Can The Spectra Of Hermitian Operator Be Invariant Under x $\rightleftharpoons$ p ? :Case Study :1D General Oscillator

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Abstract

We address an intriguing question on spectral invariance in quantum mechanics on exchange of co-ordinate and momentum x $\rightleftharpoons$ p considering general oscillator as an example.

Keywords

exchange of co-ordinate and momentum , spectral invariance , general oscillator .

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I.Introduction

In a recent paper Rath and Mallick [1] proposed a generalised model on co-ordinate and momentum transformation in the case of Harmonic Oscillator to reflect the spec-tral invariance. Further it is well known that commutation relation

$$[x, p] = i \tag{1}$$

between co-ordinate (x) and momentum (p) on exchange(x $\rightleftharpoons$ p) becomes

$$[p, x] = -i \tag{2}$$

.Now question arrises whether spectra of Hermitian operator (more precisely self-adjoint operator) be invariant under exchange of co-ordinate and momentum ?. If the answer to this case is yes , then why not address this to some model Hermitian operator. . In this context we would like to state that in the past there was a considerable interest among many others to study spectra of anharmonic oscillator [2-9] . In any way we consider a more general type of oscillator [2-9]

$$h = \mu p^2 + \lambda_1 x^2 + \lambda_2 x^4 + \lambda_3 x^6 \tag{3}$$

and study its spectra on exchanhe of co-ordinate and momentum.

II.New Operator and Spectra

Here we consider the operator

$$H = \mu x^2 + \lambda_1 p^2 + \lambda_2 p^4 + \lambda_3 p^6 \tag{4}$$

In order to solve it we use the eigenvalue relation[2,6,8,10,11]

$$H|\Psi> = \in |\Psi> \tag{5}$$

where

$$|\psi >= \sum A_m|m > \quad (6)$$

In the above |m > stands for standard harmonic oscillator wave function[10,11] sat-isfying the relation

$$[p^2 + x^2]|m >= (2m + 1)|m > \quad (7)$$

Now using the above relation one will notice that A_M satisfies the following recurrence relation

$$P_m A_{m-6} + Q_m A_{m-4} + R_m A_m + S_m A_m + T_m A_{m+2} + U_m A_{m+4} + V_m A_{m+6} = 0 \quad (8)$$

where

$$P_m =< m - 6|H|m > \quad (9)$$

$$Q_m =< m - 4|H|m > \quad (10)$$

$$R_m =< m - 2|H|m > \quad (11)$$

$$S_m =< m|H|m > \quad (12)$$

$$T_m = R_{m+2} \quad (13)$$

$$U_m = Q_{m+4} \quad (14)$$

$$V_m = P_{m+4} \quad (15)$$

The eigen values calculated using this relation using matrix diagonalisation method [8,10,11] are tabulated in table-1.

Tale-1: Eigenvalues of New Operator and Comparision.

n	$H = x^2 - 100p^2 + p^4$	$h = p^2 - 100x^2 + x^4$ [7]
0	- 2485.867 880 343	-2485.867 880 343
1	- 2485.867 880 343	-2485.867 880 343
2	-2457.643 822 699	-2457.643 822 699
3	-2457.643 822 699	-2457.643 822 699
n	$H = x^2 - 2p^2 - 2p^4 + p^6$	$h = p^2 - 2x^2 - 2x^4 + x^6$[2,8]
0	-1.000 000	-0.999 987
1	-0.154 110	-0.154 093
2	3.629 625	3.629 880
3	8.007 560	8.007 742
n	$H = x^2 + p^4$	$h = p^2 + x^4$[9,8]
0	1.060 362 090	1.060 362 090
1	3.799 073 029	3.799 073 029
2	7.455 697 937	7.455 697 973
3	11.644 745 511	11.644 745 511

III.Conclusion

In one-dimensional general oscillator considered above we notice that hermitian operator has an equivalent operator whose eigenspectra remain invariant . Further we plot the $|\Psi_{N=0-3}|^2$ corresponding to Hamiltonian

$$H = x^2 - 2p^2 - 2p^4 + p^6 \tag{16}$$

in fig-1 . Similarly we plot the $|\Phi_{N=0-3}|^2$ corresponding to Hamiltonian

$$h = p^2 - 2x^2 - 2x^4 + x^6 \tag{17}$$

in fig-2. From the figs it is claer that eventhough two systems are iso-spectral in nature but different from each other.

