ISSN 2347-3487

Calculation the Cross Sections for 6Li(α,p)9Be Reaction by Reverse Reaction

Firas Mahmood Hady
Physics Department, College of Education for Pure Sciences (Ibn Al-Haitham),
Baghdad University, Baghdad-Iraq.
firas_1962@yahoo.com

ABSTRACT:

In this study light elements ^{6}Li , ^{9}Be , ^{10}Be for $^{6}Li(\alpha , p)^{9}Be$ reaction with proton energy from (27.5) MeV to (67.5) MeV with threshold energy (2.3626) MeV are used according to the available data of reaction cross sections. The Q-value is equale (2.125MeV) and parity of (^{9}Be =3/2^{+}) ,(B =3^{+}) and (Li = 1^{+}) for the ground state .The more recent cross sections data of $^{6}Li(\alpha,p)^{9}Be$ reaction is reproduced in fine steps and by using (Matlab-7.6-2008a) program and get the equation from 3-degree for plotted. We deduced that the high probability to produced ^{9}Be by bombard ^{6}Li by alpha particle .

Keywords: The cross sections , nuclear reactions , reverse reaction , compound nucleus, ^{6}Li(α,p)^{9}Be reaction , ^{9}Be(p,α)^{6}Li reaction.

Council for Innovative Research

Peer Review Research Publishing System

Journal: Journal of Advances in Physics

Vol 4, No. 2

editor@cirjap.com, japeditor@gmail.com

www.cirjap.com

ISSN 2347-3487

1-THEORY:

The cross section of compound nucleus is given by [1]:

$$\sigma = \frac{\pi}{k^2} g \frac{\Gamma^2}{(\mathrm{E}-\mathrm{E}_R)^2 + \Gamma^2/4} \qquad \text{-----------------------(1)}$$

Where g is a statistical factors.

E **is the kinetic energy of an incident particle.**

E_R **is a single isolated resonance energy.**

Γ **is the width of the state.**

k is the wave number which is given by:

$$k = \frac{1}{\lambda\!\!\!^{-}} = \frac{p}{\hbar} = \frac{mv}{\hbar} = \frac{\sqrt{2mE}}{\hbar} \qquad \text{------------------(2)}$$

Where $\lambda\!\!\!^{-}$ **is the de-Broglie wavelength divided by 2π of incident particle.**

$\hbar$ **is the Plank constant divided by 2π.**

p **is the momentum of an incident particle.**

$m \, and \, v$ **are the mass and velocity of an incident particle.**

The statistical g-factors is given by [1]:

$$g = \frac{2I_c + 1}{(2S_a + 1)(2S_X + 1)} \qquad \text{-------- (3)}$$

Where I is the total angular momentum of the resonance (compound nucleus) which is given by:

$$I_c = S_a + S_X + \ell_a \qquad \text{-------- (4)}$$

Where S_a **is the spin of the incident particle.**

S_X **is the spin of the target.**

ℓ_a **is the orbital angular momentum of incident particle.**

The total width of the state is the sum of the partial widths [2,3]:

$$\Gamma = \sum_i \Gamma_i \qquad \text{------- (5)}$$

or

$$\Gamma = \frac{\hbar}{\tau} \qquad \text{-------- (6)}$$

Where τ **is the lifetime of any decay state;**

At resonance $\mathrm{E} = \mathrm{E}_R$

The Γ^2 **in equation (1) is directly related to the formation of the resonance and to its probability to decay into a particular exit channel. That is, for the reaction a+X = b+Y, a different exit width must be used [4]:**

$$\sigma = \frac{\pi}{k^2} g \frac{\Gamma_{aX}\Gamma_{bY}}{(E-E_R)^2 + \Gamma_{aX}\Gamma_{bY}/4} \qquad \text{------- (7)}$$

Where Γ_{aX} is the partial width for decay into a+X.

Γ_{bY} is the partial width for a different exit.

