The Von Neumann and Double Slit Paradoxes Lead to a New Schrödinger Wave Mathematics

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Abstract

John von Neumann states a paradox. Why does measuring something disrupt the smooth Schrödinger wave, causing it to collapse for no mathematical reason? This paradox is embedded in the double slit experiment. When a dot appears on the target screen, how does that cause the Schrödinger wave to collapse everywhere else, faster than the speed of light? Von Neumann didn’t follow his mathematics to its logical conclusion. If wave function collapse irreversibly changes reality, then the math is telling us that the timing and location of that event cannot be at the target screen. An event fitting that description happens only once: at the gun. A gunshot CAN change history. We propose a new mathematics of Schrödinger waves. Zero energy waves from the target screen pass backwards through the double slits and impinge on the gun prior to the gun firing. A particle randomly chooses one to follow backwards. The particle’s choice of wave is proportional to the amplitude squared of that wave at the gun, determined by the superposition of the two waves moving backwards through the two slits. Why follow a wave of zero energy? Because Schrödinger waves convey amplitudes determining the probability density of that path.

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Type (Method/Approach): This is the seventh in a series of articles published in JAP on the subject of the Theory of Elementary Waves. Elementary waves are that part of physical nature where quantum equations live. Those equations provide a roadmap to the world of elementary waves, but unfortunately the map is written in hieroglyphs The double slit experiment provides a Rosetta stone, allowing us to begin to decipher the map.

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Introduction

In his book, *Mathematical Foundations of Quantum Mechanics*, John von Neumann speaks of a discontinuous transition of a time dependent Schrödinger equation from $\psi$ into one of the eigenstates $\phi_1, \phi_2, \phi_3, \ldots$ [1]. This abrupt jump is not described by the Schrödinger equation, nor is there any explanation of how this jump occurs. This collapse of $\psi$ is caused by measurement, von Neumann says, and such measurement is irreversible. Something in the physical world changes permanently when the wave function collapses. How and why a subjective observation would cause $\psi$ to collapse is a paradox that he cannot solve. This dependence on the observer introduces a troublesome subjectivity into the natural world [1].

The double slit experiment is a superb laboratory within which to grapple with von Neumann’s paradox. Einstein pointed out that QM has no explanation for how and why the appearance of a single dot on a target screen causes collapse of the wave function everywhere else instantaneously, faster than the speed of light. If the wave function did not collapse instantaneously, then some other part of the wave function might produce a second dot. Once again we see that when $\psi$ collapses, it appears to occur when an irreversible measurement is made, at which time human awareness could be the decisive factor. But how and why would human consciousness interfere in the physical world. Is the physical world dependent on human observers? Einstein asked a question no one could answer: would observation by a mouse cause wave function collapse [2]?

Von Neumann didn’t follow his mathematics to its logical conclusion. If wave function collapse involves a change in reality and an irreversible collapse of the wave equation, then the math is telling us that the timing and location of wave function collapse cannot be at the target screen. Only if the collapse of $\psi$ occurs when a particle is emitted from the gun, does such an abrupt change make sense. Consider an analogy.

We know from World War I that a gunshot can cause the abrupt and irreversible collapse of $\psi$, if we take $\psi$ to represent all the smooth probabilities of commerce and diplomacy. On June 28, 1914 a student named Gavrilo Princip assassinated Archduke Franz Ferdinand by firing a pistol shot in Sarajevo, Bosnia, at which time $\psi$ collapsed into one specific eigenstate, which we could call $\phi_3$. Austria declared war on Serbia and World War I inexorably began as a chain reaction. All the other eigenstates, such as $\phi_1, \phi_2$, or $\phi_4$ involving reason and negotiated compromise were no longer available as options after that gunshot of June 28, 1914. If $\psi$ collapses at the particle gun in the double slit experiment, that would require that the waves travel in the opposite direction as what QM assumes, because otherwise a particle emitted from the gun could not produce the wave pattern on the target screen. We show below that if waves do travel from the screen to the gun, the pattern on the target screen can easily be explained, and a different mathematics applies than the QM mathematics of the double slit experiment. The Theory of Elementary Waves (TEW) is based on the assumption that waves travel in the opposite direction as particles, penetrate the two slits backwards, add into a superposition, and impinge on the gun. Such waves are called “elementary waves” and later in this article we will show that they are Schrödinger waves [3-19].

In the TEW model of the double slit experiment the elementary waves precede the emission of a particle, and the particle then randomly selects which one of the many elementary waves to respond to. Although the waves carry no energy, they carry probability amplitudes, which is why particles follow them. The waves are in a superposition, but particles cannot be in a superposition, which is very different than QM. After the gun is fired the wave equation is no longer relevant. We could say it “collapses.” All the other eigenstates, such as $\phi_2$, $\phi_3$, or $\phi_4$, no longer have any influence on the particle. After emission from the gun, the particle is subject to no further interference. The particle then follows its specific wave, $\phi_3$, backwards with a probability of one, through one and only one slit (it doesn’t matter which slit) and makes a dot on the target screen at that point $\alpha$, from which its wave emanated from the screen.

If waves travel from the target screen to the gun, that contradicts Thomas Young’s idea, published in 1801 [20]. In order to understand this article, the reader needs to temporarily forget Young’s picture. Assume that Young was wrong.
Time always goes forwards in TEW. When we say that particles follow waves backwards, for both waves and particles the vector of time is pointed toward the future. For example, if a wave is traveling east to west while time is going forwards, a particle follows the wave west to east with time going forwards. The direction of the vector of time differentiates TEW from Transactional Interpretation of John Cramer or Ruth Kastner. Furthermore, Transactional Interpretation is an “interpretation” of QM, whereas TEW is not, for reasons discussed below.

