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Yang-Mills Families

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ABSTRACT

The Yang-Mills theory structure is based on group theory. It rules the symmetry relationship where the number of potential fields transforming under a same group must be equal to the number of group generators. It defines the group valued expression $A_\mu = A_{\mu a}\, t_a$ from where the corresponding non-abelian symmetry properties are derived.

Nevertheless based on different origins as Kaluza-Klein, fibre bundles, supersymmetry, σ-model , BRST and anti-BRST algorithm, counting degrees of freedom leads to a Yang-Mills extension under the existence of different potential fields rotating under a same single group. They establish for SU(N) the relationship $A'_{\mu I} = U\, A_{\mu I}\, U^{-1} + \frac{i}{g_I}\partial_\mu U \cdot U^{-1}$ where $U = e^{i\omega_a t_a}$ and I is a flavor index, $I = 1 \ldots N$. Physically, it says that different Yang-Mills families can share a common symmetry group. They develop a whole non-abelian gauge theory.

The effort in this work is to explore such non-abelian extension. First, to build up it on the so-called $\{D_\mu, X^i_\mu\}$ constructor basis where gauge symmetry is more available for expressing the corresponding fields strengths, Lagrangian and classical equations. After that, given that the physical fields are those associated to the poles of two-point Green functions, one derives the physical Lagrangian $\mathcal{L}\,(G_I)$ written in the physical basis $\{G_{\mu I}\}$.

A new physical Lagrangian associated to $SU(N)$ symmetry is generated. The meaning of Yang-Mills families appears. A symmetry of difference is realized. Where every quanta is distinguished from each other. It yields a quanta diversity associated to corresponding Yang-Mills families. There are N-spin 1 and N-spin 0 quanta separated by different quantum numbers through a whole N-dynamics. An extension to QCD becomes possible.

1 Introduction

Since 1954 Yang-Mills have been guiding physics [1, 2, 3, 4]. Until the twenty century first two decades, physics performances had been determined by geometry [5]. However, the Weyl introduction of group theory in physics changed the route [6]. Quantum Mechanics became a theory bound on the Plank constant plus group theory. Following that, the Standard Model [7, 8, 9], Supersymmetry [10], String Theory [11] and other models became physical insertions based on gauge symmetry.

This work intends to enlarge on non-abelian gauge symmetry. Beyond to Yang-Mills theory develop the so called symmetry of difference. Introduce under a common gauge symmetry different field families, $A^a_{\mu I}$, where the index I means the inserted families, $I = 1, \ldots, N$. It gives an initial fields set transforming under a same gauge symmetry

$$A'_{\mu I} = U A_{\mu I} U^{-1} + \frac{i}{g_I}\partial_\mu U \cdot U^{-1}, \tag{1}$$

where $U = e^{i\omega_a t_a}$, where t_a are the group generators of $SU(N)$. The index a is an internal index and run according to the group's choice.

Eq.(1) moves physics beyond Yang-Mills. It introduces mass term without pursuing the Goldstone Theorem [12]. Its symmetry of difference offers an alternative to the spontaneous symmetry breaking [13, 14, 15] and dynamical symmetry breaking

[16]. For this new symmetry approach to happen, different origins based on Kaluza-Klein [17, 18], fibre bundle [19], supersymmetry [20, 21, 22], σ-model [23], BRST and anti-BRST algorithm [24, 25] have been studied in order to prove on eq.(1) existence.

Thus eq.(1) provides a new performance for generating physical laws. It introduces the antireductionist approach. The principle where physical laws are generated from a set of fields transforming under a same gauge symmetry. A physics which primordials are in the whole as the fundamental unity. In other words, understands nature as a whole.

A non-abelian whole gauge symmetry appears to be understood. Considering that these fields satisfy the Kamefuchi-O'Raifeartaigh-Salam theorem [26], one can change their variables preserving physics. So, in order to get a better transparence on symmetry, one should write the model in terms of $\{D_\mu, X_{\mu i}\}$ field basis, which is called as the constructor basis where $D_\mu \equiv D_\mu^a t_a$ is defined as

$$D_\mu = \sum_I A_{\mu I} \tag{2}$$

showing just one field transforming inhomogeneously and $X_{\mu i} \equiv X_{\mu i}^a t_a$ are $(N-1)$ potential fields

$$X_{\mu 1} = A_{\mu_1} - A_{\mu_2} \tag{3}$$

$$X_{\mu(N-1)} = A_{\mu 1} - A_{\mu N} \tag{4}$$

with $i = 2, \ldots, N$ and transforming covariantly. Nevertheless, the $\{D_\mu, X_{\mu i}\}$ basis is not the physical basis. By definition, the physical fields are that ones which physical masses are the poles of two-points Green functions. For this, one has to diagonalize the transverse sector by introducing the matrix Ω [27]. Thus, the physical basis $\{G_{\mu I}\}$ is obtained by rotating as

$$D_\mu = \Omega_{1I} G_\mu^I \qquad X_{\mu i} = \Omega_{iI} G_\mu^I \tag{5}$$

under the Ω matrix invertible condition

$$\Omega_{IK}\Omega_{KJ}^{-1} = \delta_{IJ} \tag{6}$$

The notation to be followed is $Y_\mu = Y_\mu^a t_a$, where t_a are the matrices which satisfy the Lie algebra for $SU(N)$.

Thus, the objective of this work is to study classically on this whole non-abelian gauge theory. The paper organizes such new non-abelian performance as follows. In section 2, one studies the symmetry composition at constructor basis $\{D, X_i\}$. Section 3 rewrites the corresponding fields strengths in the physical basis $\{G_I\}$. At section 4, classical properties are derived from the physical Lagrangian $\mathcal{L}(G_I)$. It is left for conclusion a possible extension to QCD.

2 Constructor Basis $\{D, X_i\}$

Eq. (1) should be rewritten by expressing a connection with Yang-Mills as boundary condition. Thus, one defines the field D_μ as the usual gauge field and the fields X_μ^i as a kind of vector-matter fields transforming in the adjoint representation. It gives

$$\begin{aligned} D_\mu &\longrightarrow D'_\mu = U D_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \\ X_\mu^i &\longrightarrow X_\mu^i = U X_\mu^i U^{-1} \end{aligned} \tag{7}$$

Eq. (7) propitiates a more easy understanding on the symmetry properties that the model carries. Geometrically, the origin of the potential fields can be treated back to the vielbein, spin-connection and potential fields of higher-dimensional gravity-matter coupled theory spontaneously compactified for an internal space with torsion [17, 18, 19]. Also by relaxing supersymmetry constraints [20, 21, 22].

Thus the most general expression for the Lagrangian is

$$\mathcal{L} = \operatorname{tr}(Z_{\mu\nu}Z^{\mu\nu}) + \operatorname{tr}(z_{\mu\nu}z^{\mu\nu}) + \operatorname{tr}(Z_{\mu\nu}z^{\mu\nu}) + \eta\operatorname{tr}(Z_{\mu\nu}\tilde{Z}^{\mu\nu}) + \eta\operatorname{tr}(z_{\mu\nu}\tilde{z}^{\mu\nu}) - \frac{1}{2}{m_{ij}}^2\operatorname{tr}(X^i_\mu X^{\mu j}) \tag{8}$$

where $Z_{\mu\nu}$ is the most general covariant field strengh with an unique dependence on fields,

$$Z_{\mu\nu} \longrightarrow Z'_{\mu\nu} = UZ_{\mu\nu}U^{-1} \tag{9}$$

which can be decomposed as

$$Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)}. \tag{10}$$

The granular antisymmetric field strength is

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X^i_{[\mu\nu]} \tag{11}$$

with

$$\begin{aligned} D_{\mu\nu} &= \partial_\mu D_\nu - \partial_\nu D_\mu + ig[D_\mu, D_\nu] \\ X^i_{[\mu\nu]} &= \partial_\mu X^i_\nu - \partial_\nu X^i_\mu + ig([D_\mu, X^i_\nu] - [D_\nu, X^i_\mu]) \end{aligned} \tag{12}$$

Considering the symmetric sector, one gets as the granular counterpart

$$Z_{(\mu\nu)} = \beta_i X^i_{(\mu\nu)} + \rho_i g_{\mu\nu} X^{\alpha i}_\alpha \tag{13}$$

where

$$X^i_{(\mu\nu)} = \partial_\mu X^i_\nu + \partial_\nu X^i_\mu + ig([D_\mu, X^i_\nu] + [D_\nu, X^i_\mu]). \tag{14}$$

Another type of field strength is $z_{\mu\nu}$. It is a collective field which does not depend on derivatives and does not appear in usual Yang-Mills theories.

$$z_{\mu\nu} = z_{[\mu\nu]} + z_{(\mu\nu)} \tag{15}$$

where

$$z_{[\mu\nu]} = a_{(ij)}[X^i_\mu, X^j_\nu] + b_{[ij]}\{X^i_\mu, X^j_\nu\} + \gamma_{[ij]} X^i_\mu X^j_\nu \tag{16}$$

and

$$\begin{aligned} z_{(\mu\nu)} =& a_{[ij]}[X^i_\mu, X^j_\nu] + u_{[ij]} g_{\mu\nu}[X^i_\alpha, X^{\alpha j}] \\ &+ b_{(ij)}\{X^i_\mu, X^j_\nu\} + v_{(ij)} g_{\mu\nu}\{X^i_\alpha, X^{\alpha j}\} + \gamma_{(ij)} X^i_\mu X^j_\nu. \end{aligned} \tag{17}$$

It also transforms covariantly

$$z_{\mu\nu} \to z'_{\mu\nu} = Uz_{\mu\nu}U^{-1}. \tag{18}$$

At this work we are not going to explore on $\tilde{t}_{\mu\nu}$ term. Notice that $Z_{\mu\nu}$ and $z_{\mu\nu}$ are not Lie algebra valued as it is $F_{\mu\nu}$ in the QCD. However, in order to explore the abundance of gauge scalars that such extended model offers one should also consider contributions from decomposing the fields strengths in terms of group terms t_a, t_b, $t_a t_b$, $[t_a, t_b]$, $\{t_a, t_b\}$. It yields an expansion where each coefficient transforms covariantly the non-irreducible sector.

