



Introduction to The Spiral Dynamics and The Spiral Coriolis Force

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Abstract

The spiral dynamics of a point-like body of mass m in spiral differential geometry are introduced. New ideal motions have been studied, the uniform spiral motion and the uniform spiral-polar motion. The analysis is proposed by comparing the ideal spiral motions with the ideal circular motions. The theoretical forces acting on a point-like body of mass m moving in spiral frames were analyzed. The spiral and polar components of the Coriolis forces were compared.

Indexing terms/Keywords: Coriolis force; Newton's laws; Schwarz-Christoffel conformal mapping; spiral coordinates; relative motions; inertial systems

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Introduction

To find explicit solutions to physical phenomena and engineering problems it is necessary to choose an appropriate coordinate system. In fact, the choice of the coordinates must represent the geometry of the problem so that the corresponding mathematical formulation is simplified as much as possible. For example, elliptical geometry is used to explain the phenomena associated with galaxies and circular geometry is used to explain tropical cyclones. However, the limits of circular and elliptic geometries in explaining Nature are more than evident and the resulting models are not adapted to the reality of many events.

The term theory derives from the philosophy of the Ancient Greeks, and was associated with the meaning of "looking at, viewing, beholding" that originates from the word "thaumas" or wonder [1].

After the works of many scientists such as Copernicus, Brahe, Galileo, Descartes, Newton, it became accepted that the physics of the Greeks was not viable for practical purposes.

Galileo used practical experiments as a research tool and presented his treatise in the form of mathematical demonstrations [2].

In formulating his physical scientific theories, Newton [3] introduced new mathematical tools, "the method of fluxions and fluents" [4], which we know today in their formulation of the differential calculus proposed by Leibniz [5].

The Coriolis effect is a deflection of moving objects when the motion is described relative to a rotating frame. The mathematical expression of the Coriolis force appeared [8] in connection with the theory of water wheels.

Nowadays, the term Coriolis force is mainly used in connection with meteorology.

A particularly important problem in this category is the description of the movement of water particles of tropical storms with respect to the coordinated axis that rotate with the Earth. In this case the coordinates in which the Newton's laws are valid is that of fixed stars as an ideal inertial system [9].

The behaviour of hurricanes is explained in terms of classical physics using the Newton's second law and the thermodynamics of the moist air. Around a low-pressure centre, the pressure-gradient force directed inward balances the Coriolis force and the centrifugal force, both directed outward.

However, the hurricane consists of a characteristic spiral formed by dense clouds that surround a cloud-free eye also in the form of a spiral that can range from a few kilometers to 100 km across.

Starting from these simple empirical observations, the correct inductive scientific method to analyze the phenomena of the hurricanes, as would be taught by the philosopher Francis Bacon [6], can not disregard the appropriate mathematical tools. As depicted in the frontispiece of the "Novum Organum" [6], the spiral differential geometry can be imagined as a new galleon passing between the mythical Pillars of Hercules standing at the sides of the Strait of Gibraltar, marking the exit of the well-protected Mediterranean waters in the Atlantic Ocean, opening a new world for exploration. Galileo, the first pioneer of the scientific method wrote (see pag. 4 in [7]) "

Philosophy is written in this grand book -I mean the Universe- which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth."

Since the Nature is mostly spiral-shaped, it can not be well understood unless one first learns to understand the language of spiral geometry.



Consequently, the motion of the air around the central axis of rotation of the hurricane and its inward and outward flows should be described in the most appropriate geometry, that is, in the spiral geometry.

The force on a passenger who moves on a bus traveling along a trajectory depends on the geometry of the trajectory itself (see for example [10]). If the path of the bus is circular, we refer to the rotating frame, but if the path is spiral-shaped, we must introduce the spiral reference system.

In this pioneering paper, the orthogonal spiral differential geometry [11, 12] is used to introduce new ideal motions, namely the uniform spiral motion and the spiral-polar motion.

These ideal spiral motions are then used as methods for introducing the concept of fictitious spiral forces.

Materials and Methods

The polar and spiral geometry

Let's start considering the Schwarz Christoffel conformal mapping

$$f(z) = G \int_{z_0}^z \prod_{j=1}^{n-1} (\eta - x_j)^{-k_j} d\eta + F, \quad (1)$$

And let's consider the simplest case $x_j = 0, k_j = 1, j = 1, k_j = 0, j \neq 1$.

In terms of Cartesian coordinates¹, if $G \in \mathbb{R}, F = 0$ we obtain the polar coordinates

$$(1) \begin{cases} x = r \cos \varphi, \\ y = r \sin \varphi. \end{cases} \quad (2)$$

Whilst, if we choose $G \in \mathbb{C}, G = 1 - ig$, we obtain the spiral coordinates [11, 12].

$$(2) \begin{cases} x = e^{\frac{\delta}{g}} \cos(\delta + \theta + \psi), \\ y = e^{\frac{\delta}{g}} \sin(\delta + \theta + \psi), \psi \in \mathbb{R}. \end{cases} \quad (3)$$

The spiral coordinates, like the polar ones are curvilinear, their basis vectors change at all points in space.