TEW differs from the pilot waves of de Broglie and Bohm in terms of the direction of the waves. Pilot waves travel in the same direction as particles; elementary waves travel in the opposite direction. Furthermore, the pilot wave interpretation is an “interpretation” of QM, whereas TEW is not.

1. Elementary Wave Model of the double slit experiment

Figure 2 shows the apparatus we will discuss.

It is useful to think of the target screen in two ways, as if there were two screens. One we will call the “Origin-screen,” and the other we will call the “Target-screen.” We propose to take every point $\alpha$ on the “Origin-screen” as the point of origin of elementary waves moving toward the two slits. When a particle is emitted from the particle gun at a later time, that particle will trace backwards one of these elementary waves, through one of the two slits (it doesn't matter which slit) and hit the “Target-screen” at the corresponding point $\alpha$. Thus on the right hand side of our diagrams we have a two-dimensional surface serving two purposes, for which we will use two names.
It is like a ping-pong game. First you hit the ball with a paddle, then it comes back at you. As an offensive player you are like the Origin-Screen. As a defender you are like the Target-Screen.

1. Elementary Wave Model of the double slit experiment

1a. Explaining how the model works

The TEW model is that zero energy waves start at any point $\alpha$ on the Origin-screen, and move in the opposite direction as particles (Figure 3). We have chosen this particular $\alpha$ such that the peak of one wave goes through one slit just as the peak of the next wave goes through the other slit. Therefore the waves are in phase as they penetrate the two slits. They remain in phase as they converge on the particle gun. Therefore there is constructive interference at the gun, meaning a lot of amplitude of the waves incident to the gun.

![Figure 3. Waves emanate from point $\alpha$ on the Origin-screen.](image)

The probability of a particle being triggered is directly proportional to the square of the amplitude of the incident wave at the gun. The particle triggering is random. If a particle is triggered, it will follow that specific wave backwards. The particle only uses one slit: it doesn’t matter which. The particle follows its wave backwards with a probability of one, and strikes the Target-screen at exactly that point $\alpha$ from which its wave is emanating. This is shown in Figure 4.

![Figure 4. A particle follows its wave backwards to the Target-screen.](image)

Because point $\alpha$ produces constructive interference at the particle gun, therefore there is an increased probability of a dot appearing on the screen at point $\alpha$, as shown in Figure 5.
The waves in our model travel in the opposite direction of what Thomas Young thought. Nevertheless, Young’s double slit equation applies to our model. Let “m” be an integer, $\lambda$ is the wavelength, $d$ is the distance between the slits, and $\rho$ is the angle shown in Figure 6. If $L$ is the distance between the Origin-screen and the double slit barrier, we assume $d << L$. Young’s equation:

$$m\lambda = d \sin \rho$$

(1)

This tells us the difference in angle $\rho$ between the peaks of constructive interference on the target screen.

For each point $\alpha$ on the Origin-screen there is a difference in path length ($d \sin \rho$) between that point and each of the slits. That difference determines the difference in phase of the waves ($\phi - \theta$) as they penetrate the two slits. Since the path length from each slit to the gun is the same, therefore the difference in phase that is determined at the slits ($\phi - \theta$), is invariant as the waves impinge on the gun. It is that phase difference ($\phi - \theta$) that determines whether the interference at the gun is constructive ($\phi - \theta = 0$ or $\pi$ or $2\pi$ . . .) or destructive ($\phi - \theta = \pi/2$ or $3\pi/2$ or $5\pi/2$ . . .).
The phase difference \((\varphi - \theta)\) of the waves impinging on the gun determines the amplitude with which that wave hits the gun. The amplitude squared at the gun is proportional to the probability that a particle will randomly be triggered by that wave. That in turn is the probability of a dot appearing in the final dataset at point \(\alpha\) on the Target-screen.

According to our model every point \(\alpha\) on the Origin-screen is emitting waves in all directions and at all wavelengths. The overwhelming majority of such waves can be ignored as having no effect on our experiment. The only waves that matter are those traveling in the right direction, with a wavelength \(\lambda = h/c/E_1\) where \(E_1\) is the energy of a particle that will subsequently be emitted. A particle will only follow a wave of de Broglie wavelength corresponding to the energy of the particle.

Now let’s consider waves emanating from point \(\beta\) (see Figure 7). This point is located at a position on the Origin-screen where the peak of one wave hits the upper slit just as the trough (valley) of some wave hits the other slit. Therefore the waves are completely out of phase as they pass through the two slits \((\varphi - \theta = \pi/2\) or \(3\pi/2\) or \(5\pi/2\) . . .), and remain out of phase when they impinge on the particle gun.

There is destructive interference at the gun. The peaks of one wave fill in the valley of the other wave, so there is “flat water”: NO AMPLITUDE of the impinging wave at the gun, with the result that there is zero probability that a particle will be triggered. Therefore no particle will trace those waves back to point \(\beta\). Therefore in the final data, point \(\beta\) will remain black, as shown in Figure 8.
The amount of constructive or destructive interference at the gun varies depending on the location of the starting point on the Origin-screen. This is what we described above:

A. The path length difference \(d \sin \rho\) as the waves pass through the two slits

B. determines the wave phase difference \(\varphi - \theta\),

C. and that determines the type of interference at the gun.

The red sinusoidal curve on Figure 9 shows the type of interference at the gun caused by different points of origin on the screen. If there is strong constructive interference at the gun then there is a larger probability of a particle following the wave backward and leaving a white dot on the target screen. If there is strong destructive interference at the gun then there is a smaller probability of a white dot appearing on the target screen. It all depends on where the starting point of the elementary wave was located at the Origin-screen to the right.

Figure 9. Type of interference at the gun, depending on position on the Origin-screen.