Working out the total antisymmetric field tensor, one gets

$$\begin{aligned} Z_{[\mu\nu]} &= A^a_{\mu\nu} t_a \\ z_{[\mu\nu]} &= B_{\mu\nu}\mathbb{1} + A'^a_{\mu\nu} t_a + C^{ab}_{\mu\nu} t_a t_b \end{aligned} \tag{19}$$

where

$$\begin{aligned} A^a_{\mu\nu} &= dD^a_{\mu\nu} + \alpha_i X^{ia}_{[\mu\nu]} \\ B_{\mu\nu} &= \frac{1}{N} b_{[ij]} X^i_{\mu a} X^{ja}_{\nu} \\ A'^a_{\mu\nu} &= C^{abc}_{(ij)} X^i_{\mu c} X^j_{\nu b} \\ C^{\mu\nu}_{(ij)} &= \gamma_{[ij]} X^{ia}_{\mu} X^{jb}_{\nu} \end{aligned} \tag{20}$$

with

$$C^{abc}_{(ij)} = -i a_{(ij)} f^{abc} + b_{[ij]} d^{cba} \tag{21}$$

Similarly, one expands the symmetric field strength

$$\begin{aligned} Z_{(\mu\nu)} + z_{(\mu\nu)} &= (\beta X^{ia}_{(\mu\nu)} + \rho_i g_{\mu\nu} X^{ia\alpha}_{\alpha}) t_a \\ &+ (a_{[ij]} [X^{ia}_{\nu}, X^{jb}_{\nu}] + u_{[ij]} g_{\mu\nu} [X^{i\alpha}_{\alpha}, X^{ib\alpha}] \\ &+ b_{(ij)} \{X^i_{\mu}, X^j_{\nu}\}) [t_a, t_b] + v_{(ij)} g_{\mu\nu} \{X^i_{\mu}, X^j_{\nu}\} \{t_a, t_b\} \\ &+ \gamma_{(ij)} X^i_{\mu} X^j_{\nu} t_a t_b \end{aligned} \tag{22}$$

We should now split the Lagrangian in antisymmetric and symmetric parts. It gives,

$$\mathcal{L}(D, X_i) = \mathcal{L}_A + \mathcal{L}_S - \frac{1}{2} {m_{ij}}^2 X^i_{\mu}, X^{\mu j} \tag{23}$$

where

$$\mathcal{L}_A = \lambda_1 Z_{[\mu\nu]} Z^{[\mu\nu]} + \lambda_2 z_{[\mu\nu]} z^{[\mu\nu]} + \lambda_3 Z_{[\mu\nu]} z^{[\mu\nu]} \tag{24}$$

and

$$\mathcal{L}_S = \xi_1 Z_{(\mu\nu)} Z^{(\mu\nu)} + \xi_2\, z_{(\mu\nu)} z^{(\mu\nu)} + \xi_3 Z_{(\mu\nu)} z^{(\mu\nu)} \tag{25}$$

It yields the following equations of motion:

$$\begin{aligned} &\lambda_1 \left(4d\, \partial_{\nu} Z^{[\mu\nu]} t_a + 4i \frac{g}{N} (d\, D^b_{\nu} + \alpha_i X^{ib}_{\nu}) Z^{[\mu\nu]} [t_a, t_b] \right) \\ &- 4\xi_1 i\, g \left(\beta_i X^{ib}_{\nu} Z^{(\mu\nu)} + \rho_i X^{\mu i b} Z_{(\nu}{}^{\nu)} \right) [t_a, t_b] \\ &+ \lambda_3 \left(2d\, \partial_{\nu}\, z^{[\mu\nu]} t_a + 2i \frac{g}{N} (d\, D^b_{\nu} + \alpha_i X^{ib}_{\nu}) z^{[\mu\nu]} [t_a, t_b] \right) \\ &- 2\xi_3 i\, g\ \left(\beta_i X^{ib}_{\nu} z^{(\mu\nu)} + \rho_i X^{\mu i b} z_{(\nu}{}^{\nu)} \right) [t_a, t_b] = 0 \end{aligned} \tag{26}$$

for D^a_{μ} -field. And,

$$
\begin{aligned}
&4\lambda_1\alpha_i\left(\partial_\nu Z^{[\mu\nu]}t_a + i\,g\,D^b_\nu Z^{[\mu\nu]}[t_a,t_b]\right)\\
&+\lambda_3\left(2a_{(ij)}X^{jb}_\nu Z^{\mu\nu}[t_a,t_b] + (2b_{[ij]}+\gamma_{[ij]})X^{jb}_\nu Z^{[\mu\nu]}\{t_a,t_b\}\right)\\
&+4\xi_1\begin{pmatrix} -\beta_i\partial_\nu Z^{(\mu\nu)}t_a - \rho_i\partial^\mu Z_{(\nu}{}^{\nu)}t_a \\ +i\,g\left(\beta_i\,D^b_\nu Z^{(\mu\nu)} + \rho_i D^{\mu b}Z_{(\nu}{}^{\nu)}\right)[t_a,t_b]\end{pmatrix}\\
&+2\xi_3\begin{pmatrix} \left(a_{[ij]}X^{jb}_\nu Z^{(\mu\nu)} + u_{[ij]}X^{\mu jb}Z_{(\nu}{}^{\nu)}\right)[t_a,t_b] \\ +\left(b_{(ij)}X^{jb}_\nu Z^{(\mu\nu)} + v_{(ij)}X^{\mu jb}Z_{(\nu}{}^{\nu)}\right)\{t_a,t_b\}\end{pmatrix}\\
&+2\lambda_2\left(2a_{(ij)}X^{jb}_\nu\,z^{[\mu\nu]}[t_a,t_b] + (2b_{[ij]}+\gamma_{[ij]})X^{jb}_\nu\,z^{[\mu\nu]}\{t_a,t_b\}\right)\\
&+\lambda_3\left(2\alpha_i(\partial_\nu\,z^{[\mu\nu]}t_a + i\,g\,D^b_\nu\,z^{[\mu\nu]}[t_a,t_b])\right)\\
&+4\xi_2\begin{pmatrix} \left(a_{[ij]}X^{jb}_\nu\,z^{(\mu\nu)} + u_{[ij]}X^{\mu jb}\,z_{(\nu}{}^{\nu)}\right)[t_a,t_b] \\ +\left(b_{(ij)}X^{jb}_\nu\,z^{(\mu\nu)} + v_{(ij)}X^{\mu jb}\,z_{(\nu}{}^{\nu)}\right)\{t_a,t_b\}\end{pmatrix}\\
&+2\xi_3\begin{pmatrix} -\beta_i\partial_\nu z^{(\mu\nu)}t_a - \rho_i\partial^\mu z_{(\nu}{}^{\nu)}t_a \\ +i\,g\left(\beta_i\,D^b_\nu z^{(\mu\nu)} + \rho_i D^{\mu b}z_{(\nu}{}^{\nu)}\right)[t_a,t_b]\end{pmatrix} = 0
\end{aligned}
\tag{27}
$$

for X^{ia}_μ-fields.

Taking the trace in above equations, one gets for D^a_μ

$$
\begin{aligned}
&-2\lambda_1\left(d\,\partial_\nu Z^{[\mu\nu]a} + g\,f_{abc}(d\,D^b_\nu - \alpha_i X^{ib}_\nu)Z^{[\mu\nu]c}\right)\\
&+2\xi_1\,g\,f_{abc}\left(\beta_i X^{ib}_\nu Z^{(\mu\nu)c} + \rho_i X^{\mu ib}Z_{(\nu}{}^{\nu)c}\right)\\
&+\lambda_3\left(d\,\partial_\nu\,z^{[\mu\nu]a} + g\,f_{abc}(d\,D^b_\nu - \alpha_i X^{ib}_\nu)z^{[\mu\nu]c}\right)\\
&+\xi_3\,g\,f_{abc}\left(\beta_i X^{ib}_\nu\,z^{(\mu\nu)c} + \rho_i X^{\mu ib}\,z_{(\nu}{}^{\nu)c}\right) = 0
\end{aligned}
\tag{28}
$$