For the polar coordinates, the covariant basis vectors are identified as

$$(1) \begin{cases} \hat{g}_r = \frac{\partial x}{\partial r} \hat{i} + \frac{\partial y}{\partial r} \hat{j} = \cos \varphi \hat{i} + \sin \varphi \hat{j}, \\ \hat{g}_\varphi = \frac{\partial x}{\partial \varphi} \hat{i} + \frac{\partial y}{\partial \varphi} \hat{j} = -r \sin \varphi \hat{i} + r \cos \varphi \hat{j}, \end{cases} \quad (4)$$

Where \hat{i}, \hat{j} are the Cartesian unit basis vectors.

For the spiral case

¹ Henceforth (1) indicates the formulas in polar coordinates and (2) in spiral coordinates.



$$\left. \begin{aligned}
 \hat{g}_\delta(\psi) &= \frac{\partial x}{\partial \delta} \hat{i} + \frac{\partial y}{\partial \delta} \hat{j} = e^{\frac{\delta}{g} - g\theta} \left\{ \begin{aligned}
 &\left[\frac{1}{g} \cos(\delta + \theta + \psi) - \sin(\delta + \theta + \psi) \right] \hat{i} + \\
 &\left[\frac{1}{g} \sin(\delta + \theta + \psi) + \cos(\delta + \theta + \psi) \right] \hat{j}
 \end{aligned} \right\}, & g = \tan G, \\
 \hat{g}_\delta(\psi) &= \frac{e^{\frac{\delta}{g} - g\theta}}{\sin G} \{ \cos(\delta + \theta + \psi + G) \hat{i} + \sin(\delta + \theta + \psi + G) \hat{j} \}, \\
 \hat{g}_\theta(\psi) &= \frac{\partial x}{\partial \theta} \hat{i} + \frac{\partial y}{\partial \theta} \hat{j} = e^{\frac{\delta}{g} - g\theta} \left\{ \begin{aligned}
 &\left[-g \cos(\delta + \theta + \psi) - \sin(\delta + \theta + \psi) \right] \hat{i} + \\
 &\left[-g \sin(\delta + \theta + \psi) + \cos(\delta + \theta + \psi) \right] \hat{j}
 \end{aligned} \right\}, \\
 \hat{g}_\theta(\psi) &= \frac{e^{\frac{\delta}{g} - g\theta}}{\cos G} \{ -\sin(\delta + \theta + \psi + G) \hat{i} + \cos(\delta + \theta + \psi + G) \hat{j} \}.
 \end{aligned} \right\} \tag{5}$$

Where G represents the angle between the spiral and the polar basis vectors.

In polar coordinates, the contravariant dual basis vectors are identified as

$$(1) \left\{ \begin{aligned}
 \hat{g}^r &= \frac{\partial r}{\partial x} \hat{i} + \frac{\partial r}{\partial y} \hat{j} = -\frac{\sin \varphi}{r} \hat{i} + \frac{\cos \varphi}{r} \hat{j}, \\
 \hat{g}^\varphi &= \frac{\partial \varphi}{\partial x} \hat{i} + \frac{\partial \varphi}{\partial y} \hat{j} = \cos \varphi \hat{i} + \sin \varphi \hat{j}.
 \end{aligned} \right. \tag{6}$$

While, the spiral contravariant dual basis is

$$(2) \left\{ \begin{aligned}
 \hat{g}^\delta(\psi) &= \frac{\partial \delta}{\partial x} \hat{i} + \frac{\partial \delta}{\partial y} \hat{j} = \frac{g e^{-\frac{\delta}{g} + g\theta}}{1 + g^2} \left\{ \begin{aligned}
 &\left[\cos(\delta + \theta + \psi) - g \sin(\delta + \theta + \psi) \right] \hat{i} + \\
 &\left[\sin(\delta + \theta + \psi) + g \cos(\delta + \theta + \psi) \right] \hat{j}
 \end{aligned} \right\}, \\
 \hat{g}^\delta(\psi) &= e^{-\frac{\delta}{g} + g\theta} \sin G \{ \cos(\delta + \theta + \psi + G) \hat{i} + \sin(\delta + \theta + \psi + G) \hat{j} \}, \\
 \hat{g}^\theta(\psi) &= \frac{\partial \theta}{\partial x} \hat{i} + \frac{\partial \theta}{\partial y} \hat{j} = \frac{e^{-\frac{\delta}{g} + g\theta}}{1 + g^2} \left\{ \begin{aligned}
 &\left[-g \cos(\delta + \theta + \psi) - \sin(\delta + \theta + \psi) \right] \hat{i} + \\
 &\left[-g \sin(\delta + \theta + \psi) + \cos(\delta + \theta + \psi) \right] \hat{j}
 \end{aligned} \right\}, \\
 \hat{g}^\theta(\psi) &= e^{-\frac{\delta}{g} + g\theta} \cos G \{ -\sin(\delta + \theta + \psi + G) \hat{i} + \cos(\delta + \theta + \psi + G) \hat{j} \}.
 \end{aligned} \right. \tag{7}$$

The position vector in polar coordinates can be written as

$$(1) \quad \vec{r} = r \hat{g}_r = r \hat{g}^\varphi, \tag{8}$$

Whilst, in spiral coordinates

$$(2) \quad \vec{r} = \frac{g}{1 + g^2} [\hat{g}_\delta - \hat{g}_\theta] = \frac{e^{\frac{2\delta}{g} - 2g\theta}}{g} [\hat{g}^\delta - g \hat{g}^\theta], \tag{9}$$