Figure 10 shows how the red curve from the previous diagram relates to the final dataset etched on the Target-screen. Constructive interference at the gun leads to a white area on the screen. Destructive interference at the gun leads to an area where the screen remains black.

Figure 10. Resulting pattern on the target screen.
The mechanism described above explains how the double slit experiment works. This is more complicated than Thomas Young's idea, but it also fits the empirical data better.

1b. Elementary waves with NO interference

So far we made an unstated assumption about wave interference. We assumed that waves from any two points $\alpha$ and $\gamma$ on the Origin-screen do not interfere with one another. Those waves cannot add together into a superposition. This "no-superposition rule" is true no matter where the two points are located, even if they are located a nanometer away from one another (see Figure 11). For that reason, there are no "plane waves" traveling from the target screen over to the two slits.

![Figure 11](image)

Figure 11. Waves from two points on Origin-screen do not interfere with each other. This drawing is distorted: in reality the waves overlap in space, but they don't add together into a superposition. Therefore we have diagramed them as if they don't overlap.

In other words, these elementary waves act like waves in some respects, but in other respects they behave like no waves we have ever seen before. Furthermore, the weirdness of wave behavior gets worse, a lot worse, as shown in Figure 12.

![Figure 12](image)

Figure 12. A light is used to discover which slit the particle used.
In Figure 12 we place a light near the barrier, as shown. For the past century scientists have wondered “which slit did the particle go through?” Therefore they placed a lamp in the position we show in Figure 12. This light emits a small amount of energy, which changes the nature of the zero energy elementary waves in its vicinity, as symbolized by the waves turning from black to purple in color. The result is that the elementary waves going through the two slits are no longer capable of interfering with one another. They lose their brotherhood, or sisterhood quality. The purple waves can no longer add together in a superposition.

As a result of the inserted light, with its energy, the probability of a particle going through each of the slits is calculated by the probability of one slit plus the probability of the other slit. It is like a different experiment is being conducted, one experiment using slit A and a second experiment using slit B. Therefore the total probability is calculated by squaring the amplitude for slit A, and squaring the amplitude for slit B, then adding them together. This “no superposition” rule does not hold, however, if the light is sufficiently in the red part of the spectrum (low energy).

1c. Elementary waves with and without a superposition

We learn from the double slit experiment that elementary waves have heir own peculiar behavior patterns, unlike any waves we ever previously encountered. If the waves come from two points $\alpha$ and $\gamma$ then they are independent of one another: cannot add together to form a superposition. If the waves all come from the same point and then pass through a double slit barrier, they acquire an ability to interfere with one another: i.e. add together in a superposition. However, this superposition capability is lost if a light shines on the waves. That destructive power of light occurs only above a certain threshold for the energy of the light. A light with a long enough wavelength does not have a toxic effect on the waves.

So far we have learned that there are two unrelated types of elementary waves: non-superposition and superposition, in terms of their capability for addition or interference.

We also learn about the relationship of waves and particles from studying the double slit experiment. Particles in the target screen appear to be the point of origin for elementary waves traveling to the left. These waves are intrinsic to space, flowing through the apparatus like a zero energy aether. But a particle acts as a disruption to the flow, so that the waves scatter after encountering a particle. Thus particles are like rocks in a riverbed, causing downstream waves visible on the river’s surface.

The waves also interact with a particle in the gun. Let’s say it’s an electron gun. When an electron is energized and is about to be emitted, it is bombarded by a zillion elementary waves coming from every point on the target screen, each with its own amplitude. The particle at random makes a choice which wave to respond to. The particle is like an airplane about to take off. The pilot is given a hundred different flight plans and must choose one of them. Each elementary wave constitutes a “flight plan” for the trajectory of the particle flying backwards.

2. Wave particles in a double slit experiment

We will show later that the Schrödinger wave equation is very different in TEW than in QM. The fact that there are different equations (Equations 19 and 20) is reflected in the drawings or graphs of how a Schrödinger wave passes through a double slit experiment (Figures 13, 14 and 15).

2a. The QM complicated model

According to QM a wave particle is unbelievably complicated as it moves through a double slit experiment: see Figures 13 and 14. These are diagrams of the movement of probability densities over time for a wave particle. There are two diagrams because different computer models generate different videos. Both Figures 13 and 14 are snapshots (sketched by the author) of one time in the video, not necessarily the same time.

In these diagrams the red areas represent regions of high probability density, followed by pink, purple, acqua and blue-green. The gray-white areas have the lowest probability density. The wave particle starts at the gun
on the left, then spreads out in a cloud. As the cloud moves to the right, it diffuses and widens. Then it bumps into the double slit wall. Two things happen. More than half the wave particle bounces off the wall and forms a reverse wave: a “back splash.” Although the larger part of the wave particle remains on the left, QM experts ignore that part, and focus only on the wave particle on the right.

Less than half the wave particle penetrates the two slits, where it forms a diffuse wave particle that moves to the right. As it moves it changes shape, as shown on the right side of Figures 13 and 14. All this intricate choreography is best understood by watching a video. As noted above, Figures 13 and 14 are snapshots from the middle of the computer generated videos.

At the target screen everything abruptly changes. The entire cloud vanishes (even the cloud to the left of the double slit barrier vanishes instantly) and a single dot appears on the target screen. The wave particle becomes a particle.

Einstein said that one of the illogical parts of this picture is that when the wave function collapses and a dot appears at only one place, all other parts of the wave function vanish instantly, which is faster than the speed of light. No one has ever provided an explanation of how the appearance of a single dot could instantaneously cause wave function collapse everywhere else. This is a superb illustration of the von Neumann paradox.