and for X^{ia}_μ

$$
\begin{aligned}
&2\lambda_1\alpha_i\left(\partial_\nu Z^{[\mu\nu]a} - g\, f_{abc}\, D_\nu^b Z^{[\mu\nu]c}\right)\\
&+\lambda_3\left(i\, a_{[ij]}\, f_{abc} + b_{[ij]}\, d_{abc} + \frac{1}{2}\gamma_{[ij]}\, d_{abc}\right) X_\nu^{jb} Z^{[\mu\nu]c}\\
&-2\xi_1\left(\begin{array}{c}\beta_i\partial_\nu Z^{(\mu\nu)a} + \rho_i\partial^\mu Z_{(\nu}{}^{\nu)a}\\ +g\, f_{abc}\left(\beta_i D_\nu^b Z^{(\mu\nu)c} + \rho_i D^{\mu b} Z_{(\nu}{}^{\nu)c}\right)\end{array}\right)\\
&+\xi_3\left(\begin{array}{c}(i\, u_{[ij]} f_{abc} + \nu_{(ij)} d_{abc}) X^{\mu jb} Z_{(\nu}{}^{\nu)c}\\ +(i\, a_{[ij]} f_{abc} + b_{(ij)} d_{abc}) X_\nu^{jb} Z^{(\mu\nu)c}\end{array}\right)\\
&+\lambda_2\left(2ia_{(ij)} f_{abc} + 2b_{[ij]} d_{abc} + \gamma_{[ij]} d_{abc}\right) X_\nu^{jb} z^{[\mu\nu]c}\\
&\qquad+\lambda_3\alpha_i\left(\partial_\nu z^{[\mu\nu]a} + g f_{abc}\, D_\nu^b\, z^{[\mu\nu]c}\right)\\
&+2\xi_2\left(\begin{array}{c}(ia_{[ij]} f_{abc} + b_{(ij)} d_{abc}) X_\nu^{jb} z^{(\mu\nu)c})\\ +(iu_{[ij]} f_{abc} + v_{(ij)} d_{abc}) X^{\mu jb} z_{(\nu}{}^{\nu)c}\end{array}\right)\\
&-\xi_3\left(\begin{array}{c}\beta_i\partial_\nu z^{(\mu\nu)a} + \rho_i\partial^\mu z_{(\nu}{}^{\nu)a}\\ +g f_{abc}\left(\beta_i D_\nu^b z^{(\mu\nu)c} + \rho_i D^{\mu b} z_{(\nu}{}^{\nu)c}\right)\end{array}\right)\\
&=0.
\end{aligned}
\tag{29}
$$

Multiplying the equations of motion by t_k and taking again the corresponding trace, we finally have the following equations of motion

$$
\begin{aligned}
&\lambda_1\left(\begin{array}{c} d(d_{aek} - if_{aek})\partial_\nu Z^{[\mu\nu]a} +\\ g(dD_\nu^b + \alpha_i X_\nu^{ib})(if_{abc}f_{cek} - f_{abc}d_{cek}) Z^{[\mu\nu]e}\end{array}\right)\\
&+\xi_1\left(\begin{array}{c} g\rho_i(f_{abc}d_{cek} - if_{abc}d_{cek}) X^{\mu ib} Z_{(\nu}{}^{\nu)e}\\ +g\beta_i(f_{abc}d_{cek} - if_{abc}d_{cek}) X_\nu^{ib} Z^{(\mu\nu)e}\end{array}\right)\\
&+\frac{\lambda_3}{2}\left(\begin{array}{c} d(d_{aek} - if_{aek})\partial_\nu z^{[\mu\nu]a}\\ +g(dD_\nu^b + \alpha_i X_\nu^{ib})(if_{abc}f_{cek} - f_{abc}d_{cek}) z^{[\mu\nu]e}\end{array}\right)\\
&+\frac{\xi_3}{2}\left(\begin{array}{c} g\rho_i(f_{abc}d_{cek} - if_{abc}d_{cek}) X^{\mu ib} z_{(\nu}{}^{\nu)e}\\ +g\beta_i(f_{abc}d_{cek} - if_{abc}d_{cek}) X_\nu^{ib} z^{(\mu\nu)e}\end{array}\right)\\
&=0
\end{aligned}
\tag{30}
$$

for D_μ^a. And,

$$\begin{aligned}
&\lambda_1 \begin{pmatrix} \alpha_i(d_{aek} - i f_{aek})\partial_\nu Z^{[\mu\nu]e} \\ +g\alpha_i(i f_{abc} f_{cek} - f_{abc} d_{cek}) D^b_\nu Z^{[\mu\nu]e} \end{pmatrix} \\
&+ \frac{\lambda_3}{2} \begin{pmatrix} a_{(ij)}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +(b_{[ij]} + \frac{1}{2}\gamma_{[ij]})(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{jb}_\nu Z^{[\mu\nu]e} \\
&+ \xi_1 \begin{pmatrix} \beta_i(i f_{aek} - d_{cek})\partial_\nu Z^{(\mu\nu)e} + \rho_i(i f_{aek} - d_{aek})\partial^\mu Z_{(\nu}{}^{\nu)e} \\ +g\beta_i(i f_{abc} f_{cek} - f_{abc} d_{aek}) D^b_\nu Z^{(\mu\nu)e} \\ +g\rho_i(i f_{abc} f_{cek} - f_{abc} d_{cek}) D^{\mu b} Z_{(\nu}{}^{\nu)e} \end{pmatrix} \\
&+ \xi_3 \begin{pmatrix} \frac{1}{N} b_{(ij)} X^{ja}_\nu Z^{(\mu\nu)k} + \frac{1}{N} v_{(ij)} X^{\mu ja} Z_{(\nu}{}^{\nu)k} \\ +\frac{1}{2}\begin{pmatrix} a_{[ij]}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +b_{(ij)}(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{jb}_\nu Z^{(\mu\nu)e} \\ + \begin{pmatrix} u_{[ij]}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +v_{(ij)}(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{\mu jb} Z_{(\nu}{}^{\nu)e} \end{pmatrix} \\
&+ \lambda_2 \begin{pmatrix} \frac{1}{N}(2b_{[ij]} + \gamma_{[ij]}) X^{ja}_\nu z^{[\mu\nu]k} \\ + \begin{pmatrix} a_{(ij)}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +(b_{[ij]} + \frac{1}{2}\gamma_{[ij]})(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{jb}_\nu z^{[\mu\nu]e} \end{pmatrix} \\
&+ \frac{\lambda_3}{2} \begin{pmatrix} \alpha_i(d_{aek} - i f_{aek})\partial_\nu z^{[\mu\nu]e} \\ +g\alpha_i(i f_{abc} f_{cek} - f_{abc} d_{cek}) D^b_\nu z^{[\mu\nu]e} \end{pmatrix} \\
&+ \xi_2 \begin{pmatrix} \frac{2}{N} b_{(ij)} X^{ja}_\nu z^{(\mu\nu)k} + \frac{2}{N} v_{(ij)} X^{\mu ja} z_{(\nu}{}^{\nu)k} \\ + \begin{pmatrix} a_{[ij]}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +b_{(ij)}(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{jb}_\nu z^{(\mu\nu)e} \\ + \begin{pmatrix} u_{[ij]}(i f_{abc} d_{cek} + f_{abc} f_{cek}) \\ +v_{(ij)}(d_{abc} d_{cek} - i d_{abc} f_{cek}) \end{pmatrix} X^{\mu jb} z_{(\nu}{}^{\nu)e} \end{pmatrix} \\
&+ \frac{\xi_3}{2} \begin{pmatrix} \beta_i(i f_{aek} - d_{aek})\partial_\nu z^{(\mu\nu)e} + \rho_i(i f_{aek} - d_{aek})\partial^\mu z_{(\nu}{}^{\nu)e} \\ +g\beta_i(i f_{abc} f_{cek} - f_{abc} d_{cek}) D^b_\nu z^{(\mu\nu)e} \\ +g\rho_i(i f_{abc} f_{cek} - f_{abc} d_{cek}) D^{\mu b} z_{(\nu}{}^{\nu)e} \end{pmatrix} \\
&= 0
\end{aligned} \tag{31}$$

for X^{ia}_μ.

The corresponding Bianchi identities are

$$D_\mu Z_{[\nu\rho]} + D_\nu Z_{[\rho\mu]} + D_\rho Z_{[\mu\nu]} = 0 \tag{32}$$

where

$$D_\mu Z_{[\nu\rho]} = \partial_\mu Z_{[\nu\rho]} + i\,g\,[d\,D_\mu + \alpha_i X^i_\mu, Z_{[\nu\rho]}] \tag{33}$$

and

$$D_\mu z_{[\nu\rho]} + D_\nu z_{[\rho\mu]} + D_\rho z_{[\mu\nu]} = 0 \tag{34}$$

where

$$D_\mu z_{[\nu\rho]} = \partial_\mu z_{[\nu\rho]} + i\,g\,[d\,D_\mu + \alpha_i X^i_\mu, z_{[\nu\rho]}] \tag{35}$$

Notice that they are Bianchi identities with sources.