The covariant $[g_{\mu\nu}] \equiv [\hat{g}_\mu \cdot \hat{g}_\nu]$ and contravariant $[g^{\mu\nu}] \equiv [\hat{g}^\mu \cdot \hat{g}^\nu]$ metric coefficients in polar coordinates are identified as

$$(1) \quad \begin{cases} [g_{\mu\nu}] = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}, \\ [g^{\mu\nu}] = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{r^2} \end{bmatrix}. \end{cases} \quad (10)$$

Whilst, in spiral coordinates

$$(2) \quad \begin{cases} [g_{\mu\nu}] = e^{\frac{2\delta}{g} - 2g\theta} (1 + g^2) \begin{bmatrix} \frac{1}{g^2} & 0 \\ 0 & 1 \end{bmatrix} = e^{\frac{2\delta}{g} - 2g\theta} \begin{bmatrix} \frac{1}{\sin^2 G} & 0 \\ 0 & \frac{1}{\cos^2 G} \end{bmatrix}, \\ [g^{\mu\nu}] = \frac{e^{-\frac{2\delta}{g} + 2g\theta}}{1 + g^2} \begin{bmatrix} g^2 & 0 \\ 0 & 1 \end{bmatrix} = e^{-\frac{2\delta}{g} + 2g\theta} \begin{bmatrix} \sin^2 G & 0 \\ 0 & \cos^2 G \end{bmatrix}. \end{cases} \quad (11)$$

Uniform spiral and circular motions with constant phases

Uniform circular motion describes the motion of a point-like body of mass m along a circular path at constant speed. The distance of this body from the axis of rotation remains constant at all times.

Similarly, the uniform spiral motion of a point-like body of mass m is characterized by constant kinetic energy along a spiral path.

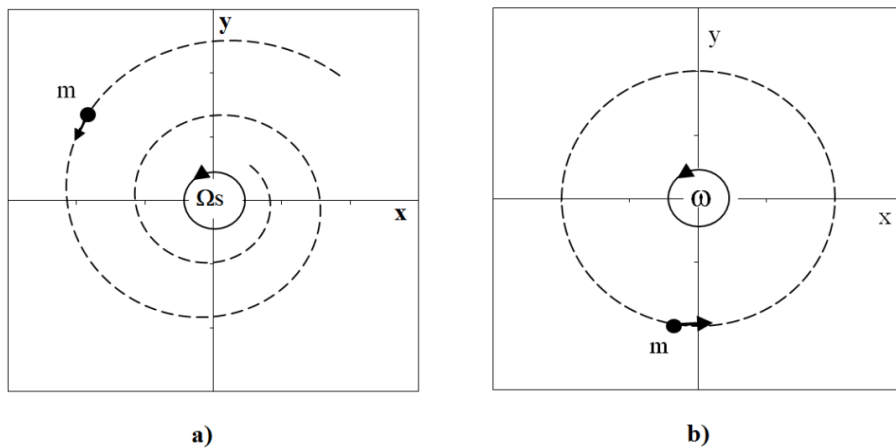


Figure 1: a) The spiral uniform motion, b) the circular uniform motion.

In terms of coordinates, if we identify the trajectory of the body with $(r(t), \varphi(t))$ we have



$$(1) \begin{cases} x(t) = r(t) \cos \varphi(t), \\ y(t) = r(t) \sin \varphi(t). \end{cases} \quad (12)$$

while, in spiral coordinates $(\delta(t), \theta(t))$, i.e., $\dot{\psi} = 0, \psi = 2n\pi$

$$(2) \begin{cases} x(t) = e^{\frac{\delta(t)}{g} - g\theta(t)} \cos \alpha(t), \\ y(t) = e^{\frac{\delta(t)}{g} - g\theta(t)} \sin \alpha(t), \\ \alpha(t) = \delta(t) + \theta(t). \end{cases} \quad (13)$$

In polar coordinates, the velocities are identified by²,

$$(1) \begin{cases} \dot{x} = \dot{r} \cos \varphi - r\dot{\varphi} \sin \varphi, \\ \dot{y} = \dot{r} \sin \varphi + r\dot{\varphi} \cos \varphi. \end{cases} \quad (14)$$

By using the following relations

$$(1) \begin{cases} \dot{\hat{g}}_r = \frac{\dot{\varphi}}{r} \hat{g}_\varphi, \\ \dot{\hat{g}}_\varphi = \frac{\dot{r}}{r} \hat{g}_\varphi - r\dot{\varphi} \hat{g}_r. \end{cases} \quad (15)$$

eq. (14) can be rewritten as $\dot{\vec{r}} = \dot{r}\hat{g}_r + \dot{\varphi}\hat{g}_\varphi$ or as

$$(1) \begin{cases} \dot{\vec{r}} = \frac{\dot{r}\vec{r}}{r} + \vec{\omega} \times \vec{r}, \\ \vec{\omega} = \dot{\varphi}\hat{k}, \\ r = |\vec{r}|, \\ T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2). \end{cases} \quad (16)$$