Equation (7) is the Breit-Wigner formula for the shape of a single, isolated resonance.

At resonance $E = E_R$ and $\Gamma_{aX}\Gamma_{bY} = \Gamma^2$ since $\Gamma_{aX} = \Gamma_{bY} = \Gamma$, we call Γ_{aX} the partial width for decay into a+X and Γ_{bY} the partial width for any other channels energetically allowed, then equation (7) becomes:

$$\sigma = \frac{4\pi}{k^2} g \qquad \text{------- (8)}$$

The basic assumption of the compound nucleus model is that the compound nucleus has been formed in such a complicated set of interactions that it does not remember the initial stage of formation. The cross sections for the reaction X(a,b)Y can be split into a formation cross section of the compound nucleus [C.N.]* corresponding to the process:

a + X → [C.N.]* → Y + b -------- **(9)**

And the fractional probability that [C.N.]* breaks up into particles b+Y. We can therefore write [5].

$$\sigma(a,b) = \sigma_{a,c}(T_0)Pb(E) \qquad \text{------- (10)}$$

Where T_0 : bombarding energy in center of mass.

E : corresponding excitation energy of the compound nucleus.

Pb(E) : Fractional probability of [C.N.]* to break up into Y+b.

2-REVERSE REACTION :

If the cross-sections of the reaction A(α,p)B are measured as a functions of Tα (T_α = Kinetic energy of α-particle) the cross –sections of the inverse reaction B(p,α)A can be calculated as a function of Tp (T_p = Kinetic energy of proton) using the reciprocity theorem [6] which states that :

$$\frac{\sigma_{(\alpha,p)}}{g_{\alpha,p}\,\lambda_{\alpha}^{2}} = \frac{\sigma_{(p,\alpha)}}{g_{p,\alpha}\,\lambda_{p}^{2}} \qquad \text{--------- (11)}$$

Where σ(α,p) and σ(n,p) represent cross- sections of (α,p) and (p,α) reactions respectively , g is a statistical factor and λ is the de–Broglie wave length divided by 2π and is given by

$$\lambda = \frac{\hbar}{MV} \qquad \text{----------- (12)}$$

Where ħ is Dirac constant (h /2π) , h is plank constant , M and V are mass and velocity of alpha or proton .

From eq.(12) ,we have

$$\lambda^2 = \frac{\hbar^2}{2\,MT} \qquad \text{--------- (13)}$$

ISSN 2347-3487

The statistical g-factors are givens by [6]

$$g_{\alpha,p} = \frac{2J_c + 1}{(2I_A + 1)(2I_\alpha + 1)} \quad \text{--------- (14)}$$

and

$$g_{p,\alpha} = \frac{2J_c + 1}{(2I_B + 1)(2I_p + 1)} \quad \text{---------- (15)}$$

The conservation low of the momentum and parity implique that:

$$I_A + I_\alpha = J_c = I_B + I_p \quad \text{---------- (16)}$$

$$\pi_A \,.\, \pi_\alpha (-1)^{\ell\alpha} = \pi_c = \pi_B \,.\, \pi_p (-1)^{\ell p} \quad \text{---------- (17)}$$

J_c and π_c are total angular momentum and parity of the compound nucleus .

I_A and π_A are total angular momentum and parity of nucleus A.

I_B and π_B are total angular momentum and parity of nucleus B.

I_α and π_α are total angular momentum and parity of α-particle.

