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A Tiny, Counterintuitive Change to the Mathematics of the Schrödinger Wave Packet and Quantum ElectroDynamics Could Vastly Simplify How We View Nature

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Abstract: This article proposes that an unexpected approach to the mathematics of a Schrödinger wave packet and Quantum Electro-Dynamics (QED), could vastly simplify how we perceive the world around us. It could get rid of most if not all quantum weirdness. Schrödinger's cat would be gone. Even things that we thought were unquestionably true about the quantum world would change. For example, the double slit experiment would no longer support wave particle duality. Experiments that appeared to say that entangled particles can communicate instantaneously over great distances, would no longer say that. Although the tiny mathematical change is counterintuitive, Occam's razor dictates that we consider it because it simplifies how we view Nature in such a pervasive way. The change in question is to view a Schrödinger wave packet as part of a larger Elementary Wave traveling in the opposite direction. It is known in quantum mechanics that the same wave can travel in two countervailing directions simultaneously. Equivalent changes would be made to QED and Quantum Field Theory. A lively 18 minute YouTube video ("Mathematics of hope despite COVID-19") explains this article in layperson's terms.

1 Introduction

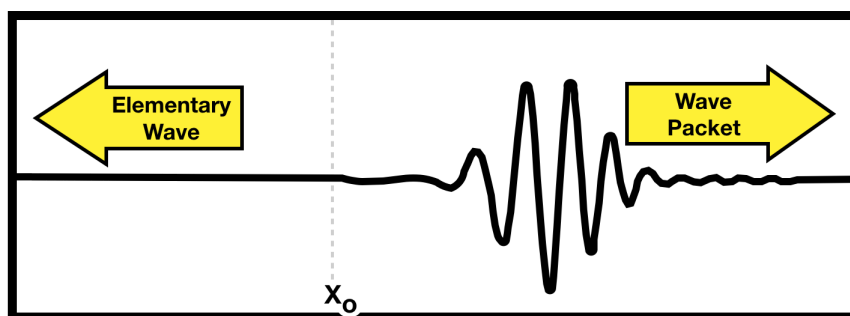


Figure 1: A one dimensional Elementary Wave (\mathcal{E}) moves to the \leftarrow left, while a Schrödinger wave packet (Ψ) moves to the right \rightarrow . They are two aspects of the same thing.

The Schrödinger wave packet in Fig. 1 consists of a wiggly line. Although the wave packet is moving to the right, in what direction is the line itself moving? One might assume that it is stationary. But consider a model in which the line itself is moving rapidly to the left. The Theory of Elementary Waves (TEW) proposes that the left side of the line in Fig. 1 is a one dimensional plane wave $\Psi(x) = e^{-i(kx - \omega t)}$ moving at light speed to the left. Under certain specialized circumstances a Schrödinger wave packet would pop out from inside this plane wave, and move to the right.[Boyd 2012 through 2020c; Little 1996, 2000 & 2009]

Such a plane wave that carries within it a trigger mechanism for the sudden emergence of a Schrödinger wave packet



moving in the opposite direction, is called an “Elementary Wave” which we denote by the letter \mathcal{A} (pronounced “ash”). It is a zero energy wave. Equations will be given in Section 2.

We know cousins of these waves from quantum mechanics (QM). For example, if we picture a one dimensional plane wave coming from the right, and moving to the left until it hits a barrier of infinite potential, what would happen? It would bounce off and double back on itself. It then appears to be a standing wave bobbing up and down. We can still think of it as a single wave traveling in countervailing directions. The Elementary Wave in Fig. 1 is similar in that it consists of one wave traveling in two opposite directions. With the \mathcal{A} the Schrödinger wave packet is usually absent, so we normally would see only a one dimensional plane wave moving to the left. We are omitting the infinite barrier from this example.

Under specialized circumstances, usually involving collision with a particle, a Schrödinger wave packet abruptly emerges from inside the \mathcal{A} and carries the particle to the right. In this model there is no wave particle duality. There is however one wave moving in two opposite directions simultaneously. It is a zero energy wave.

Although it is often said that waves always carry energy, that is naive. Schrödinger waves carry no energy. They don't push or pull particles, nor can they do any work. Schrödinger waves carry amplitudes, which are the square root of the probability of a particle being at that location (the Bohm rule). So if Schrödinger waves are zero energy waves, we are simply expanding the boundaries of that concept to include a plane wave moving in the opposite direction. Like many new ideas in mathematics, our model may sound preposterous.

The corresponding changes in Quantum Electro-Dynamics will be discussed in Section 7. If you wonder what this article is saying, Fig. 20 in Section 7 is a perfect illustration. If you understand Figs. 19 & 20, then you will understand the heart and soul of this article. This author, by the way, tends to think in pictures, so the graphics in this article are abundant. His friends view this author as a cartoonist, not as a serious mathematician.

1.1 Why being counterintuitive is an advantage

This model is counterintuitive. Why would we be interested in something that sounds absurd to QM experts? Because in sections 3 to 7 of this article we intend to demonstrate that this peculiar model has astonishing effects when applied to those experiments that allegedly “prove” quantum weirdness. Our overall goal is to preserve quantum math, which we regard as the most powerful and accurate science that humans have ever had, but vanquish quantum weirdness.

The fact that our model is counterintuitive is a strength rather than a weakness. Why? Insofar as our model is preposterous, that explains why Einstein and some of the greatest geniuses of all time could not figure this out over the past century. Logical thinking was the downfall of the founders of QM.

If you start with reasonable assumptions you end up with a quantum world that is weird. If you start with weird assumptions you end up with a quantum world that is reasonable. We pay our “weirdness tax” up front. QM does not pay a “weirdness tax” and is therefore penalized forever with a misperception that the quantum world is weird.

Below we will demonstrate that this model changes the meaning of five quantum experiments. Where there was previously quantum weirdness, there is no longer weirdness. The five experiments are:

- Double slit experiments;
- The Purcell effect;

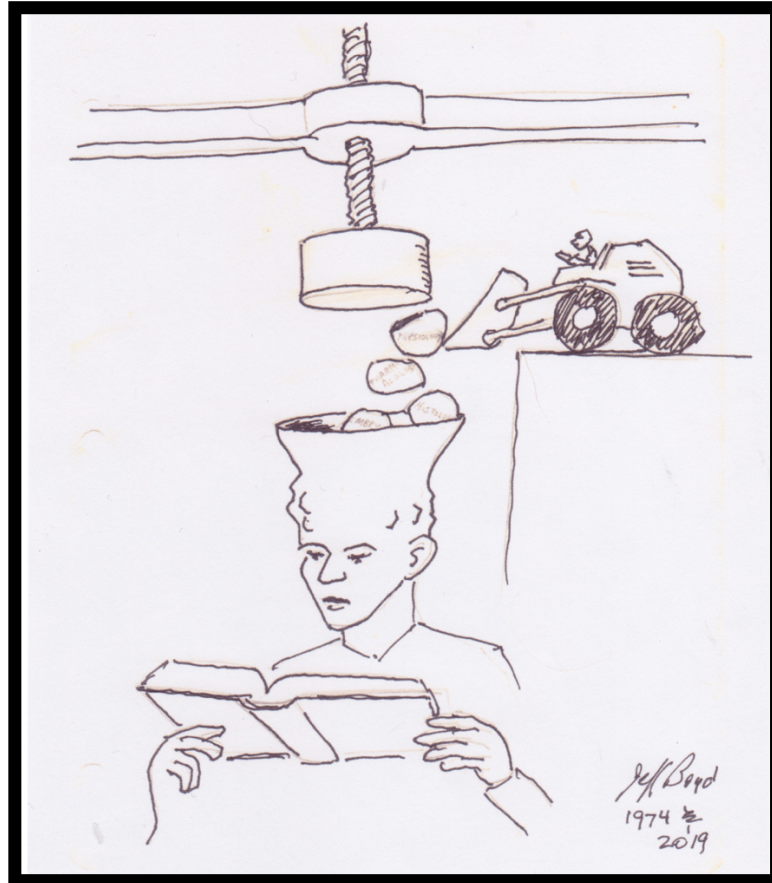


Figure 2: Graduate school is exciting.

- A quantum eraser experiment;
- Bell test experiments; and
- Quantum Electro-Dynamic experiments.

If we succeed in showing a vast transformation of the quantum landscape, that would not constitute proof that the peculiar model described in this article should be accepted. All it would mean is that we have discovered an interesting mathematical imbalance. On one side of the scales we would have a small counterintuitive change in how we approach quantum math. On the other side of the scales we would have a pervasive simplification of how Nature appears to work.

Occam's razor might then come to mind.

2 Equations of an elementary wave ($\mathcal{A}\mathcal{E}$)

We will divide our one dimensional Elementary Wave in Fig. 1 into the part traveling left, which we will call Ψ_L , and the part traveling right, which we will call Ψ_R . The point x_0 is where we divide left from right.

Our thinking is guided by an asymmetry. While a wave function might flow in both directions (symmetrical), energy

and momentum only flow to the right, not the left. Thus we anticipate a tiger (Schrödinger Wave) moving to the right, but an elongated tail moving left with high speed but no energy. In many ways we are more interested in the tail, because if you control the tail, you control the tiger.

We define x_L to be a location to the left of x_0 (Fig. 1) and x_R to be a position to the right. The vertical axis is amplitude. We define the height and slope of the wave functions to be equal at $x = 0$:

$$x_L = x_R \tag{1}$$

$$\frac{\partial \Psi_L}{\partial x_L} = \frac{\partial \Psi_R}{\partial x_R} = 0 \tag{2}$$

Time, frequency and angular frequency are equal on the two sides:

$$f_L = f_R \equiv f \tag{3}$$

$$\omega_L = \omega_R \equiv \omega = 2\pi f \tag{4}$$

The entire graph in Fig. 1 consists of a wiggly line. The speed of the line itself is tricky. We claim the line moves to the left at light speed, while the wave packet (if it exists) moves to the right at v_R (often less than light speed). We will attribute light speed c to the Ψ_L and v_R to Ψ_R , remembering in the back of our mind that they are comprised of the same substrate, and the substrate is the line moving to the left at c .

Therefore the two wavelengths can be different. We will define

$$\lambda_L = \frac{c}{f} \quad \text{and} \quad \lambda_R = \frac{v_R}{f} \tag{5}$$

Note that $\lambda_L > \lambda_R$ for wave packets moving slower than light.

$$\frac{v_R}{c} = \frac{\lambda_R}{\lambda_L} \tag{6}$$

The substrate Ψ_L carries zero energy. The wave packet Ψ_R also carries zero energy, as we said earlier, but it carries amplitudes for momentum. Variables such as E_R , p_R and k_R exist only on the right side of Fig. 1.

In other words $k_L \neq k_R$. We define

$$k_L = 2\pi/\lambda_L \quad \text{but} \quad k_R = p_R/\hbar \quad \text{where } p_R \text{ is momentum.} \tag{7}$$

Note that p_L and E_L are undefined. As we said before, velocity $v_L = c$, the speed of light. On the other hand, $v_R \neq c$ unless the wave packet moves at light speed, which would only happen if the experiment involved a particle of zero mass, such as a photon.

We now define our two wave functions:

$$\Psi_L = e^{-i(k_L x_L - \omega t)} \quad \text{and} \quad \Psi_R = e^{i(k_R x_R - \omega t)} \tag{8}$$

$$Re(\Psi_L) = \cos(k_L x_L - \omega t) \quad \text{and} \quad Re(\Psi_R) = \cos(k_R x_R - \omega t) \tag{9}$$

$$Re(\Psi_L) = \cos\left(\frac{2\pi x_L}{\lambda} - \omega t\right) \quad \text{and} \quad Re(\Psi_R) = \cos\left(\frac{p_R x_R}{\hbar} - \omega t\right) \tag{10}$$

Note that the ingredients with which to build a Schrödinger wave equation only exist to the right of x_0 .

2.1 Deriving the Time Independent Schrödinger Equation:

We define

$$E_R = \text{kinetic energy} + \text{potential energy} \tag{11}$$

$$= \frac{1}{2}mv_R^2 + u = \frac{p_R^2}{2m} + u \tag{12}$$

Taking the second derivative $\partial^2/\partial x_R^2$ of the wave function $\Psi_R = e^{i(k_R x_R - \omega t)}$ (Eq. 8 Right), we get:

$$\frac{\partial^2 \Psi_R}{\partial x_R^2} = \frac{\partial^2}{\partial x_R^2} (e^{i(k_R x_R - \omega t)}) = (ik_R)^2 \Psi_R = -k_R^2 \Psi_R = \frac{p_R^2}{\hbar^2} \Psi_R \tag{13}$$

$$\hbar^2 \frac{\partial^2 \Psi_R}{\partial x_R^2} = p_R^2 \Psi_R \tag{14}$$

Multiplying both sides of Eq. 12, $\left[E = (p_R^2/2m) + u \right]$, by Ψ_R , we get:

$$E\Psi_R = \frac{p_R^2 \Psi_R}{2m} + u\Psi_R \tag{15}$$

and inserting Eq. 14, we get the **Time Independent Schrödinger Equation:**

$$E\Psi_R = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_R}{\partial x_R^2} + u\Psi_R = \text{TISE} \tag{16}$$

2.2 Deriving the Time Dependent Schrödinger Equation:

The time dependent equation can be easily derived by differentiating our wave equation

$$\Psi_R = e^{i(k_R x_R - \omega t)} \quad \text{by } \partial/\partial t:$$

$$\frac{\partial \Psi_R}{\partial t} = -i\omega \Psi_R \tag{17}$$

$$\text{We define } E_R = \hbar\omega. \text{ Multiplying that by } \Psi_R \text{ we get:} \tag{18}$$

$$E_R \Psi_R = \hbar\omega \Psi_R \tag{19}$$

$$-\frac{i}{\hbar} E_R \Psi_R = -i\omega \Psi_R = \frac{\partial \Psi_R}{\partial t} \tag{20}$$

$$E_R \Psi_R = -\frac{\hbar}{i} \frac{\partial \Psi_R}{\partial t} = i\hbar \frac{\partial \Psi_R}{\partial t} \tag{21}$$

We can substitute that into the Eq. 16:

$$i\hbar \frac{\partial \Psi_R}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_R}{\partial x_R^2} + u\Psi_R \tag{22}$$

which is the **Time Dependent Schrödinger Equation, TDSE.**

2.2.1 Wave Packet

Until now we have been focusing on waves of a single frequency f and momentum p_R . We now change that to a model that includes a cluster of frequencies Δf and momenta Δp_R . The reason we do so is because Fig. 1 shows a wave

packet moving to the right. In order to construct a wave packet we need a cluster of frequencies that we add into a superposition that exhibits constructive interference in a narrow range of distance (Δx_R).