In spiral coordinates

$$(2) \begin{cases} \dot{x} = e^{\frac{\delta}{g} - g\theta} \left\{ \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \cos \alpha - \dot{\alpha} \sin \alpha \right\}, \\ y = e^{\frac{\delta}{g} - g\theta} \left\{ \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \sin \alpha + \dot{\alpha} \cos \alpha \right\}, \\ \dot{\alpha} = \dot{\delta} + \dot{\theta}. \end{cases} \quad (17)$$

² $\dot{x} = dx / dt$



By using the following relations

$$(2) \quad \begin{cases} \dot{\hat{g}}_{\theta} = \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \hat{g}_{\theta} - g(\dot{\delta} + \dot{\theta}) \hat{g}_{\delta}, \\ \dot{\hat{g}}_{\delta} = \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \hat{g}_{\delta} + \frac{(\dot{\delta} + \dot{\theta})}{g} \hat{g}_{\theta}. \end{cases} \quad (18)$$

Eq. (17) can be rewritten as $\dot{\vec{r}} = \dot{\delta} \hat{g}_{\delta} + \dot{\theta} \hat{g}_{\theta}$ or as

$$(2) \quad \begin{cases} \dot{\vec{r}} = \Omega_1 \vec{r} + \Omega_2 \times \vec{r}, \\ \Omega_1 = \frac{\dot{\delta}}{g} - g\dot{\theta}, \\ \Omega_2 = (\dot{\delta} + \dot{\theta}) \hat{k}. \end{cases} \quad (19)$$

The kinetic energy can be written as

$$(2) \quad T = \frac{1}{2} g_{\mu\nu} \frac{dx_{\mu}}{dt} \frac{dx_{\nu}}{dt} = \frac{1}{2} m e^{\frac{2\dot{\delta}}{g} - 2g\theta} (1 + g^2) \left(\frac{\dot{\delta}^2}{g^2} + \dot{\theta}^2 \right). \quad (20)$$

The uniform circular motion is obtained for $T = const.$, which means, $r = const. = R, \dot{\phi} = \omega$, where $\omega \in \mathbb{R}^+$ is a constant.

Whilst, the uniform motion along the spiral path $\delta = const. = \delta_0$ is obtained for

$$(2) \quad \begin{cases} e^{-g\theta(t)} \dot{\theta} = K_1, & K_1, K_2 \in \mathbb{R}, K_2 - K_1 t > 0, \\ \theta(t) = -\frac{1}{g} \ln(K_2 - K_1 t) - \frac{1}{g} \ln g, \\ \alpha^*(t) = \delta_0 - \frac{1}{g} \ln(K_2 - K_1 t) - \frac{1}{g} \ln g, \\ \Omega_s(t) = \dot{\theta}(t) = \frac{K_1}{g(K_2 - K_1 t)}, \\ \ddot{\theta}(t) = \frac{K_1^2}{g(K_2 - K_1 t)^2} = g\dot{\theta}^2. \end{cases} \quad (21)$$

For $r = const.$, eq. (16) can be simplified as $\dot{\vec{r}} = \vec{\omega} \times \vec{r}$ and for $\delta = const.$ eq. (19) as $\dot{\vec{r}} = -g\Omega_s \vec{r} + \vec{\Omega}_s \times \vec{r}$, where $\vec{\Omega}_s = \dot{\theta} \hat{k}$, is defined the spiro-angular velocity.

The acceleration in polar coordinates is identified as³,

³ $\ddot{x} = d^2x / dt^2$



$$(1) \begin{cases} \ddot{x} = \ddot{r} \cos \varphi - 2\dot{r}\dot{\varphi} \sin \varphi - r\ddot{\varphi} \sin \varphi - r\dot{\varphi}^2 \cos \varphi, \\ \ddot{y} = \ddot{r} \sin \varphi + 2\dot{r}\dot{\varphi} \cos \varphi + r\ddot{\varphi} \cos \varphi - r\dot{\varphi}^2 \sin \varphi. \end{cases} \quad (22)$$

or in terms of the basis polar convectors

$$(1) \quad \ddot{\vec{r}} = (\ddot{r} - r\dot{\varphi}^2) \hat{g}_r + \left(2\frac{\dot{r}\dot{\varphi}}{r} + \ddot{\varphi} \right) \hat{g}_\varphi, \quad (23)$$

In spiral coordinates, the acceleration becomes

$$(2) \begin{cases} \ddot{x} = e^{\frac{\delta}{g} - g\theta} \left\{ \left[\left(\frac{\ddot{\delta}}{g} - g\ddot{\theta} \right) + \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right)^2 - (\dot{\delta} + \dot{\theta})^2 \right] \cos \alpha - \left[(\ddot{\delta} + \ddot{\theta}) + 2(\dot{\delta} + \dot{\theta}) \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \right] \sin \alpha \right\}, \\ \ddot{y} = e^{\frac{\delta}{g} - g\theta} \left\{ \left[\left(\frac{\ddot{\delta}}{g} - g\ddot{\theta} \right) - (\dot{\delta} + \dot{\theta})^2 + \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right)^2 \right] \sin \alpha + \left[(\ddot{\delta} + \ddot{\theta}) + 2(\dot{\delta} + \dot{\theta}) \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \right] \cos \alpha \right\}, \\ \alpha = \delta + \theta. \end{cases} \quad (24)$$

or in terms of the basis spiral convectors

$$(2) \quad \ddot{\vec{r}} = \left[\ddot{\delta} + \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \dot{\delta} - g\dot{\theta}(\dot{\delta} + \dot{\theta}) \right] \hat{g}_\delta + \left[\ddot{\theta} + \dot{\delta} \frac{\dot{\delta} + \dot{\theta}}{g} + \dot{\theta} \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \right] \hat{g}_\theta, \quad (25)$$

