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Slow seepage of polar fluids through porous media

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Abstract

A study of slow seepage of polar fluids through porous media is made using smoothed continuity equation and Darcy's equation in a porous medium. A transformation and approximate solution of the governing equations was carried out and its analysis showed that increase in both porosity and permeability result in an increase in the pressure of the fluid. Comparison with other studies also showed reasonable agreement.

Keywords

Polar fluids; Darcy's equation; Smoothed continuity equation; Porous media; Permeability; Fluid pressure

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1. Introduction

Fluids are sometimes found in the void spaces of earth materials. The amount of void space available for fluid flow is the effective porosity. The grain size, sorting, grain shape and clay and organic content of the earth materials determine the void spaces available for fluids to flow. The nature of the void spaces also gives an insight to the possible availability of natural resources. Porosity greatly affects fluid movement and exchange which are important for organisms that live in the soil. The transport of nutrients and contaminants will also be affected by porosity and because flow of fluids through porous media is common in nature and has many applications in engineering and earth sciences, its study cannot be overemphasized. The study of fluid flow through porous medium has been in conjunction with other parameters [1], [2], [3], [4], [5], [6], [7] [8], [9] and [10]. In the listed studies, a combination of continuity, Navier-Stokes and energy equations were used which in our view diminished the effect or otherwise of porosity and permeability in the various studies. Derivation of Darcy's law from Navier-Stokes equation through homogenization [11] is still not good enough for fluid flow description in porous medium in conjunction with other parameters. To effectively study fluids in porous media, a combination of smoothed continuity equation and Darcy's equation for fluid through porous media is of necessity [12]. Darcy's law is a phenomenologically derived constitutive equation that describes the flow of fluid through porous media. Studies such as [13], [14], [15], [16] used Darcy's equation with modification in the study of fluids through porous media. Our aim in this study is to determine the effect of porosity and permeability on fluid flow. This in our view will broaden the study of fluid flow through porous media and also add to existing literatures.

2. Formalism

For flow of fluid through porous medium, the smoothed continuity equation and the Darcy's equation respectively are

$$\xi\frac{\partial\rho}{\partial t}=-\left(\nabla.\rho v_0\right) \tag{1}$$

$$v_0=-\frac{\kappa}{\mu}\left(\nabla p-\rho g\right) \tag{2}$$

where $\xi,\kappa,\rho,\mu,v_0,p,t,g$ are respectively the porosity, permeability, density of fluid, fluid viscosity, superficial velocity, pressure of fluid, time and acceleration due to gravity.

Combination of Equations (1) and (2) results in

$$\left(\frac{\xi\mu}{\kappa}\right)\frac{\partial\rho}{\partial t}=\left(\nabla.\rho\left(\nabla p-\rho g\right)\right) \tag{3}$$

We write the equation of state for this study following the argument of [2] as

$$\rho=\rho_0 p^m e^{\beta p} \tag{4}$$

Where ρ_0 is the fluid density at unit pressure, m and β are integers.

For polar fluids, m = 0 , $\beta\neq 0$ and equations (3) and (4) reduced to

$$\left(\frac{\xi\mu\beta}{\kappa}\right)\frac{\partial\rho}{\partial t}=\nabla^2\rho-\left(\nabla.\rho^2\beta g\right) \tag{5}$$

With the boundary conditions

$$\rho(0)=0 \quad \text{and} \quad \rho(-1)=1 \tag{6}$$

3. Method of Solution

We approximate ρ^2 by ignoring powers of ρ greater than unity using Taylor series expansion about 0 and reduce equation (5) into the form

$$\left(\frac{\xi\mu\beta}{\kappa}\right)\frac{\partial\rho}{\partial t}=\frac{\partial^2\rho}{\partial z^2}-2\beta g\frac{\partial\rho}{\partial z} \tag{6}$$

To solve equation (6) we assume a solution of the form

$$\rho=\theta(z)e^{-\lambda t} \tag{7}$$

where λ is a constant and the boundary conditions also modified into

$$\theta(0) = 0 \text{ and } \theta(-1) = e^{\lambda t} \quad (8)$$

If we put equation (7) into equation (6) , we get

$$-a\lambda\theta(z) = \theta''(z) - b\theta'(z) \quad (9)$$

where $a = \frac{\xi\mu\beta}{\kappa}$ and $b = 2\beta g$

The solution of (9) and imposition of the boundary conditions of equation (8) as well as substitute back into equation (7) results in

$$\rho(z) = \frac{e^{CZ}}{2} - \frac{e^{DZ}}{2} \quad (10)$$

where $C = \frac{-b+\sqrt{b^2-4a\lambda}}{2}$ and $D = \frac{-b-\sqrt{b^2-4a\lambda}}{2}$

From the equation of state in (4) and the conditions for compressible fluids, we can rewrite equation (10) as

$$p(z) = \frac{1}{\beta}\log\left[\frac{1}{2\rho_0}\left(e^{CZ} - e^{DZ}\right)\right] \quad (11)$$

4. Results and discussion

In order to get physical insight and numerical validation of the problem, an approximate value of acceleration due to gravity $\left(g = 9.8ms^{-2}\right)$, constant viscosity of fluid at $20^0 C$ (water $\mu = 1.0x10^{-3} N.sm^{-2}$) and a decay constant $(\lambda = 0.0035)$ is chosen. The values of other parameters made use of are

$\beta = 1$

$\xi = 0.3, 0.6, 0.9, 1.2, 1.5$

$\kappa = 0.5, 1.0, 1.5, 2.0, 2.5$

$\rho_0 = 1.0x10^{-3} kgm^{-3}$

Figure 1 shows the relationship between the fluid pressure and the space coordinate as porosity increases. As the effective porosity increases so also the pressure of the fluid decreases as a result of increase in the surface area of the void spaces. This result is also in agreement with an earlier study of [2]. Figure 2 shows that increase in permeability also result to a decrease in fluid pressure and this result laid credence to an earlier study of [5]

5. Conclusions

Generally, pressure of fluid reduces as porosity and permeability of materials are increased. In most of the studies listed in the literature, particularly studies that combined porosity and other parameters, permeability is not considered which confirmed our position that for effective description of porosity and permeability, the application of Darcy's model is of necessity.