In order to have a cluster of frequencies on the right side of Fig. 1, we need to have the same frequencies on the left. As we said in Eq. 3, $f_L = f_R \equiv f$. However, there is no wave packet on the left because that is an area in which a superposition of wave equations adds up with destructive interference.

In the remainder of this article we will portray Elementary-Schrödinger Waves as having a nascent wave packet but not an explicit Schrödinger wave packet in most cases. The triggering of a Schrödinger Wave Packet to suddenly appear when the elementary wave approaches a particle, is an unusual event that occurs rarely and under special conditions.

In any volume of space there are a finite number of wave packets but an infinite number of elementary waves.

2.3 Elementary Wave traveling to the left

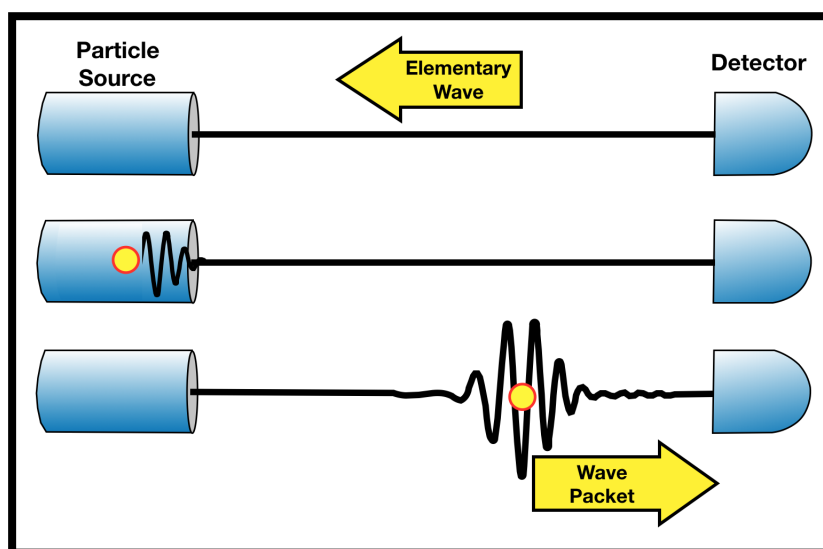


Figure 3: This diagram shows what precedes the emergence of a Schrödinger wave packet. The top row shows a zero energy plane wave moving from the detector leftward to the particle source. That is **the detector’s “invitation.”** Middle shows a particle about to be emitted from inside the particle gun. Such a particle triggers a Schrödinger wave packet to emerge from inside the incident plane wave. The bottom row shows the wave packet carrying the particle toward the detector. This diagram represents our model of how an Elementary Wave (\mathcal{E}) works.

In Eq. 8 we stated the wave function for the elementary wave traveling to the left. When that wave equation is combined with the Schrödinger equation of the wave traveling right, you get a compound equation that defines and elementary wave \mathcal{E} . Compound equations are well known in quantum mathematics. For example in a wave equation for a potential well it is commonplace to have a plane wave defined if $x < 0$ or $|A| < x$, but another wave equation for the well itself (when $0 \leq x \leq |A|$).

According to TEW the world is more inter-connected than QM knows about. **No Schrödinger wave packet can strike a detector unless the detector has “invited” it to do so.** Such an “invitation” is diagramed in the top row of Fig. 3.

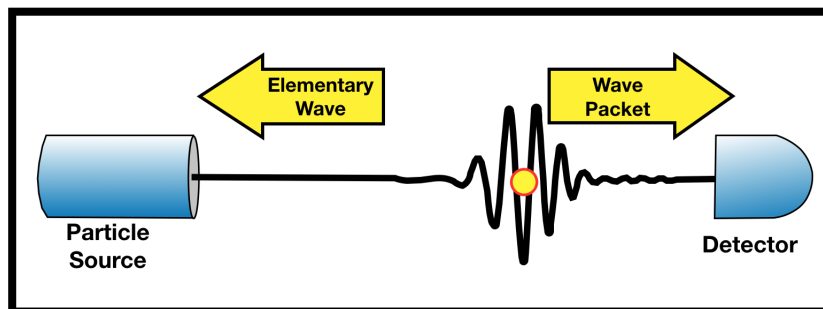


Figure 4: This is a simplification of the previous Figure. This diagram serves as a symbol or condensed version of the previous Figure.

As we said before, the defining feature of an elementary wave is that it carries this intrinsic trigger mechanism for a Schrödinger wave packet to emerge, moving in the opposite direction. Such a trigger is activated when the \mathcal{A} encounters a particle with precisely the right characteristics, usually involving a particle that has chosen to respond to that specific incident wave.

Such a trigger might be activated if the \mathcal{A} of frequency f approaches a particle whose De Broglie frequency is $f = E/2\pi\hbar$, and if the particle is about to be launched from a gun, and if the particle makes a random choice of that specific \mathcal{A} rather than the other competing \mathcal{A} 's. Under those circumstances the wave packet mechanism could be triggered as shown in the middle of Fig. 3. The wave packet would then carry the particle off toward the detector from which that specific \mathcal{A} is propagating (bottom row of Fig. 3).

In QM Hilbert space is often assumed to be highly abstract, in the stratosphere, in the “space of states.” In TEW Hilbert space is interwoven in the Euclidean space of everyday experience. This down-to-earth concept of Hilbert space was developed in one of our earlier publications[Boyd 2019a]

According to TEW, at every point in space there are an infinite number of Elementary Waves traveling in all directions and at all frequencies, at the speed of light. Because they carry no energy, most of them are invisible to our detectors. Our detectors can only see a wave particle \mathcal{A} -II (where the symbol “II” signifies a particle). There is no such thing as a particle without an elementary wave. The intrinsic nature of particles is that they must always be attached to one \mathcal{A} or another. They can jump from one elementary wave to another. But naked particles, disconnected from all elementary waves do not exist.

Particles can pop in and out of existence, transferring their mass-energy to a new particle, attached to a different \mathcal{A} . A sketch of TEW as a quantum field theory will be developed in Section 7.

We turn now to showing how this TEW technology applies to the five experiments we mentioned earlier:

3. Double slit experiments;
4. The Purcell effect;
5. A quantum eraser experiment;
6. Bell test experiments; and
7. Quantum electrodynamic experiments.

3 Double Slit Experiments

Einstein proved that the conventional view of a double slit experiment is wrong (Fig. 5). The conventional view is that a single particle leaves a gun, spreads out like a wave that penetrates both slits, and interferes with itself so an interference fringe pattern appears on the target screen.[Khalili 2013] Einstein proved that picture is absolutely impossible. Whenever a dot appears anywhere, the entire wave function would need to vanish everywhere, instantaneously, faster than the speed of light. If it did not do so, then some other part of the wave could produce a second dot, which would be impossible since only one particle left the gun.[Baggott 2011; Popper 1982]



Figure 5: This is where QM went off the rails in the 1920's. Even today this misleading diagram of a double slit experiment is the official QM picture. Einstein proved that this is absolutely impossible. As convincing as it may appear, this diagram is WRONG. Einstein proved it! See the text.

No one ever explained how the wave function function could vanish everywhere, faster than the speed of light. Therefore Einstein proved that the double slit experiment showed in Fig. 5 is untenable. It is a picture that must be abandoned, no matter how fond of it you might be.

Let's say that the reader is committed to the idea that a particle fired from an electron gun in a double slit experiment, takes both paths so that the same particle (in a superposition) penetrates BOTH slits A and B (Fig. 5). The problem is that Einstein's criticism is a brick wall that prevents you from saying that. There is one and only one way in which the reader could defend the conventional view of the double slit experiment. The reader would need to formulate a credible response to Einstein. If you can devise a theory for how the entire cloud of amplitudes could instantaneously vanish, faster than the speed of light, then you might be able to defend the conventional view about the particle superposition. You might even get an Abel prize for your effort, because no one has been able to answer Einstein's criticism.

Unfortunately the double slit experiment is the primary evidence of wave particle duality. When someone explains wave particle duality, they usually start by saying there is no other explanation for the double slit experiment. But they

are wrong. **Because of what Einstein said, the double slit experiment provides zero support for wave particle duality.** We will discuss below the question whether other experiments could rescue the doctrine of wave particle duality from extinction.

3.1 The TEW explanation of the double slit experiment.

The TEW model can explain the double slit experiment. Our model is immune to Einstein's complaint.

Briefly, zero energy plane waves from every point of the target screen travel toward the double slit barrier, as shown in Fig. 6. The waves through slit A interfere with its sibling waves through slit B as they impinge on the gun. The phase difference ($\theta_B - \theta_A$) determines whether interference at the gun is constructive, intermediate, or destructive.

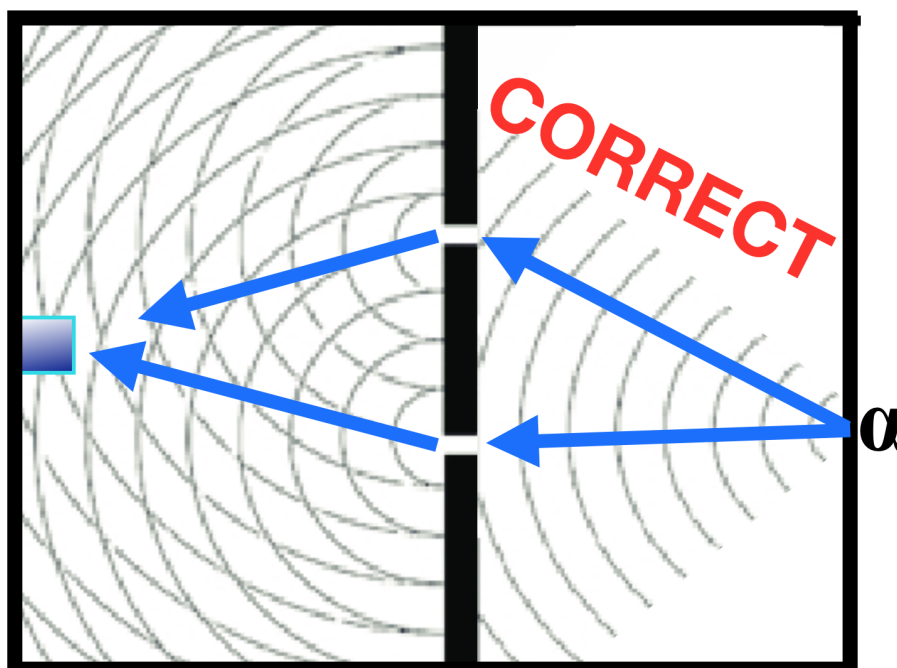


Figure 6: This is the TEW explanation of the double slit experiment.

A particle in the gun randomly selects one incident wave to respond to. That decision triggers the emergence of a Schrödinger wave packet from inside that one incident wave (as in the center of Fig. 3). The wave packet travels in the opposite direction, following its plane wave backwards as its trajectory. The particle is carried by the wave packet with a probability of one. It goes through only one of the slits (it doesn't matter which slit) and makes a dot at that point of the target screen from which its specific plane wave is emanating.

This mechanism would produce the observed interference fringe pattern on the target screen. That is proved in the caption of Fig. 7. Wave function collapse occurs when the particle leaves the gun. After that the experiment is deterministic. Nothing changes when the particle hits the target screen. Unlike QM, the particle has a trajectory before it hits the detector. This illustrates one of the Axioms of TEW, "Wave function collapse occurs **before** something is measured."

As far as we know, ours is the only possible explanation of the double slit experiment. Prior to the discovery of TEW,

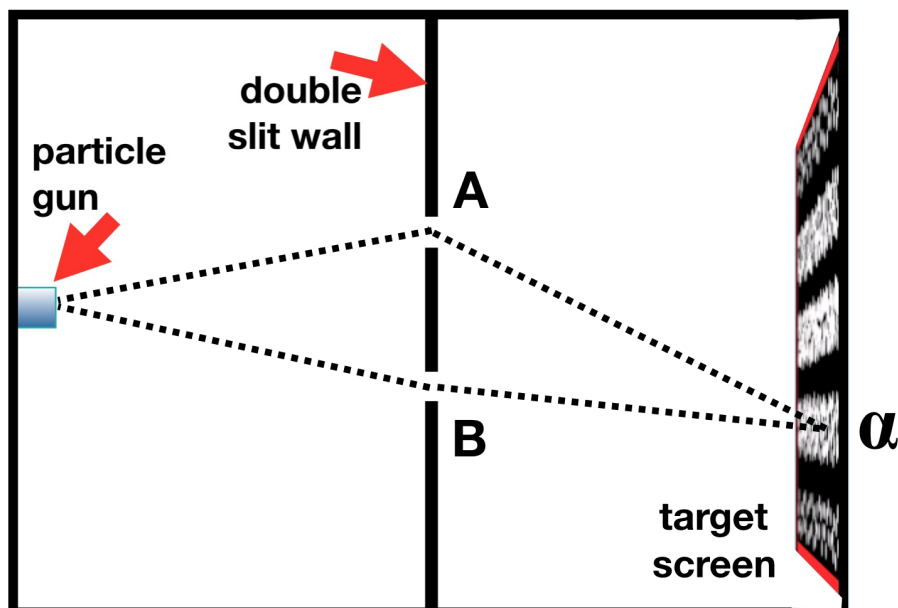


Figure 7: This diagram explains why QM and TEW produce the same results. There are two pathways (dotted lines): from the gun to slit A to point α on the target screen; and from the gun to slit B to point α . If we measure them both, subtract one from the other, then divide by the wavelength (λ), the modulo divided by 2π is the phase difference. Our point is that the phase difference ($\theta_B - \theta_A$) is the same whether the waves move from left to right, or right to left. The phase difference produces the wave pattern in the final dataset.

