation, at interface between two media with different refractive indices and at photon destruction (absorption). The linear acceleration at the interface is given by

$a = \frac{v-c}{t}$ or $a = \frac{(v-c)c}{\lambda}$ or $a = \frac{(\frac{c}{n}-c)c}{\lambda}$ or $a = \frac{(\frac{1}{n}-1)c^2}{\lambda}$

Which reduces to

$a = -\frac{N'}{n} * \frac{c^2}{\lambda}$ (32)

Equation 32 is the linear acceleration of a photon at interface between vacuum and a medium. The negative sign implies de-acceleration. The linear acceleration is positive when photon travels from medium to vacuum. Furthermore, a photon linearly accelerates at creation from a velocity of zero to the velocity value in vacuum as expressed below.

$a = \frac{c-0}{t}$ or $a = \frac{(c)c}{\lambda}$ or $a = \frac{c^2}{\lambda}$ (33)

At photon absorption (destruction) the final velocity is zero. Therefore, the centripetal acceleration is zero, whilst its linear acceleration is given by

$a = \frac{0-c}{t}$ or $a = \frac{-(c)c}{\lambda}$ or $a = \frac{-c^2}{\lambda}$ (34)

Notice the value of the acceleration is equal but opposite at creation and absorption.

Torque of a Photon

It is important to know the torque of the photon because it is responsible for driving the linear motion of a photon. The electric field, magnetic field and the torque are mutually perpendicular to each other. We can determine the torque of a photon using the centripetal force (as it is equivalent in magnitude with the tangential force at the boundary of the photon) and the radius of circular motion

$\tau = F\text{x}R = \frac{2\pi hc}{\lambda^2}\frac{\lambda}{2\pi} Sine(\theta)$ or $\tau = \frac{hc}{\lambda} sine(\theta)$ (35)

In vacuum $\theta = 90^0$, where θ is the angle between the radius and the tangential force. Hence the above torque equals the energy of the photon.

$\tau = \frac{hc}{\lambda}$ (36)

From this equation we see that the whole energy of a photon goes to drive its linear motion.

Surface Area and Volume of a Photon

Using the Lorentz force which is a centripetal force,

$$F = qv \times B \quad \text{but} \quad q = ER^2/k \tag{37}$$

Where E is the electric field, R is the radius and $k = 1/4\pi\varepsilon_0$. Substituting we get

$$F = 4\pi\varepsilon_0 vR^2 E \times B \tag{38}$$

For a photon in vacuum *v* = c and the angle between the electric field and magnetic field is 90^0. The equation above reduces to

$$F = 4\pi\varepsilon_0 cR^2 EB \tag{39}$$

We further note that $cB = E$. Hence

$$F = 4\pi R^2(\varepsilon_0 E^2) \quad \text{or} \quad F = 4\pi R^2\left(\frac{e}{V}\right) \tag{40}$$

Where $\varepsilon_0 E^2$ is the energy density or energy per unit volume, *e* is the energy and *V* is the volume. Since the Lorentz force is a centripetal force equivalent to the force of the photon, we equate the two.

$$\frac{2\pi hc}{\lambda^2} = 4\pi R^2\left(\frac{e}{V}\right) \quad \text{or} \quad \frac{2\pi hc}{\lambda^2} = 4\pi R^2\left(\frac{hc}{\lambda V}\right) \quad \text{or} \quad V = 4\pi R^3 \tag{41}$$

Where we took $e = \frac{hc}{\lambda}$

Substituting R for the case of a photon we get

$$V = \frac{4\pi\lambda^3}{(2\pi)^3} \quad \text{or} \quad V = \frac{\lambda^3}{2\pi^2} \tag{42}$$

From the preceding equations we note that a photon is not spherical in shape as its volumetric formula is different. On the contrary, this volume can be taken to be that of a cylinder (solenoid) with radius *r* and length 4*r* (Fig. 3) with its spin about the central axis. The solenoid shape is further supported by the helical spring model presented in the subsequent sections.