Fig. 13 This is the first computer model of the QM wave particle (position representation): It is a snapshot from halfway through a video of the QM wave particle movement. The wave particle starts at the particle gun. When it hits the wall in the middle, two things happen. More than half the wave sloshes off the wall and moves to the left. A smaller part slithers through the two slits, and continues moving to the right.
The QM Schrödinger wave equation is a first degree differential equation in time, and a second degree differential equation in space. With an equation as complex as the QM Schrödinger wave moving across a double slit apparatus, different computer models will diverge, as evident in Figures 13 and 14. Each computer model makes certain assumptions to simplify and approximate the insoluble equation. That Figures 13 and 14 look different is not so important. The main point is that the QM Schrödinger wave is intricate and confusing, moving in several directions simultaneously. Later in this article we will discuss a mathematical equation that might have generated these diagrams (see equation 20).

2b. TEW model of a wave particle is a one dimensional flat line: a constant $E = E_1$

Whereas QM pictures one Schrödinger wave, TEW pictures hundreds of very simple wave equations, one for each point $\alpha$ in the Origin-screen. Each wave starts at a different point $\alpha$ on the screen, penetrates backwards through the two slits, and forms a superposition in proximity to the gun. Figures 3 through 10 show diagrams of these waves. The particle randomly chooses which wave to link up to and follow backwards.

In section 3 of this article we will provide equations and diagrams of these hundreds of waves moving to the left.

An energy representation must start with the idea that none of these waves convey any energy. Prior to wave function collapse the wave (moving to the left) carries no energy itself, but it carries an amplitude for the amount of energy of a particle at a later date. This is typical of Schrödinger waves: they convey probability amplitudes for energy, but not energy itself.

That information is encoded in the wavelength $\lambda$ of the elementary wave, according to the equation:

$$\lambda = \frac{hc}{E_1}$$ (1)
where $h$ is Planck's constant, $c$ is the speed of light, and $E_1$ is the energy of the particle that will subsequently follow that ray backwards. All elementary waves, from all parts of the screen have the same wavelength, or else they are irrelevant to this experiment and are therefore ignored.

Since the elementary waves are conceived as being real waves in TEW, they have a phase as they penetrate the two slits. Figure 6 shows that the phase difference $(\phi - \theta)$ of the two waves (in slit A and B) is determined by the difference in path length $(d \sin \rho)$ from the Origin-screen to the slits. The phase of the two waves penetrating backwards through the slits will be diagrammed later in Figure 17.

The phase difference acquired at the two slits $(\phi - \theta)$ is maintained as the waves impinge on the gun, where they form constructive or destructive interference, or something in between. The phase difference at the slits determines the type of interference at the gun. This interference determines the amplitude with which the waves impinge on the gun, and therefore determine the probability of a particle being randomly triggered to follow that wave backwards. This is summarized in Figure 10.

Only when one wave links up with one particle, and the particle starts moving to the right, is there wave function collapse. Since all the energy is in the particle (none in the wave) therefore the wave particle has energy and therefore it is the first wave that can be described using the energy representation of a Schrödinger equation. But it is not a robust Schrödinger equation. Rather it is a single eigenstate $(\phi_3)$ that emerges after wave function collapse:

$$E = E_1$$

(2)

The diagram for this eigenstate is Figure 15. This eigenstate is the first time we have a wave particle. Figures 13 and 14 were about a QM wave particle. Figure 15 shows an infinitely simpler picture of the same thing.

In Figure 15 the X axis represents a one dimensional line from the gun, through the midpoint between the two slits, then it bends so it ends up at point $\alpha$ on the Target-screen.

If you compare Figures 13, 14 and 15, you can see that QM has an bewildering picture of the movement of a wave particle in the double slit experiment, whereas TEW has a simple, one dimensional picture. At the end of
the day the QM experts tell you they don’t understand it, whereas the TEW experts tell you they do understand it.

Some readers are troubled with our model because of the Heisenberg uncertainty principle. How can we know both the position and momentum of a particle? The answer is that we don’t know both. We embrace the Heisenberg uncertainty equation. It is at the core of quantum mathematics, and TEW is built on quantum math.

3. A mathematical description of the double slit experiment

For QM the probability of a dot appearing on the target screen is the square of the amplitude that the particle came through slit A plus the amplitude that it came through slit B. Prior to detection the particle was allegedly in a superposition and had no specific location: as shown in Figures 13 and 14. (In Figure 16 below we will violate that assumption and draw red arrows as if the wave particles had a straight line trajectory prior to hitting the target screen.)

In the TEW model wave function collapse occurs at the gun, and consists of the particle randomly choosing to latch onto one of the incident waves coming from various points on the target screen. Although the waves are in a superposition, particles are not. The probability of a particle being triggered is the square of the sum of the amplitudes that the wave came through slit A and B. If a particle is triggered, it will map itself, with a probability of one, onto that point of the Origin-screen from which its wave is emanating. Therefore the TEW model yields the same math as the QM model.

Our goal is to find an equation for the waves.

We define the symbol $\text{Æ}$ as the symbolic representation of an elementary ray. $\text{Æ}$ is the first letter of the term Ælementary Ray. It is from Old English and Old Norse, pronounced “ash.” The corresponding Anglo-Saxon rune, carved into ancient stones unearthed around the North Sea by archaeologists, was meant to resemble an ash tree:

\[
\text{
\begin{align*}
\text{Æ}
\end{align*}
}
\]

The advantage of using $\text{Æ}$ is that we emphasize that Ælementary rays exist outside QM. Greek letters are used in QM. No one would accuse $\text{Æ}$ of being a Greek letter.

To reiterate: we define $\text{Æ} \equiv \text{elementary ray}$.

We adopt $\Pi$ as the symbol for a particle. TEW particles differ from QM particles because we split off the concept of a wave from a particle. In a system restricted to one particle, we say $\psi = \text{Æ} + \Pi$. A wave and particle cannot be identical if they travel in opposite directions. We defined the term “wave particle” to be $\text{Æ} + \Pi$. This is different than the “wave particle” of QM which can morph from a wave to a particle depending on how you look at it. The TEW “wave particle” cannot morph in that way.