For the uniform circular motion, eq. (23) reduces to

$$(1) \quad \begin{cases} \ddot{\vec{r}} = -R\omega^2 \hat{g}_R, \\ \hat{g}_R \equiv \hat{g}_r = \cos \varphi \hat{i} + \sin \varphi \hat{j}, \\ a_c = |\ddot{\vec{r}}| = R\omega^2 = \frac{v_c^2}{R}, \\ v_c = |\dot{\vec{r}}| = R\omega. \end{cases} \quad (26)$$

Whilst, for the uniform spiral motion eq. (25) reduces to $\ddot{\vec{r}} = -g \left(\dot{\theta}^2 \hat{g}_{\delta_0}(\delta_0) + (\ddot{\theta} - g\dot{\theta}^2) \hat{g}_\theta(\delta_0) \right)$.

It appears clear that the spiro-centrifugal acceleration has two components, the first along the direction of movement \hat{g}_θ and the second one orthogonal to it.

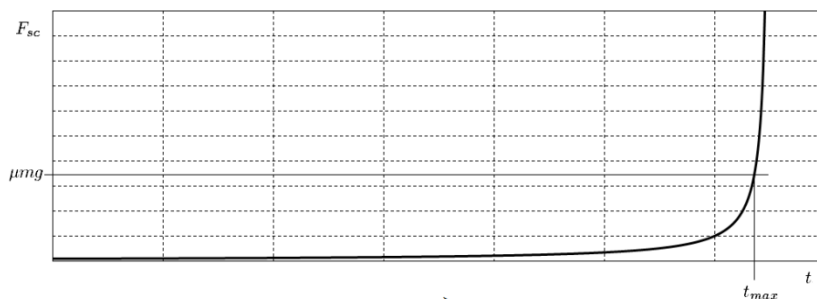
According to eq. (21), acceleration for uniform spiral motion is reduced to



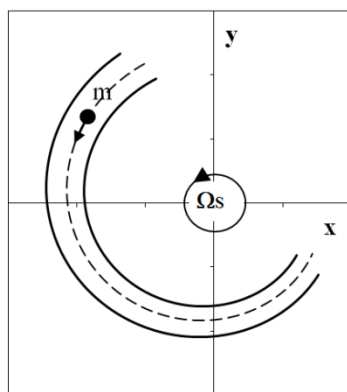
$$(2) \begin{cases} \ddot{\vec{r}} = -g\dot{\theta}^2 \hat{g}_{\delta_0} = -\frac{K_1^2}{g(K_2 - K_1 t)^2} \hat{g}_{\delta_0}, \\ \hat{g}_{\delta_0} = g(K_2 - K_1 t) e^{\frac{\delta_0}{g}} \left\{ \left[\frac{1}{g} \cos \alpha^*(t) - \sin \alpha^*(t) \right] \hat{i} + \left[\frac{1}{g} \sin \alpha^*(t) + \cos \alpha^*(t) \right] \hat{j} \right\}, \\ \alpha^*(t) = \delta_0 - \frac{1}{g} \ln(K_2 - K_1 t) - \frac{1}{g} \ln g, \\ a_{sp} = |\ddot{\vec{r}}| = \frac{K_1^2 \sqrt{1+g^2} e^{\frac{\delta_0}{g}}}{g(K_2 - K_1 t)}, \\ v_{sp} = |\dot{\vec{r}}| = K_1 e^{\frac{\delta_0}{g}} \sqrt{1+g^2}. \end{cases} \quad (27)$$

Taking into account for eq. (21) and eq. (25), it is possible to rewrite the spirocentrifugal acceleration for uniform spiral motions as

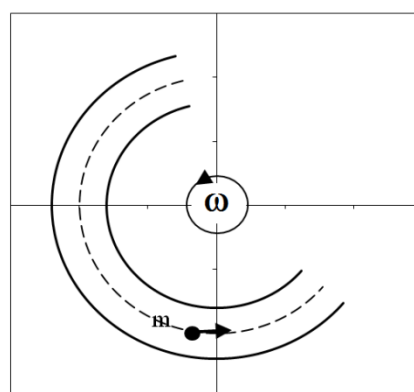
$$(2) \begin{cases} \ddot{x} = e^{\frac{\delta_0}{g}} \frac{K_1^2}{g^2 (K_2 - K_1 t)} [g \sin \alpha^*(t) - \cos \alpha^*(t)], \\ \ddot{y} = e^{\frac{\delta_0}{g}} \frac{K_1^2}{g^2 (K_2 - K_1 t)} [-g \cos \alpha^*(t) - \sin \alpha^*(t)]. \end{cases} \quad (28)$$



a)



b)



c)

Figure 2: a) centrifugal Force versus time b) mass m moving on a stretch of spiral curve and c) circular curve.