The associated energy-momentum tensor is

$$\theta_{\mu\nu} = T_{\mu\nu} + \frac{1}{2}\partial^{\rho}(S_{\rho\mu\nu} + S_{\mu\nu\rho} - S_{\nu\rho\mu}) \tag{36}$$

where

$$\begin{aligned} T_{\mu\nu} &= \frac{\partial \mathcal{L}}{\partial(\partial^{\mu} D_{\alpha a})}\partial_{\nu} D_{\alpha a} + \frac{\partial \mathcal{L}}{\partial(\partial^{\mu} X^{i}_{\alpha a})}\partial_{\nu} X^{i}_{\alpha a} - \eta_{\mu\nu}\mathcal{L} \\ S_{\rho\mu\nu} &= \frac{\partial \mathcal{L}}{\partial(\partial^{\rho} D_{\alpha a})}\Sigma_{\mu\nu}{}^{\alpha\beta} D_{\beta a} + \frac{\partial \mathcal{L}}{\partial(\partial^{\rho} X^{i}_{\alpha a})}\Sigma_{\mu\nu}{}^{\alpha\beta} X^{i}_{\beta a} \end{aligned} \tag{37}$$

with

$$\Sigma_{\mu\nu}{}^{\alpha\beta} = i(\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - \delta^{\alpha}_{\nu}\delta^{\beta}_{\mu}) \tag{38}$$

Splitting the energy-momentum tensor in antisymmetric and symmetric sectors,

$$\theta_{\mu\nu} = \theta^{A}_{\mu\nu} + \theta^{S}_{\mu\nu} \tag{39}$$

it yields for the antisymmetric sector the following expression:

$$\theta^{A}_{\mu\nu} = 4Z_{[\mu\rho]a}Z_{[\nu}{}^{\rho]a} + 4z_{[\mu\rho]a}z_{[\nu}{}^{\rho]a} + 2z_{[\mu\rho]a}Z_{[\nu}{}^{\rho]a} + 2Z_{[\mu\rho]a}z_{[\nu}{}^{\rho]a} - \eta_{\mu\nu}\mathcal{L}^{A} \tag{40}$$

with

$$\eta_{\mu\nu}\mathcal{L}^{A} = \eta_{\mu\nu}(Z_{[\alpha\beta]}Z^{[\alpha\beta]} + z_{[\alpha\beta]}z^{[\alpha\beta]} + Z_{[\alpha\beta]}z^{[\alpha\beta]}) \tag{41}$$

where $\mathcal{L}^{A}$ is defined at eq. (24).

For the symmetric sector,

$$\begin{aligned} \theta^{S}_{\mu\nu} = &\, 4\beta_i Z_{(\mu\rho)a}X_{(\nu}{}^{\rho)ia} + 4\rho_i Z_{(\rho}{}^{\rho)}{}_{a}X_{(\mu\nu)}{}^{ia} \\ &+ 2\beta_i z_{(\mu\rho)a}X_{(\nu}{}^{\rho)ia} + 2\rho_i z_{(\rho}{}^{\rho)}{}_{a}X_{(\mu\nu)}{}^{ia} \\ &+ 4z_{(\mu\rho)a}a_{[ij]}[X_{\nu}{}^{i}, X^{\rho j}]^{a} + 4z_{(\rho}{}^{\rho)}{}_{a}u_{[ij]}[X_{\mu}{}^{i}, X_{\nu}{}^{j}]^{a} \\ &+ 4z_{(\mu\rho)a}b_{[ij]}\{X_{\nu}{}^{i}, X^{\rho ja}\} + 4z_{(\rho}{}^{\rho)}{}_{a}v_{[ij]}\{X_{\mu}{}^{i}, X_{\nu}{}^{ja}\} \\ &+ 2Z_{(\mu\rho)a}a_{[ij]}[X_{\nu}{}^{i}, X^{\rho j}]^{a} + 2Z_{(\rho}{}^{\rho)}{}_{a}u_{[ij]}[X_{\mu}{}^{i}, X_{\nu}{}^{j}]^{a} \\ &+ 2Z_{(\mu\rho)a}b_{[ij]}\{X_{\nu}{}^{i}, X^{\rho ja}\} + 2Z_{(\rho}{}^{\rho)}{}_{a}v_{[ij]}\{X_{\mu}{}^{i}, X_{\nu}{}^{ja}\} \\ &- 4\beta_i\partial^{\rho}(Z_{(\mu\rho)a}X_{\nu}{}^{ia}) + 4\beta_i\partial^{\rho}(Z_{(\mu\nu)a}X_{\rho}{}^{ia}) \\ &- 4\rho_i\partial_{\mu}(Z_{(\rho}{}^{\rho)}{}_{a}X_{\nu}{}^{ia}) - 4\beta_i\partial^{\rho}(Z_{(\nu\rho)a}X_{\mu}{}^{ia}) \\ &- 4\rho_i\partial_{\nu}(Z_{(\rho}{}^{\rho)}{}_{a}X_{\mu}{}^{ia}) + 2\beta_i\partial^{\rho}(z_{(\mu\nu)a}X_{\rho}{}^{ia}) \\ &- 2\beta_i\partial^{\rho}(z_{(\mu\rho)a}X_{\nu}{}^{ia}) - 2\beta_i\partial^{\rho}(z_{(\nu\rho)a}X_{\mu}{}^{ia}) \\ &- 2\rho_i\partial_{\mu}(z_{(\rho}{}^{\rho)}{}_{a}X_{\nu}{}^{ia}) + 4\rho_i\partial^{\rho}(Z_{(\alpha}{}^{\alpha)}{}_{a}X_{\rho}{}^{ia})\delta^{\nu}_{\mu} \\ &- 2\rho_i\partial_{\nu}(z_{(\rho}{}^{\rho)}{}_{a}X_{\mu}{}^{ia}) + 2\rho_i\partial^{\rho}(z_{(\alpha}{}^{\alpha)}{}_{a}X_{\rho}{}^{ia})\delta^{\nu}_{\mu} \\ &- \eta_{\mu\nu}\mathcal{L}^{S} \end{aligned} \tag{42}$$

where $\mathcal{L}^{S}$ is defined at eq.(25). Eq.(39) provides the symmetry relationship

$$\theta^{\mu\nu} = \theta^{\nu\mu} \tag{43}$$

and the conservation law

$$\partial_\mu \theta^{\mu\nu} = 0 \tag{44}$$

3 Physical Basis $\{G_\mu^{Ia}\}$

We should now to study such non-abelian whole gauge symmetry in terms of physical fields. Changing basis, one gets

$$\begin{pmatrix} D\mu \\ \\ X_{\mu i} \end{pmatrix} = \Omega \begin{pmatrix} G_{\mu 1} \\ \vdots \\ G_{\mu N} \end{pmatrix} \tag{45}$$

where Ω matrix is defined in eq.(5). Consequently, the physical fields suffer the following gauge transformation

$$G_{\mu I} \to G'_{\mu I} = U G_{\mu I} U^{-1} + \frac{i}{g_I} \partial_\mu U \cdot U^{-1} \tag{46}$$

where $G_{\mu I} = G_{\mu I}^a t_a$ and the presence of different coupling constants, $g_I = \frac{g}{\Omega_{I1}^{-1}}$, means the possibility for coupling with different currents. Eq.(45) yields the following fields strengths redefinition

$$D_{\mu\nu} = \Omega_{1I}(\partial_\mu G_\nu^I - \partial_\nu G_\mu^I - ig\Omega_{1J}[G_\mu^I, G_\nu^J]) \tag{47}$$

$$X_{[\mu\nu]}^i = \Omega_I^i(\partial_\mu G_\nu^I - \partial_\nu G_\mu^I) - ig(\Omega_{1I}\Omega_{iJ} + \Omega_{1J}\Omega_{iI})[G_\mu^I, G_\nu^J] \tag{48}$$

Thus we obtain the following antisymmetric granular and collectives fields strengths,

$$Z_{[\mu\nu]} = a_I(\partial_\mu G_\nu^I - \partial_\nu G_\mu^I) - iga_{(IJ)}[G_\mu^I, G_\nu^J] \tag{49}$$

$$z_{[\mu\nu]} = b_{IJ}[G_\mu^I, G_\nu^J] + c_{IJ}\{G_\mu^I, G_\nu^J\} + \gamma_{[IJ]} G_\mu^I G_\nu^J \tag{50}$$

Notice that eq.(49) reproduces the usual Yang-Mills field strenght for the symmetric sector,

$$Z_{(\mu\nu)} = \beta_I G_{(\mu\nu)}^I + \rho_I g_{\mu\nu} G_{(\alpha}{}^{\alpha)I} \tag{51}$$

where

$$G_{(\mu\nu)}^I = \partial_\mu G_\nu^I + \partial_\nu G_\mu^I - ig_J([G_\mu^I, G_\nu^J] + [G_\nu^I, G_\mu^J]) \tag{52}$$

and

$$z_{(\mu\nu)} = b_{[IJ]}[G_\mu^I, G_\nu^J] + c_{(IJ)}\{G_\mu^I, G_\nu^J\} + u_{[IJ]} g_{\mu\nu}[G_\alpha^I, G^{\alpha J}] + v_{(IJ)} g_{\mu\nu}\{G_\alpha^I, G^{\alpha J}\} \tag{53}$$

with the following relationships with respect to constructor basis

$$\begin{array}{ll} a_I = d\Omega_{1I} + \alpha_i \Omega_I^i & a_{(IJ)} = a_I \Omega_{1J} + \alpha_i \Omega_{1I} \Omega_J^i \\ b_{(IJ)} = a_{(ij)} \Omega_I^i \Omega_J^j & b_{[IJ]} = b_{[ij]} \Omega_I^i \Omega_J^j \\ c_{[IJ]} = b_{[ij]} \Omega_I^i \Omega_J^j & c_{(IJ)} = c_{(ij)} \Omega_I^i \Omega_J^j \\ u_{[IJ]} = u_{[ij]} \Omega_I^i \Omega_J^j & u_{(IJ)} = u_{(ij)} \Omega_I^i \Omega_J^j \\ \gamma_{[IJ]} = \gamma_{[ij]} \Omega_I^i \Omega_J^j & g_I = g_{(ij)} \Omega_I^1 \\ \beta_I = \beta_i \Omega_I^i & \rho_I = \rho_i \Omega_I^i \end{array} \tag{54}$$

Physics does not depend on the fields basis. The symmetry results obtained in the constructor basis can be transposed to the physical basis [28]. Notice that the above fields strengths are also preserving the covariance property

$$\begin{aligned} Z_{\mu\nu}(G_I) &\to Z_{\mu\nu}(G_I)' = UZ_{\mu\nu}(G_I)U^{-1} \\ z_{\mu\nu}(G_I) &\to z_{\mu\nu}(G_I)' = Uz_{\mu\nu}(G_I)U^{-1} \end{aligned} \tag{55}$$

A further consistency on how such non-abelian whole symmetry implementation at $SU(N)$ gauge group was studied in a previous work [25]. Analyzing through the constructor basis one derives eight aspects attached to group generators and gauge parameters. They are from groups generators: algebra closure through Jacobi identities and Bianchi identities. From gauge parameters: Noether Theorem, gauge fixing, BRST symmetry, global transformations (BRST, ghost scale, gauge global); charges algebra, covariant equations of motion plus Poincaré lemma.