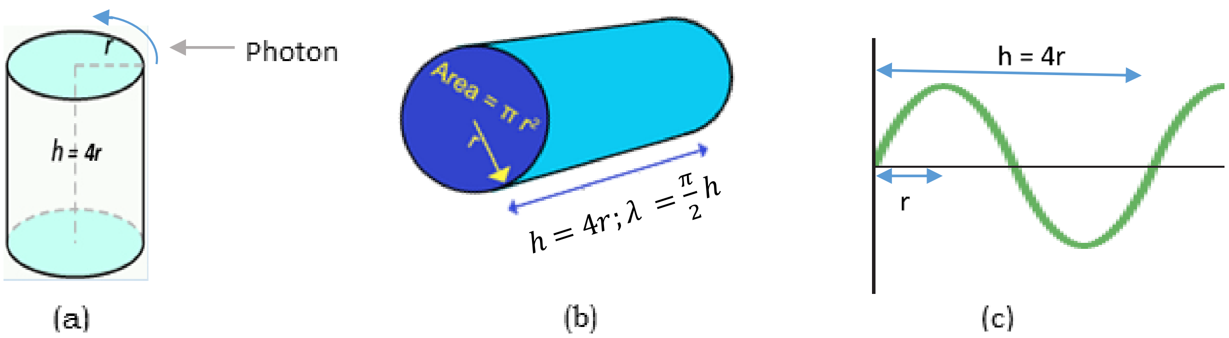


Figure 3: The solenoidal (cylindrical) shape of a photon. Showing (a) the height and spin, and (b) the cross-section area and length (h), and (c) the location of h in the sine wave function.

From the given shape, the Surface area can be taken as

$$SA = 2\pi R^2 + h(2\pi R) = 2\pi R^2 + 4R(2\pi R) \quad \text{or} \quad SA = 10\pi R^2 = \frac{10\pi\lambda^2}{4\pi^2} \tag{43}$$

This leads to $SA = \frac{5\lambda^2}{2\pi}$ (44)

By interpretation, since a photon has volume, then *energy has volume and occupies space*. This is in addition to earlier inference that energy has mass and weight. An object that gains energy increases in volume to accommodate the energy, and conversely reduces in volume when it emits that energy. This implies that there is simultaneous increase in weight and volume when energy is absorbed. However, the increase in weight is negligible compared to the increase in the volume. This explains the expansion of gasses, liquids and solids when they absorb energy. The crushing-can experiment can be taken as a good illustration of this principle.

Further to the assertion that energy has volume and occupies space, we look at the ideal heat equation and the ideal gas law.

$$Q = mCT \quad (45)$$

$$PV = nRT \quad (46)$$

Where Q is heat, m is mass, C is specific heat capacity, T is temperature, P is pressure, V is volume, n is number of moles, and R is the gas constant.

If we substitute T from eqn. 45 into 46 we get the following equation

$$Q = \frac{PVCm}{nR} \quad (47)$$

From eqn. 47, it can be seen that an increase in heat energy leads to an increase in both mass and volume. This is in agreement with our earlier assertion that energy has volume and mass.

The expansion of the universe could equally be related to an increase in ordinary energy. Since energy cannot be created nor destroyed, but converted from one form to another, we can assume the gained ordinary energy is converting from dark energy leading to increase in ordinary energy and expansion of the universe.

The Helical Spring Model

The behaviour of a photon can be modeled after a helical spring (Fig. 4). The helical spring forms a solenoidal shape which conforms to the earlier shape of a cylinder. A solenoid can be defined as a coil wound into a tightly packed helix. The helical spring can be compressed or streched by an applied external force. A photon behaves in the similar manner under a refractive force. Experiments have revealed the action of the combination of the electric and magnetic fields to a solenoid and helical path [15 - 20].

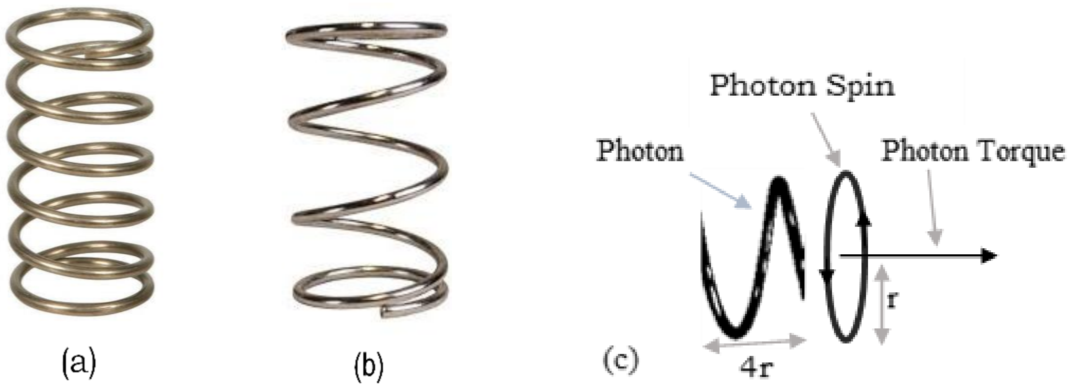


Figure 4: The helical spring depicting the shape of a photon. Part (a) is a normal unstreched spring, part (b) is a streched spring. Part (c) relates the spring to photon parameters.