I_p and π_p are total angular momentum and parity of proton .

$$\pi_\alpha = \pi_p = +1 \quad \text{---------- (18)}$$

$$I_\alpha = s_\alpha + \ell_\alpha \quad \text{---------- (19)}$$

Where

I_α is the total angular momentum of alpha particle s_α is spin of α-particle = 0

ℓ_α is the orbital angular momentum of α-particle

$$I_p = s_p + \ell_p \quad \text{---------- (20)}$$

Where

I_p is the total angular momentum of the proton

s_p is spin of proton = 1/2

ℓ_p is the orbital angular momentum of proton

From eq.(16) ,we have :

$$| Jc - IA | \leq I\alpha \leq | Jc + IA | \quad \text{---------- (21)}$$

$$| Jc - IB | \leq Ip \leq | Jc + IB | \quad \text{---------- (22)}$$

ISSN 2347-3487

The reactions A(α,p)B and B(p, α) can be represented with the compound nucleus C as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the proton and alpha particle . One can calculate the separation energies of α-particle (S_α) and proton (S_p) using the following relations:

$$E = S_\alpha + \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{---------- (23a)}$$

$$E = S_p + \frac{M_B}{M_B + M_p} T_p \quad \text{---------- (23b)}$$

$$S_\alpha = 931.5 \left[M_A + M_\alpha - M_c \right] \quad \text{---------- (24)}$$

$$S_p = 931.5 \left[M_B + M_p - M_c \right] \quad \text{---------- (25)}$$

Combining (23a) , (23b) , (24) and (25)

and as the Q- value of the reaction A(α,p)B is given by :

$$Q = 931.5 \left[MA + M\alpha - MB - Mp \right] \quad \text{---------- (26)}$$

$$Q = \frac{M_B}{M_B + M_p} T_p - \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{---------- (27)} \qquad \text{Or :}$$

$$T_p = \frac{M_B + M_p}{M_B} \left[\frac{M_A}{M_A + M_\alpha} T_\alpha + Q \right] \quad \text{-------- (28)}$$

The threshold energy E_{th} is given by

$$E_{th} = - Q \frac{M_A + M_\alpha}{M_A} \quad \text{-------- (29a)}$$

ISSN 2347-3487

$$Q = -\frac{M_A}{M_A + M_\alpha} E_{th} \quad \text{-------- (29b)}$$

Then

$$T_p = \frac{M_B + M_p}{M_B} * \frac{M_A}{M_A + M_\alpha} (T_\alpha - E_{th}) \quad \text{-------- (30)}$$

Thus eq . (11) can be written as follows :

$$\sigma (p,\alpha) = \frac{g_{p,\alpha} \; M_\alpha \; T_\alpha}{g_{\alpha,p} \; M_p \; T_p} \sigma (\alpha,p) \quad \text{-------- (31)}$$

3- RESULTS AND DISCUSSION:

By using semi empirical formula the evaluated cross sections as a function of proton energy from (27.5)MeV to (67.5)MeV of present work are listed in table (1). From these data which were plotted and we get the mathematical equation representing the cross sections distribution in the indicated range of proton energy Fig.(1) and percentage error ($\pm$ 0.1819) mbarn as follows :

y = - 6.1e-010*x^{3} + 1.5e-007*x^{2} - 1.3e-005*x + 0.0004

y=cross sections of(p,α) x=proton energy(T_p)

In fig.(1)[7] we observed that the cross sections were smoothly decreased and the maximum cross section(0.4492mbarn) when proton energy is equal (T_p=27.5MeV).

By using the compound theory we derived the mathematical formula from $^9Be(p,\alpha)^6Li$ reaction for ground state to get the cross sections of $^6Li(\alpha,p)^9Be$ reaction:

$$\sigma_{\alpha,p} = 0.2685 \frac{T_p}{T_\alpha} \sigma_{p,\alpha} \quad \text{--------- (32)}$$

We calculated the cross sections of alpha energy with energy range between (37.6533MeV) to (96.0714MeV) are (0.8452mbarn)to (0.027mbarn) respectively . These data are plotted in fig.(2) and listed in table(2) . We observed that the high probability to produced 9Be by bombard 6Li with alpha energy is (37.6533MeV) and we get semi empirical formula with three-degree as follow:

y = - 6.1e-010*x^{3} + 1.5e-007*x^{2} - 1.3e-005*x + 0.0004

Table 1 :The cross sections of $^{9}Be(p,\alpha)^{6}Li$ reaction with threshold energy of (2.3626MeV) [7]