Comparing the eq. (26) and eq. (27) it is possible to notice that, while in the first case the centrifugal acceleration is constant, in the second it depends on the time. Consequently, a mass in a circular curve will experience a constant centrifugal force perpendicular to the direction of uniform motion, while a mass in a spiral curve will experience a spiro-centrifugal force perpendicular to the direction of motion as a function of time.

A constant centripetal force is required to maintain uniform circular motion, whereas a time-dependent spiro-centripetal force is required to maintain uniform spiral motion. In many real physical cases, such as the motion of a car in a turn [13], the centrifugal force F_c is compensated by the friction force $F_{cp} = \mu mg$ which is generated between the tire and the road surface and depends on the mass itself and on the gravitational acceleration g . When a vehicle is cornering, the centrifugal force acting outwards must be balanced by lateral forces on the wheels so that the vehicle is able to follow the curve of the road,

$$(1) \quad F_c = m\omega^2 R \leq \mu mg,$$

$$(2) \quad F_{sc} = me^{\frac{\delta_0}{g}} \frac{K_1^2 \sqrt{1+g^2}}{g(K_2 - K_1 t)} \leq \mu mg. \quad (29)$$

Observing eq.(29) (2), we notice that there is always a maximum time to exit a spiral curve for a "safe turning" avoiding lateral slippage, defined by

$$(2) \quad t_{\max} = \frac{K_2}{K_1} - \frac{K_1 \sqrt{1+g^2}}{\mu g^2} e^{\frac{\delta_0}{g}}. \quad (30)$$

The spiral-polar uniform motion

We now consider a point-like body of mass m which is characterized by constant spiral path $\delta = \delta_0, \dot{\psi} \neq 0$, and the constant kinetic energy $T = T_0$ where T_0 is a constant.

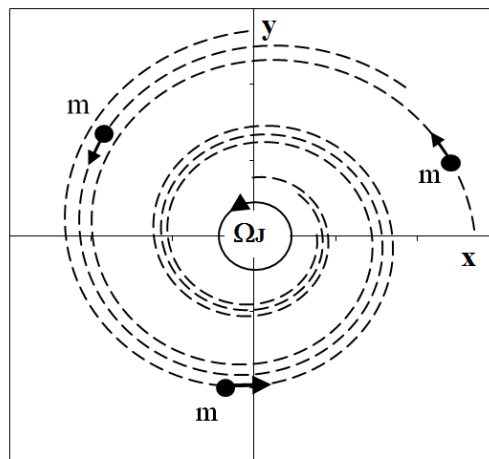


Figure 3: The spiral-polar motion of a mass m .

The spiral-polar motion is composed of the spiral motion $\theta(t)$ superimposed to the rotation $\psi(t)$. In terms of coordinates, if we identify the trajectory of the body with $(\delta(t), \theta(t))$ we have



$$(2) \begin{cases} x(t) = e^{\frac{\delta(t)}{g} - g\theta(t)} \cos \gamma(t), \\ y(t) = e^{\frac{\delta(t)}{g} - g\theta(t)} \sin \gamma(t), \\ \gamma(t) = \delta(t) + \theta(t) + \psi(t). \end{cases} \quad (31)$$

The velocity is characterized by

$$(2) \begin{cases} \dot{x} = e^{\frac{\delta}{g} - g\theta} \left\{ \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \cos \gamma - \dot{\gamma} \sin \gamma \right\}, \\ \dot{y} = e^{\frac{\delta}{g} - g\theta} \left\{ \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \sin \gamma + \dot{\gamma} \cos \gamma \right\}, \\ \dot{\gamma} = \dot{\delta} + \dot{\theta} + \dot{\psi}. \end{cases} \quad (32)$$

using the following relationships

$$(2) \begin{cases} \dot{\hat{g}}_{\theta}(\psi) = \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \hat{g}_{\theta}(\psi) - g\dot{\gamma} \hat{g}_{\delta}(\psi), \\ \dot{\hat{g}}_{\delta}(\psi) = \left(\frac{\dot{\delta}}{g} - g\dot{\theta} \right) \hat{g}_{\delta}(\psi) + \frac{\dot{\gamma}}{g} \hat{g}_{\theta}(\psi). \end{cases} \quad (33)$$

Eq. (32) can be rewritten as

$$(2) \begin{cases} \dot{\vec{r}} = \Omega_1 \vec{r} + \Omega_J \times \vec{r}, \\ \Omega_1 = \frac{\dot{\delta}}{g} - g\dot{\theta}, \\ \vec{\Omega} = (\dot{\delta} + \dot{\theta}) \hat{k}, \\ \vec{\Omega}_L = \dot{\psi} \hat{k}, \quad \vec{\Omega}_J = \vec{\Omega} + \vec{\Omega}_L, \\ \dot{\vec{r}} = \left[\dot{\delta} + \frac{g^2 \dot{\psi}}{1+g^2} \right] \hat{g}_{\delta} + \left[\dot{\theta} + \frac{\dot{\psi}}{1+g^2} \right] \hat{g}_{\theta}. \end{cases} \quad (34)$$

Results and Discussion

The polar and the spiral Coriolis forces

Imagine that we have two right-handed frames, the inertial frame F with origin O and the basis vectors $\hat{i}, \hat{j}, \hat{k}$ and frame F' with origin O' and the basis vectors $\hat{i}', \hat{j}', \hat{k}'$.