Maxwell's Equations of Wave Propagation

The nature and propagation of electromagnetic waves are more frequently described by Maxwell's Equations [17];

$$\nabla . \boldsymbol{E} = 0 \quad (48\text{ a})$$

$$\nabla . \boldsymbol{B} = 0 \quad (48\text{ b})$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (48\text{ c})$$

$$\nabla \times \boldsymbol{B} = \mu\varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \quad (48\text{ d})$$

Equations 48 a and 48 b imply that the electric and magnetic fields in an EM wave is in the form of a solenoid, whilst 48 c and 48 d describe the solenoid as rotatable or a helix [18]. This agrees with our Helical Spring Model. If we further divide 48 c by 48 d, and taking the speed of light

$c = 1/\sqrt{\mu\varepsilon}$, we get another eqution.

$$\frac{\nabla \times \boldsymbol{E}}{\nabla \times \boldsymbol{B}} = -\frac{1}{\mu\varepsilon} * \frac{\partial \boldsymbol{B}}{\partial t} * \frac{\partial t}{\partial \boldsymbol{E}} \quad \text{or} \quad \frac{\nabla \times \boldsymbol{E}}{\nabla \times \boldsymbol{B}} = -\frac{1}{\mu\varepsilon} * \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{E}} \tag{49}$$

Which further reduces to

$$\nabla \times \boldsymbol{E} = -c\nabla \times \boldsymbol{B} \tag{50}$$

Equation 50 implies that the electricfield in a photon rotates the magnetic field in the opposite direction at the speed of light (c) in a vacuum.

Magnitude of Electric and Magnetic Fields of a Photon

The force produced by the photon's electric field is obtained from the energy-density equation (1) also known as energy-volume ratio equation [5-6] below,

$$\frac{e}{V} = \varepsilon_0 E^2 \quad \text{or} \quad E^2 = \frac{e}{\varepsilon_0 V} \quad \text{or} \quad E^2 = \frac{hc}{\lambda\varepsilon_0}\frac{2\pi^2}{\lambda^3} \quad \text{or} \quad E = \pm\frac{\pi}{\lambda^2}\sqrt{\frac{2hc}{\varepsilon_0}} \tag{51}$$

The magnetic field is given as

$$B = \pm\frac{\pi}{c\lambda^2}\sqrt{\frac{2hc}{\varepsilon_0}} \quad \text{or} \quad B = \pm\frac{\pi}{\lambda^2}\sqrt{\frac{2h}{c\varepsilon_0}} \quad \text{or} \quad B = \pm\frac{\pi}{\lambda^2}\sqrt{2c\mu_0 h} \tag{52}$$

Where E is the electric field, B is the magnetic field, h is plank constant, ε_0 is permittivity of free space and μ_0 is permeability of free space. The above equations can also be derived from the Lorentz force equation without having to use the energy - volume ratio expression. The outcome is the same.

From these equations, the electric and magnetic fields of a photon are independent of charge, rather depend on wavelength, velocity and permittivity/permeability of the media. In general, photons with shorter wavelengths have stronger electric fields.

Photon Elastic Constant

A photon can be compressed or stretched by a refractive index. Compression produces reduced wavelengths while stretching increases photon's wavelength. The photon elastic constant is the measure of the stiffness of a photon to applied external force. A stiffer photon cannot easily be compressed or stretched. Stiffer photons suffer less distortion due to the effect of refractive index. This principle is supported by the helical spring model discussed area. The derivation of the elastic constant of a photon is from the relationship between angular frequency and the elastic constant as given below.

$$\omega = \sqrt{\frac{k}{m}} \quad \text{or} \quad \frac{(2\pi)^2 c^2}{\lambda^2} = \frac{kc\lambda}{h} \quad \text{or} \quad k = \frac{(2\pi)^2 hc}{\lambda^3} \tag{53}$$

In terms of the force of the photon,

$$k = \frac{2\pi F}{\lambda} \quad \text{and} \quad F = \frac{k\lambda}{2\pi} \tag{54}$$

Which is a form of Hooke's Law on elastic springs.