Proton Energy(MeV)	Cross Sections(mbarn)	Proton Energy(MeV)	Cross Sections(mbarn)
27.5	0.4492	49.5	0.0674
28.5	0.4159	50.5	0.0622
29.5	0.3825	51.5	0.057
30.5	0.3491	52.5	0.0519
31.5	0.3157	53.5	0.048
32.5	0.2824	54.5	0.0442
33.5	0.2646	55.5	0.0403
34.5	0.2468	56.5	0.0365
35.5	0.2291	57.5	0.0326
36.5	0.2113	58.5	0.0305
37.5	0.1935	59.5	0.0284
38.5	0.1776	60.5	0.0263
39.5	0.1617	61.5	0.0242
40.5	0.1458	62.5	0.0221
41.5	0.1299	63.5	0.0202
42.5	0.114	64.5	0.0182
43.5	0.1068	65.5	0.0163
44.5	0.0995	66.5	0.0144
45.5	0.0923	----	----
46.5	0.085	----	----
47.5	0.0777	----	----
48.5	0.0726	----	----

Table 2 :The cross sections $^{6}Li(\alpha,p)^{9}Be$ reaction (the present work)

Alpha Energy(MeV)	Cross Sections(mbarn)	Alpha Energy(MeV)	Cross Sections(mbarn)
37.6533	0.8452	78.0966	0.0831
39.1512	0.7824	79.5945	0.0758
40.6491	0.7196	81.0924	0.0686
42.147	0.6568	82.5903	0.0614
43.6449	0.5941	84.0882	0.0574
45.1428	0.5313	85.5861	0.0535
46.6407	0.4978	87.084	0.0495
48.1386	0.4644	88.5819	0.0456
49.6365	0.431	90.0798	0.0416
51.1344	0.3975	91.5777	0.038
52.6323	0.3641	93.0756	0.0343
54.1302	0.3342	94.5735	0.0307
55.6281	0.3043	96.0714	0.027
57.126	0.2744	----	----

58.6239	0.2445	----	----
60.1218	0.2146	----	----
61.6197	0.2009	----	----
63.1176	0.1872	----	----
64.6155	0.1736	----	----
66.1134	0.1599	----	----
67.6113	0.1463	----	----
69.1092	0.1365	----	----
70.6071	0.1268	----	----
72.105	0.117	----	----
73.6029	0.1073	----	----
75.1008	0.0976	----	----
76.5987	0.0903	----	----

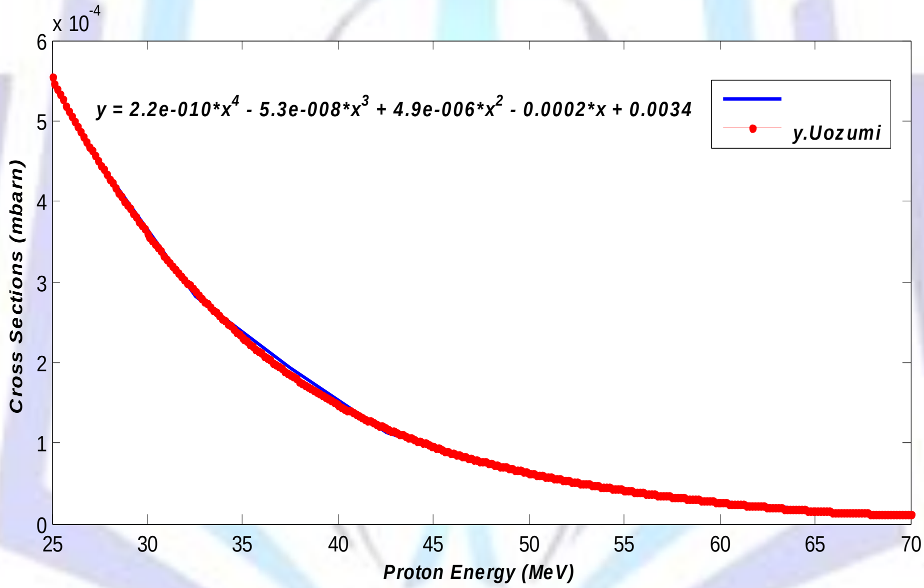

Fig 1: The cross sections of ^{9}Be(p, α)^{6}Li [7].