As example to express the consistency for introducing gauge families, let us take the Jacobi identity unification. Taking the infinitesimal transformation from eq.(46)

$$\begin{aligned} \delta G^a_{\mu I} &= \partial_\mu \omega^a(x) + g f^a_{bc} G^b_{\mu I}\omega^c(x) \\ &= [D_{\mu I}\omega(x)]^a \end{aligned} \tag{56}$$

one has to verify Jacobi identity acting on the field $G^a_{\mu I}$

$$\{[\delta_1,[\delta_2,\delta_3]] + [\delta_3,[\delta_1,\delta_2]] + [\delta_2,[\delta_3,\delta_1]]\}\, G^a_{\mu I} = 0 \tag{57}$$

Showing that

$$[\delta_1,\delta_2]G^a_{\mu I} = g f^{abc}[(D_{\mu I}\alpha_2)^b\alpha_1^c - (D_{\mu I}\alpha_1)^b\alpha_2^c] = g f^a_{bc} D_{\mu I}(\alpha_2^b\alpha_1^c) \tag{58}$$

one verifies eq.(56) similarly to the YM case. Consequently, eq.(46) is saying that it is possible to derive a Lagrangian where the number of potential fields is not necessarily equal to the number of group generators as ruled by Yang-Mills theory. The introduction of Yang-Mills families in a same $SU(N)$ group becomes a realistic whole physics to be understood.

The Yang-Mills extension becomes realistic. Eq.(46) underly the basis for deriving a whole non-abelian gauge theory. It contains scalar and vector sectors to be understood. This fact is already predicted from Lorentz Group representation $\left(\frac{1}{2},\frac{1}{2}\right)$. A next step is to open it through the corresponding dynamics.

4 Classical Equations

The objective of this work is to promote a model where nature works as grouping. At this way from physical fields set $\{G_I\}$ sharing a common gauge transformation as eq.(46) says, a next step is to build up the wholeness principle through $(2N+7)$ interconnected classical equations. From eq.(55), one gets a systemic Lagrangian

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_S \tag{59}$$

$$\mathcal{L} = \lambda_1 Z_{[\mu\nu]}Z^{[\mu\nu]} + \lambda_2 z_{[\mu\nu]}z^{[\mu\nu]} + \lambda_3 Z_{[\mu\nu]}z^{[\mu\nu]} + \xi_1 Z_{(\mu\nu)}Z^{(\mu\nu)} + \xi_2 z_{(\mu\nu)}z^{(\mu\nu)} + \xi_3 Z_{(\mu\nu)}z^{(\mu\nu)} \tag{60}$$

Eq.(59) provides the following whole non-abelian Lagrangian

$$\mathcal{L} = \mathcal{L}_K + L_I + L_m \tag{61}$$

Eq.(60) shows that Yang-Mills should not be more considered as the unique Lagrangian performed from $SU(N)$. At Apendice C, didactically, one studies each building block for eq.(60). The kinetic term is given by

$$\begin{aligned} \mathcal{L}_K &= 2(\lambda_1 a_I a_J + \xi_1 \beta_I \beta_J)(\partial_\mu G^I_\nu)(\partial^\mu G^{\nu J}) \\ &- 2(\lambda_1 a_I a_J - \xi_1 \beta_I \beta_J)(\partial_\mu G^I_\nu)(\partial^\nu G^{\mu J}) \\ &+ 8\xi_1 \beta_I \rho_J (\partial_\alpha G^{\alpha I})(\partial_\beta G^{\beta J}) \\ &+ 4\xi_1 \rho_I \rho_J g_{\mu\nu} g^{\mu\nu} (\partial_\alpha G^{\alpha I})(\partial_\alpha G^{\alpha J}) \end{aligned} \tag{62}$$

Working out eq.(62), one gets

$$\mathcal{L}_K = A_{IJ}\partial_\mu G^I_\nu \cdot \partial^\mu G^{\nu J} + B_{IJ}\partial_\mu G^I_\nu \cdot \partial^\nu G^{\mu J} + C_{IJ}\partial_\alpha G^{\alpha I} \cdot \partial_\beta G^{\beta I} \tag{63}$$

Decomposing eq.(63) in terms of transverse and longitudinal operators,

$$\mathcal{L}_K = -A_{IJ}G^I_\mu \Box \theta^{\mu\nu} G^J_\nu - (A_{IJ}+B_{IJ}+C_{IJ})G^I_\mu \Box \omega^{\mu\nu} G^I_\nu + M^2_{IJ} G^I_\mu G^{\mu J} \tag{64}$$

where $A_{IJ} = \delta_{IJ}$ and $M^2_{IJ} = m^2_{II}\delta_{IJ}$ are the relationships that redefines the constructor basis $\{D, X_i\}$ in terms of the physical basis $\{G_I\}$. It diagonalises the transverse sector.

Eq.(1) contains N-spin 1 and N-spin 0 quanta involving a diversity on quantum numbers. Eq.(64) splits the quanta diversity. From group theory one knows that a quadrivector carries information about different spins states. For this understanding one has to separate in transverse and longitudinal sectors. Eq.(64) makes that. It yields different poles where the transverse and longitudinal masses are related by the expression

$$\det m_L^2 = \frac{\det m_T^2}{\det(\mathbb{1} + B_{IJ} + C_{IJ})} \tag{65}$$

where $m^2_{T,L}$ means the transverse and longitudinal masses. Notice that if m^2_T contains zero mass it will yield zero masses on the longitudinal sector.

The interaction Lagrangian is composed of a trilinear term and a quadrilinear term. They are

$$L_I = \mathcal{L}^{(3)}_{\text{int}} + \mathcal{L}^{(4)}_{\text{int}} \tag{66}$$

where

$$\begin{aligned}
\mathcal{L}^{(3)}_{\text{int}} = & \begin{pmatrix} -4\lambda_1 i g a_K a_{(IJ)} + 2\lambda_3 a_K b_{(IJ)} + \\ +4\xi_1 i(g_I\beta_J\beta_K - g_J\beta_I\beta_K) + 2\xi_3\beta_K b_{[IJ]} \end{pmatrix} \partial_\mu G^K_\nu [G^{\mu I}, G^{\nu J}] \\
& + 2\lambda_3 a_I c_{[JK]} + \partial_\mu G^I_\nu \{G^{\mu J}, G^{\nu K}\} + 2\lambda_3 a_I \gamma_{[JK]} + (\partial_\mu G^I_\nu) G^{\mu J} G^{\nu K} \\
& + \begin{pmatrix} -8\xi_1 i(g_J\beta_I\rho_K + g_J\beta_K\rho_I) + \\ +2\xi_3(\rho_K b_{[IJ]} + \beta_K u_{[IJ]}) \end{pmatrix} \partial_\alpha G^{\alpha K}[G^I_\beta, G^{\beta J}] \\
& - (8\xi_1 i g_J \rho_I \rho_K - 2\xi_3\rho_K u_{[IJ]}) g_{\mu\nu} g^{\mu\nu} \partial_\alpha G^{\alpha K} [G^I_\alpha, G^{\alpha J}] \\
& + 2\xi_3(\beta_I v_{(JK)} + \rho_I c_{(JK)}) \partial_\alpha G^{\alpha I}\{G^J_\beta, G^{\beta K}\} \\
& + 2\xi_3 \rho_I v_{(JK)} g_{\mu\nu} g^{\mu\nu} \partial_\alpha G^{\alpha I}\{G^J_\alpha, G^{\alpha K}\}
\end{aligned} \tag{67}$$