Feynman said that no one could explain the double slit experiment. He also said that experiment contains the “central mystery of QM.”[Feynman 2010]

Our proposal for how a double slit experiment works not only solves Einstein’s complaint, it also solves a complaint of John von Neumann. He had said that the Schrödinger equation is deterministic. How then did the randomness get into QM? According to our model the randomness is attributable to the particle, which unpredictably chooses any one of a myriad of incident waves (one from each point of the target screen). After the particle has made its random decision, only one Schrödinger wave packet blossoms out of the selected Elementary Wave. All the other incident plane waves impinging on the particle fizzle out and never produce a wave packet. They are duds.

So our answer to von Neumann is that the particle, not the Schrödinger equation, is the origin of randomness in the double slit experiment. Brownian motion shows that particles wiggle and jiggle erratically. To say that a particle brings randomness into the double slit experiment, is not surprising.

3.2 Wave particle duality: evidence from other experiments

Since the double slit experiment is incompatible with wave particle duality, are there other experiments that would rescue the doctrine of wave particle duality from death?

It was said that the Davisson and Germer experiments “proved wave particle duality.” However, that was not true. The only thing those experiments proved was that waves and particles interact. We showed in previous publications

that the Davisson and Germer experiments can be explained by TEW. We showed why there would be what Davisson and Germer called a “spur” in their data at angle 50° in Azimuth A for electrons fired at 54 volts.[Davisson & Germer 1927; Davisson 1928a & 1928b]

Here is a thumbnail sketch of the TEW model of the Davisson Germer experiments. Elementary waves from the detector are more intense when they refract at the Bragg angle (50° for Azimuth A) through the nickel crystal, i.e. at the angle for constructive interference for that crystal lattice at that wavelength λ . The electrons are more likely to select incident Elementary Waves that have greater intensity as the waves impinge on the electron gun. Once the electron selects one specific incident wave, a Schrödinger wave packet pops out of that ray and carries the electron backwards through the nickel crystal, to the detector from which the wave originated. As the experiment sweeps through different detector angles, there would be a surge in the data at the Bragg angle. This is if the Elementary Waves go from the detector through the crystal to the gun, and an electron follows that specific Schrödinger wave packet back.

In summary: the Davisson Germer experiments can be explained by TEW, without wave particle duality.[Boyd 2013b]

Louis de Broglie is often cited as the person who proposed wave particle duality, but that is not true. De Broglie said that waves and particles interact, not that they were identical. De Broglie applauded and encouraged Franco Selleri’s proposal that wave particle duality is wrong. Selleri cited experiments of Pfleegor and Mandel, and de Broglie applauded.[Feire 2003; Selleri 1982, 2002]

Pfleegor and Mandel’s attenuated laser experiments **disprove** wave particle duality. They crossed two laser beams so there was wave interference. This was like each photon in a double slit experiment coming through only one slit, because each photon came from only one of the two lasers. Therefore when the target screen showed wave interference it could not be a photon interfering with itself, because each photon came from only one of the two lasers. Could it be that the wave interference was caused by a photon interfering with the photon ahead of it or behind it? No! They turned down the intensity, so the laser beams were attenuated. There were usually zero photons in the equipment. One photon would cross the equipment, followed by 200 times as much empty time before another photon was emitted. Yet the final data still showed a wave interference fringe pattern. They concluded that there was interference of zero energy waves, and the photons simply made that interference visible.

Selleri said the Pfleegor and Mandel experiments **disproved** wave particle duality. De Broglie agreed with Selleri.[Pfleegor & Mandel 1967, 1968; Selleri 1982, 2002]

In summary, the disproof of wave particle duality that we learn from the double slit experiment is consistent with other experiments and a theme in quantum literature that has been ignored.

3.3 Complementarity explained

Our model provides a simple explanation of complementarity. “Complementarity” means that if you know which slit a particle uses in a double slit experiment then the interference fringe pattern on the target screen vanishes. QM says this is an unexplained mystery. With TEW this is no longer a mystery.

When we look at the target screen in a double slit experiment, what are we looking at? **According to TEW we are looking at a picture of wave interference in proximity to the particle gun. An image of the zero energy waves is displayed on the screen!** If we see an interference fringe pattern in the final data, that means there was wave interference at the gun. If the target screen shows no interference fringe pattern, that means there was

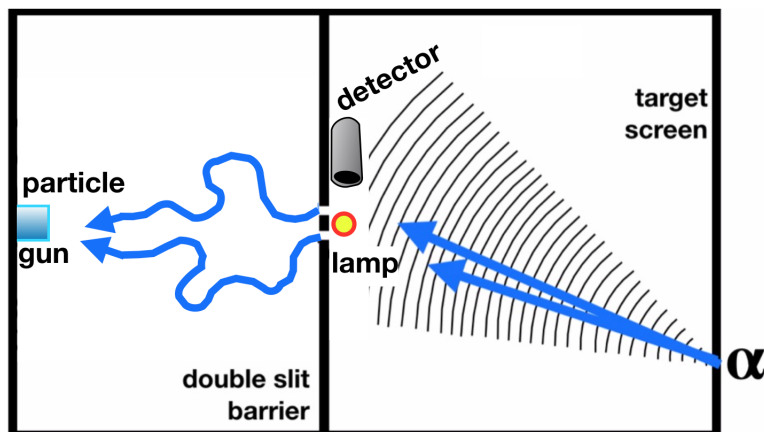


Figure 8: **Complementarity explained:** If the lamp is turned “ON” it destroys the superposition additivity of the zero energy waves through slits A and B. Therefore the sibling waves don’t recognize one another as they approach the gun. They don’t add together. There is no wave interference. The waves on the left do not actually follow crooked paths. In this diagram the meandering blue lines on the left symbolize two waves that are strangers to one another. They have lost their way because they were blinded by the light.

no wave interference at the gun.

In order to know which slit a particle used, we need to introduce a lamp (or low energy source) into the experiment, along with a detector as shown in Fig. 8. Whenever the light is switched “ON” that energy is infinitely more than the zero energy elementary waves passing through that neighborhood. The light’s energy destroys the superposition additivity of the elementary waves. If two waves cannot be added together into a superposition, then they cannot interfere with one another when they impinge on the particle gun. In other words, waves from point α on the target screen, when they travel through slit A will not interfere with waves from point α that travel through slit B.

The final data on the target screen simply tell us the truth, which is that “You have destroyed the wave interference at the gun when you turned that little light ON!”

4 The Purcell effect shows that quantum experts already know about elementary waves

Many experiments have established the Purcell effect, which is that an excited atom will decay more rapidly and emit a photon (to carry off the excess energy) if the excited atom is in a micro-cavity whose diameter is a multiple of $\lambda/2$ where λ is the wavelength of the photon about to be emitted. This was discovered in the 1946 by Edward Purcell.[Purcell 1946; Goy, Raimond, Gross et.al. 1983; Haroche & Klepper 1989; Hulet, Hilfer, & Kleppner 1985]

Information about the size of the cavity must be carried somehow into the excited atom, before it decays and emits a photon. Otherwise how does the atom know that it is a hospitable environment? That information enters the atom with zero energy. QM experts give this phenomenon the names “available states” or “modes of the cavity.” TEW agrees and simply uses a different word. What the experts call an “available state,” we call an “Elementary Wave.”

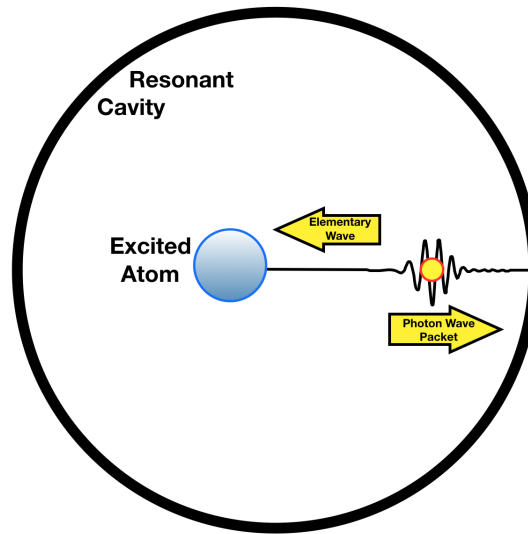


Figure 9: The elementary wave travels left carrying into the atom information about the diameter of the cavity. A photon wave packet travels in the opposite direction if the diameter is a multiple of $\lambda/2$.

For example, if a Rydberg atom is heated in an oven, then a laser is used to excite the outer electron to a higher energy state, and the excited atom is put into a resonant micro-cavity (●), the excited atom will decay hundreds of times faster and lose its excess energy (as a photon) if the width of the cavity is a multiple of the wavelength λ of the photon which the atom wants to emit.

This experiment demonstrates that quantum experts are familiar with elementary waves. They simply give them a different name than we do.

5 A quantum eraser experiment re-analyzed

In order to demonstrate our peculiar model of a Schrödinger wave, we will re-analyze the data from a famous QM experiment called the “Quantum Eraser” experiment. This experiment was published by Kim, et.al. in the year 2000. After we explain the QM viewpoint, we will re-analyze this experiment to show the TEW viewpoint. A quick summary is that QM claims that wave function collapse occurs at the detectors, whereas TEW claims wave function collapse occurs a dozen nanoseconds earlier, at the laser. That dozen nanoseconds means that the conclusions we draw from this experiment are totally different.[Boyd 2013a, 2019b; Kim, Yu, Kulik, et.al. 2000; Scully & Drühl 1982]

5.1 The experiment as explained by QM

In a double slit experiment you see an interference fringe pattern on the target screen if and only if you are ignorant of which slit was used by the particle. As we said earlier, this is called “complementarity.” If you discover which slit was used, then the pattern vanishes. John Wheeler wondered whether the same thing would be true if there were delayed choice. In other words, suppose you build an experiment in which an interference fringe pattern is etched on the target screen at a time when you do not know which slit was used. But then at a later time you discover that slit A was used.

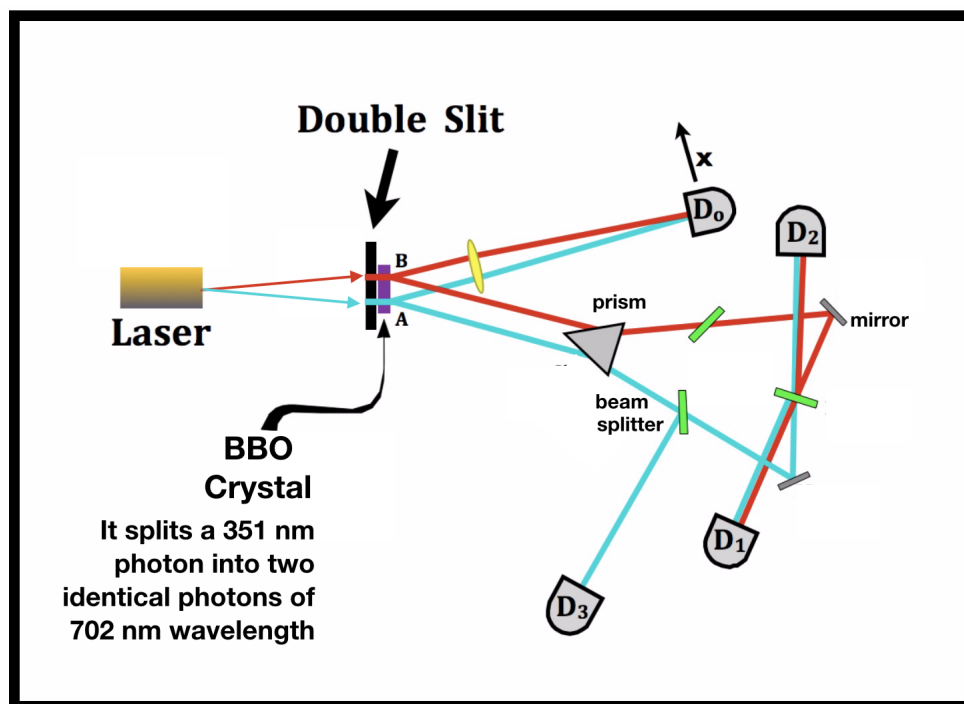


Figure 10: Apparatus used in quantum eraser experiment, color coded for whether a photon went through the upper slit (B = red) or lower slit (A = aqua blue) at the double slit barrier. After going through one slit or the other, each photon is split into two photons by a BBO crystal. A 351 nm wavelength photon classified by us as “red” is split into two 702 nm photons also coded as “red.” Similarly the 351 nm photon from the laser and penetrating the lower slit, coded “blue,” is also split into two 702 nm photons that we classified as “blue.”

Wheeler’s hypothesis was that the interference fringe pattern on the target screen would be erased, backwards in time. The effect would precede the cause.

The experiment by Kim, et. al. appears to confirm Wheeler’s idea about backwards-in-time erasure of data (Fig. 10).

In the experiment each photon goes through a double slit barrier, then immediately encounters a BBO Crystal ($\beta - BaB_2O_4$) which splits it into two identical photons. One of these photons is sent up into a double slit experiment (Fig. 10) where an interference fringe pattern is made on the screen. That photon goes less distance, so the pattern on the screen is established BEFORE the lower photon randomly chooses which other detector to “click.”

Fig. 10 is color coded according to whether a photon went through the upper slit (red) or lower slit (blue). If a red and blue line enter a detector (as in detectors D_1 and D_2) then we don’t know which slit the original photon used. If only an aqua blue line enters a detector (as in detector D_3), then we DO KNOW that the photon came through slit A (the lower slit, color coded in blue).

When the computer assembles data, it connects data from two detectors: from D_0 paired with one of the other detectors. The final results show that if the lower photon subsequently “clicked” D_1 or D_2 then there is an interference fringe pattern visible on the target screen (in the upper area). But if the lower photon subsequently “clicked” detector D_3 then the interference fringe pattern on the target screen is blank. The experimenters are confident that this means that data can be “erased” backwards in time if you discover at a later date which slit was used in a double slit experiment.