There are two varieties of Ælementary rays: those to which a particle is, versus those to which no particle is attached. The only Ælementary rays that we study are those attached to a particle, and even then, it is the particle and not the ray that is visible to our detectors. Only the particle has energy, momentum, angular momentum, charge, etc. What the particle and ray have in common is frequency and inverse directions. Intrinsic spin is also a property of Ælementary rays, as evident in Stern Gerlach experiments.

Ælementary rays with no attached particle are invisible to our detectors. No one has ever seen an unattached $\Pi$ either. The only particles that appear in our experiments are $\text{Æ} + \Pi = $ wave particles. There is no such thing as a particle with no wave. There are a finite number of particles in any volume of space, but an infinite number of waves.
TEW defines the term $\psi$ the same way as QM defines it: for a one particle system it is a wave–particle. Comparing the behavior of a wave–particle in TEW versus QM, with one exception the wave particle moves and acts the same in either system. It moves in the direction of the particle and is detected only when and where the particle is detected. The rest of the wave is much longer, but is invisible and undetectable.

The difference between $\psi$ in TEW versus QM is that a TEW wave–particle responds to information from the detector ahead, since the detector is the origin of the ray $\mathcal{AE}$ embedded inside the wave particle. That which Einstein called “spooky action at a distance,” we call $\mathcal{AE}$, thereby bringing it into the scientific enterprise so it can be studied, classified and quantified. $\mathcal{AE}$ is a specific real thing that replaces the vague QM term “nonlocality.” The $\mathcal{AE}$ can be modeled mathematically, whereas “nonlocality” is too vague and fluffy to calibrate, define, or use as a variable in future experiments.

3a. Equations that can be added as a superposition

In Figure 16-top, let $A = |A|e^{i\theta}$ be the amplitude that an particle impinging on “$\alpha$” at the target screen comes through slit A, and $B = |B|e^{i\phi}$ the amplitude that it comes through slit B. Then the combined amplitude is $A + B$. The square of that is proportional to the probability of a dot appearing at “$\alpha$” on the screen:

$$P(x) = |A + B|^2$$
$$= |A|^2 + |B|^2 + \text{Re}(AB^* + A^*B)$$
$$= |A|^2 + |B|^2 + |A||B| \text{Re}(e^{i(\phi-\theta)} + e^{i(\theta-\phi)})$$
$$= |A|^2 + |B|^2 + 2 |A||B| \cos(\phi - \theta)$$

The phase difference $(\phi - \theta)$ is determined by the distance from slit A to “$\alpha$” versus slit B to “$\alpha$” measured in wavelengths. One wavelength equals $2\pi$ rotations of phase. For example, if we move “$\alpha$” laterally so that there is $\frac{1}{4}$ wavelength more between slit B and “$\alpha$”, than between slit A and X, then the phase difference $(\phi - \theta)$ will be changed by $\pi/2$.

![Figure 16. The QM and TEW models of the double slit experiment. In the top half the red arrows represent wave particles. In the bottom half the green arrows represent waves (ælementary rays) without particles.](image-url)
On the bottom half of Figure 16, let \( A = |A|e^{i\theta} \) be the amplitude an elementary ray emanating from point "\( \alpha \)" comes through slit A to reach the particle gun. Let \( B = |B|e^{i\phi} \) the amplitude that it comes through slit B. Then the overall amplitude at the particle gun is \( A + B \). The square of that is proportional to the probability of an particle being triggered in response to that elementary ray. If a particle is triggered then a dot will appear on the target screen at point "\( \alpha \)." Once triggered an particle follows its ray with a probability of one. It uses only one slit: it doesn't matter which slit. No further interference has any impact. So the probability of a dot appearing at "\( \alpha \)" is:

\[
P(x) = |A + B|^2 = |A|^2 + |B|^2 + 2|A||B|\cos(\phi - \theta)
\]

This equation is consistent with a helical structure of \( \mathcal{E} \). Figure 17 shows that \( \mathcal{E}_A \) is a cylindrical helix traveling at the speed of light from "\( \alpha \)" to slit A and then bending so it travels towards the particle gun. \( \mathcal{E}_A \) has amplitude \( A \) which is a complex number. The amplitude is defined as the square root of the probability of a particle following that ray backwards. The radius of \( \mathcal{E}_A \) is \( r \equiv |A| \) and \( \theta \) is the angle of rotation at any location and time. Figure 17 shows \( \mathcal{E}_A \) and \( \mathcal{E}_B \) as they penetrate the double slit barrier. There is no electromagnetism associated with these helices.

![Figure 17. Two elementary rays in the form of helices, as they pass through the double slit barrier. The cylindrical helix (corkscrew) is an artistic device that allows us to see the phase of the wave. It also allows us to see that with each phase change of 2\( \pi \) the wave advances one wavelength \( \lambda \).](image)

As the phase \( \theta \) spins around the axis X of \( \mathcal{E}_A \), that phase is related to the position of the elementary ray, in the sense that every time \( \theta \) rotates 2\( \pi \), the position \( x \) is advanced by one wavelength \( \lambda \). That reflects the helical shape of the elementary ray. We will show later that \( x \) and \( \lambda \) are very small, because the frequency is determined by \( E_1/\hbar \) and \( \hbar \) is extremely small.
Figure 17 shows the X axis contiguous with the main axis of $\mathcal{E}_A$. We will define an imaginary plane $(Y, iZ)$ at the particle gun, orthogonal to $X$, with $Y$ being the real and $Z$ the imaginary component. We define $\theta$ as the angle of rotation of the corkscrew at the $(X, iZ)$ plane. $\theta$ is a function of time: it spins at incredible speeds. $\mathcal{E}_A$ can be described as follows: The energy of $\mathcal{E}_A$ is zero: $E(\mathcal{E}_A) = 0$. The energy is all located inside the particle, not inside the elementary wave. The spiral can be described as follows:

\[
\begin{align*}
    x &= |A| \cos(\theta) \\
    z &= |A| i \sin(\theta) \\
    y &= c \beta t
\end{align*}
\]

where $c$ is the speed of light along axis $X$, $\beta$ is a constant of our choice, $\omega$ is the angular frequency and $f$ is the frequency, so:

\[
\begin{align*}
    \theta &= \omega t = 2\pi f t \\
    f &= \frac{\theta}{2\pi t} = \frac{\omega}{2\pi}
\end{align*}
\]