and

$$\mathcal{L}_{\text{int}}^{(4)} = \begin{pmatrix} -\lambda_1 g^2 a_{(IJ)}a_{(KL)} + \lambda_2 b_{(IJ)}b_{(KL)} - \lambda_3 iga_{(IJ)}b_{(KL)}+ \\ +2\xi_1(g_J g_K \beta_I \beta_L - g_J g_L \beta_I \beta_K) + \xi_2 b_{[IJ]}b_{[KL]}+ \\ +\xi_3 i(g_I \beta_J b_{[KL]} - g_J \beta_I b_{[KL]}) \end{pmatrix} [G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}]$$
$$+ \begin{pmatrix} 2\lambda_2 b_{(IJ)}c_{[KL]} - \lambda_3 iga_{(IJ)}c_{[KL]} \\ +\xi_3 i(g_I\beta_J c_{(KL)} - g_J\beta_I c_{(KL)}) \end{pmatrix} [G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\}$$
$$+ \left(2\lambda_2\gamma_{[IJ]}b_{(KL)} - \lambda_3 ig\gamma_{[IJ]}a_{(KL)} \right) G_\mu^I G_\nu^J [G^{\mu K}, G^{\nu L}]$$
$$+ \left(\lambda_2 c_{[IJ]}c_{[KL]} + \xi_2 c_{(IJ)}c_{(KL)} \right) \{G_\mu^I, G_\nu^J\}\{G^{\mu K}, G^{\nu L}\}$$
$$+ 2\lambda_2\gamma_{[IJ]}c_{[KL]}G_\mu^I G_\nu^J\{G^{\mu K}, G^{\nu L}\}$$
$$+ \lambda_2\gamma_{[IJ]}\gamma_{[KL]}G_\mu^I G_\nu^J G^{\mu K}G^{\nu L}$$
$$+ \begin{pmatrix} -8\xi_1 g_J g_L \beta_I \beta_K + 2\xi_2 b_{[IJ]}u_{[KL]}+ \\ -2\xi_3 i(g_J\rho_I b_{[KL]} + g_J\beta_I u_{[KL]}) \end{pmatrix} [G_\alpha^I, G^{\alpha J}][G_\beta^K, G^{\beta L}] \tag{68}$$
$$+ \begin{pmatrix} -4\xi_1 g_J g_L \rho_I \rho_K + \xi_2 u_{[IJ]}u_{[KL]}+ \\ -2\xi_3 ig_J\rho_I u_{[JK]} \end{pmatrix} g_{\mu\nu}g^{\mu\nu}[G_\alpha^I, G^{\alpha J}][G_\alpha^K, G^{\alpha L}]$$
$$+ \begin{pmatrix} 2\xi_2(b_{[IJ]}v_{(KL)} + u_{[IJ]}c_{(KL)})+ \\ -2\xi_3 i(g_J\beta_I v_{(JK)} + g_J\rho_I c_{(JK)}) \end{pmatrix} [G_\alpha^I, G^{\alpha J}]\{G_\beta^K, G^{\beta L}\}$$
$$+ 2\xi_2 b_{[IJ]}c_{(KL)}g_{\mu\nu}[G^{\mu I}, G^{\nu J}]\{G^{\mu K}, G^{\nu L}\}$$
$$+ (2\xi_2 u_{[IJ]}v_{(KL)} - 2\xi_3 ig_J\rho_I v_{(JK)})g_{\mu\nu}g^{\mu\nu}[G_\alpha^I, G^{\alpha J}]\{G_\alpha^K, G^{\alpha L}\}$$
$$+ \xi_2 v_{(IJ)}v_{(KL)}g_{\mu\nu}g^{\mu\nu}\{G_\alpha^I, G^{\alpha J}\}\{G^{\mu K}, G^{\nu L}\}$$
$$+ 2\xi_2 v_{(IJ)}c_{(KL)}g_{\mu\nu}\{G_\alpha^I, G^{\alpha J}\}\{G^{\mu K}, G^{\nu L}\}$$

There is a wholeness principle to be understood classically. The first step is to understand how it provides a whole interconnected dynamics. The corresponding equations of motion derived from eq.(61) are

$$\operatorname{tr}\lambda_1(4a_I\partial_\nu Z^{[\mu\nu]}t_a - 4iga_{(IJ)}G_\nu^{Jb}Z^{[\mu\nu]}[t_a,t_b])+$$
$$+ \operatorname{tr}\lambda_2\left(4b_{(IJ)}G_\nu^{Jb}z^{[\mu\nu]}[t_a,t_b] + 4c_{[IJ]}G_\nu^{Jb}z^{[\mu\nu]}\{t_a,t_b\} + 2\gamma_{[IJ]}G_\nu^{Jb}z^{[\mu\nu]}[t_a,t_b]\right)$$
$$+ \operatorname{tr}\lambda_3\begin{pmatrix} 2a_I\partial_\nu z^{[\mu\nu]}t_a - 2iga_{(IJ)}G_\nu^{Jb}z^{[\mu\nu]}[t_a,t_b] + 2b_{(IJ)}G_\nu^{Jb}Z^{[\mu\nu]}[t_a,t_b] \\ +2c_{[IJ]}G_\nu^{Jb}Z^{[\mu\nu]}\{t_a,t_b\} + 2\gamma_{[IJ]}G_\nu^{Jb}Z^{[\mu\nu]}\{t_a,t_b\} \end{pmatrix}+$$
$$+ \operatorname{tr}\xi_1\begin{pmatrix} -4\beta_I\partial_\nu Z^{(\mu\nu)}t_a - 4\rho_I g_{\rho\sigma}\partial^\mu Z^{(\rho\sigma)}t_a+ \\ +4i(g_I\beta_J - g_J\beta_I)G_\nu^{Jb}Z^{(\mu\nu)}[t_a,t_b]+ \\ +4i(g_I\rho_J - g_J\rho_I)g_{\rho\sigma}G^{\mu Jb}Z^{(\rho\sigma)}[t_a,t_b] \end{pmatrix}+$$
$$+ \operatorname{tr}\xi_2\begin{pmatrix} +4b_{[IJ]}G_\nu^{Jb}z^{(\mu\nu)}[t_a,t_b] + 4c_{(IJ)}G_\nu^{Jb}z^{(\mu\nu)}\{t_a,t_b\} \\ +4u_{[IJ]}G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}[t_a,t_b] + 4u_{(IJ])}G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}\{t_a,t_b\} \end{pmatrix}+$$
$$+ \operatorname{tr}\xi_3\begin{pmatrix} -2\beta_I\partial_\nu z^{(\mu\nu)}t_a - 2\rho_I\partial^\mu g_{\rho\sigma}z^{(\rho\sigma)}t_a + 2b_{[IJ]}G_\nu^{Jb}Z^{(\mu\nu)}[t_a,t_b] + 2c_{(IJ)}G_\nu^{Jb}Z^{(\mu\nu)}\{t_a,t_b\}+ \\ +2u_{[IJ]}G^{\mu Jb}g_{\rho\sigma}Z^{(\rho\sigma)}[t_a,t_b] + 2v_{(IJ)}G^{\mu Jb}g_{\rho\sigma}Z^{(\rho\sigma)}\{t_a,t_b\}+ \\ +2i(g_I\beta_J - g_J\beta_I)G_\nu^{Jb}z^{(\mu\nu)}[t_a,t_b] + 2i(g_I\rho_J - g_J\rho_I)G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}[t_a,t_b] \end{pmatrix}$$
$$= 0 \tag{69}$$

Eq.(69) shows the systemic dynamics involving the fields set $\{G_{\mu I}^a\}$ evolution at space-time. Notice that the term λ_1 rewrites the Yang-Mills for N-potential fields. Nevertheless the other terms are adding on new contributions. They are showing that $SU(N)$ symmetry should not be limited to Yang-Mills theory. Beyond the inclusion of other fields it develops an expressian in terms of group generators.

Taking the trace from the above equations, one gets

$$
\begin{aligned}
&\lambda_1\left(2a_I\partial_\nu Z^{[\mu\nu]a}+2ga_{(IJ)}f_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}\right)+\\
&+\lambda_2\left(2ib_{(IJ)}f_{abc}G_\nu^{Jb}z^{[\mu\nu]c}+2c_{[IJ]}d_{abc}G_\nu^{Jb}z^{[\mu\nu]c}+\gamma_{[IJ]}d_{abc}G_\nu^{Jb}z^{[\mu\nu]c}\right)+\\
&+\lambda_3\begin{pmatrix} a_I\partial_\nu z^{[\mu\nu]a}+ga_{(IJ)}f_{abc}G_\nu^{Jb}z^{[\mu\nu]c}+ib_{(IJ)}f_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}\\ +c_{[IJ]}d_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}+\gamma_{[IJ]}d_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}\end{pmatrix}\\
&+\xi_1\begin{pmatrix} \beta_I\partial_\nu Z^{(\mu\nu)a}-2\rho_I\partial^\mu Z_{(\nu}{}^{\nu)a}-2(g_I\beta_J+g_J\beta_I)f_{abc}G_\nu^{Jb}Z^{(\mu\nu)c}\\ -2(g_I\rho_J+g_J\rho_I)f_{abc}G^{\mu Jb}Z_{(\nu}{}^{\nu)c}\end{pmatrix}\\
&+\xi_2\begin{pmatrix} 2ib_{[IJ]}f_{abc}G_\nu^{Jb}z^{(\mu\nu)c}+2c_{(IJ)}d_{abc}G_\nu^{Jb}z^{(\mu\nu)c}\\ +2iu_{[IJ]}f_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}+2u_{(IJ)}d_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}\end{pmatrix}\\
&+\xi_3\begin{pmatrix} -\beta_I\partial_\nu z^{(\mu\nu)a}-(g_I\beta_J-g_J\beta_I)f_{abc}G_\nu^{Jb}z^{(\mu\nu)c}\\ -\rho_I\partial^\mu z_{(\nu}{}^{\nu)a}-(g_I\rho_J-g_J\rho_I)f_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}\\ +ib_{[IJ]}f_{abc}G_\nu^{Jb}Z^{(\mu\nu)c}+c_{(IJ)}d_{abc}G_\nu^{Jb}Z^{(\mu\nu)c}\\ +iu_{[IJ]}f_{abc}G^{\mu Jb}Z_{(\nu}{}^{\nu)c}+v_{(IJ)}d_{abc}G^{\mu Jb}Z_{(\nu}{}^{\nu)c}\end{pmatrix}
\end{aligned}
\tag{70}
$$

Comparing with the usual Yang-Mills dynamics, eq.(70) is showing how from $SU(N)$ symmetry one can enlarge the dynamics. As consequence, we would say that just from symmetry one can not defines physics. It is necessary to implement an observational physical principle.