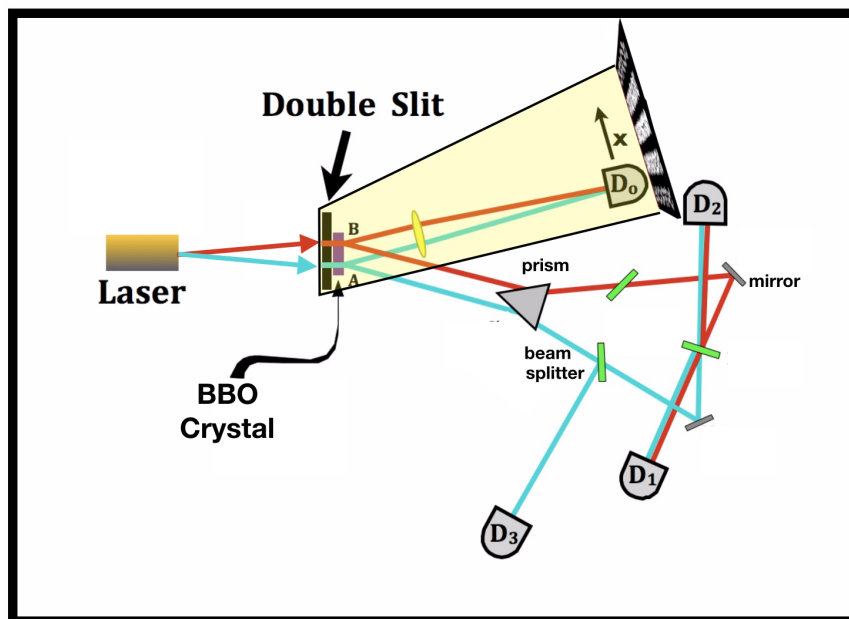


Figure 11: The upper photon in the yellow area is in a double slit experiment. The purpose of the experiment is to inscribe an interference wave pattern by means of the upper photon(s) and then (a nanosecond later) the lower red and blue photons determine whether we know or do not know which slit was used.

“Backwards in time” means a nanosecond earlier.

If you limit yourself to the mathematics of QM then you are forced to say that this experiment proves that data can be erased backwards in time. That is an example of “quantum weirdness.” You know you have encountered quantum weirdness when you get a migraine. Fortunately, if we view the experiment with the mathematics of TEW, then the same experiment reaches different conclusions.

The original research team thought the laser was so irrelevant that they omitted it from their diagrams. Their focus was on non-parametric down conversion in the BBO crystal. To the left of the double slit barrier their diagram referred to a “pump” sending photons into the BBO crystal.

5.2 TEW explanation of the quantum eraser experiment

According to TEW this is a simple experiment (Fig. 12). All decisions are made at the laser, dozens of nanoseconds earlier than QM believes that decisions were made. Time does not go backwards. Data are not erased from the target screen. The target screen shows us a picture of wave interference in proximity to the laser.

TEW says an elementary wave travels from each detector to the photon source (laser) (Fig. 13). The elementary waves from different detectors compete with each other at the laser for the attention of the photon about to be launched. The photon randomly selects one of them. After the photon leaves the laser this becomes a deterministic experiment. Each photon is carried by its Schrödinger wave packet, and that wave packet follows backwards the trajectory of the Elementary Wave to which it is tethered, back to the detector(s) from which that Elementary Wave is coming. A Schrödinger wave packet of 351 nm wavelength is split into two wave packets, each having wavelength 702 nm, by the

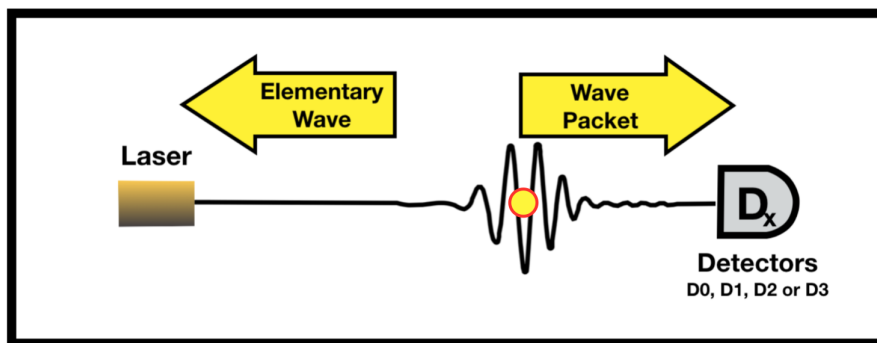


Figure 12: Elementary Wave travels left and Schrödinger wave packet travels right, two aspects of the same thing.

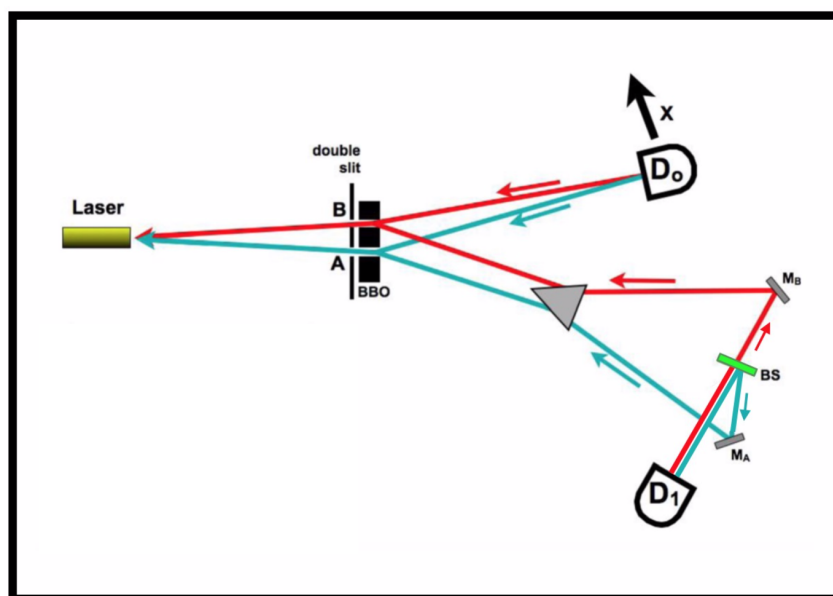


Figure 13: Simplified TEW model. Some detectors are omitted from this diagram, for simplicity. Elementary rays of 702 nm (red or aqua) originate from the detectors and move to the BBO crystal, where they combine into red or aqua rays of 351 nm heading toward the laser. Since two rays (red and aqua) impinge on the laser, there *is wave interference* at the laser, which is why there is wave interference reported by detector D_0 . What the target screen displays in the final dataset is the wave interference incident to the laser.

BBO crystal.

What is reported on the target screen is **reality**. You cannot erase reality. The reality is that there is interference of two Elementary Waves at the laser iff there are two waves (red and blue) impinging on the laser. If there is only one Elementary Wave impinging on the laser, then of course there will be no wave interference to report on the target screen, because you cannot have interference with only one wave impinging on the laser.

So how could there be an interference pattern on the target screen at one time, and then it is erased? The answer is, that does not happen. Nothing is ever erased. What happens is that if data from the target screen is paired with data

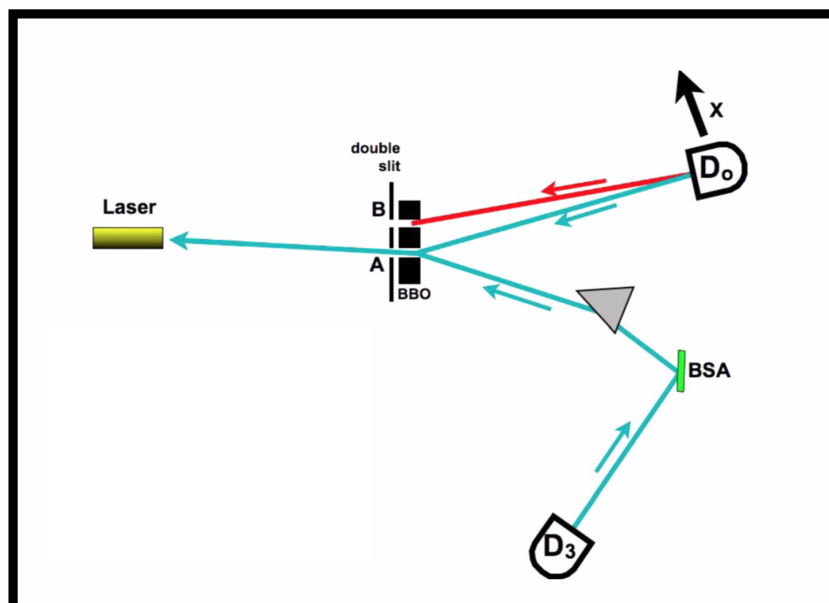


Figure 14: Simplified TEW model: Detector D_3 puts out an aqua elementary ray of 702 nm, but no red ray, for the simple reason that it cannot “see” slit B (the upper slit) because of how the equipment is constructed. The BBO crystal cannot manufacture a red Elementary Wave of 351 nm to send toward the laser, because the BBO crystal lacks half the ingredients. This means that a red wave of 702 nm from detector D_0 is not matched with a red wave of 702 nm from detector D_3 impinging on the BBO crystal. Since the BBO crystal cannot manufacture a red wave of 351 nm, therefore there is no wave interference at the laser, because it takes two waves of 351 nm to make interference. The red line we might expect to impinge on the laser is missing. Therefore the final dataset reports “No wave interference.”

from detectors D_1 or D_2 , then there are both red and blue Elementary Waves impinging on the laser (Fig. 13), and therefore there is wave interference at the laser. But if data from detector D_0 is paired with data from detector D_3 , then only a blue wave (see Fig. 14) and therefore only one blue elementary wave is incident to the laser, and the final data show **no interference** because you cannot have interference with only one wave.

TEW proposes that the data describe reality. The D_0 data report wave interference iff there is wave interference at the laser. If Detector D_3 is involved then there is no red Elementary Wave of 351 nm impinging on the laser. Why? Because the BBO crystal is missing half the ingredients needed to manufacture a red wave of 351 nm wavelength. The red wave of 702 nm coming down from above is not joined with a red wave of 702 nm coming up from detector D_3 because no such wave is coming from detector D_3 because of how the equipment is built. Detector D_3 can see the lower slit, but cannot see the upper slit.

In that case the blank screen does not mean something was erased, because there never was any data on that screen. The blank screen when detector D_3 is involved means there is only one elementary wave (a blue one) of 351 nm impinging on the laser, but the second (blue) wave is absent. You cannot have wave interference with only one wave.

The decision maker in this experiment is the photon as it is about to exit the laser. It is confronted with three incident elementary waves:

1. Waves from D_0 and D_1 coming through both slits and interfering;

2. Waves from D_0 and D_2 coming through both slits and interfering;
3. Waves from D_0 and D_3 coming only through slit A: no interference;

When the photon makes that random choice the Schrödinger wave packet mechanism is triggered and that wave packet sweeps the photon off its feet and carries it away from the laser. When the wave packet of 351 nm wavelength reaches the BBO crystal it splits into two wave packets of 702 nm wavelength. Once the photon leaves the laser the ball game is over. The final data are determined. Nothing changes from that moment on. The photons simply follow their Schrödinger waves packet, which follow the Elementary Waves backwards to the detectors from which those waves are coming.

If the photon randomly chooses # 1 or # 2 (from the list above) then the final data will show an interference pattern on the target screen (D_0). If the photon randomly chooses # 3 (from the list above), then the final data will show no interference pattern on the target screen (D_0). That decision is made before the photon leaves the gun.

6 The Bell test experiments re-analyzed

The Bell test experiments are alleged to be a fountain of quantum weirdness. That weirdness disappears when we apply TEW technology.[Arndt, Nairz, Vos-Andreae, et. al 1999; Aspect, Grangier & Rogers, 1981, 1982; Aspect, Dalibard & Rogers 1982; Becker 2018; Bell 1964, 1981; Boyd 2019c, 2015f, 2013b; Cirel'son 1980; Clauser, Horne, Shimony et al. 1969; Einstein, Podolsky & Rosen 1935; Giustina, Mech, Ramelow et. al. 2013; Hensen, Bernien, Dréau, et.al. 2015]

It is well known that when Alice and Bob test their entangled photons at random angles θ_1 and θ_2 , their results obey this equation: $P = \cos^2(\theta_2 - \theta_1)$ where P is the probability of both Alice and Bob seeing a photon simultaneously. That equation contradicts Einstein's prediction. We will use the word "probability" instead of "coincidence rate," which is the word used in QM discussions of this phenomenon.

6.1 Bi-Rays defined

As we said earlier, TEW proposes that at every point in space there are an infinite number of Elementary Rays traveling in all directions and at all frequencies. This implies that every Elementary Ray has a mate, namely an identical ray traveling coaxially in the opposite direction. The pair is called a "bi-ray."

A pair of entangled photons is defined as two photons moving in opposite directions on such a bi-ray (Figs. 15-16). Each photon has an amplitude for following each of the two train tracks that constitute the bi-ray. Each photon is like a railroad engine, providing the energy and momentum for the photon to follow that train track. What bind the two countervailing Elementary Rays together and makes them coherent is the photon attached to them both.

TEW can explain the Bell test results based on Bi-Rays (Fig. 15-16). A pair of photons, when emitted, is already embedded in such a Bi-Ray that extends from Alice's equipment, across fiberoptic cable to Bob's equipment. The relationship $P = \cos^2(\theta_2 - \theta_1)$ is intrinsic to that Bi-Ray. Wave function collapse occurs when the pair of photons are born into that environment, which can be dozens of nanoseconds before they are observed. Wave function collapse does not occur when Alice observes a photon. Therefore Alice's equipment does not send a "signal" to Bob's equipment.

The ideas stated in the last two paragraphs, which are few in number, are the **only assumptions** that are needed for the Theory of Bi-Rays to explain the Bell test experiments!