The wavelength is

\[
\gamma = \frac{\lambda}{c}
\]

and the wave number is

\[
k = \frac{2\pi}{\lambda}
\]

Although this helical object travels at the speed of light on the $X$ axis, we are primarily interested in a cross section at the particle gun. In that plane $A = |A| e^{i\theta}$. And of course, if there were a particle following the ray backwards then the energy $E$ of the object $\mathcal{E} + \Pi$ would be

\[
E(\mathcal{E} + \Pi) = \frac{hc}{\lambda} = fc
\]

It is important to distinguish equation 6 from the next two equations that some readers confuse with it.

\[
\begin{align*}
    E(\mathcal{E}) &= 0 \\
    E(\mathcal{E}) &= \mathcal{E}
\end{align*}
\]

Equation 7 means that an elementary ray with no particle attached carries no energy. Only a wave-particle carries energy. Equation 8 means that there is no such thing as a particle without an elementary ray. Every particle in nature is attached to at least one elementary ray. It might sometimes jump from one elementary ray to another one, but there are no “rayless” particles in nature.

Amplitudes from the two elementary rays in Figure 17 can be added at the particle gun, where $\mathcal{E}_A$ overlaps with $\mathcal{E}_B$ and you get a superposition. The combined amplitude is $A + B$. Amplitudes can be multiplied by a scalar.

The elementary ray amplitudes form a vector space $V$. An adjoint of $A$ is defined $A^* \equiv |A| e^{-i\theta}$. Elementary rays $\mathcal{E}$ with a right handed helix have adjoints $\mathcal{E}^*$ with a left handed helix. The adjoint form an vector space $V^*$. We define an inner product:
The vector space \( V \) is complete, and is a Hilbert space.

We need to avoid confusion. It is not the elementary rays that form a Hilbert space. It is only the amplitudes of the elementary that constitute of Hilbert space. That confusion is easy to avoid if we remember that the symbol \( \Lambda \) is not part of the Greek alphabet, and the mathematics of Hilbert space is written in Greek letters.

3b. Wave equations that cannot be added as a superposition

We showed earlier that a light introduced into a double slit experiment abolishes the ability of the waves through slit A to interfere with the waves through slit B. In other words the amplitudes of \( \Lambda_A \) and \( \Lambda_B \) cannot be added together when they impinge on the gun. Therefore the amplitudes do not form a Hilbert space. The same non-interference prevails between elementary waves originating at two points (\( \alpha \) and \( \gamma \)) of the Origin-screen. The elementary waves \( \Lambda_{\alpha} \) and \( \Lambda_{\gamma} \) cannot be added together. Therefore elementary waves originating on the Origin-screen do not form plane waves.

4. Schrödinger wave equations in the double slit experiment

4a. The TEW mathematics of Schrödinger wave equations

A central axiom of TEW is that elementary waves are that aspect of physical nature where quantum mathematics lives. This is different than the Copenhagen assumption that such equations are simply mathematics, living in a Hilbert space that is abstract and disconnected from the physical world of everyday experience. We will now show that the Schrödinger equation is the basis for elementary waves \( \Lambda_A \) and \( \Lambda_B \).

Here is the time dependent Schrödinger equation that we will apply to the double slit experiment:

\[
\frac{\psi(t,x)}{\hbar} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \psi(t,x)
\]

(10)

There will be two stages in our argument; first we will use an approach taught by Ramamuti Shankar [21] to find a solution to a specific variety of Schrödinger equation, then, in part “b” we will apply that solution to the TEW elementary waves \( \Lambda_A \) and \( \Lambda_B \).

4a. A solution to an unusual Schrödinger equation

If the solution of a Schrödinger wave function \( \psi(t,x) \) can be segregated into a product of two functions \( \xi(t) \) and \( \psi(x) \), one dependent only on time \( \xi(t) \), the other dependent only on position \( \psi(x) \), then the Schrödinger equation can be solved. So our premise is that the solution to the TEW Schrödinger equation will be of the form:

\[
\psi(t,x) = \xi(t) \psi(x)
\]

(11)

We now take equation 11 and plug it into equation 10, and get the following:

\[
\hbar \frac{\partial \xi(t)}{\partial t} = \xi(t) \left[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi(x) \right]
\]

(12)

We define a symbol “\( H\psi(x) \)” to be:
\[ H\psi(x) \equiv \left[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x) \right] \] (13)

and therefore equation 12 can be rewritten:

\[ i\hbar \psi(x)\frac{\partial \xi(t)}{\partial t} = \xi(t)H\psi(x) \] (14)

Now we divide both sides of equation 14 by \( \xi(t)\psi(x) \):

\[ i\hbar \frac{1}{\xi(t)} \frac{\partial \xi(t)}{\partial t} = \frac{1}{\psi(x)} H\psi(x) \] (15)

Equation 15 has time as the only variable on the left, and position \( x \) as the only variable on the right. The only possible way the right and left sides of equation 15 could be equal is if they are both equal to a constant, which we will call \( E \).

