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A LESS-CONSTRAINED (2,0) SUPER-YANG-MILLS MODEL: THE COUPLING TO NON-LINEAR σ-MODELS

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Abstract

Considering a class of (2,0) super-Yang-Mills multiplets characterised by the appearance of a pair of independent gauge potentials, we present here their coupling to non-linear σ-models in (2,0)-superspace. Contrary to the case of the coupling to (2,0) matter superfields, the extra gauge potential present in the Yang-Mills sector does not decouple from the theory in the case one gauges isometry groups of σ-models.Considering a class of (2,0) super-Yang-Mills multiplets characterized by the appearance of a pair of independent gauge potentials, we present here their coupling to non-linear σ-models in (2,0)-superspace. Contrary to the case of the coupling to (2,0) matter superfields, the extra gauge potential present in the Yang-Mills sector does not decouple from the theory in the case one gauges isometry groups of σ-models.

Key-words: Extended gauge theories.

The rise of interest in the investigation of geometrical aspects and quantum behavior of two-dimensional systems, such as Yang-Mills theories and non-linear σ-models, especially if endowed with supersymmetry, has been broadly renewed in connection with the analysis of superstring background configurations [1,2] and the study of conformal field theories and integrable models.

As for supersymmetries defined in two space-time dimensions, they may be generated by p left-handed and q right-handed independent Majorana charges: these are the so-called (p, q) supersymmetries [1,3] and are of fundamental importance in the formulation of the heterotic superstrings [4].

Motivated by the understanding of several features related to the dynamics of world-sheet gauge fields [5] and the possibility of finding new examples of conformal field theories, we have recently considered the superspace formulation of a (2,0) Yang-Mills model [6,7] enlarged by the introduction of a second gauge potential transforming under the same *simple* gauge group as the ordinary Yang-Mills field of the theory.

In the works of refs. [6,7], we have attempted an understanding of a further gauge potential based on discussing constraints on field-strength superfields in the algebra of gauge-covariant derivatives in (2,0) superspace. The minimal coupling of this sort of less-superspace. The minimal coupling of this sort of less-constrained Yang-Mills model to matter superfields has been contemplated and it has been ascertained that the additional gauge potential corresponds to non-interacting degrees of freedom in the Abelian case. For non-Abelian symmetries, the further Yang-Mills field still decouples from matter though it presents self-interactions with the gauge sector.

It is therefore our purpose in this letter to find out a possible dynamical role for the additional gauge potential discussed in refs. [6,7], through its coupling to matter superfields that describe the coordinates of the Kähler manifold adopted as the target space of a (2,0) non-linear σ-model [8]. To pursue such an investigation, we shall gauge the isometry group of the σ-model under consideration while working in the (2,0) superspace; then, all we are left with is the task of coupling the (2,0) Yang-Mills extended supermultiplets of ref. [6] to the superfields that define the (2,0) σ-model in question.

The coordinates we choose to parametrise the (2,0) superspace are given by

$$z^A \equiv (x^{++}, x^{--}, \theta, \overline{\theta}) \tag{1}$$

where x^{++}, x^{--} denote the usual light-cone coordinates, whereas θ, $\bar{\theta}$ stand for complex right-handed Weyl spinors. The supersymmetry covariant derivatives are taken as:

$$D_+ \equiv \frac{\partial}{\partial\theta} + i\overline{\theta}\partial_{++} \tag{2a}$$

and

$$\overline{D}_+ \equiv \frac{\partial}{\partial\overline{\theta}} + i\theta\partial_{++}, \tag{2b}$$

where ∂_{++} (or ∂_{--}) represents the derivative with respect to the space-time coordinate x^{++} (or x^{--}). They fulfil the algebra:

$${D_+}^2 = {\overline{D}_+}^2 = 0 \qquad (3a); \qquad \{D_+, \overline{D}_+\} = 2i\partial_{++}. \qquad (3b)$$

The coordinates that parametrise the Kähler target manifold are given by complex scalar superfields, Φ^i and $\overline{\Phi}^i \equiv \Phi^i$, constrained according to:

$$\overline{D}_i\Phi^i = 0 \qquad (4a); \qquad D_i\Phi^i = 0, \qquad (4b)$$

$$i = 1, 2, \ldots, n.$$

This sort of "chirality" constraints yields the following component-field expansion for Φ^i:

$$\Phi^i(x; \theta, \overline{\theta}) = \phi^i(x) + \theta\, \eta^i(x) + i\theta\overline{\theta}\, \partial_{++}\phi^i(x), \tag{5}$$

where the ϕ^i's are complex scalars and the η^i's are complex left-handed Weyl spinors. The (2,0)-supersymmetric σ-model action written in (2,0)-superspace reads [8]:

$$S = -\frac{i}{2}\int d^2x\, d\theta\, d\overline{\theta} \left[K_i(\Phi, \overline{\Phi})\, \partial_{--} - \text{c.c.}\right], \tag{6}$$

where the target space vector $K_i(\Phi, \overline{\Phi})$ can be expressed as the gradient of a *real scalar* (Kähler) potential, $K(\Phi, \overline{\Phi})$, wherever the Wess-Zumino term is absent (i.e., torsion-free case) [1]:

$$K_i(\Phi, \overline{\Phi}) = \partial_i K(\Phi, \overline{\Phi}) \equiv \frac{\partial}{\partial\Phi^i} K(\Phi, \overline{\Phi}). \tag{7}$$

We shall draw our attention to Kählerian target manifolds of the coset type, G/H. The generators of the isometry group, G, are denoted by Q_α ($\alpha = 1, \ldots,$dim H). The transformations of the isotropy group are linearly realized on the superfields Φ and $\overline{\Phi}$, and act as matrix multiplication, just as on flat manifolds. The isometry transformations instead are non-linear and their infinitesimal on the points of G/H can be written as:

$$\delta\Phi^i = \lambda^\alpha\, k^i_\alpha(\Phi) \tag{8a}$$

and

$$\delta\overline{\Phi}_i = \lambda^\alpha\, \bar{k}_{\alpha i}(\overline{\Phi}) \tag{8b}$$

where $k_{\alpha i}$ and $\bar{k}_{\alpha i}$ are (holomorphic and anti-holomorphic) Killing vectors of the target manifold. The finite versions of the isometry transformations above reads:

$$\Phi'^{i} = \exp(\mathrm{L}_{\lambda,k})\Phi^{i} \tag{9a}$$

and

$$\overline{\Phi}'_{i} = \exp\left(\mathrm{L}_{\lambda,\bar{k}}\right)\overline{\Phi}_{i} \tag{9b}$$

with

$$\mathrm{L}_{\lambda,k}\,\Phi^{i} \equiv \left[\lambda^{\alpha}k^{j}_{\alpha}\frac{\partial}{\partial\Phi^{j}},\Phi^{i}\right] \equiv \delta\Phi^{i}. \tag{10}$$

Though the Kälher scalar potential can always be taken H-invariant, isometry transformations indices on K a variation given by:

$$K = \lambda^{\alpha}[(\partial^{i}K)k_{\alpha i} + (\overline{\partial^{i}K})\bar{k}_{\alpha i}] = \lambda^{\alpha}[\eta_{\alpha}(\Phi) + \bar{\eta}_{\alpha}(\overline{\Phi})], \tag{11}$$

where the holomnrphic and anti-holomorphic functions η_{α} and $\bar{\eta}_{\alpha}$ can be determined up to a purely imaginary quantity as below:

$$(\partial_{i}K)k^{i}_{\alpha} \equiv \eta_{\alpha} + iM_{\alpha}(\Phi,\overline{\Phi}) \tag{12a}$$

and

$$(\overline{\partial^{i}K})\bar{k}_{\alpha i} \equiv \bar{\eta}_{\alpha} - iM_{\alpha}(\Phi,\overline{\Phi}). \tag{12b}$$

The real functions M_{α} and the holomorphic and anti-holomorphic functions η_{α} and $\bar{\eta}_{\alpha}$ play a crucial role in discussing the gauging of the isometry group of the target manifold [9,10]. Therefore, by the transformation equation (11) and the constraints imposed on Φ and $\overline{\Phi}$, it can be readily checked that the superspace action (6) is *invariant under global isometry transformations.*

Proceeding further with the study of the isometries, a relevant issue in the framework of (2,0)-supersymmetric σ-models is the gauging of the isometry group G of the Kälherian target manifold. This in turn means that one should contemplate the minimal coupling of the (2,0)-σ-model to the Yang-Mills supermultiplets of (2,0)-supersymmetry [11]. An eventual motivation for pursuing such an analysis is related to 2-dimensional conformal fields theories. It is known that 2-dimensional σ—models define conformal field theories provided that suitable constraints are imposed upon the target space geometry [1,2]. Now, the coupling of these models to the Yang-Mills sector might hopefully yield new conformal field theories of interest.

