The Mularz Paradox—Relativistic Discrepancies in The Resistivity of Cylindrical Electrical Conductors Travelling at Near Light-Speed—A Thought Experiment

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Abstract

Most of us are familiar with Einstein's now-famous relativistic thought experiments—his stationary-versus-moving 'light clock' and the interstellar twin astronauts aging at different rates especially stand out.[1] In this simple thought experiment the author wishes to propose a heretofore unrecognized set of relativity paradoxes involving the predicted electrical resistivity of long cylindrical conductors when moving at or near light-speed.

Keywords: Relativity; Lorentz Contraction; Electrical Resistivity; Paradox

Introduction:

The electrical resistivity of a cylindrical conductor is classically described by the equation in **Figure 1** where R is resistivity, the Greek letter *rho* is the specific resistance of the material, l is length of the wire in meters and A is the cross-sectional area of the wire in m^2.[2]

For the purposes of our thought experiment, it is important to note here that the cross sectional area of a typical wire is described by a circle—thus the cross sectional area of a conventional wire may then be expressed by finding the area of the circle created by the wire's radius. The length of the wire is a less complicated measurement.

For a stationary wire of length 10m, a radius of 0.01m and circular cross-sectional area $3.14 \times 10^{-4} m^2$ via $A = \pi r^2$—and assuming the wire is made of copper with material-specific resistance of 1.68×10^{-8} Ohms per meter—we would arrive at a calculated electrical resistivity of 5.35×10^{-4} Ohms in standard conditions.

The thought experiment:

As noted above, the resistivity of a conventional cylindrical conductor (wire) is a function of its length and cross-sectional area. On board a vessel traveling at near light-speed an electrical engineer would observe our hypothetical wire to be round with a circular cross section. However, a stationary observer not on board that vessel would notice that the wire, with its long axis perpendicular to the direction of travel, now has a cross sectional shape resembling that of an ellipse vis-à-vis Lorentz contraction. The stationary observer would be compelled to predict the resistivity of this wire utilizing the cross sectional area of an ellipse in his or her calculations, giving rise to a relativistic discrepancy between the on board electrical engineer's calculations and their own.

For our hypothetical wire with dimensions described above (as observed from on board the moving vessel) we arrive at an inherent electrical resistivity of 5.35×10^{-4} Ohms. The outside stationary observer would see this wire's cross section as an ellipse ($\pi a b$ where a is the height and b is the contracted diameter) with a contracted horizontal diameter (via Lorentz's equation in **figure 2**) of 0.0014m while retaining its original vertical diameter.[3] Our stationary observer calculates the moving wire's resistivity to be 3.8×10^{-3} Ohms if moving at 99% the speed of light.

If the hypothetical wire is oriented instead with its long axis parallel to the direction of the ship's travel, the Lorentz contraction will alter the wire's measured length when observed from afar, affecting calculations at a speed of 0.99c thusly to give a resistivity of 7.54×10^{-5} Ohms.

$$R = \rho \frac{\ell}{A}$$

Figure 1: Equation for Resistivity

$$L = L_0 \sqrt{1 - v^2/c^2}$$

Figure 2: Equation for Lorentz Contraction

Discussion:

Interestingly, the new resistivity values for the same wire in varying orientation to the direction of travel at the same speed vary by a fixed ratio from the stationary (on board reference frame) measurement. The wire now parallel to the direction of travel (its length shortened by Lorentz) will have a resistivity of 7.54×10^{-5} Ohms. When divided by our stationary wire's resistivity of 5.35×10^{-4} Ohms, we obtain a ratio of 0.14. When the stationary wire's resistivity is divided by the perpendicular wire's observed resistivity, we also arrive at 0.14.

Flow rates for fluids in cylindrical pipes also rely on their typically circular cross-sectional area—perhaps a stationary plumber would witness an overflowing toilet or two on our light-speed vessel as it passes?

Further imagine a metallic wire loop spinning on a wand and connected to a stationary galvanometer. If this metal loop was spinning in the presence of a magnetic field and, as the angular velocity approached the speed of light, what would the galvanometer display as the loop's surface area was compressed into an ellipse? If a ship with a metallic hull accelerated toward light-speed through a uniform magnetic field—would the EMF across the hull be measured differently by someone on board versus a stationary observer?

The utility of this paradox, aside from an amusing thought experiment, is not abundantly clear. However, as the saying goes, a paradox is a hidden truth standing on its head to garner attention. It is the author's hope that those readers with a more robust knowledge of relativistic physics and mathematics can use aspects of this paradox to help uncover greater underlying truths of our reality.

References

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