\[ i\hbar \frac{1}{\xi(t)} \frac{\partial \xi(t)}{\partial t} = \frac{1}{\psi(x)} H\psi(x) = E \]

Therefore we divide equation 15 into two equations:

\[ i\hbar \frac{\partial \xi(t)}{\partial t} = E\xi(t) \] (16)

and

\[ H\psi(x) = E\psi(x) \] (17)

The left hand side of equation 17 is a symbol (see equation 13) and when we plug that symbol in, we get:

\[ \left[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_E}{\partial x^2} + V(x)\psi_E(x) \right] = E\psi_E(x) \] (13)

which is the equation for a state of definite energy. Therefore we can add "E" as a subscript in equation 13:

\[ \left[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_E}{\partial x^2} + V(x)\psi_E(x) \right] = \psi_E(x) \] (14)

We can also simplify equation 16:

\[ \xi(t) = \xi(0)e^{-iEt/\hbar} \] (15)

where \( \xi(0) \) is a constant, representing the value of \( \xi(t) \) at time zero. Our original premise (equation 11) was:
so by choosing our starting time as zero, we can say that for a state of definite energy $E$:

$$\hat{\mathbf{A}}(t',\mathbf{x}) = c(t')\hat{\mathbf{A}}^E(\mathbf{x}) = c(-iE\hbar/\hbar)\hat{\mathbf{A}}^E(\mathbf{x})$$  \hspace{1cm} (16)

This is the solution to the Schrödinger equation that we were seeking. This solution is well known. We will now apply equation 16 to the TEW model of a double slit experiment.

4b. Applying that Schrödinger solution to TEW elementary waves

Figure 17 shows two elementary waves, each with a corkscrew structure, penetrating the two slits. They both start at some point $\alpha$ on the Origin-screen, and both are converging on the gun. This pair can be considered to be a Schrödinger wave, as we will now show.

The amplitude of the wave from $\alpha$ at the gun is $A + B$, and so it is the partnership of both waves that determines the probability of a particle being emitted in response. When a particle is triggered, the particle follows only one of the helices backwards, not both of them. Which one is a random choice by the particle. As we said before, the particle only goes through one of the slits, not both of them. It does not matter which slit is used. The Schrödinger wave is the pair of corkscrews.

Set aside for a moment the question which direction the Schrödinger wave is traveling. If we use equation 16, and take $E_1$ to be the well defined energy, then equation 16 can be written:

$$\psi(t,x) = \xi(t)\psi_{E_1}(x) = e^{-iE_1\hbar/\hbar}\psi_{E_1}(x)$$

The term

$$e^{-iE_1\hbar/\hbar}$$

describes a unit vector spinning rapidly around the origin in a complex plane, i.e. a rotating phase. This is like the spinning vector that defines a corkscrew, or cylindrical helix. The frequency of spinning is large, determined by the ratio of $E_1$ to $\hbar$ and $\hbar$ is very small.

We will define $\psi_{E}(x)$ to be a constant.

$$\mathbf{A}^E(\mathbf{x}) \equiv |A + B|$$  \hspace{1cm} (17)

where $|A + B|$ is a constant, which is the sum of the radii of the two elementary rays $\mathbf{A}_A$ and $\mathbf{A}_B$. Then equation 16 (the Schrödinger wave equation) $\psi(x,t)$ defines two helices identical to the helix penetrating slit “A” plus the helix penetrating slit “B” in Figure 17. The probability of finding the particle at any position $x$ (along the X axis) is constant in the position representation.

$$P(x) = |\psi(t,x)|^2$$

$$= \left| e^{-iE_1\hbar/\hbar} |A + B| \right|^2$$

$$= \left| e^{-iE_1\hbar/\hbar} \right|^2 \cdot |A + B|^2 = |A + B|^2$$

$$= |A|^2 + |B|^2 + \text{Re}(AB* + A*B)$$

$$= |A|^2 + |B|^2 + |A||B|\text{Re}(e^{i(\theta-\phi)} + e^{-i(\theta-\phi)})$$

$$= |A|^2 + |B|^2 + 2|A||B|\cos(\phi - \theta)$$  \hspace{1cm} (18)
By the term “well defined energy” we mean an eigenstate \( \varphi_3 \) of the energy of the particle leaving the gun. We have therefore shown the pair of helices from Figure 17 to fit a Schrödinger wave equation:

\[
\hbar (i^2 \chi) = \mathcal{E}_{(-|E_1|\hbar \chi)} | \chi + \chi |
\]

(19)

What about the direction of the wave? In QM it is alleged that Schrödinger waves travel in the same direction as a particle. But that is false, as Figures 13 and 14 show. The QM wave travels in all directions, as it sloshes around the box, sometimes going sideways or in the opposite direction (toward the gun, away from the Target-screen). In the TEW model the wave at first travels in the opposite direction as the particle, then, if it links up with a particle, the wave particle travels from the gun to the particle screen. So it is reasonable to say this is a Schrödinger wave.

This model differs from the QM picture of a double slit experiment because of WHEN and WHERE wave function collapse occurs. In the QM model the middle of the double slit apparatus is full of Schrödinger waves, as shown in Figures 13 and 14. The waves only collapse when any tiny piece of it hits the Target-screen. However, in the TEW model the only thing in the middle of a double slit experiment are invisible elementary waves, and a single particle representing a collapsed Schrödinger wave, which is to say a single eigenstate, which we have been calling \( \varphi_3 \). As we saw with Figures 13, 14 and 15 the QM Schrödinger wave is very complicated, the TEW Schrödinger wave can be represented by a simple constant: \( E_1 \) in the energy representation, and \( |A + B| \) in the position representation.

