New Theorem and Formula for Circle Arc Length Calculations with Trigonometric Approach Application in Astrophysics

Sayed Ali El-Mongy

Former V. Chairman of Egypt Nuclear Regulatory Authority (ENRRA)

Board member of *the Egyptian Nuclear Physics Association (ENPA)*

sayedelmongy@hotmail.com

Abstract:

The circle and sphere have been studied since the ancient Egyptians and Greeks before the Common Era (BCE). The recent scientific renaissance has also used them in different fields. It is also mentioned in the Prophet Mohamed`s Holy Quran. This article introduces a new Theorem (S. El-Mongy`s Theorem) as an empirical formula to correlate the constant (e) with circle and sphere. It states that "the arc length is correlated as a direct function in *{(e π r s^A)}*, whatever the central angle (Ө) and radius (r). The factor *s^A* is *(Ө/10ϕ)*. The formula can also be written as; A_L = *{(0.0174533185 r Ө)}*. Where the value 0.0174533185 is a constant called Sayed`s number (I_s) and equals *(eπ/10ϕ)*. This factor is very close to value (π/180 = 0.0174532925) with ~1.5x10^{-4} % difference. The formula was applied for calculation the arc length (A_L) of circles of different radii and angles. The results of this formula were validated and verified for very wide range; from 0.5 cm to 4.4x10^{23} km (46.5x10^9 light-years; ly) and compared with the old published arc length formula results. The difference is from 0.000% to 0.002% only. The formula was also used as trigonometric functions of circular orbits for calculation the distances between the Earth and Sun, Moon, planets, stars and EH-M87 Black hole with relatively small error; the difference is from 0.26% to maximum ~ 2.27%. The error was 0.29% for ~54 x 10^6 ly distance to the M87 black hole. The S. El-Mongy formula may open the door for further scientific and engineering applications.

Keywords: S. El-mongy`s theorem, (eπr s^A) formula, I_s constant, arc length, astronomical distance.

I.   Introduction

The circle has been known since before the beginning of recorded history. Natural circles would have been observed, such as the Moon and Sun. The circle was first observed in one of Pharos papyrus (1). The philosopher and mathematician Thales (624-546 BCE), who transferred the science and geometry from the Egyptians to Greeks has a theorem. The word *circle* in Greek, means "hoop" or "ring" (1,2,3). A circle is a plane figure bounded by one curved line and all the bounding line is equal as given by Euclid, Elements Book, 300 BCE (1,3).

The circle is 360° all the way around. The circle may also be defined as a special kind of ellipse in which the two foci are coincident (zero eccentricity) (1,3). Both circles and spheres are circular. They are 2D figure (plane) and 3d object (space) respectively. The sphere is a dual geometrical surface, fully symmetrical, resulting from a circle rotation around one of its diameters. The sphere and the circle arc, sector and segment are given in (Fig. 1).
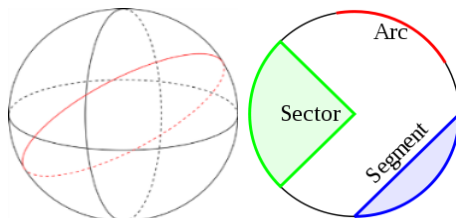

Figure1: The Sphere and Circle arcs, chord and sector (3).

The three major established circle theorems and concepts indicate that the (i) circle arc length is directly proportional to its radius. (ii) If central angles (Ө) congruent, then arcs congruent. (iii) The chord lengths dividing the circumference of a circle into equal number of segments.

The circle is the basis for the wheel, which, with related inventions such as gears, makes much of modern machinery possible. In mathematics, the study of the circle has helped inspire the development of geometry, astronomy and calculus (1,3).

Our galaxy revolves on its own axis which is its center with the result that the Sun revolves around the same center in a circular orbit. Recently, 1917, Shapley estimated the distance between the Sun and the center of our galaxy at 10 kilo-parsecs, i.e., 10×10^{17} km. The orbital movement of the Sun was already referred to by the Prophet Mohamed`s Quran (4,5).