Tho study of (2,0)-supersymmetric Yang-Mills theories have been carried out in ref. [11] and the gauging of σ-model isometries in (2,0)-superspace has been recently considered in ref. [12] in the absence of a Wess-Zumino term. However, an alternative and so to say *less constrained* version of (2,0) gauge multiplets have been proposed and discarded in refs. [6,7]. It has been shown that the relaxation of some constraints on the gauge superconnections and on-field strength superfields leads to the appearance of an extra gauge potential that shares a common gauge transformation with the Yang-Mills field. Nevertheless, this extra potential is shown to decouple from the (2,0) matter superfields whenever they are minimally coupled to the Yang-Mills sector.

It is our main purpose henceforth to carry out the coupling of a (2,0)-σ-model to the *more relaxed* gauge superfields of ref. [6] and show that the extra gauge degrees of freedom do not now decouple from the matter fields (that is, the target, space coordinates). The extra gauge potential obtained upon relaxing constraints can therefore acquire a dynamical significance employing the coupling between the σ-model and the Yang-Mills fields of ref. [6]. Moreover, this system might provide another example of a gauge-invariant conformal field theory.

The Yang-Mill supermultiplets are introduced by means of the gauge covariant derivatives which, according to the discussion of ref. [6], are defined as below

$$\nabla_+ \equiv D_+ - ig\Gamma_+, \quad (13a) \qquad \overline{\nabla}_+ \equiv \overline{D}_+, \quad (13b)$$

$$\nabla_{++} \equiv \partial_{++} - ig\Gamma_{++}, \quad (13c) \quad \text{and} \quad \nabla_{--} \equiv \partial_{--} - ig\Gamma_{--} \quad (13d)$$

where the gauge superconnections Γ_+, Γ_{++} and Γ_{--} are all Lie-algebra-valued and g stands for the gauge coupling parameter, Γ_+ and Γ_{++} can be both expressed in terms of a *real scalar* superfield, $V(x;\theta,\bar{\theta})$ according to

$$\Gamma_+ = e^{-gV}(D_+ e^{gV}) \tag{14a}$$

and

$$\Gamma_{++} = -\frac{1}{2}\overline{D}_+[e^{-gV}(D_+ e^{gV})]. \tag{14b}$$

Therefore, the gauging of the σ-model isometry group shall be achieved by minimally coupling the action (6) to the gauge superfields V and Γ_{--}, as we shall see in the sequel.

To establish contact with a component-field formulation and to identify the presence of an additional gauge potential, we write down the θ-expansion for V and Γ_{--}:

$$V(x;\theta,\bar{\theta}) = C + \theta\xi - \bar{\theta}\bar{\xi} + \theta\bar{\theta}\, v_{++} \tag{15}$$

and

$$\begin{aligned}\Gamma_{--}(x;\theta,\bar{\theta}) &= (A_{--} + iB_{--}) + i\theta(\rho + i\eta) \\ &+ i\bar{\theta}(\chi + i\omega) + \theta\bar{\theta}(M + iN).\end{aligned} \tag{16}$$

A_{--}, B_{--} and v_{++} are the light-cone components of the gauge potential fields; ρ, η, χ and ω are left-handed Majorana spinors; M, N and C are real scalars and ξ is a complex right-handed spinor.

The gauge transformations of the component fields above can be found in ref. [6] and they suggest that the component v_{++} should be identified as the light-cone partner of A_{--},

$$v_{++} \equiv A_{--}, \tag{17}$$

so that we end up with two component-field gauge potentials: $A^\mu \equiv (A^0; A^i)$ and $B(x)$.

To write down the local version of the isometry transformations (8), we have to replace the global parameter λ^α by a pair of *chiral* and *antichiral* superfields, $\Lambda^\alpha(x;\theta,\bar{\theta})$ and $\overline{\Lambda}^\alpha(x;\theta,\bar{\theta})$, by virtue of the constraints satisfied by Φ and $\overline{\Phi}$. This can be realized according to:

$$\Phi'^i = \exp(\mathrm{L}_{A,k})\Phi^i \tag{18a}$$

and

$$\overline{\Phi}'^i = \exp(\mathrm{L}_{\bar{A},\bar{k}})\overline{\Phi}^i. \tag{18b}$$

In order to get closer to the case of global transformations and to express all gauge variations exclusively in terms of the superfield parameters Λ^α, we propose a field redefinition that consists in replacing $\overline{\Phi}$ by a new superfield, $\widetilde{\Phi}$, as it follows:

$$\widetilde{\Phi}_i \equiv \exp(i\mathrm{L}_{V,\bar{k}})\overline{\Phi}^i. \tag{19}$$