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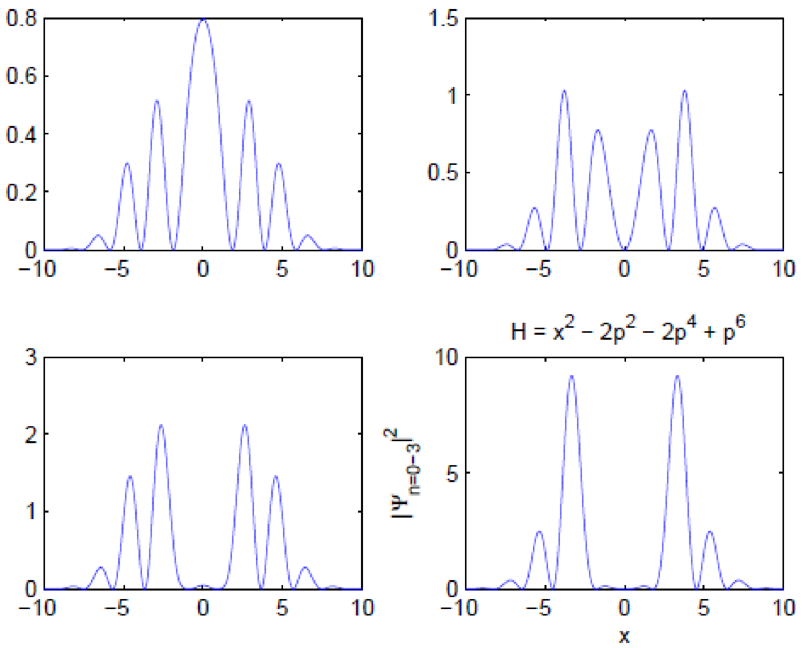

Figure 1: $H = x^2 - 2p^2 - 2x^4 + x^6$

: $|\Psi_{n=0-3}|^2$ of Equivalent Sextic Oscillator

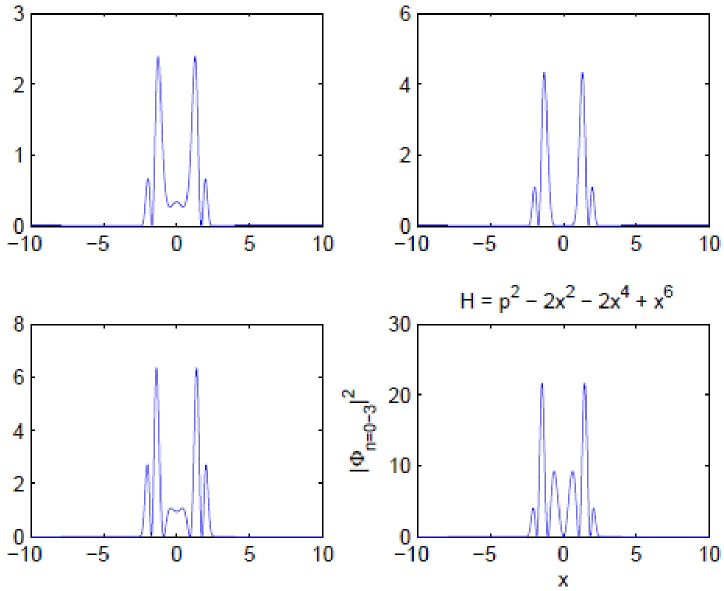

Figure 2: $h = p^2 - 2x^2 - 2x^4 + x^6$
: $|\Phi_{n=0-3}|^2$ of Sextic Well Oscillator