II.   Theoretical Aspects of the Formula:

II.A. S. El-mongy`s Theorem: "The circle arc length is a function in (e π r s^A)".

A new empirical formula for calculation of the circle arc length (A_L) was introduced in this article. It correlated the circle circumstance with other mathematical constants. The old formula for arc length can be given as follows (2,3):

The circle circumstance = *2 π r*   (1)

Arc length = *2 π r Ө/360*   (2)

Where, (r) is the circle radius and Ө is the central angle in degree. The (π) is the irrational constant equal to (22/7) or 3.14159265 (π is also called Archimedes constant (3, **6)**. The arc length can also be calculated by the other known formula; ($A_L = r. Ө$). Where, Ө is in Radian.

The proposed empirical formula was practically found as mentioned and is mathematically given and expressed as follows:

Arc length *(A$_L$) = {(e π r s^A)}*   (3)

This empirical formula is called S. El-Mongy`s theorem. The factor (s^A) represents the ratio {(Ө/ 10ϕ)}. Where, Ө is the central angle in degree and ϕ corresponding 48.929^0. This formula was applied tested and validated for any circular orbit whatever the radius length and Ө.

The constant e is the base of the natural logarithm (Euler`s number) and equal to 2.7182818 (7).

This formula number 3 can also be rewritten to be;

Arc length *(A$_L$) = {(0.0174533185 r Ө)}*   (4)

Where, the value 0.0174533185 is a constant equals (*e π/10ϕ*) and called Sayed`s number (I$_s$).

III.   Results and discussion:

III.A. Validation of S.El-Mongy`s Theorem: The arc length is a function in (*e π r s^A*).

The arc length can be calculated using our abovementioned empirical formula (eq.3). It was validated for different circle radii (0.5 cm – 4.4×10^{23} km) and central angles Ө (0.0005^0 – 360^0). The precise results compared with the old published formula are given in Table 1.

Table 1: Results of Arc Lengths Calculation for Different Circles as a Function of (*e π r s^A*)

Circle Radius	Central Angle (Ө) (Degree)	**S. El-mongy`s formula** $A_L = \{(e\ π\ r\ s^A)\}$	**old formula** $A_L = 2\ π\ r\ Ө/360$	% diff.
1 cm	0.0005^0	8.72666×10^{-6}	8.72665×10^{-6}	0.0001
0.5 cm	0.05^0	0.00043633	0.00043633	0.0000
0.5 cm	0.56^0	0.0048869	0.0048869	0.0000
0.5 cm	5^0	0.043633	0.043633	0.0000
1 cm	5^0	0.0872666	0.087266	0.0000

1 cm	10^0	0.17453	0.17453	0.0000
1 cm	15^0	0.261799	0.261799	0.0000
1 cm	30^0	0.52359	0.52359	0.0000
8 cm	40^0	5.58506	5.58505	0.0002
5 cm	45.8^0	3.99681	3.99680	0.0003
100 cm	45^0	78.5399	78.5398	0.0001
4.3 cm	60^0	4.50295	4.50295	0.0000
6 cm	70^0	7.33039	7.33038	0.0001
1000 cm	90^0	1,570.798	1,570.796	0.0001
8.599 cm	120^0	18.0097	18.0097	0.0000
10 cm	135^0	23.5619	23.5619	0.0000
8 cm	180^0	25.1327	25.1327	0.0000
13 cm	300^0	68.0679	68.0678	0.0001
1 cm	360^0	6.28319	6.28318	0.0002
100 cm	360^0	628.3195	628.3185	0.0002
48 m	10^0	8.37776	8.37758	**0.002**
25 m	90^0	39.2699	39.2699	0.0000
5 m	120^0	10.47199	10.47197	0.0002
200 m	200^0	698.1327	698.1317	0.0001
100 m	330^0	575.9595	575.9586	0.0002
1000 Km	360^0	6283.19459	6283.1853	0.0001
10^6 Km	180^0	3,141,597.299	3,141,592.65	0.0001
10^6 km	250^0	4,363,329.583	4,363,323.125	0.0001
10^6 Km	360^0	6,283,194.598	6,283,185.3	0.0002
149597870 km	180^0	469,976,264.388	469,975,568.848	0.0002
40×10^{12} km	15^0	10.47199×10^{12}	10.47197×10^{12}	0.0002
10×10^{17} km	180^0	3.141597×10^{18}	3.14159×10^{18}	0.0002
4.40×10^{26} m	180^0	1.3823×10^{27}	1.3823×10^{27}	0.0000
4.40×10^{26} m	250^0	1.919865×10^{27}	1.91986×10^{27}	0.0003
4.40×10^{23} km*	360^0	2.76460×10^{24}	2.7646×10^{24}	0.0000
600 ly**	30^0	314.1597	314.15926	0.0001
10^9 ly***	180^0	3.141597×10^9	3.14159×10^9	0.0002