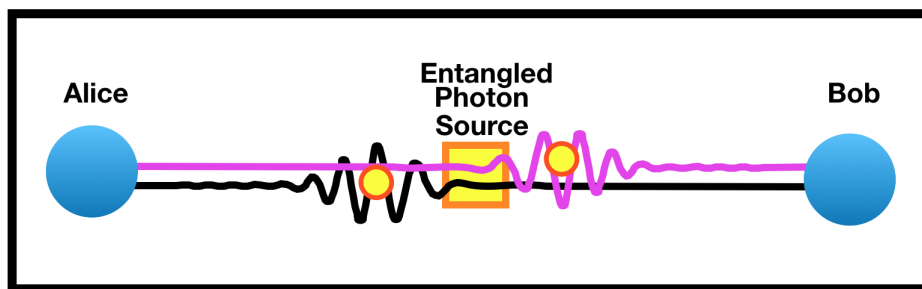


Figure 15: A Bi-Ray consists of two elementary waves traveling coaxially in opposite directions.

Here is a more detailed explanation of what we just said.

Einstein proposed in 1935 that something was missing from QM. His thought experiment was to imagine that an atom produces two particles with equal but opposite spin traveling in opposite directions. If you do an experiment on one, you learn something about the other. The term “local realism” has become a code word meaning “Einstein’s idea of how the Nature works.”

John Bell proposed in 1964 that Einstein’s thought experiment contradicted QM. Clauser, Horne, Shimony and Holt (CHSH) designed experiments using photon polarizers that would test whether Einstein or Bell was correct. Over the next fifty years experiments proved Einstein was wrong and QM was correct. Experiments that closed all loopholes that might allow Einstein’s concept of Nature to escape the trap.

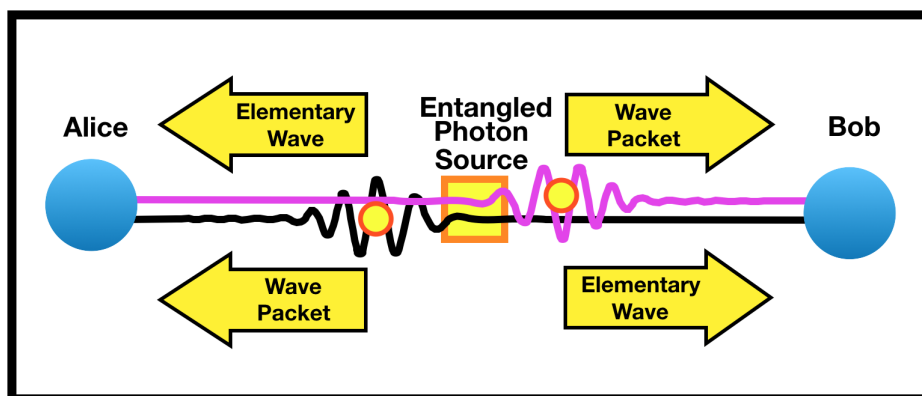


Figure 16: This is the previous Figure with yellow labels added.

From the Bell test experiments it was allegedly “proved” that the quantum world is weird. Allegedly when Alice makes a measurement, that sends a “signal” to Bob that travels faster than the speed of light. The basic assumption is that wave function collapse occurs when something is measured. Prior to measurement the photon heading toward Alice had no specific characteristics, it was in a “superposition.” Alice’s alleged signal is allegedly instantaneous. This leads to various theories of “entanglement” that involve instantaneous contact, but not transmission of information faster than the speed of light.

Although Einstein’s view of quantum reality has been defeated, that does not prove that the QM view is correct,

because there are other theories which fit the Bell test experimental data. Specifically TEW is such a theory. Bell and CHSH would classify TEW as a “nonlocal” theory. It has been known for decades that nonlocal theories can explain the Bell test experiments.

What makes TEW a “nonlocal” theory is that the same Bi-Ray stretches from Alice’s equipment, through the fiberoptic cable, across the 2-photon source, through more fiberoptic cable, and into Bob’s equipment. The environment inside that Bi-Ray is the same for Alice and for Bob. When a pair of entangled photons is born, it is born into that environment. All the components of the Bi-Ray are limited by the speed of light. Nothing is transmitted instantaneously.

6.2 Bi-Ray trigonometry

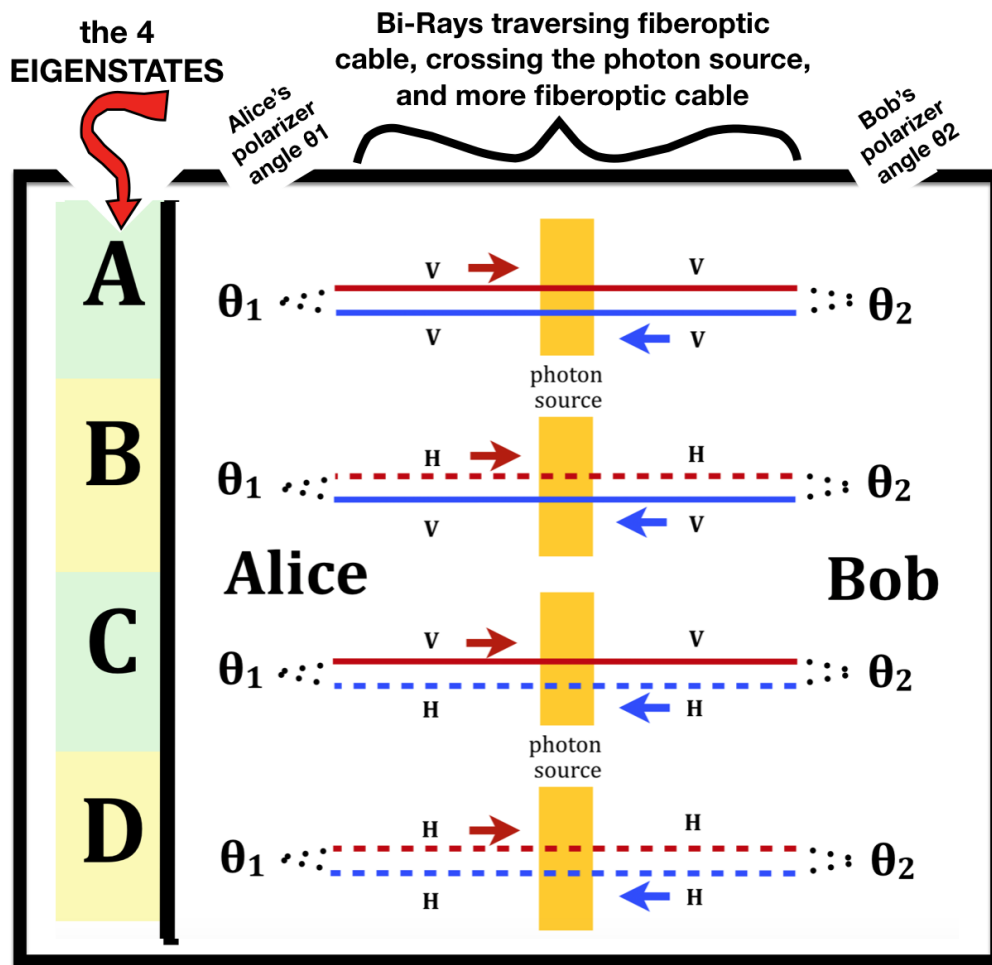


Figure 17: Each row of this diagram represents a different Eigenstate of the bi-ray. Alice and Bob look at incident photons through polarizers set at random angles θ_1 and θ_2 and record whether they do, or do not see a photon: YES / NO. At a later time a computer analyzes the data after coordinating the time of the two citings.

The key to understanding why Bi-Rays explain the Bell test experiments is that the two mono-rays relate to each other in complex ways. There are four eigenstates of the elementary rays individually (“V”=Vertical and “H”= Horizontal)

$(\vec{V}, \vec{H}, \overleftarrow{V}, \overleftarrow{H})$. We use the color red to signify that a ray is moving to the right; and blue means left. Bi-Rays are more complicated than monorays and have these four Eigenstates:

- Eigenstate A $\vec{V} \overleftarrow{V}$,
- Eigenstate B $\vec{H} \overleftarrow{V}$,
- Eigenstate C $\vec{V} \overleftarrow{H}$,
- Eigenstate D $\vec{H} \overleftarrow{H}$.

Fig. 17 shows the complicated situation that exists inside a Bell test experiment. On the left is Alice who randomly sets her polarizer to angle θ_1 . On the right is Bob who randomly sets his polarizer to angle θ_2 . Between Alice and Bob is fiberoptic cable and a 2-photon-source (see yellow rectangles). The research question is, “What is the probability of Alice and Bob both seeing a photon simultaneously?”

QM asks that same question but uses the term “coincidence rate” instead of “probability.”

Fig. 17 is divided into four layers, one for each Eigenstate of the Bi-Ray. For example, the top layer consists of Eigenstate A, which we defined as: $\vec{V} \overleftarrow{V}$. If Alice places her polarizer at angle θ_1 , what is that amplitude of a photon being visible to Alice from the Mono-Ray moving to the left with vertical polarization? Answer: $\cos(\theta_1 - V)$. What is the amount due to the Mono-Ray moving to the right? Answer: $\cos(\theta_1 - V)$. We have previously defined the probability of a photon following the Bi-Ray is the amplitude of it following one ray times the amplitude of it following the countervailing ray, or $\cos(\theta_1 - V)$ times $\cos(\theta_1 - V)$.

A law of probability is that the probability of Alice and Bob both seeing a photon is a product of the probability of each of them seeing a photon in Eigenstate A. Therefore the top layer of Fig. 17 will give us:

$$P = [\cos(\theta_1 - V)\cos(\theta_1 - V)] \text{ TIMES } [\cos(\theta_2 - V)\cos(\theta_2 - V)]$$

The probability of both people seeing a photon simultaneously is the sum of the probabilities in each of the four Eigenstates. When we turn the crank of the trigonometry machinery the trigonometry does the work for us. The probability of both Alice and Bob seeing a photon simultaneously is:

$$\begin{aligned} & \text{EigenstateA } [\cos(\theta_1 - V)\cos(\theta_1 - V)] \text{ X } [\cos(\theta_2 - V)\cos(\theta_2 - V)] \\ & \text{EigenstateB} + [\cos(\theta_1 - H)\cos(\theta_1 - V)] \text{ X } [\cos(\theta_2 - H)\cos(\theta_2 - V)] \\ & \text{EigenstateC} + [\cos(\theta_1 - V)\cos(\theta_1 - H)] \text{ X } [\cos(\theta_2 - V)\cos(\theta_2 - H)] \\ & \text{EigenstateD} + [\cos(\theta_1 - H)\cos(\theta_1 - H)] \text{ X } [\cos(\theta_2 - H)\cos(\theta_2 - H)] \end{aligned}$$

If we use polar coordinates, so the angle V is zero, and H is $\pi/2$, then we get:

$$\begin{aligned} P = & [\cos(\theta_1)\cos(\theta_1)] \text{ X } [\cos(\theta_2)\cos(\theta_2)] \\ & + [\sin(\theta_1)\cos(\theta_1)] \text{ X } [\sin(\theta_2)\cos(\theta_2)] \\ & + [\cos(\theta_1)\sin(\theta_1)] \text{ X } [\cos(\theta_2)\sin(\theta_2)] \\ & + [\sin(\theta_1)\sin(\theta_1)] \text{ X } [\sin(\theta_2)\sin(\theta_2)] \end{aligned}$$

which can be factored:

$$\begin{aligned} = & [\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)] \\ & \bullet [\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)] \end{aligned}$$

for which there is a trigonometry equation, which gives us:

$$= [\cos(\theta_2 - \theta_1)] \bullet [\cos(\theta_2 - \theta_1)] \tag{23}$$

$$= \cos^2(\theta_2 - \theta_1) \tag{24}$$

The result, $P = \cos^2(\theta_2 - \theta_1)$, is the probability of both Alice and Bob seeing a photon simultaneously. In the literature about Bell test experiments, this is called a ‘‘Coincidence Rate.’’ It is exactly the answer found by QM if the two photons are emitted with the same orientation.

Fig. 18 shows a three dimensional graph of the equation $Z = \cos^2(\theta_2 - \theta_1)$. It looks like blue ocean waves. If Alice chooses angle θ_1 , that can be graphed as the red line undulating across the waves (Fig. 18). That leaves a full range of possible values of θ_2 for Bob to choose from. Alice’s choice does not constrain or influence Bob’s choice.

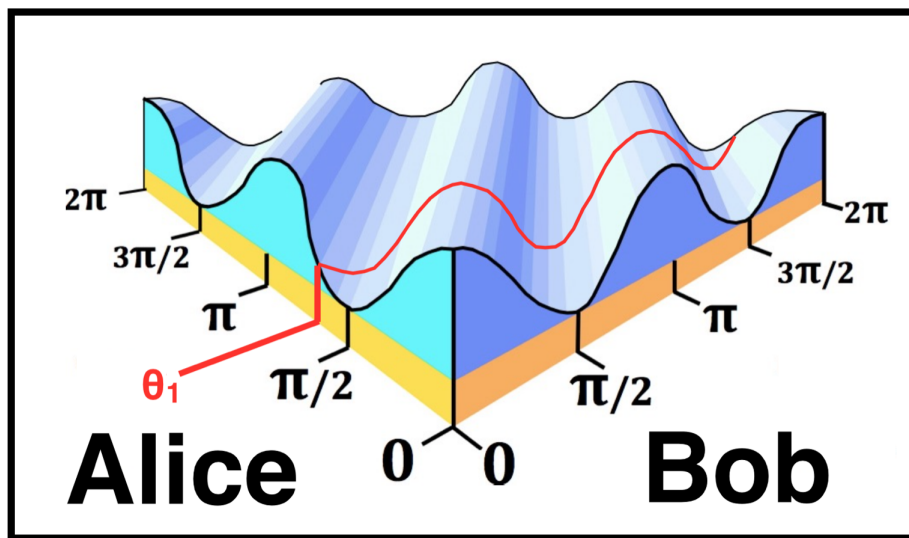


Figure 18: This graph of $Z = \cos^2(\theta_2 - \theta_1)$ has an undulating red line indicating that Alice chose angle $\theta_1 = 0.3\pi$. Bob can choose angle θ_2 at random and the height of the curve shows the probability that they each see a photon at that pair of polarizer angles. Alice’s choice places no constraints on Bob’s choice. This three dimensional graph is intrinsic to the bi-ray. When Alice makes her observation that sends no signal to Bob.