In summary, the elementary wave pictured in Figure 17 is a Schrödinger wave. In the TEW model there are hundreds of Schrödinger waves, one coming from each point \( \alpha \) of the Origin-screen. The particle randomly chooses one Schrödinger wave to respond to. This explains something von Neumann couldn’t figure out: where does randomness come from? Quantum mechanics is famous for being random, yet the two equations \( \psi \) and \( \varphi \) are deterministic equations. The answer from TEW is that every particle is a random number generator: it randomly chooses one of the incident elementary rays \( \varphi_n \).

4c. Some QM approximations of Schrödinger wave equations

If you compare Figures 13, 14 and 15 it is evident that the TEW approach to the double slit Schrödinger equation is infinitely simpler than the QM approach. These diagrams reflect the mathematics that is entered into a computer program such as Python or Mathematica. The different computerized models are not Schrödinger equations, because the Schrödinger equation for the QM model cannot be solved. They are various mathematical approximations, such as one using Green’s functions. Equation 20 shows one of many possibilities, an approximation using Green’s functions:

\[
| \psi(x, y, z, t) | = \frac{(a^2)^{3/2}}{(a^2 + 4t^2)^{3/2}} \left[ \exp \left( -a \frac{h^2 + 2hz + x^2 + y^2 + z^2}{a^2 + 4t^2} \right) + \exp \left( -a \frac{h^2 - 2hz + x^2 + y^2 + z^2}{a^2 + 4t^2} \right) + 2 \cos \left( \frac{4hz}{a^2 + 4t^2} \right) \exp \left( -a \frac{h^2 + x^2 + y^2 + z^2}{a^2 + 4t^2} \right) \right]
\]

(20)

In one computer model from Physics Stack Exchange, a=0.7. This illustrates that equation 19 from TEW is vastly simpler than equation 20 from a QM model of the double slit experiment.

Conclusions

John von Neumann spoke of the “measurement problem” in QM. The paradox is how to understand wave function collapse, which in Neumann’s thinking occurs when something is measured. This problem is illustrated in the double slit experiment. Why is there wave function collapse when a dot appears on the
Target-screen? The TEW answer is that wave function collapse occurs long before that, and at a different location.

TEW listens to what the math is telling us. If we assume the waves travel in the opposite direction as the particles, the von Neumann paradox and double slit paradoxes are solved. The reason for the abrupt change in the wave equation at the instant of wave function collapse, is the collapse occurs when the gun is fired. Something actually happens in nature when a gun is fired. Physical reality is different after a gun is fired.

The gunshot of June 28, 2914, is an analogy. If \( \psi \) represented the probabilities of world commerce and diplomacy before that date, after that date only one eigenstate, which we could call \( \varphi_3 \), existed. That eigenstate involved Austria declaring war on Serbia which led inexorably to World War I and the death of thirty seven million people. All the other eigenstates, such as \( \varphi_1, \varphi_2, \) or \( \varphi_4 \) involving reason, dialog, negotiation and compromise, were no longer available after the gunshot of June 28, 2914.

If reality abruptly changes when a Schrödinger wave equation \( \psi \) collapses, then it is an event that only happens once. It is impossible that a measurement would have caused that collapse, which is why von Neumann was stumped. It is easy to understand that a gunshot would have that effect. The von Neumann and double slit paradoxes point toward a physics different than that of QM. When we develop this TEW physics in this article, it is logical and self-coherent, and it explains the double slit data better than QM does. As Feynman said, no one using QM can explain the double slit experiment, which contains what he calls the “central mystery” of QM, which Feynman said was an unsolved mystery.

The waves which particles follow backwards are called elementary waves. These are real Schrödinger waves, but they have their own rules and behavior patterns. There are three peculiar rules.

A. Sometimes you can, and other times you cannot add two waves together into a superposition.

B. The wave carries a probability flux pointing in the opposite direction as the direction of the wave.

C. Like all Schrödinger waves, these waves convey no energy. They convey quantum amplitudes.

TEW is not another “interpretation” of QM. This is evident in the fact that the wave particle for the double slit experiment looks different between QM and TEW, as evident equations 19 versus 20, and in the difference pictures in Figures 13, 14 and 15. Furthermore, experiments can be designed for which QM and TEW predict divergent outcomes, as shown in a previous publication [12].

If the competition between the TEW and QM equations were based on Occam’s razor, then TEW would win the competition. Occam’s razor means the simpler explanation is better. At the present time the Copenhagen interpretation remains dominant, and Occam’s razor is not something scientists care about. With powerful computers doing the heavy lifting, scientists don’t find Occam’s razor a compelling issue. For this author, it is the decisive issue.

In the end the QM model of a double slit experiment is not based on a Schrödinger equation, but on a crude approximation. By contrast, the TEW model is based on the actual Schrödinger equation (equation 10), as we have demonstrated.

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References


Author’s biography with Photo

Jeffrey H. Boyd was born into a working class family in 1943 in New Jersey, USA. He is the first in his family to graduate from college. Boyd’s undergraduate degree in mathematics was from Brown University in 1965. Subsequently he became a polymath. Boyd has advanced degrees from Harvard University, Yale University and Case Western Reserve Universities. He served on the faculty of the National Institutes of Health in Bethesda, Maryland. His day job is as a physician taking care of sick people. He has published research in the areas of epidemiology, psychiatric diagnoses, firearms suicide, and wrote a book about how to live with a chronic illness: Being Sick Well. He has published research in the Journal of Advances in Mathematics, New England Journal of Medicine, Journal of Advances in Physics and Physics Essays. He gave scholarly lectures on TEW at the American Physical Society more than a dozen times. Boyd was in dialogue with his cousin, Lewis E. Little, for six decades. It was Little who obtained a PhD in physics, then discovered TEW: he was the first person to conceive the idea that particles follow elementary waves backwards. Boyd built mathematics on the foundation that Little established.