*The commoving distance from Earth to the edge of the observable universe is about 14.26 gigaparsecs (46.5 $\times 10^9$ ly) or 4.40×10^{23} km in any direction. ** A huge circular arc called Ursa Major arc (8). ***Radius of a galactic gaseous halo (9.461×10^{21} km) around our Milky Way galaxy (9).   It should be mentioned that the correlation between e to power π was only given by Euler`s identity; ($e^{i\pi}$ = -1) (10).

It can be observed that the results arc length calculation achieved by El-Mongy`s formula are almost identical with these obtained by the old published formula. Our formula was accurately and sharply validated using circles radii (r) from 0.5 cm to 4.4×10^{23} km (46.5×10^9 ly). The highest difference does not exceed 0.002%.

III.B. Applications of S. El-Mongy`s Theorem and Formula in Astrophysics:

The small-angle approximation is a useful simplification of the basic trigonometric functions which is approximately true in the limit where the angle approaches zero (11, 12, 13, 14, 15). Figure 2 shows clarification how to calculate the distance of object (e.g. Sun) as seen from Earth based on small angle approximation (5,14,15). For example, the arc that runs through the Sun's diameter has an angle (angle subtended by an object) and an arc length equal to actual diameter (the arc-length formula). This is presented for a right triangle with the trigonometric relationship as follows:

Tan Θ = diameter (d)/ distance (D)   (4)

Where, D>>d, Tan Θ ≈ Θ and angular diameter (Θ) is in radian. For conversion, it is
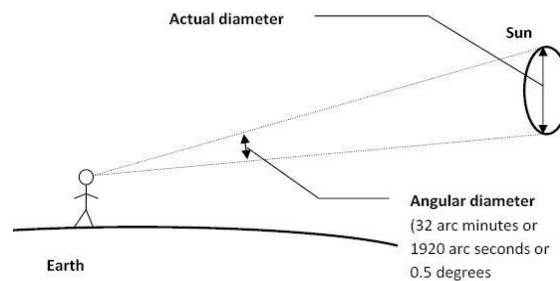
Diameter/distance = *Θ / 57.3*   (5)

Figure 2: Angular diameter and Arc length approach for distance calculation (15).

By using our formula; eq.3: $\boldsymbol{A_L=\{(e \, \pi \, r \, s^A)\}}$**,** that can be rewritten to be:

$$D = (A_L \times \phi \times 10) / (e \, \pi \, \Theta)$$ (6)

This formula was also and simply used for astronomical distances (D) calculation. It can also be written as follows;

$$D = 57.295695 (A_L / \Theta^0)$$ (7)

III.B.1 Validation and verification of the formula:

Validation of this formula was also carried out by calculating the distance (D) between the Earth and Sun, Moon, planets, Stars and M87 Black hole in the center of our Milky Way galaxy (5,16). The Table 2 shows the calculated results compared with the literature values. Where, the arc length **(**A_L**)** equals to diameter (the arc-length approach) and Θ is the sighting angle in degree (angular diameter) from the Earth (14,15)**.**

Table 2: The Calculated Distance from Earth to Moon, Sun, Planets, Stars and a Black Hole.