To reiterate, if Bob chooses a random angle θ_2 , when Alice has already chosen θ_1 , then the height of the graph will give the probability that they will both simultaneously see a photon at angles θ_1 and θ_2 .

The results reported so far are based on a 2-photon source that emits photons with a correlated polarization. For example, the famous Aspect, Dalibard and Roger experiment of 1982 used a calcium-40 source that produced two photons with correlated polarization and obtained similar results as ours.

There would be different results if the two photons were orthogonal to one another at birth. That would happen for example if the pair of photons was produced by a Wollaston prism. Then the final probability would be $Z = \sin^2(\theta_2 - \theta_1)$.

6.3 What do the Bell test experiments tell us about the quantum world?

In experiment after experiment QM has asserted that the quantum world is weird. Yet when we change to the TEW mathematics we find that the quantum world is not weird. The quantum world is very similar to the classical world of everyday experience. Our view is that the world of everyday experience is like a plate glass window. When you look at the world, what you see is the quantum world, staring right back at you! You don't see the world of Alice in Wonderland.

In the Bell test experiments QM asserts that wave function collapse occurs when something is measured. Thus when Alice observes her photon at angle θ_1 , that reality comes into existence. Prior to her observation that photon had no specific characteristics or attributes. When Bob observes his photon at angle θ_2 that reality also comes into existence. So how quickly would we expect there to be an equation showing that the correlation rate of Alice and Bob's observation is $\cos^2(\theta_2 - \theta_1)$?

From the viewpoint of QM it is astonishing that this result emerges instantaneously, without time for a signal from Alice's equipment to reach Bob at the speed of light. That is why they conclude that instantaneous communication has occurred.

When we shift to the TEW mathematics, things look entirely different. The fact that the correlation rate between Alice's data and Bob's data is $P = \cos^2(\theta_2 - \theta_1)$ is what we would expect, once we know that there is a Bi-Ray stretching from Alice to Bob. Nothing else could happen, unless you change the 2-photon source to a source which emits photons orthogonal to one another. In TEW nothing travels faster than the speed of light, and wave function collapse occurs **before** you measure something. Wave function collapse happens when a pair of photons is born into that bi-ray environment instead of being born into another bi-ray environment.

There is a different way of defining the words "local" and "non-local." If Alice is performing any experiment in her laboratory, can she ignore the non-local affect of the Andromeda galaxy? With the TEW definition of how a Bell test experiment works, Alice's experiment is only affected by things that are within a light-sphere around her experiment. She can ignore the effect of the Andromeda galaxy on her local experiment.

We demonstrated in an earlier publication that TEW is able to explain quantum computers.[Boyd 2019c]

7 Quantum Electro-Dynamic (QED)

There is a famous myth about Richard Feynman. Allegedly when he was a student the professor taught the conventional view of the double slit experiment. Feynman asked what happens if you drill a third hole in the barrier screen, and then a fourth, and you add a second barrier screen and then a third. You keep drilling holes and adding barrier screens until there is nothing but empty space between the particle gun and the detector. By this method Feynman has introduced his path integral approach, in which each particle has an infinite number of paths it can take between the gun and the detector.[O'Dowd 2017]

We will use this vignette in two ways. It will become the central focus of our discussion about why TEW provides a better way approach to the path integral mathematics than Feynman's ideas. Basically Feynman thought of paths going from the gun to the detector, but when he adds up the amplitudes of all those paths the final total is self-contradictory. The precision of QED requires that all possible pathways be included, but what makes no sense is that any particle takes only one pathway, not an infinite number of pathways.

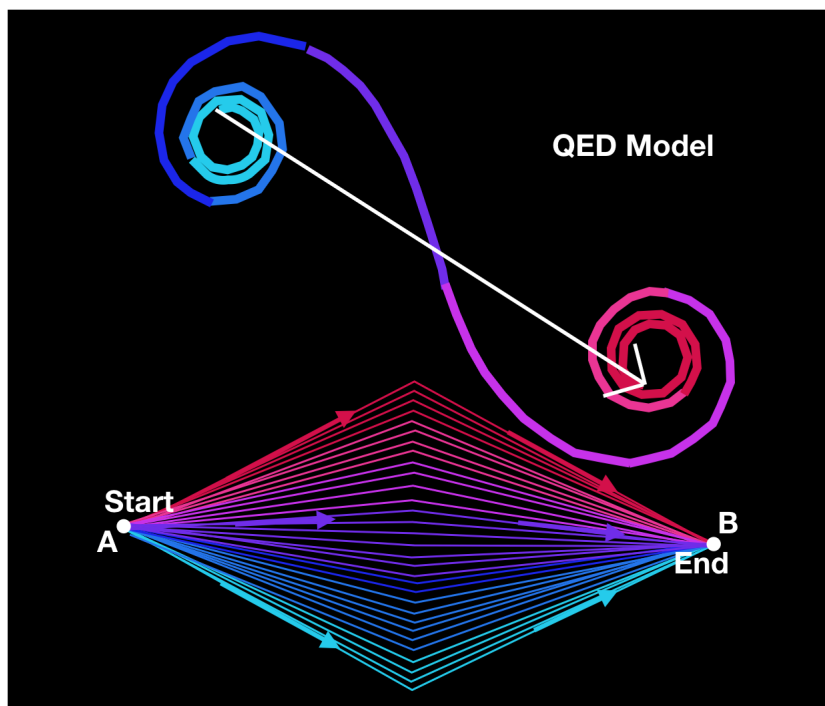


Figure 19: If photons travels from point A to B, as shown in the bottom part of this diagram, they could take many different paths (which are color coded). The top of this diagram shows the total probability amplitude in Hilbert space, after adding up all the vectors for each path. The vertical axis is real; the horizontal axis is imaginary. (From Matthew O'Dowd, who gives credit to Jan Dowd)

TEW uses the exact same paths but diagrams them as going from the detector to the gun. At the gun the particle selects only one pathway to follow backwards to the detector, which gets rid of the quantum weirdness. When we analyze the entire model, TEW is able to explain it, but Feynman cannot. That is what we will seek to prove in this section. It illustrates our theme that TEW banishes quantum weirdness.[Boyd 2015b, 2015g; Feynman 1985]

The other idea we will harvest from Feynman's path integral approach is that we will develop a far more robust picture of Elementary Waves than we had before.

Fig. 18 shows the Feynman method for calculating the total amplitude of a particle traveling from point A to B, along the different paths shown in the bottom of the diagram. We think of a photon as taking the straight path from A to B, but Feynman considered ALL paths (in Fig. 19 notice all the angled paths of different colors from red to aqua blue). We will assign a probability amplitude vector to each path, and add up all the vectors (placing them end to end to produce the curve at the top of the diagram).

The white arrow is the total amplitude for a photon going from A to B. The vertical axis is the real amplitude and the horizontal is imaginary. It is the length of the white arrow that is important. When we add all the vectors together (in Hilbert space, top) we find that the middle paths contribute most to the total amplitude, whereas the weird, indirect paths end up going around in circles in Hilbert space and cancelling each other out. This is shown in the spirals at each end of the curve at the top of the diagram.

The take home message from Fig. 19 is that Feynman may be interested in an infinite number of paths from A to

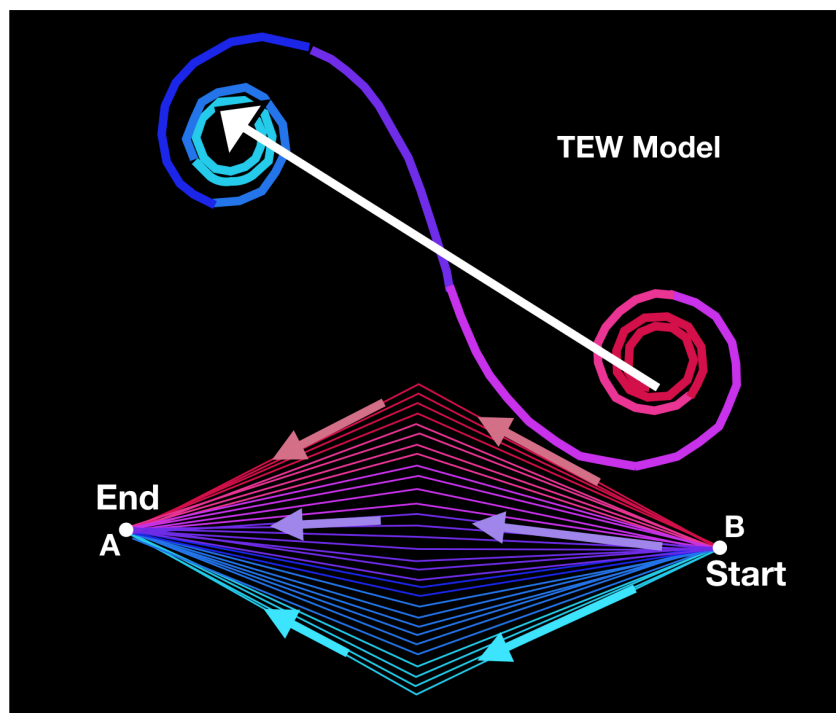


Figure 20: If you wonder what this article is saying, this diagram is a perfect illustration. TEW reverses the direction of the previous Figure. If Elementary Waves travel from point B to A, as shown here, they could take many different paths (which are color coded). The top of this diagram shows the total probability amplitude (the white arrow) in Hilbert space. After the Elementary Waves arrive at point A, the photon chooses one \mathcal{A} at random, which triggers from inside that \mathcal{A} the emergence of one single Schrödinger wave packet, which will carry the photon backwards along its specific \mathcal{A} pathway to point B, which is the detector. This results in the same math as QED, because the total amplitude = length of white arrow which is the same length as the white arrow in Fig. 19. But TEW gets rid of the weirdness, because the **photon only takes one path, not an infinite number of paths.**

B, some of which are very circuitous and indirect, but the weird paths have amplitude vectors that point in opposite directions, so they cancel each other out. The total amplitude from A to B is primarily determined by the straight or almost straight paths (shown in the bottom of Figure 18) as we would expect.[Tong 2017]

Fig. 20 shows the same thing in the reverse direction. An infinity of \mathcal{A} travel from B to A, which explains why the Feynman math integrates across all possible pathways. **However the TEW model gets rid of quantum weirdness because the photon only takes ONE of those paths from A to B.**

7.1 Feynman diagrams

Feynman diagrams are an elegant way of dealing with particle physics. In Figure 19 two electrons go in toward the center, and two electrons come out. There is an exchange of energy between the electrons. This diagram shows one of the ways in which that could happen. The photon γ in the center of this diagram is called a “virtual particle” which has questionable existence. We never see it and cannot measure it. The only parts of this diagram which are visible to our detectors are the periphery: two electrons going in and two electrons coming out. What happens in the middle is

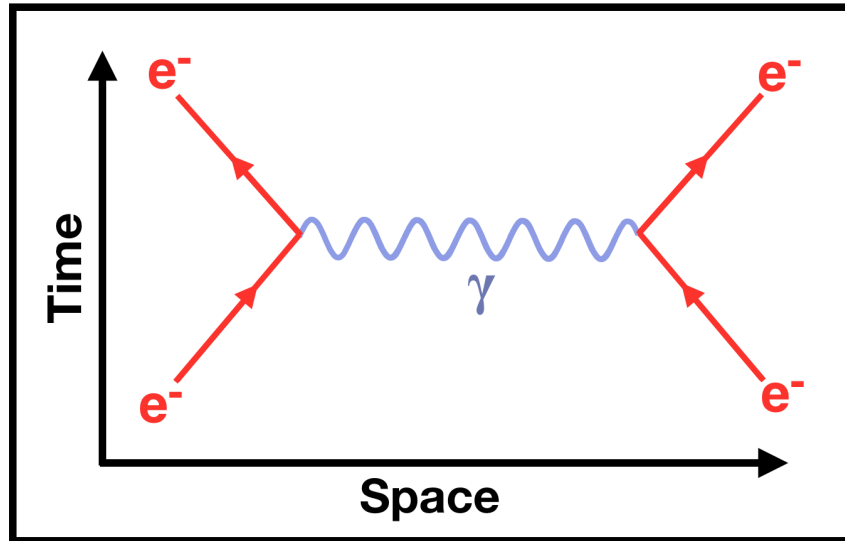


Figure 21: Feynman diagram with two vertices. On the lower left an electron comes in, emits a photon γ and a lower energy electron leaves to the upper left. The photon γ is then absorbed on the right by another electron, and a higher energy electron exits in the upper right.

shrouded in mystery. Feynman’s approach is to speculate about what happens in the center, and his speculation takes the form of an infinite number of Feynman diagrams.

This Feynman diagram is a pictorial way of symbolizing some of the complex equations of these reactions. In this case the equation is:

$$\text{Probability Amplitude} = \bar{u}_1 i e \gamma^\mu u_1 \frac{-i g_{\mu\nu}}{p^2} \bar{u}_2 i e \gamma^\nu u_2 \tag{25}$$

Figs. 22 and 23 show how the Feynman diagram and the equation are related. The Feynman diagrams provide an elegant and simple way of cataloging intricate equations.

Figs. 21 and 24 are only two of an infinite number of ways these two electrons could interact. Feynman’s method for calculating the total probability amplitude is to figure out all possible Feynman diagrams and then add together all the amplitudes. At first the task sounds impossible. But it turns out that there is only one kind of vertex in such a diagram, and that when you count the number of vertices, you determine the order of magnitude contribution. For example, Fig. 21 has two vertices. The next more intricate Feynman diagram would need to have four vertices, and for each vertex you reduce the magnitude of the amplitude contribution by 1/100. Thus a Feynman diagram with four vertices would contribute 0.0001 as much to the total probability amplitude, compared with the contribution from a Feynman diagram with one vertex.