Where ω is the angular frequency, k is the elastic constant and m is the mass of photon. Equation 53 defines the elastic constant of a photon. We see that photons with shorter wavelengths are stiffer compared to those with longer wavelengths. As an example, an X-ray is stiffer compared to a visible light ray.

Power of a Photon

The power *(P)* of a photon is defined in terms of its centripetal force and angular velocity.

$$P = F_c\omega \quad \text{or} \quad P = \frac{2\pi hc}{\lambda^2} * \frac{2\pi c}{\lambda} \quad \text{or} \quad P = \frac{(2\pi)^2 hc^2}{\lambda^3} \tag{55}$$

Equation 55 defines the power of a photon and can be expressed in terms of the photon elastic constant as below.

$$P = kc \tag{56}$$

Equation 56 implies the power of a photon is directly proportional to the elastic constant.

Photon Interaction with Refractive Index

The refractive index of a medium is synonymous to optical drag or resistance factor. It impedes the flow of electromagnetic field lines. In a medium other than vacuum, a photon self-assembling process slows down as the photon gains inertia. Refractive index can also be defined in terms of ratio of speeds of photon in vacuum to that of the given medium.

$$n = \frac{c}{v} \quad \text{or} \quad nv = c \tag{57}$$

Where *n* is the refractive index, *v* is the velocity of photon in a medium, and *c* is the velocity of photon in vacuum.

By dividing both sides of eqn. 57 by wavelength we get

$$\frac{nv}{\lambda} = \frac{c}{\lambda} \quad \text{or} \quad \frac{v}{\lambda'} = \frac{c}{\lambda} \tag{58}$$

Where λ is wavelength in vacuum and λ' is wavelength in medium.

Equation 58 leads to two other equations below.

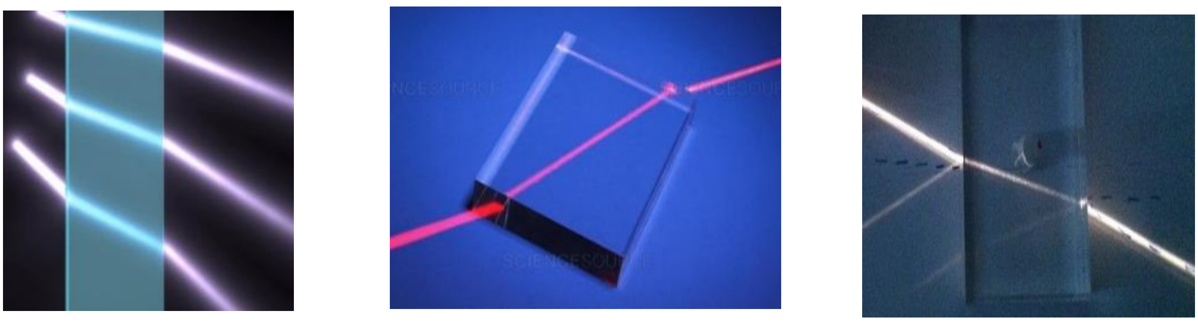
$$\frac{1}{\lambda'} = \frac{n}{\lambda} \quad \text{or} \quad n = \frac{\lambda}{\lambda'} \quad \text{or} \quad \lambda = n\lambda' \tag{59}$$

$$f' = f \tag{60}$$

Where $f' = \frac{v}{\lambda'}$ is the frequency of the photon in a medium.

From eqn. 59 we see that the wavelength of a photon in a medium reduces compared to its length in vacuum. This is so because in eqn. 59, n > 1 in media other than vacuum. Further, from eqn. 60 we see that the frequency of photon in the medium is equal to that in the vacuum. This is the conservation of energy of a photon in media that are characterized mainly by refractive index. Such is a transparent medium.

The interpretation of eqn. 6 together with eqn. 60 is that since the wavelength is directly proportional to the amplitude of the photon, a reduced wavelength leads to reduced amplitude of the photon. This means the photon reduces in brightness (becomes dimmer) and narrows (becomes thinner) in a media compared to when it is in vacuum. Observations from experiments with different class prisms (Fig. 5) supports this assertion.