Diameter ≈ (A_L)	(Θ) Angular Diameter	Distance by Sayed Formula $D = (A_L \times \phi \times 10) / (e \, \pi \, \Theta)$	**Reference** distance (km)	% diff.
Earth: 12742 km	2.048^0	356475.46	363104 perigee	1.83
Sun: 1.3927×10^6 km	0.53^0	150.5579×10^6	151.84×10^6	0.84
Sun: 1.3927×10^6 km	0.53^0	150.5579×10^6	149.59787×10^6	0.64
Moon: 3474.2 km	0.5^0	398113.41	405696 apogee	1.87
Venus: 12104 km	0.01572^0	44.1162×10^6	43.926×10^6	0.43

Mercury: 4879.4 km	0.0033^0	84.7178 x10^6	85.927x10^6	1.43
Mars: 6790 km	0.00699^0	55.6563 x 10^6	55.8 x10^6	0.26
Jupiter: 142984 km	0.01306^0	627.2869 x 10^6	625.48 x 10^6	0.29
Saturn: 116460 km	0.0051235^0	1.3024 x 10^9	1.277 x 10^9	1.98
Uranus: 50724 km	0.000972^0	2.98999 x 10^9	3.0511 x 10^9	2,04
Neptune: 49244 km	0.0006127^0	4.6049x10^9	4.5x10^9	2.27
Pluto: 2290 km	2.627x10^{-5}	4.9946 x 10^9	4.9693 x 10^9	0.51
Proxima Centauri star: 214550 km	3.153 x 10^{-7} Degree	3.899 x 10^{13}	3.974 x 10^{13} (4.2 ly)	1.92
Alphard Star: 70.266 x 10^6 km	2.388 x 10^{-6} Degree	1.6859 x 10^{15}	1.675 x 10^{15} (177 ly)	0.65
EH, M87 Black Hole*: 38x10^9 km	4.314 x 10^{-9} Degree	5.0469 x 10^{20}	5.0616x10^{20} (53.5 x 10^6 ly)	0.29

*Event horizon of M87 black hole in the center of our Milky Way galaxy (16). Note: The distance between Earth and the Sun orbit is an average 149597870 km. ** One of the nearest stars to our Earth is Alpha Centauri at 4.3 light years (ly) away (40.68 x10^9 km away from the Earth). ***The distance between the Sun and our Galaxy center (10^5 ly circular) equal 9.5x10^{17} km.

The relatively small difference (0.26% to maximum ~ 2.27%) with the reference values is mainly due to the variation of these values (e.g. angular diameter and distances) given in different **references** and literature (14,15).

The formula was also validated by calculating the radius of some planets and stars. The result between the certified published values and the proposed formula are given in the following Table 3.

Table 3: Results of some Radius Calculations using Sayed`s Formula.

Planet	**Published radius (Km)**	**r = 10L$_A$φ/eπθ (or, r=0.15915470 L$_A$)**	% diff.
Sun	6.963x10^5	6.969x10^5	0.09
Alphard Star	35.13x 10^6	35.13x10^6	----
Neptune	24622	24764.5	0.058
Earth	6.4x10^3	6.53x10^3	2.03
Jupiter	71492.0	69910.9	2.21

The results given in table 3 are based on spherical shape of the planets and stars (Ɵ =360^0). The formula stated in the table 3 can be expreesed to be; r=0.15915470 L$_A$.

It should be mentioned that the small angle approximation is useful in many areas of engineering and physics, including mechanics, electromagnetics, optics, cartography, astronomy, computer science (14,15). S. El-Mongy formula is currently developed to be used in nuclear physics fields.