Now multiplying by t_k and taking the trace one derives the relation

$$
\begin{aligned}
&\lambda_1\left(a_I(d_{aek}-if_{aek})\partial_\nu Z^{[\mu\nu]e}+ga_{(IJ)}f_{abc}(d_{ekc}-if_{ekc})G_\nu^{Jb}Z^{[\mu\nu]e}\right)+\\
&+\lambda_2\begin{pmatrix} b_{(IJ)}f_{abc}(id_{eck}-f_{eck})G_\nu^{Jb}z^{[\mu\nu]e}+\frac{2}{N}c_{[IJ]}G_\nu^{Ja}z^{[\mu\nu]k}\\ +c_{[IJ]}d_{abc}(d_{eck}+if_{eck})G_\nu^{Jb}z^{[\mu\nu]e}+\frac{1}{N}\gamma_{[IJ]}G_\nu^{Ja}z^{[\mu\nu]k}\\ +\frac{1}{2}\gamma_{[IJ]}d_{abc}(d_{eck}+if_{eck})G_\nu^{Jb}z^{[\mu\nu]e}\end{pmatrix}+\\
&+\lambda_3\begin{pmatrix} \frac{1}{2}a_I(d_{aek}-if_{aek}\partial_\nu z^{[\mu\nu]e}+\frac{g}{2}a_{(IJ)}f_{abc}(d_{ekc}-if_{ekc})G_\nu^{Jb}z^{[\mu\nu]e}\\ +\frac{1}{2}b_{(IJ)}f_{abc}(id_{eck}-f_{eck})G_\nu^{Jb}Z^{[\mu\nu]e}+\frac{1}{N}c_{[IJ]}G_\nu^{Ja}Z^{[\mu\nu]k}\\ +\frac{1}{2}c_{[IJ]}d_{abc}(d_{eck}+if_{eck})G_\nu^{Jb}Z^{[\mu\nu]e}+\frac{1}{N}\gamma_{[IJ]}G_\nu^{Ja}Z^{[\mu\nu]k}\\ +\frac{1}{2}\gamma_{[IJ]}d_{abc}(d_{eck}+if_{eck})G_\nu^{Jb}Z^{[\mu\nu]e}\end{pmatrix}+\\
&+\xi_1\begin{pmatrix} \beta_I(if_{aek}-d_{aek})\partial_\nu Z^{(\mu\nu)e}+\rho_I(if_{aek}-d_{aek})\partial^\mu Z_{(\nu}{}^{\nu)e}\\ +(g_I\beta_J+g_J\beta_I)f_{abc}(if_{aek}-d_{aek})G_\nu^{Jb}Z^{(\mu\nu)e}\\ +(g_I\rho_J+g_J\rho_I)f_{abc}(if_{aek}-d_{aek})G^{\mu Jb}Z_{(\mu}{}^{\mu)e}\end{pmatrix}+\\
&+\xi_2\begin{pmatrix} b_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G_\nu^{Jb}z^{(\mu\nu)e}+\frac{2}{N}c_{(IJ)}G_\nu^{Ja}z^{(\mu\nu)k}\\ +c_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G_\nu^{Jb}z^{(\mu\nu)e}\\ +u_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{\mu Jb}z_{(\nu}{}^{\nu)e}+\frac{2}{N}v_{(IJ)}G^{\mu Ja}z_{(\nu}{}^{\nu)k}\\ +v_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{\mu Jb}z_{(\nu}{}^{\nu)e}\end{pmatrix}+\\
&+\xi_3\begin{pmatrix} \frac{1}{2}\beta_I(if_{aek}-d_{aek})\partial_\nu z^{(\mu\nu)e}+\frac{1}{2}\rho_I(if_{aek}-d_{aek})\partial^\mu z_{(\nu}{}^{\nu)e}\\ +\frac{1}{2}(g_I\beta_J+g_J\beta_I)f_{abc}(if_{aek}-d_{aek})G_\nu^{Jb}z^{(\mu\nu)e}\\ +\frac{1}{2}(g_I\rho_J+g_J\rho_I)f_{abc}(if_{aek}-d_{aek})G^{\mu Jb}z_{(\mu}{}^{\mu)e}\\ +\frac{1}{2}b_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G_\nu^{Jb}Z^{(\mu\nu)e}+\frac{1}{N}c_{(IJ)}G_\nu^{Jb}Z^{(\mu\nu)k}\\ +\frac{1}{2}c_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G_\nu^{Jb}Z^{(\mu\nu)e}\\ +\frac{1}{2}u_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{\mu Jb}Z_{(\nu}{}^{\nu)e}+\frac{1}{N}v_{(IJ)}G^{\mu Jb}Z_{(\nu}{}^{\nu)k}\\ +\frac{1}{2}v_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{\mu Jb}Z_{(\nu}{}^{\nu)e}\end{pmatrix}
\end{aligned}
\tag{71}
$$

The local Noether equations associated to the $SU(N)$ symmetry are

$$\partial_\mu J_N^{\mu a}(G) = 0 \tag{72}$$

$$\frac{i}{g_I}\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu G_{\nu I}^a)} = J_N^{\mu a} \tag{73}$$

$$\frac{i}{g_I}\frac{\delta \mathcal{L}}{\delta(\partial_\mu G_{\nu I}^a)}\partial^\mu \partial^\nu \omega^a = 0 \tag{74}$$

where

$$J_N^{\mu a} = [\frac{\delta \mathcal{L}}{\delta(\partial_\mu G_{\nu I}^a)}, G_\nu^I]^a \tag{75}$$

Eq.(73) is understood as the symmetry equation involving a set of fields.

The granular Bianchi identities are

$$D_\rho G_{[\mu\nu]I} + D_\nu G_{[\rho\mu]I} + D_\mu G_{[\nu\rho]I} = 0 \tag{76}$$

The antisymmetric collectives ones are

$$\partial_\mu z_{[\nu\rho]} + \partial_\nu z_{[\rho\mu]} + \partial_\rho z_{[\mu\nu]} = \gamma_{[IJ]} G_\nu^I G_{[\mu\rho]}^J + \gamma_{[IJ]} G_\rho^I G_{[\nu\mu]}^J + \gamma_{[IJ]} G_\mu^I G_{[\rho\nu]}^J \tag{77}$$

The collective symmetric Bianchi identity is

$$\partial_\mu z_{(\nu\rho)} + \partial_\nu z_{(\rho\mu)} + \partial_\rho z_{(\mu\nu)} = \gamma_{(IJ)} G_\mu^I G_{(\nu\rho)}^J + \gamma_{(IJ)} G_\nu^I G_{(\rho\mu)}^J + \gamma_{(IJ)} G_\rho^I G_{(\mu\nu)}^J \tag{78}$$

and

$$\partial_\mu z_{(\nu}{}^{\nu)} + 2\partial_\nu z_{(\nu}{}^{\nu)} = \gamma_{(IJ)} G_\mu^I G_\nu^{\nu J} + 2\gamma_{(IJ)} G_\nu^I G_\mu^{\nu J} \tag{79}$$

The above classical equations are showing that from $SU(N)$ symmetry it is possible to generate a physics with Yang-Mills families interlaced. Something that move us to understand that QCD is not the only one theory related to $SU(3)_c$ [29, 30].

5 Conclusion

Physical laws are being derived from symmetry. Three performances classify the symmetry behavior. They are the phase factor–gauge transformation [31, 32, 33], spontaneous symmetry breaking [13, 14, 15] and the symmetry of difference [34, 35]. Another possibility is the dynamical symmetry breaking [16]. Most of present physical models are developed through the first two aspects. A mechanism where the first one generates interactions and the SSB masses. At this context the standard model unifies the electroweak interaction with Higgs [36].

Eq.(1) assumes as physical principle that nature contains diversity and systemic behaviors. This leads us to look for a symmetry performance able to sustain both aspects. So the corresponding fields set introduces a step forward in the symmetry notion. It provides the symmetry of difference. Historically, symmetry comes out through the multiplet degeneracy [6]. For instance, as degeneracy examples on energy and mass, take the hydrogen atom described through $SU(4)$ symmetry or the proton and neutron associated through a same multiplet. Nevertheless, eq.(1) incorporates a quanta diversity. It says that it is also possible to introduce different quantum numbers under a same symmetry. So, instead of Weyl degeneracy, it develops a non-degeneracy called as pluriformity.