(a) (b) (c)

Figure 5: Light passing through different transparent glass prisms (a), (b) and (c) showing the dimming and thinning of light inside the glass media. (Courtesy: Russell Kightley, 2016).

Strain on Photon and Net Refractive Index

As the photon travels from vacuum into a medium, there is a change in wavelength.

$$\Delta\lambda = \lambda^{'} - \lambda \quad \text{or} \quad \Delta\lambda = \frac{\lambda}{n} - \lambda \quad \text{or} \quad \Delta\lambda = -\lambda\left(\frac{n-1}{n}\right) \tag{61}$$

Which gives

$$\Delta\lambda = -\lambda\left(\frac{N^{'}}{n}\right) \tag{62}$$

Where $N^{'}$ is called the net refractive index, with $N^{'} = n - 1$. The negative sign on eqn. 62 shows that the photon gets compressed as it passes through the medium.

The strain on the photon due to the refractive index is given by

$$Strain = \frac{\Delta\lambda}{\lambda} \quad \text{or} \quad Strain = \frac{-\lambda\left(\frac{N^{'}}{n}\right)}{\lambda} = -\left(\frac{N^{'}}{n}\right). \tag{63}$$

Or generally,

$$Strain = \left(\frac{N^{'}}{n}\right) \tag{64}$$

The strain is negative when a photon passes from vacuum to medium and positive when passing from medium to vacuum. Hence the equation for the magnitude of strain on photon is expressed by eqn. 64. The change in wavelength can therefore be expressed in terms of strain on the photon.

$$\Delta\lambda = -\lambda * strain \tag{65}$$

Force of Photon Inside a Medium

In a medium the force of a photon is given by

$$F^{'} = \frac{2\pi hc}{\lambda} * \frac{1}{\lambda^{'}} \quad \text{or} \quad F^{'} = \frac{2\pi hc}{\lambda} * \frac{n}{\lambda} \quad \text{or} \quad F^{'} = \frac{2\pi nhc}{\lambda^2} \quad \text{or} \quad F^{'} = nF \tag{66}$$

Where $F^{'}$ and $\lambda^{'}$ are the force and wavelength of photon in the medium, respectively.

We note from the above equation that the force inside the medium increases as the amplitude reduces. This is the conservation of energy principle.

Work Done by Media on the Photon

$$W = F.\Delta r \quad \text{or} \quad W = \frac{2\pi hc}{\lambda^2}.(-)\frac{\lambda}{2\pi}\frac{N^{'}}{n} \quad \text{or} \quad W = -\frac{hc}{\lambda}.\frac{N^{'}}{n} \tag{67}$$

Where F is the force of photon, Δr is change in amplitude and W is work done.

The work done by the photon is negative implying the work is done by the media on the photon, and is equivalent to the energy of the photon multiplied by the strain.

Refractive Force of the Media

The refractive force is the force responsible for compressing a photon at the interface of two media. It is a form of a drag or frictional force on a photon. It originates from the refractive index of the media. By applying Hooke's Law the refractive force of the medium for a photon moving from vacuum to medium is given by

$$F_n = -k\Delta R \quad \text{or} \quad F_n = -\frac{(2\pi)^2 hc}{\lambda^3} * \Delta R \quad \text{or} \quad F_n = -\frac{(2\pi)^2 hc}{\lambda^3} * -\frac{N^{'}\lambda}{2\pi n} \quad \text{or} \quad F_n = \frac{N^{'}F}{n} \tag{68}$$

Where, F is the force of the photon, F_n is the refractive force of the media, $N^{'}$ is the net refractive index and n is the refractive index.

From equation 68, we note that as the refractive index approaches infinity, the refractive force approaches, but is not equal to the force of a photon. Hence *refractive force cannot stop a photon propagation*, but can only slow it down. In other words, a photon never comes to rest, and thus cannot acquire a rest mass. In a nutshell, the rest mass of a photon is zero. This assertion is also supported by the equation below which indicates that the velocity is never zero in a refractive media.

$v = \frac{c}{n}$ and $v = \frac{c}{n} > 0$ (69)

Since refractive force is a form of drag or frictional force, the strain becomes equal to the coefficient of friction.

$f = \mu F_{normal} = \frac{N'F}{n}$ or $\mu = \frac{N'}{n} = strain$ (70)

Effective Mass of Photon Inside Refractive Media

The mass of a photon inside a refractive medium can be determined by using eqn. 19 and 57.