Conclusion

It can be observed that **S.El-Mongy`s theorem**, is a new empirical formula for calculating the circle arc length as a function in **(eπrs$^A$)**. This formula can also be given as; **A$_L$ = {(0.0174533185 r Ɵ)}.** Where, the value 0.0174533185 is called **Sayed`s number (I$_s$)** and equal to (eπ/10φ). The difference between the calculated arc lengths using the old well-known formula and our formula does not exceed 0.002% whatever the central angle and circular radius.

The distances between the Earth and sun, moon, planets, stars and EH-M87 black hole were precisely calculated using the S. El-Mongy`s formula with differences from 0.26 to ~ 2.27%. The formula was also used for radius calculations of some planets and stars.

Finally, it should be stated that the our new formula that correlates **(eπrs^A)** with circle arc length and its applications may dramatically leads to new and developed concept in many scientific fields. Previously, the study of the circle has helped inspire the development of many fields; geometry, astronomy, physics and calculus.

Conflicts of Interest

The Author declares that there is no conflict of interests with any other author.

Acknowledgment

References

1. David Joice, (2013) "Euclids Elements Book I","Department of Mathematics and Computer Science, Clark University, Worcester, MA 01610.
2. Pat Collingwood, Ruth Emond &Rona Woodward (2007)."The Theory Circle: A Tool for Learning and for Practice", journal Social Work Education. https://doi.org/10.1080/02615470601141409 ,P 70-83 ,
3. The circle, sphere and astronomy websites; www.wikipedia.org. updated on 2020
4. Astronomy in the Quran. www.islam-guide.com › bqs ›.
5. Andrew Norton, (2016). "An Introduction to Astrophysics and Cosmology". http://www.open.ac.uk/science/physical-science/sites/www.open.ac.uk.science.physical-science/files/files/book.pdf.
6. R.D. Sarva Jagannadha Reddy (2014) "An Alternate Formula in terms of Pi to find the Area of a Triangle and a Test to decide the True Pi value (Atomic Energy Commission Method", IOSR Journal of Mathematics, e-ISSN: 2278-5728, Volume 10, Issue 4.
7. https://en.wikipedia.org/wiki/E_(mathematical_constant)
8. Monica Young,(2020)."Astronomers Discover Huge Circular Arc near the Big Dipper", https://skyandtelescope.org/astronomy-news/astronomers-discover-huge-circular-arc-near-the-big-dipper/.
9. Tumlinson et al., (2018). "Annual Reviews of Astronomy & Astrophysics", .USA, SJR, Annual review Inc. ISSN 15454282, 00664146.
10. https://www.engineeringtoolbox.com/chord-length-circumference-circle-segments
11. Kim Plof'ker, (2002). "Spherical Trigonometry and the Astronomy of the Medieval Kerala School". Brown University,https://link.springer.com/chapter/10.1007%2F978-94-015-9862-08
12. Wladimir Lyra, "Fundamentals of Astronomy and Astrophysics , www.wladimirlyra.com.
13. Howard E. Bond, Edmund P. Nelan, Nancy Remage Evans , Gail H. Schaefer , and Dianne Harmer,(2018). "Hubble Space Telescope Trigonometric Parallax of Polaris B, Companion of the Nearest Cepheid". The Astrophysical Journal, 853:55 (8pp),
14. http://astro.physics.uiowa.edu/ITU/glossary/small-angle-formula/
15. https://en.wikipedia.org/wiki/Angular_diameter
16. Sky and telescope eBook,(2006)." Black Holes: Spinning hearts of darkness light the universe".

Biography

Prof. Sayed A. El-Mongy is currently nuclear affairs consultant and scientific supervisor. He has more than 100 papers published in national and international journals and conferences. He participated in national, regional and international meetings in USA, Russia, EU states, IAEA, S.Korea and Arab states. He has many newspaper articles and TV show concerning nuclear affairs and nuclear non-proliferation. He has private studies in the field of astronomy.