ISSN 2347-3487

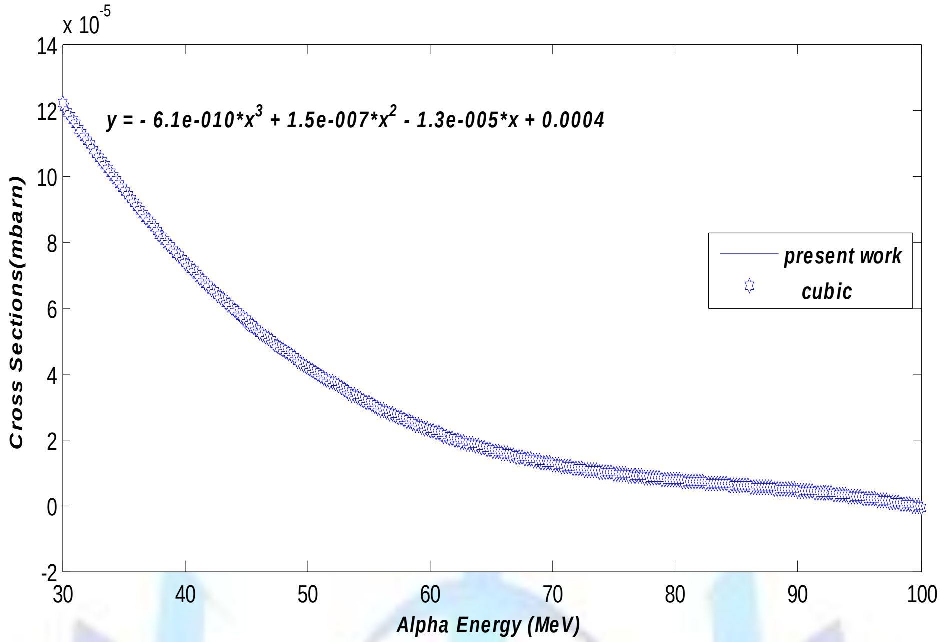

Fig 2:The cross sections of $^{6}Li(\alpha,p)^{9}Be$ reaction P.W.

REFERENCES :

[1] Krane K.S. , 1988. Introductory Nuclear Physics, John Wiley and Sons, PP.378-379.

[2] Cattingham W.N. and Green Wood D.A., 2004. An Introduction to Nuclear Physics, 2nd Edition, Cambridge, P.236.

[3] Burcham W.E., 1973. Nuclear Physics an Introduction, 2nd Edition, William Clowes and Sons, PP.344-395.

[4] Samuel S.M. Wong ,1990. Introductory Nuclei Physics, Prentice-Hall, PP.331-381 .

[5] Meyerhof W. E., 1967. Elements of Nucl. Phys. Mc Graw- Hill Book Inc.

[6] Macklin R.L. and Gibbons J.H. , 1968. Phys. Rev. 165 , 1147.

[7] Y.Uozumi ,P.Evtoukhovitch ,(2007) . Magnitude factor systematic of kalbach phenomenology for reactions emitting helium and lithium ions.

Author Biography

Firas Mahmood Hady(Assistant professor)

Tel: 009647702526869
firas_1962@yahoo.com

I am researcher in field of nuclear physics and have experience in teaching, researching in subjects of both applied and theoretical atomic physics and nuclear physics for more than 29 years .