7.2 TEW and Feynman diagrams

How does QED approach these Feynman diagrams? We claim that in the 1920’s QM made a mistake (SEE FIGURE 4) by not listening to Einstein’s criticism of the conventional view of the double slit experiment. Therefore we need to reject wave-particle duality. Furthermore, we describe the zero energy waves as having a complicated structure. Each

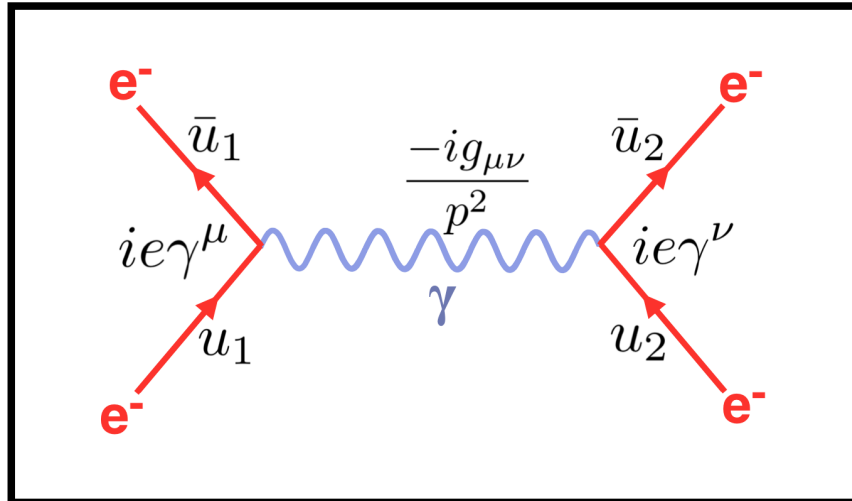


Figure 22: The Feynman diagram is actually a shorthand for organizing complicated equations. There are symbols ($\bar{u}_1, u_1, \bar{u}_2,$ and u_2) marking the ingoing and outgoing electrons, equations for the vertices, and an equation for the photon in the center.

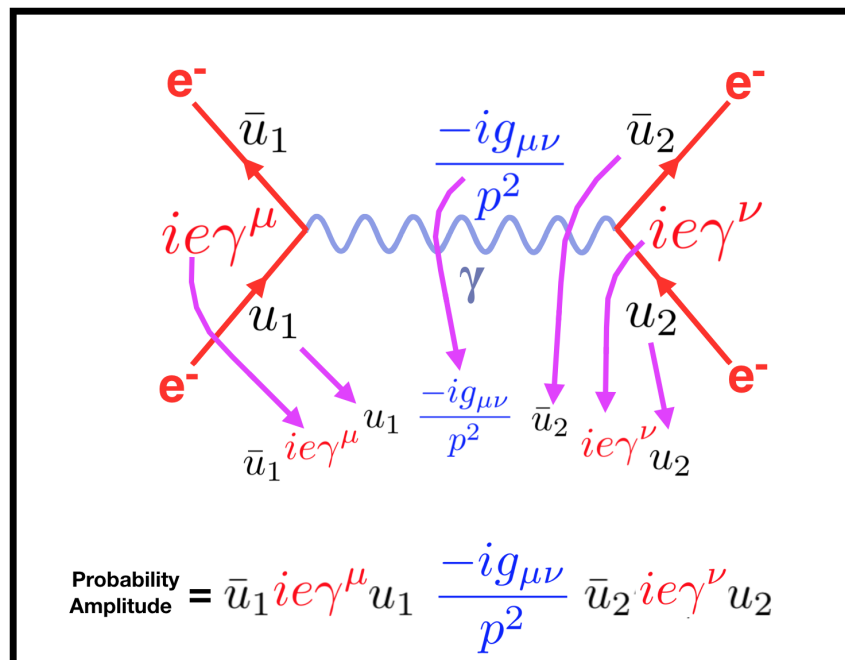


Figure 23: This shows how the symbolism of a Feynman diagram is reduced to a single equation at the bottom.

Elementary Ray \mathcal{A} is traveling in a specific direction, and when it encounters a particle there may be a Schrödinger wave packet that pops out from inside, and carries the particle in the opposite direction as the plane wave.

This makes it difficult to graph the direction of an Elementary Ray. It is somewhat like trying to graph a fish hook, which has a main shaft traveling in the same direction as the fishing line, and in the opposite direction as a fish. Then

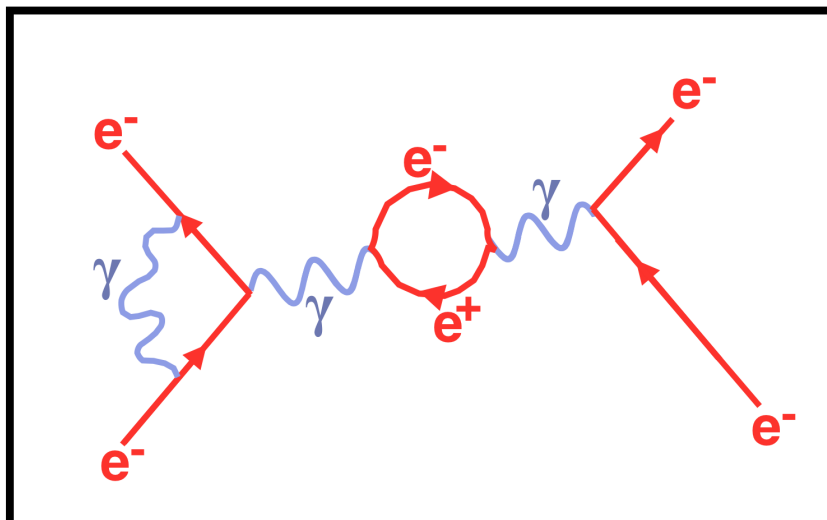


Figure 24: A more complex Feynman diagram for the same reaction. This diagram has six vertices, and therefore would not contribute much to the total probability amplitude.

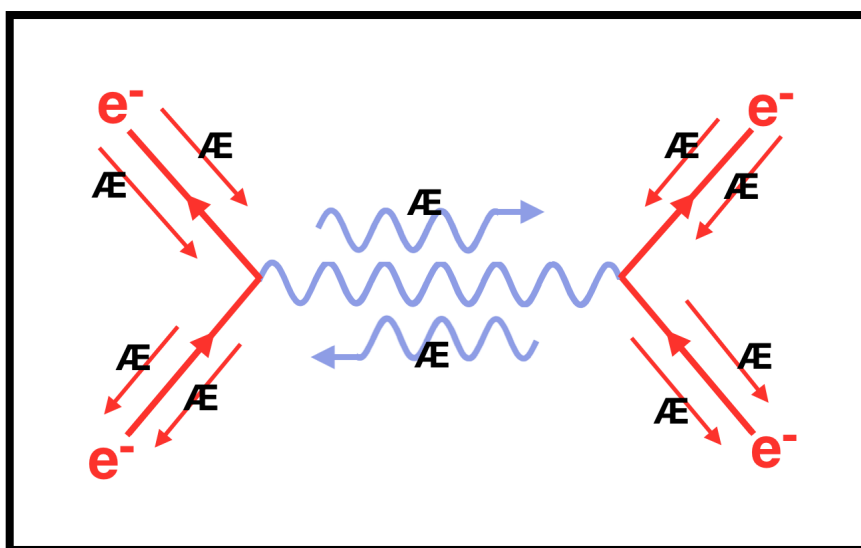


Figure 25: This shows what is happening with Elementary Waves $\mathcal{A}E$ vis-à-vis the Feynman diagram. Each particle is following an Elementary Ray backwards. Here there are two kinds of Elementary Rays, one for electrons and the other for photons.

there is a “J” which reverses direction and snags the fish. Similarly, under ideal circumstances, an $\mathcal{A}E$ snags a particle and the particle travels toward the fishing pole.

To simplify our drawings we will graph each $\mathcal{A}E$ as traveling in a direction opposite that of a particle. We will ignore the “J” part of the fish hook. Our Feynman diagrams will show only the main shaft of the fish hook. Fig. 25 shows how Elementary Rays are traveling relative to the original Feynman diagram (compare Figs. 21 and 25).

According to Fig. 25 there are an infinite number of \mathcal{A} traveling in all directions, and particles such as electrons and photons follow their type of \mathcal{A} backwards. We learn from Quantum Field Theory (QFT) that there are 17 different kinds of \mathcal{A} , one for each particle in the Standard Model of particle physics. TEW proposes that the Elementary Waves form seventeen overlapping fields. Our fields however differ from those of QFT, because we reject wave-particle duality. Thus our fields may vibrate as do the fields of QFT, but our fields lack energy and momentum. In our model, as in QFT, particles often pop out of and into existence, transferring their mass-energy to one another.

7.3 Why TEW is better than QED

We have sketched out the bare bones of Feynman's Quantum Electro-Dynamics (QED) from a TEW point of view. If we adopt all of Feynman's work but reverse the direction of integration, we would end up with the same absolute value of the total amplitude. We claim that TEW commutes with QED in the following respect:

$$\left| \int_A^B \phi(x) dx \right| = \left| \int_B^A \phi(x) dx \right| \quad (26)$$

Eq. 26 means that when we integrate over a Feynman path from A to B, we would get the same absolute value if we switched from QED to TEW and integrated over exactly the same path in the opposite direction. This is illustrated by the length of the two white arrows in Figs. 19 and 20. They are the same length whether you integrate from left to right, or right to left. Therefore the accuracy of QED should carry over to TEW.

QED is the most accurate science that humans have ever had. Here are several examples.

- Experiments have determined that the magnetic moment of an electron is 1.00115965221. According to QED the predicted value is 1.00115965246. How accurate is that? If you measured the distance from Los Angeles to New York City with this accuracy, it would be accurate to within the width of a human hair.
- QED predicts the fine structure constant of the anomalous magnetic dipole moment to an accuracy of within ten parts in a billion (10^{-8}).
- QED predicts the Hall effect to be $\alpha^{-1} = 137.0359979$, whereas in experiments it is $\alpha^{-1} = 137.0359970$.

As noted earlier, this astonishing accuracy of QED is achieved by using computers to add up probability amplitudes across thousands of different Feynman diagrams. You only achieve so much accuracy by having a vast number of different paths and then obtaining the total probability amplitude by adding together the amplitudes of each of those paths. This fact is pivotal when we argue that TEW is better than QED.

Consider Fig. 19. Suppose we do an experiment in which we send exactly one particle from A to B. QED cannot explain how that one particle took an infinite number of different paths from A to B. Yet the accuracy of QED requires that the one particle took every one of the infinite paths. It is a glaring contradiction, and therefore is an example of the quantum weirdness that TEW is designed to banish.

TEW can rescue QED from the dilemma. According to TEW the Elementary Waves are infinite in number and take every possible path from B to A (see Fig. 20). This is exactly like the double slit experiment. Every point on the target screen sends \mathcal{A} 's to the electron gun. But the electron selects **only one** of the incident paths. When the particle does that, the decision triggers a Schrödinger wave packet to pop up from inside the plane wave. The wave packet will carry

the particle back to the detector from which the \mathcal{A} arises. Therefore according to TEW there are an infinite number of paths (each path being an \mathcal{A}), but only one Schrödinger wave packet.

Therefore in the TEW model there are indeed an infinite number of possible pathways, but the particle only uses one of them to go from A to B. This is what we meant when we said that TEW can explain QED experiments, but Feynman cannot explain them.

7.4 Unfinished work

Since we claim that QFT should be assimilated into TEW, therefore we are left with a mess to clean up. QFT assumes that the different fields represent wave particles. The Feynman diagrams in Figs. 19 to 21 are said to represent vibrations in the electron field causing vibrations in the photon field, which then transfers its energy back to the electron field.

TEW however denies the existence of wave particle duality. How can we assimilate QFT, given the disagreement about that duality? Both QED and QFT would need to be reorganized or tweaked in order to accommodate these issues. The need is to fit a square peg (QFT) into a round hole (TEW).

Another piece of unfinished business is the idea of time reversal, which Feynman is so fond of. We have a resource that was not available to Feynman, which is Elementary Waves that go forward in time but backwards relative to what people expect. The “reverse waves” of TEW are NOT time reverse waves.

As we seek to put the square peg into the round hole, we hope that by using \mathcal{A} we can eliminate Feynman’s need for time reversal in QED.

8 Conclusions

This article presents a new way of thinking about the mathematics of a Schrödinger wave packet and QFT. Specifically we view that wave packet as part of the larger context of a zero energy plane wave $\left[\Psi(x) = e^{-i(kx-\omega t)} \right]$ traveling in the opposite direction. Almost always there is a plane wave with no Schrödinger wave packet.

The wave packet only pops out from inside the plane wave if it is triggered by the right conditions, usually a collision with a particle. The wave packet is like a backwash from that collision. If a wave packet does emerge from inside the plane wave, it will carry the particle in the direction opposite that of the plane wave. That combination of a plane wave carrying inside it the trigger for a Schrödinger wave to emerge and move in the opposite direction, is called an Elementary Wave, symbolized by the letter \mathcal{A} .

Our proposal is simple. We claim that re-thinking the Schrödinger wave packet and QED along these lines is a relatively small change in our mathematics. We admit that it is counterintuitive. Surprisingly, this small change in how we approach the Schrödinger wave packet and Feynman diagrams produce vast changes in how we interpret the results of quantum experiments. A small twist to the mathematics vastly simplifies how Nature appears to operate. When a mathematician is given a simple model and a more complex model, usually the simpler model is preferred.

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a corresponding mathematics.