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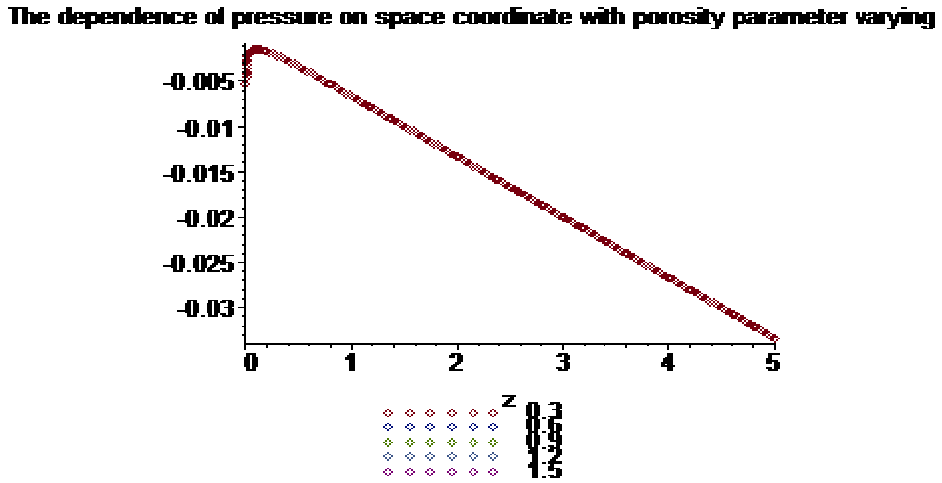

Figure 1: Pressure profile p against boundary layer z for varying porosity term. ξ

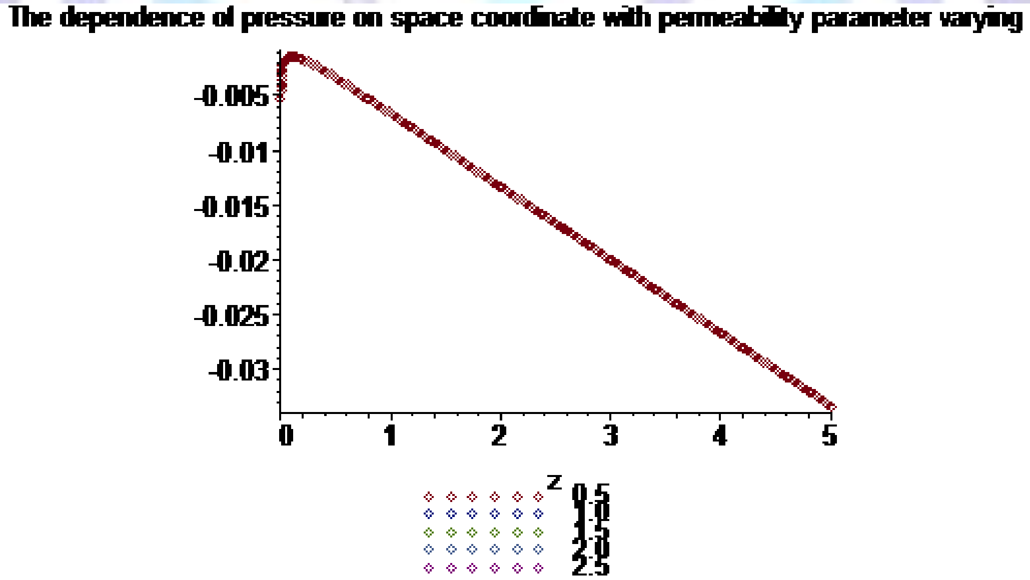

Figure 2: Pressure profile p against boundary layer z for varying permeability term (κ)

7. Appendices

Maple plot of figure 1

> **plot([(0.001)*log(exp(-6.66950650*z)-exp(-12.93049349*z)),(0.001)*log(exp(-6.6695055*z)-exp(-12.930495*z)),(0.001)*log(exp(-6.669505059*z)-exp(-12.930494*z)),(0.001)*log(exp(-6.6695061*z)-exp(-12.9304938*z)),(0.001)*log(exp(-6.6695065*z)-exp(-12.9304935*z))],z=0..5,title="The dependence of pressure on space coordinate with porosity parameter varying");**

Maple plot of figure 2

> plot([(0.001)*log(exp(-6.669506509*z)-exp(-12.93049349*z)),(0.001)*log(exp(-6.66950567*z)-exp(-12.93049433*z)),(0.001)*log(exp(-6.66950505391*z)-exp(-12.93049461*z)),(0.001)*log(exp(-6.669505251*z)-exp(-12.93049475*z)),(0.001)*log(exp(-6.669505167*z)-exp(-12.93049483*z))],z=0..5,title="The dependence of pressure on space coordinate with permeability parameter varying");

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