Now, let's consider a point P in space, its position in frame is

$$\overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}, \quad (35)$$



and in frame F'

$$\overline{O'P} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}', \quad (36)$$

we also note that $\overline{OP} = \overline{O'O} + \overline{O'P}$,

The velocity observed in the inertial frame F is

$$\vec{v} = \frac{d\overline{O'O}}{dt} + \frac{d\overline{O'P}}{dt} = \left(\frac{dx_{O'}}{dt} \hat{i} + \frac{dy_{O'}}{dt} \hat{j} + \frac{dz_{O'}}{dt} \hat{k} \right) + \left(\frac{dx'}{dt} \hat{i}' + \frac{dy'}{dt} \hat{j}' + \frac{dz'}{dt} \hat{k}' \right) + \left(x' \frac{d\hat{i}'}{dt} + y' \frac{d\hat{j}'}{dt} + z' \frac{d\hat{k}'}{dt} \right). \quad (37)$$

If the frame F' rotates uniformly with constant angular velocity $\vec{\omega}$ in respect to the inertial frame F then eq. (37), according to eq. (16), becomes

$$(1) \quad \begin{cases} \vec{v} = \vec{v}_{O'} + \vec{v}' + \vec{\omega} \times \overline{O'P}, \\ \vec{v}' = \frac{dx'}{dt} \hat{i}' + \frac{dy'}{dt} \hat{j}' + \frac{dz'}{dt} \hat{k}', \\ \vec{\omega} \times \overline{O'P} \equiv x' \frac{d\hat{i}'}{dt} + y' \frac{d\hat{j}'}{dt} + z' \frac{d\hat{k}'}{dt}. \end{cases} \quad (38)$$

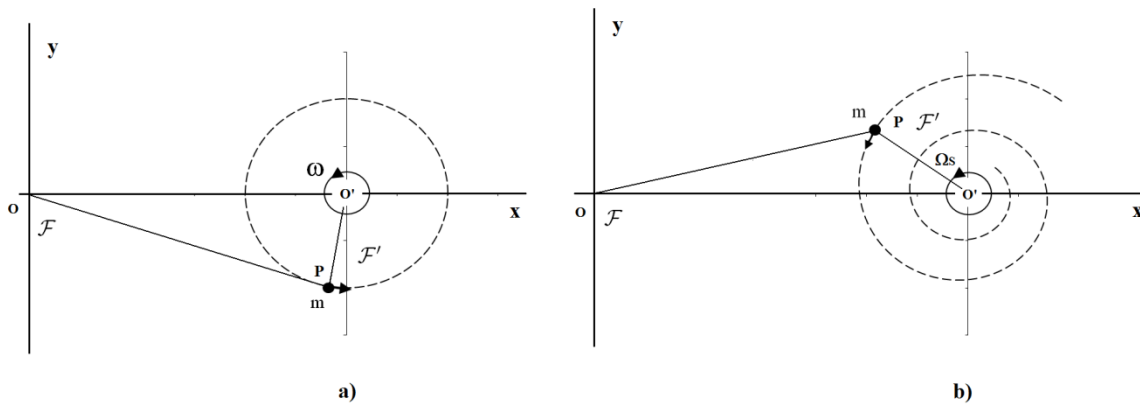


Figure 4: Coriolis relative motion a) Circular b) Spiral.

If each point of the frame F' moves with uniform spiral motion with a spirallangular velocity $\vec{\Omega}_s (\Omega_s e^{-g\theta} = K, \Omega_s = \dot{\theta})$ with respect to the inertial frame F , then the eq. (37) according to eq. (19), becomes



$$(2) \quad \left\{ \begin{array}{l} \vec{v} = \vec{v}_{O'} + \vec{v}' + \vec{\Omega}_s \times \overline{O'P} - g\Omega_s \overline{O'P}, \\ \vec{\Omega}_s \times \overline{O'P} - g\Omega_s \overline{O'P} \equiv x' \frac{d\hat{i}'}{dt} + y' \frac{d\hat{j}'}{dt} + z' \frac{d\hat{k}'}{dt}, \\ \vec{v}' = \frac{dx'}{dt} \hat{i}' + \frac{dy'}{dt} \hat{j}' + \frac{dz'}{dt} \hat{k}', \\ \frac{d\hat{i}'}{dt} = \vec{\Omega}_s \times \hat{i}' - g\Omega_s \hat{i}', \\ \frac{d\hat{j}'}{dt} = \vec{\Omega}_s \times \hat{j}' - g\Omega_s \hat{j}', \\ \frac{d\hat{k}'}{dt} = \vec{\Omega}_s \times \hat{k}' - g\Omega_s \hat{k}'. \end{array} \right. \quad (39)$$

Since δ is constant, the effect on the axis $\hat{i}', \hat{j}', \hat{k}'$ due to the uniform spiral motion is that of a partial rotation θ .