Prom the expression for the gauge transformation of the prepotential V, it can be shown that:

$$\exp\big(i\mathrm{L}_{V',\bar{k}}\big) = \exp\big(\mathrm{L}_{\Lambda}, \bar{k}\big)\exp\big(i\mathrm{L}_{V,\bar{k}}\big)\exp\big(-\mathrm{L}_{\overline{\Lambda}}, \bar{k}\big), \tag{20}$$

and $\widetilde{\Phi}_i$ consequently transforms with the gauge parameter Λ^{α}:

$$\widetilde{\Phi}'_i = \exp\big(\mathrm{L}_{\Lambda,\bar{k}}\big)\widetilde{\Phi}_i, \tag{21}$$

which infinitesimally reads:

$$\delta\widetilde{\Phi}_i = \Lambda^{\alpha}(x;\theta,\bar{\theta})\bar{k}_{\alpha i}(\widetilde{\Phi}). \tag{22}$$

Mow, an infinitesimal isometry transformation induces on the modified Kälher potential, $K(\Phi,\widetilde{\Phi})$, a variation given by:

$$\delta K(\Phi,\widetilde{\Phi}) = \Lambda^{\alpha}(\eta_{\alpha} + \widetilde{\eta}_{\alpha}), \tag{23}$$

where

$$\widetilde{\eta}_{\alpha} = (\widetilde{\partial}^{i}K)\bar{k}_{\alpha i}(\widetilde{\Phi}) + iM_{\alpha}(\Phi,\widetilde{\Phi}), \tag{24}$$

with $\widetilde{\partial}$ denoting a partial derivative with respect to $\widetilde{\Phi}$. The isometry variation δK computed above reads just like a Kähler transformation and this is a direct consequence of the existence of the real scalar $M_{\alpha}(\Phi,\widetilde{\Phi})$, as discussed in refs. [9,10].

The form of the isometry variation of $K(\Phi,\widetilde{\Phi})$ suggests the introduction of a pair of *chiral* and *antichiral* superfields, $\xi(\Phi)$ and $\bar{\xi}(\overline{\Phi})$, whose respective gauge transformations be such that they compensate the change of K under isometries. This can be achieved by means of the Lagrangian defined as:

$$\begin{aligned}\mathcal{L}_{\xi} &= \partial_i[K(\Phi,\widetilde{\Phi}) - \xi(\Phi) - \widetilde{\xi}(\widetilde{\Phi})]\nabla_{--}\Phi^{i} \\ &\quad - \widetilde{\partial}_i[K(\Phi,\widetilde{\Phi}) - \xi(\Phi) - \widetilde{\xi}(\widetilde{\Phi})]\nabla_{--}\widetilde{\Phi}^{i},\end{aligned} \tag{25}$$

where the covariant derivatives $\nabla_{--}\Phi^{i}$ and $\nabla_{--}\widetilde{\Phi}^{i}$ are defined in perfect analogy to what is done in the case of the bosonic σ-model:

$$\nabla_{--}\Phi^{i} \equiv \partial_{--}\Phi^{i} - g\Gamma^{\alpha}_{--}k^{i}_{\alpha}(\Phi) \tag{26a}$$

and

$$\nabla_{--}\widetilde{\Phi}_{i} \equiv \partial_{--}\widetilde{\Phi}_{i} - g\Gamma^{\alpha}_{--}\bar{k}_{\alpha i}(\widetilde{\Phi}). \tag{26b}$$

Finally, all we have to do so that the Lagrangian $\mathcal{L}_{\xi}$ given above is invariant under local isometrics is to fix the gauge variations of the auxiliary scalar superfields ξ and $\widetilde{\xi}$. If the latter is so chosen that:

$$(\partial_i\xi)k^{i}_{\alpha}(\Phi) = \eta_{\alpha}(\Phi) \tag{27a}$$

and

$$(\widetilde{\partial}_i\widetilde{\xi})\bar{k}^{i}_{\alpha}(\widetilde{\Phi}) = \widetilde{\eta}_{\alpha}(\widetilde{\Phi}), \tag{27b}$$

then, it can be readily verified that Kähler-transformed potential $[K(\Phi,\widetilde{\Phi}) - \xi(\Phi) - \widetilde{\xi}(\widetilde{\Phi})]$ is an isometry-invariant and the Lagrangian $\mathcal{L}_{\xi}$ of eq. (25) is indeed symmetric under the gauged isometry group.

The interesting point we would like to stress is that the extra gauge degrees of freedom accommodated in the component-field B_{--} of the superconnection $\Gamma{--}$ do not decouple from the scalars Φ and $\widetilde{\Phi}$, unlike the case of the minimal coupling of the (2,0)-Yang-Mills supermultiplets to the (2,0)-matter superfields [6]. Hence, thanks to the coupling to a σ-model, through tho gauging of the isometry group of the target manifold, we can present a situation where the extra gauge potential of (2,0)-supersymmetry can acquire a dynamical significance, as previously promised.

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