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Author Biography and Photo:



Figure 26: **Jeffrey H. Boyd**. This photo is from a YouTube video about the double slit experiment, a video of 18 minutes duration, which is the length of a Ted Talk: https://www.youtube.com/watch?v=_k9aDgDYUco&feature=youtu.be

The author was born in 1943 into the family of a factory worker in New Jersey, USA. His mother was a housewife, devoted to her children. In his childhood no one ever read books to Boyd, nor did they encourage him to read or ponder mathematics. Boyd went to public schools with 32 students per teacher and no teacher's aids. After his father was fired from his job, Boyd helped his father build houses, **digging out two basements by hand, with a pick, shovel and wheelbarrow**. It is from that grueling experience that Boyd learned that if you persist at something long enough, you might prevail, even if it means moving tons of dirt, rock and stumps by hand. Boyd is the first member of his family to graduate from college. In elementary school he and his family were astonished to discover that he had a talent in mathematics. He subsequently graduated with advanced degrees from Harvard, Yale, Brown and Case Western Reserve Universities, was ordained in the Episcopal church, and spent a decade on the faculty of the National Institutes of Health in Bethesda, Maryland. In medicine his passion was treating indigent patients with

severe chronic illnesses. He is now retired. His videos can be found at his website, *ElementaryWave.com*. The story of Boyd's involvement with TEW and his relationship with Lewis E. Little, is described elsewhere [Boyd 2020a]. Boyd feels "called" to work on TEW, which means that TEW provides him with a sense of purpose. The message of this article has been simplified, presented in the 18 minutes of a Ted Talk, and boiled down to lay people's terms, with no equations. That video can be found on YouTube, with the title "Mathematics of Hope Despite COVID-19". That video and this article were developed together as two formats for the same message.

References

- [1] Arndt, M., O. Nairz, J. Vos-Andreae, et. al. (1999) Wave-particle duality of C_{60} molecules, *Nature*, 401, 680-682. <https://doi.org/10.1038/44348>
- [2] Aspect, A., P. Grangier, and G. Roger. (1981) Experimental tests of realistic local theories via Bell's theorem. *Physical Review Letters*, 47, 460-463. <https://doi.org/10.1103/PhysRevLett.47.460>
- [3] Aspect, A., P. Grangier, and G. Roger. (1982) Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment. *Physical Review Letters*, 49, 91-94. <https://doi.org/10.1103/PhysRevLett.49.91>
- [4] Aspect, A., J. Dalibard, and G. Roger. (1982) Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters*, 49, 1804-1807. <https://doi.org/10.1103/PhysRevLett.49.1804>
- [5] Baggott, J. (2011) *The Quantum Story: a history in 40 moments*, Oxford University Press. ISBN: 978-0-19-956684-6
- [6] Becker, A. (2018) *What Is Real?* (Basic Books). ISBN: 978-1-5416-9897-0
- [7] Bell, J. S. (1981) Bertlmann's socks and the nature of reality *Journal de Physique*, 42, C2-C41. <https://doi.org/10.1051/jphyscol:1981202>
- [8] Bell, J. S. (1964) On the Einstein Podolsky Rosen paradox. *Physics* 1, 195-200. <https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195>
- [9] Boyd, J. H. (2020a) New Schrödinger wave mathematics changes experiments from saying there is, to denying there is quantum weirdness. *Journal of Advances in Mathematics* 18, 82-117. <https://doi.org/10.24297/jam.v18i.8656>
- [10] Boyd, J. H. (2020b) New Schrödinger wave math changes experiments so they deny there is quantum weirdness. https://www.youtube.com/watch?v=_k9aDgDYUco&feature=youtu.be (access date 2020-02-24).
- [11] Boyd, J. H. (2020c) A mathematical explanation for the double slit experiment of quantum mechanics. <https://www.youtube.com/watch?v=O9dpDcF6Uhs> (access date 2020-02-24).
- [12] Boyd, J. H. (2019a) Decrypting the central mystery of quantum mathematics: Part 1. New axioms explain the double slit experiment. *Journal of Advances in Mathematics* 17, 255-282. <https://doi.org/10.24297/jam.v17i0.8475>
- [13] Boyd, J. H. (2019b) Decrypting the Central Mystery of Quantum Mathematics: Part 2. A mountain of empirical data supports TEW. *Journal of Advances in Mathematics* 17, 283-314. <https://doi.org/10.24297/jam.v17i0.8489>
- [14] Boyd, J. H. (2019c) Decrypting the central mystery of quantum mathematics: Part 3. A non-Einstein, non-QM view of Bell test experiments. *Journal of Advances in Mathematics* 17, 315-331. <https://doi.org/10.24297/jam.v17i0.8490>

- [15] Boyd, J. H. (2019d) Decrypting the central mystery of quantum mathematics: Part 4. In what medium do Elementary Waves travel? *Journal of Advances in Mathematics* 17, 332-351. <https://doi.org/10.24297/jam.v17i0.8491>
- [16] Boyd, J. H. (2018a) The von Neumann and double slit paradoxes lead to a new Schrödinger wave mathematics. *Journal of Advances in Physics* 14, 5812-5834. <https://doi.org/10.24297/jap.v14i3.7820>
- [17] Boyd, J. H. (2018b) The quantum world is astonishingly similar to our world: The timing of wave function collapse according to the Theory of Elementary Waves. *Journal of Advances in Physics* 14, 5598-5610. DOI: 10.24297/jap.v14i2.7555
- [18] Boyd, J. H. (2017) A symmetry hidden at the center of quantum mathematics causes a disconnect between quantum math and quantum mechanics. *Journal of Advances in Mathematics*, 13, 7379-86. DOI: 10.24297/jam.v13i4.6413.
- [19] Boyd, J. H. (2017) Paul Dirac's view of the Theory of Elementary Waves. *Journal of Advances in Physics* 13, 4731-4734. DOI: <https://doi.org/10.24297/jap.v13i3.5921>
- [20] Boyd, J. H. (2017) The Boyd Conjecture. *Journal of Advances in Physics* 13, 4830-37. <https://doi.org/10.24297/jap.v13i4.6038> (access date 2020-02-27)
- [21] Boyd, J. H. (2015a) A paradigm shift in mathematical physics, Part 4: Quantum computers and the local realism of all 4 Bell states. *Journal of Advances in Mathematics*, 11, 5476-5493. <https://doi.org/10.24297/jam.v11i7.1224>
- [22] Boyd, J. H. (2015b) A paradigm shift in mathematical physics, Part 3: A mirror image of Feynman's quantum electrodynamics (QED). *Journal of Advances in Mathematics*, 11, 3977-3991. DOI: <https://doi.org/10.24297/jam.v11i2.1283>.
- [23] Boyd, J. H. (2015c) A paradigm shift in mathematical physics, Part 2: A new local realism explains Bell test & other experiments. *Journal of Advances in Mathematics*, 10, 3828-3839. DOI: <https://doi.org/10.24297/jam.v10i9.1884>.
- [24] Boyd, J. H. (2015d) A paradigm shift in mathematical physics, Part 1: The Theory of Elementary Waves (TEW). *Journal of Advances in Mathematics* 10, 3828-3839. <http://cirworld.com/journals/index.php/jam/article/view/4719>. (access date 2020-02-27)
- [25] Boyd, J. H. (2015e) The Theory of Elementary Waves eliminates Wave Particle Duality. *Journal of Advances in Physics* 7, 1916-1922. <https://www.rajpub.com/index.php/jap/article/view/2279>. (access date 2020-02-27)
- [26] Boyd, J. H. (2015f) A new variety of local realism explains a Bell test experiment: the Theory of Elementary Waves (TEW) with no hidden variables'. *Journal of Advances in Physics* 8, 2051-58. <https://www.semanticscholar.org/paper/A-new-variety-of-local-realism-explains-a-Bell-test-Boyd/445009d95dd80180537216f953dbf4d4ddc8af7d>. (access date 2020-02-27)
- [27] Boyd, J. H. (2015g) A proposed physical analog of a quantum amplitude: Corkscrew model from the Theory of Elementary Waves (TEW). *Journal of Advances in Physics* 10, 2774-2783. <https://rajpub.com/index.php/jap/article/view/1324> (access date 2020-02-27)
- [28] Boyd, J. H. (2013a) Re-thinking a delayed choice quantum eraser experiment: a simple baseball model. *Physics Essays*, 26, 100-109. DOI: 10.4006/0836-1398-26.1.100.
- [29] Boyd, J. H. (2013b) Re-thinking Alain Aspect's 1982 Bell test experiment with delayed choice. *Physics Essays*, 26, 582-591. <https://doi.org/10.4006/0836-1398-26.4.582>

- [30] Boyd, J. H. (2012) Rethinking a Wheeler delayed choice gedanken experiment. *Physics Essays* 25, 390-396. <https://doi.org/10.4006/0836-1398-25.3.390>
- [31] Cirel'son, B. S. (1980) Quantum generalizations of Bell's inequality *Letters in Mathematical Physics* 4, 93-100. <https://doi.org/10.1007/BF00417500>
- [32] Clauser, J. F., M. A. Horne, A. Shimony and R. A. Holt. (1969) Proposed experiment to test local hidden-variable theories. *Physical Review Letters* 23, 880-884. <https://doi.org/10.1103/PhysRevLett.23.880>
- [33] Davisson, C. J. and L. Germer (1927) Reflection of electrons by a crystal of nickel *Nature*, 119 558-560. <https://doi.org/10.1038/119558a0>
- [34] Davisson, C.J.,(1928a) The diffraction of electrons by a crystal of nickel *Bell System Technical Journal* 7 90-105. <https://doi.org/10.1002/j.1538-7305.1928.tb00342.x>
- [35] Davisson, C. J. (1928b) Are Electrons Waves? *Franklin Institute Journal* 205, 597. [https://doi.org/10.1016/S0016-0032\(28\)90979-5](https://doi.org/10.1016/S0016-0032(28)90979-5)
- [36] Einstein, A., B. Podolsky, and N. Rosen. (1935) Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47, 777-780. <https://doi.org/10.1103/PhysRev.47.777>
- [37] Feynman, R. P. (2010) *Feynman Lectures on Physics, vol. 3* (Basic Books) ISBN-13: 978-0465025015, see pages I-1 to I-11 discussion of the double slit experiment as the central mystery of QM.
- [38] Feynman, R. P. (1985) *QED: The Strange Theory of Light and Matter* (Princeton University Press). ISBN 978-0-691-12575-6
- [39] Freire, O. (2003) Interview with Dr. Franco Selleri. *American Institute of Physics: Oral History Interviews*. <https://www.aip.org/history-programs/niels-bohr-library/oral-histories/28003-1>. and <https://www.aip.org/history-programs/niels-bohr-library/oral-histories/28003-2>. (accessed July 15, 2019)
- [40] Giustina, M., A. Mech, S. Ramelow, et. al. (2013) Bell violation using entangled photons without the fair-sampling assumption. *Nature* 497, 217-230. doi:10.1038/nature12012
- [41] Goy, G., J. M. Raimond, M. Gross, et.al. (1983) Observation of cavity enhanced single-atom spontaneous emission *Physical Review Letters*, 50, 1903-1906. <https://doi.org/10.1103/PhysRevLett.50.1903>
- [42] Haroche, S. and D. Kleppner, (1989). Cavity Quantum Electrodynamics, *Physics Today*, 42, 24-30. <https://doi.org/10.1063/1.881201>
- [43] Hensen, B., H. Bernien, A. E. Dréau, et.al. (2015) Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres *Nature* 526, 682 - 686. doi: 10.1038/nature15759.
- [44] Hulet, R. G., E. S. Hilfer, and D. Kleppner (1985) Inhibited spontaneous emission by a Rydberg atom *Physical Review Letters*, 55, 2137-2140. <https://doi.org/10.1103/PhysRevLett.55.2137>
- [45] Khalili, A. (2013) Double slit experiment explained. Royal Institution. <https://www.youtube.com/watch?v=A9tKncAdlHQ> (access date 2020-02-26).
- [46] Kim, Y., R. Yu, S.P. Kulik, Y.H. Shih, Marlan O. Scully. (2000) A delayed choice quantum eraser. *Physical Review Letters* 84, 1-5. DOI: 10.1103/PhysRevLett.84.1

- [47] Little, L. E. (1996) Theory of Elementary Waves. *Physics Essays* 9, 100-134. <https://doi.org/10.4006/1.3029212>
- [48] Little, L. E. (2000) Theory of Elementary Waves. Lecture at the Jet Propulsion Labs. Go to YouTube and search for “Lewis E. Little JPL” (access date Feb 17, 2020).
- [49] Little, L. E. (2009) *Theory of Elementary Waves*. New Classics Library, New York. ISBN: 978-161694-032-1
- [50] O’Dowd, M. (2017) PBS Space Time, Quantum Field Theory playlist. <https://www.youtube.com/playlist?list=PLsPUh22kYmNBpDZPejCHGzxyfgitj26w9> (access date 2020-02-24).
- [51] Pfleeger, R. L. and L. Mandel. (1967) Interference of independent photon beams. *Physical Review*, 159,1084-1088. <https://doi.org/10.1103/PhysRev.159.1084>
- [52] Pfleeger, R. L. and L. Mandel. (1968) Further experiments on interference of independent photon beams at low light levels. *Journal of the Optical Society of America*, 58, 946-950. <https://doi.org/10.1364/JOSA.58.000946>
- [53] Popper, K. R. (1982) *Quantum Theory and the Schism in Physics* (Routledge), ISBN: 0-415-09112-8.
- [54] Purcell, E. M. (1946) Spontaneous emission probabilities at radio frequencies *Physical Review*, 69, 681.
- [55] Scully, M. O. and K. Drühl. (1982) Quantum eraser: A proposed photon correlation experiment and ‘delayed choice’ in quantum mechanics. *Physical Review A*, 25, 2208-2213. <https://doi.org/10.1103/PhysRevA.25.2208>
- [56] Selleri, F. (1982). On the direct observability of quantum waves *Foundations of Physics* 12, 1087-1112. <https://doi.org/10.1007/BF01300548>
- [57] Selleri, F. (2002) *Lezioni Di Relatività*, (Bari, Italy: Progedit), ISBN 88-88550-33-X. Translated by Dominick Scaramuzzino, 2016-2017, Bethany, CT, USA.
- [58] Tong, D. (2017) “Quantum Fields,” The Royal Institution, https://www.youtube.com/watch?v=zNVQfWC_evq (access date 2020-02-24).