$m' = \frac{h}{v\lambda'}$ (71)

Where m' is the mass of photon in the media and λ' is the wavelength of photon in the media. This equation leads to

$m' = h * \frac{n}{c} * \frac{n}{\lambda}$ or $m' = \frac{n^2 h}{c\lambda}$ or $m' = n^2 m$ (72)

With

$c = nv$ (73)

From eqn. 72 we see that the mass of a photon increases by a factor of n^2 in the media. The photon becomes massive, and thus its velocity reduces to increase the mass in conservation of kinetic energy.

The Speed of Light and Quantization of Space, Time and Matter

Of uttermost importance is to note that the quantization of energy is a consequence of the quantization of its primary constituents; space, time and mass. This study will provide proof to this effect. We start by looking at the concept of velocity.

Velocity is defined in terms of displacement and time. It should be noted that displacement and time vary independently, especially under natural environment that is dominated by entropy. For the propagation of a photon in vacuum the velocity has been found by experiment to be constant, despite the fact that velocity is a function of two independent variables. In equation form, velocity is generally given by

$v = \frac{s}{t}$ or $\Delta v = \frac{\Delta s}{\Delta t}$ (74)

and as a special case, $c = \frac{s}{t}$ (75)

Where v is the varying velocity, c is the constant velocity, s is the displacement and t is the time.

There are two assertions when v is constant under the natural environment. The first is that displacement and time are in perfect harmony such that they change together without the help of an external constant force acting on them. This assertion is less likely to be true as changes in nature are random due to entropy. The second assertion is that the changes in displacement and time reach their limits. This implies that displacement and time can no longer be subdivided any further to get smaller changes. By comparing the two assertions, we note that the second one is more plausible considering the independence of the two variables.

Further, it should be noted that velocity of a photon is independent of the internal force and energy of the photon. Thus, photons with different frequencies have the same velocity in vacuum. Therefore, velocity depends entirely on changes in displacement and time.

By applying equation 64 on equation 75 we get to see the behavior of the variables.

$\Delta c = \frac{\Delta s}{\Delta t} = 0$ or $\Delta s = o$ or $s = s_0 = constant$ (76)

Conversely,

$\Delta t = \Delta c \Delta s = 0$ or $\Delta t = o$ or $t = t_0 = constant$ (77)

Hence speed of light is correctly expressed as $c = \frac{s_0}{t_0}$ (78)

Where c is the speed of light, $s_0$is the indivisible length and t_0 is the indivisible time.

Equations 76 and 77 show that displacement is constant when *c* is constant. In this case, time also becomes constant. The inference to this behavior is that space and time have a limit to which they can be subdivided.

By looking at the primary units of energy

$$e = kg.m^2s^{-2} \quad \text{or} \quad e = \frac{[kg][L]^2}{[T]^2} = [kg]c^2 \tag{79}$$

Since energy is quantized, $e = ne_0$

Where e_0 is the indivisible energy, and *n* is an integer. Equation 79 becomes

$$ne_0 = [kg]c^2 \quad \text{or} \quad [kg] = n(\frac{e_0}{c^2}) \tag{80}$$

We note that $\frac{e_0}{c^2}$ is a constant and can be denoted by m_0

$$[kg] = nm_0 \quad \text{or} \quad mass = nm_0 \tag{81}$$

Equation 81 gives us the quantization of mass with m_0 being the indivisible mass.

This is proof that *space, time and matter are quantized*. We therefore have quantization of the four primary properties of nature; energy, space, time and matter. The quantization of space and time is the reason velocity has a maximum limit. This also implies time and space have limits. The rate of flow of energy in free space is therefore limited to the quantization of space and time. The concept of the quantization of time and space has also been proposed by other scholars [21 -23].

Effect of Gravitational Force on Wavelength and Refractive Index of Air

The gravitational force has an effect on refractive index of air and wavelength of a photon as demonstrated below. The derivation starts with the comparison of acceleration of a photon and the acceleration due to gravity. Experiment on a pendulum has proved that the acceleration of gravity is given

$$g = (2\pi)^2 f^2 L \tag{82}$$

Where, g is the acceleration due to gravity, f is the frequency and L is the length of pendulum or radius of oscillations.