If we define

$$\left\{ \begin{array}{l} \vec{a}' = \frac{d^2x'}{dt^2} \hat{i}' + \frac{d^2y'}{dt^2} \hat{j}' + \frac{d^2z'}{dt^2} \hat{k}', \\ \vec{a}_{O'} = \frac{d\vec{v}_{O'}}{dt}. \end{array} \right. \quad (40)$$

taking into account the eq. (38) for the uniform circular motion, the acceleration

$$(1) \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_{O'}}{dt} + \frac{d\vec{v}'}{dt} + \frac{d(\vec{\omega} \times \overline{O'P})}{dt}. \quad (41)$$

becomes

$$(1) \quad \left\{ \begin{array}{l} \vec{a} = \vec{a}' + \vec{a}_{O'} + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \overline{O'P}), \\ \frac{d\vec{v}'}{dt} = \vec{a}' + \vec{\omega} \times \vec{v}', \\ \frac{d\overline{O'P}}{dt} = \vec{v}' + \vec{\omega} \times \overline{O'P}, \\ \frac{d\vec{\omega}}{dt} = 0. \end{array} \right. \quad (42)$$

Taking into account the eq. (39) for the case of uniform spiral motion, the acceleration

$$(2) \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_{O'}}{dt} + \frac{d\vec{v}'}{dt} + \frac{d(\vec{\Omega}_s \times \overline{O'P})}{dt}. \quad (43)$$

becomes



$$(2) \quad \begin{cases} \vec{a} = \vec{a}' + \vec{a}_o + 2\vec{\Omega}_s \times \vec{v}' + \vec{\Omega}_s \times \vec{\Omega}_s \times \overline{O'P} - 2g\Omega_s \vec{v}' - g\Omega_s \vec{\Omega}_s \times \overline{O'P}, \\ \frac{d\vec{v}'}{dt} = \vec{a}' + \vec{\Omega}_s \times \vec{v}' - g\Omega_s \vec{v}', \\ \frac{d\overline{O'P}}{dt} = \vec{v}' + \vec{\Omega}_s \times \overline{O'P} - g\Omega_s \overline{O'P}, \\ \frac{d\vec{\Omega}_s}{dt} = g\Omega_s \vec{\Omega}_s, \\ \vec{\Omega}_s \times \vec{v}' - g\Omega_s \vec{v}' = \frac{dx'}{dt} \frac{d\hat{i}'}{dt} + \frac{dy'}{dt} \frac{d\hat{j}'}{dt} + \frac{dz'}{dt} \frac{d\hat{k}'}{dt}. \end{cases} \quad (44)$$

As can be observed from eq. (44), the fictitious forces are four:

- 1) the Coriolis force $\vec{F}_{Co} = 2m\vec{\Omega}_s \times \vec{v}'$, acting perpendicular to the direction of motion,
- 2) the spiral-centrifugal force $\vec{F}_{ce} = m\vec{\Omega}_s \times \vec{\Omega}_s \times \overline{O'P}$ with one component along the direction of motion and another perpendicular to it.
- 3) the spiral force $\vec{F}_{sp1} = 2mg\Omega_s \vec{v}'$ which acts along the direction of motion.
- 4) the spiral force $\vec{F}_{sp2} = mg\Omega_s \vec{\Omega}_s \times \overline{O'P}$ with one component along the direction of motion and another perpendicular to it.

The spiral-polar coriolis Force

If each point of the frame F' is spiro-rotating ($\delta = \delta_0$) with spiral-angular velocity $\vec{\Omega}_s$ ($\Omega_s = \dot{\theta}$) and angular velocity $\Omega_L = \dot{\psi}$ ($\Omega_J = \Omega_L + \Omega_s$) in respect to the inertial frame F then eq. (37) according to eq. (34) becomes

$$(2) \quad \begin{cases} \vec{v} = \vec{v}_o + \vec{v}' + \vec{\Omega}_J \times \overline{O'P} - g\Omega_s \overline{O'P}, \\ \vec{v}' = \frac{dx'}{dt} \hat{i}' + \frac{dy'}{dt} \hat{j}' + \frac{dz'}{dt} \hat{k}', \\ \vec{\Omega}_J \times \overline{O'P} - g\Omega_s \overline{O'P} \equiv x' \frac{d\hat{i}'}{dt} + y' \frac{d\hat{j}'}{dt} + z' \frac{d\hat{k}'}{dt}, \\ \Omega_1 = -g\dot{\theta} = -g\Omega_s. \end{cases} \quad (45)$$

and the acceleration

$$(2) \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_o}{dt} + \frac{d\vec{v}'}{dt} + \frac{d(\vec{\Omega}_J \times \overline{O'P})}{dt} - g \frac{d(\Omega_s \overline{O'P})}{dt}. \quad (46)$$

becomes



Spiral trajectories are currently used by airplanes and spacecrafts, as cars follow spiral curves in their movement. The spiral dynamics allow an analytical study of the forces acting on the moving masses along spiral trajectories.

Moreover, the spiral dynamics could be used to study the motion of stellar bodies in spiral galaxies and their gravitational waves.

Applications can be countless and many new tools can be designed.

Conflicts of Interest

The author declares that there is no conflict of interests regarding the publication of this paper.

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