The acceleration of a photon can be modeled on the acceleration of a pendulum in the above equation since both motions are centripetal in nature. We can prove this be using the above equation to derive the earlier equation for the acceleration of a photon by taking L as the amplitude of oscillation equivalent to R, where $R = \frac{\lambda}{2\pi}$ and hence

$$g = (2\pi)^2 f^2 L \quad \text{or} \quad g = (2\pi)^2 \frac{c^2}{\lambda^2} R \quad \text{or} \quad g = (2\pi)^2 \frac{c^2}{\lambda^2} \frac{\lambda}{2\pi} \quad \text{which reduces to}$$

$$g = 2\pi \frac{c^2}{\lambda} \tag{83}$$

This is the acceleration of a photon as earlier derived. From this equation we have the relationship between gravitation acceleration and wavelength,

$$\lambda = 2\pi \frac{c^2}{g} \tag{84}$$

From this equation we can see that wavelength of a photon is affected by gravitational force [24]. This is responsible for the red shift of a photon as it travels out from a massive object. Photons from the sun suffer a red shift as they get propagated [26-27]. On the other hand, photons approaching a massive object such as black holes (Fig. 6) suffer a blue shift.

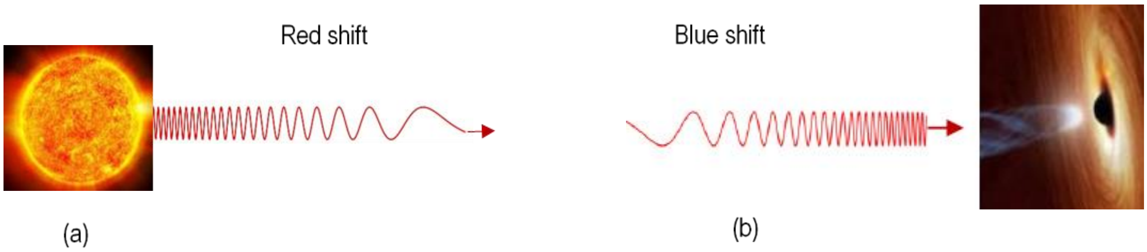


Figure 6: Part (a) illustrates the red shift after photons are emitted from the sun. Part (b) illustrates the blue shift as photons approach a black hole. (Courtesy: NASA, 2019)

Since the acceleration due to gravity is also given by Isaac Newton's equation as

$$g = \frac{GM}{y^2} \tag{85}$$

Where G is universal gravitational constant, M is mass of planet, y is distance from the center of mass of the planet.

We equate the acceleration due to gravity in eqn. 83 and 85. Furthermore,

$$2\pi \frac{c^2}{\lambda} = \frac{GM}{y^2} \quad \text{or} \quad \lambda = \frac{2\pi c^2 y^2}{GM} \tag{86}$$

The expression above gives wavelength as a function of altitude. From the above equation, the refractive index can be expressed as

$$\lambda = \frac{2\pi(nv)cy^2}{GM} \quad \text{or} \quad n = \frac{GM}{2\pi c} \cdot \frac{\lambda}{vy^2} \quad \text{or} \quad n = \frac{g\omega}{v} \tag{87}$$

This leads to $\omega = \frac{c}{g}$ and $\lambda = \frac{hc}{kg^2}$ (88)

Where n is the refractive index, ω is angular frequency, k is the photon elastic constant and v is the velocity of photon in the air.

Conclusion

The properties of photons and energy have been discussed and equations derived for quantitative and qualitative description. Photon dynamics under the influence of refractive index includes changes in wavelength and amplitude, and therefore brightness of the photon. The theory that photons are massless only applies to rest mass.

A photon has been explained in equation form to have effective mass, elastic constant, surface area, volume, momentum, force and power. A photon suffers strain and change of mass under the influence of refractive index. The frequency of a photon remains unchanged from one media to another under the influence of refractive index. The characteristic of a photon has been described as helical, and a helical spring model has been used to describe it.

Energy has been explained to have mass, weight and occupies space. The quantization of energy has been extended to the quantization of space, time and matter.

It has been theoretically proven that gravitational force has an effect on wavelength of photons and the refractive index of air. The consequence is the red and blue shifts in photons as they propagate away and towards massive bodies, respectively. It has therefore been established that refractive index has blue and red shift effects. This study has a potential to contribute to two-photon absorption which has major applications in 3D imaging of semiconductors, cryptography, optical power limiting, photodynamic therapy for cancer treatment, and optical data storage.

Conflict of Interest

Authors declare no conflict of interest

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