A new approach to make physics under symmetry appears. It opens a new relationship between physics and group theory. Based on a fields set gauge transformations, eq.(1) introduces different fields families under a common $SU(N)$ symmetry group. It says that group theory is not anymore enough for defining the physicity. From a common gauge parameter one can define different Lagrangians involving an undetermined number of potential fields. As consequence, the relationship between physics and symmetry must be restructured. Something saying that the physical principle precedes the symmetry. We follow the whole principle as basis to construct physical laws. After that depending on the whole physicity to be described one defines the group and number of fields associated. For instance, an electromagnetism defined by four bósons [37].

Our viewpoint is that nature laws are antireductionist. For this, we should center the physics behavior under a same single group. Given a certain number of fields instead of establishing relationships through a symmetry group decomposition as $SU(M)\times SU(N)\times U(1)$ or even by introducing a bigger group as $SU(5)$, it is possible to associate any branch of fields under a common $SU(N)$ group. Make them to share the same conservation laws through Noether identities and other interlaced equations.

A first consequence is on QCD. Being a model derived from the experimental result where quarks are derived with three colours, it is associated to the $SU(3)_c$ group. Then, considering the Yang-Mills interpretation, one associates eight gluons interacting with quarks. Nevertheless, without violating $SU(3)_c$ experimental results, eq.(1) introduces massive gluons. Physically massive gluons can be responsible for short interactions.

A new perspective for studying the quarks world becomes available. Quarks bring physics for a mysterious nature behavior which is confinement. Fifty years after quarks discovery, up to now physics does not have an answer to it. QCD is being the only one field theory model for describing the quarks properties. It is main result is on asymptotic freedom [38, 39], but as counterpart it has infrared problems [40], including at high energies [41]. Thus the introduction of massive gluons are welcomed for solving infrared problems since the asymptotic freedom property be preserved. Given the increasing number of non-linear terms in eq.(59) with respect to QCD, its asymptotic freedom property is expected. Another possible physical situation is on the possibility of having massive charged gluons intermediating charged quarks [42].

Concluding, a new opportunity to do physics is through antireductionistic gauge fields [25, 34, 42]. Instead of looking for new space and geometry categories as extra dimensions [43], supersymmetry [10], quantum gravity [44] or even through $SU(5)$ grand unification [45], understand physical laws through the wholeness principle.

Appendix

6 Group Theory Relationships

The following useful identities are related to the $SU(N)$ generators t_a:

(i) Commutation

$$[t_a, t_b] = i f_{abc} t^c \tag{80}$$

(ii) Anti-commutation

$$\{t_a, t_b\} = \frac{1}{N}\delta_{ab}\, id + d_{abc}t^c \tag{81}$$

(iii) Trace

$$\begin{aligned}
&\mathrm{tr}\, t_a = 0 \\
&\mathrm{tr}\, t_a t_b = \frac{1}{2}\delta_{ab} \\
&\mathrm{tr}\, t_a t_b t_c = \frac{1}{4}(i\, f_{abc} + d_{abc}) \\
&\mathrm{tr}\, t_a t_b t_c t_d = \frac{1}{4N}\delta_{ab}\delta_{cd} + \frac{1}{8}(i\, f_{abc} + d_{abc})(i\, f_{cd}{}^{e} + d_{cd}{}^{e})
\end{aligned} \tag{82}$$

7 Non-abelian Vectorial Equations at Constructor Basis

Defining

$$\phi \equiv D_0, \;\; \vec{D} \equiv -D_i, \;\; \phi^I \equiv X_0{}^I, \;\; \vec{X}^I \equiv -X_i{}^I \tag{83}$$

one gets for the anti-symmetric sector the following electromagnetic fields:

$$\vec{E} \equiv D_{0i}, \;\; \vec{B} \equiv \frac{1}{2}\epsilon_{imn} D_{nm} \vec{E}^I \equiv X_{[0i]}{}^I, \;\; \vec{B}^I \equiv \frac{1}{2}\epsilon_{ijk} X_{[kj]}{}^I \tag{84}$$

$$\vec{e} \equiv z_{[0i]}, \;\; \vec{b} \equiv \frac{1}{2}\epsilon_{ijk} z_{[kj]} \tag{85}$$

For symmetric sector:

$$\begin{gathered} \sigma \equiv \beta_I X_{(00)}{}^I, \;\; \vec{\sigma} \equiv \beta_I X_{(0i)}{}^I, \;\; \sigma_{ij} \equiv \beta_I X_{(ij)}{}^I \\ \theta \equiv \rho_I g_{00} X_{(\alpha}{}^{\alpha)i}, \;\; \theta_{ij} \equiv \rho_I g_{ij} X_{(\alpha}{}^{\alpha)i} \\ \vec{\Lambda} \equiv z_{(0i)}, \;\; \Gamma_{ij} \equiv z_{(ij)}, \tau \equiv z_{(\mu}{}^{\mu)}, \;\; \tau + \Gamma_{ii} \equiv z_{(00)} \end{gathered} \tag{86}$$

where

$$I = 2, \dots, N \qquad\qquad i = 1, 2, 3 \tag{87}$$

7.1 Non-abelian Maxwell Equations

Considering the field $D_\mu{}^a$, one derives the following equations:

For $\mu = 0$,

$$\begin{aligned} &d\vec{\nabla} \cdot (2d\vec{E}^a + 2\alpha_I \vec{E}^{Ia} + \vec{e}^a) - d\frac{g}{N} f_{abc} \vec{D}^b \cdot (2d\vec{E}^c + 2\alpha_I \vec{E}^{Ic} + \vec{e}^c) \\ &+ \frac{g}{N}\alpha_I f_{abc} \vec{X}^{Ib} \cdot (2d\vec{E}^c + 2\alpha_J \vec{E}^{Jc} + \vec{e}^c) - \frac{g}{N}\beta_I f_{abc} \vec{X}^{Ib} \cdot (2\vec{\sigma}^c + \vec{\Lambda}^c) \\ &+ \frac{g}{N}\beta_I f_{abc} \phi^{Ib} (2\sigma^c + 2\theta^c + \tau^c + \Gamma_j{}^{jc}) + 2\frac{g}{N}\rho_I f_{abc}\phi^{Ib}(\sigma^c + \theta^c) \\ &+ \frac{g}{N}\rho_I f_{abc}\phi^{Ib}(2\sigma_j{}^{jc} + 2\theta_j{}^{jc} + \Gamma_j{}^{jc}) \\ &= 0 \end{aligned} \tag{88}$$

For $\mu = i$,

$$\begin{aligned} &d\vec{\nabla} \times (2d\vec{B}^a + 2\alpha_I \vec{B}^{Ia} + \vec{b}^a) - d\frac{g}{N} f_{abc} \vec{D}^b \times (2d\vec{B}^c + 2\alpha_I \vec{B}^{Ic} + \vec{b}^c) \\ &+ \frac{g}{N}\alpha_I f_{abc} \vec{X}^{Ib} \times (2d\vec{B}^c + 2\alpha_J \vec{B}^{Jc} + \vec{b}^c) - d\frac{\partial}{\partial t}(2d\vec{E}^a + 2\alpha_I \vec{E}^{Ia} + \vec{e}^a) \\ &+ d\frac{g}{N} f_{abc}\phi^b(2d\vec{E}^c + 2\alpha_I \vec{E}^{Ic} + \vec{e}^c) - \frac{g}{N}\alpha_I f_{abc}\phi^{Ib}(2d\vec{E}^c + 2\alpha_J \vec{E}^{Jc} + \vec{e}^c) \\ &+ \frac{g}{N}\beta_I f_{abc} \vec{X}^{Ib}(2\sigma^{ijc} + 2\theta^{ijc} + \Gamma^{ijc}) - \frac{g}{N}\beta_I f_{abc}\phi^{Ib}(2\vec{\sigma}^c + \vec{\Lambda}^c) \\ &+ 2\frac{g}{N}\rho_I f_{abc}\vec{X}^{Ib}(\sigma^c + \theta^c) + \frac{g}{N}\rho_I f_{abc}\vec{X}^{Ib}(2\sigma_j{}^{jc} + 2\theta_j{}^{jc} + 2\Gamma_j{}^{jc}) \\ &= 0 \end{aligned} \tag{89}$$

Multiplying by t_k, one derives the non-abelian Gauss law:

$$
\begin{aligned}
&-\alpha_I\vec{\nabla}\cdot(2d\vec{E}^a+2\alpha_J\vec{E}^{Ja}+\vec{e}^a)+\frac{g}{N}\alpha_I f_{abc}\vec{D}^b\cdot(2d\vec{E}^c+2\alpha_J\vec{E}^{Jc}+\vec{e}^c)\\
&-\frac{1}{2}i\gamma_{[IJ]}\vec{X}^{Jb}\cdot(df_{abc}\vec{E}^c+\alpha_K f_{abc}\vec{E}^{Kc}+2d_{abc}\vec{e}^c)\\
&-ia_{[IJ]}f_{abc}\vec{X}^{Jb}\cdot(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)-b_{[IJ]}d_{abc}\vec{X}^{Jb}\cdot(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)\\
&-2ia_{(IJ)}f_{abc}\vec{X}^{Jb}\cdot\vec{e}^c-b_{(IJ)}d_{abc}\vec{X}^{Jb}\cdot(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)\\
&-2\beta_I\frac{\partial}{\partial t}(\sigma^a+\theta^a)+\beta_I\frac{\partial}{\partial t}(\tau^a+\Gamma_j^{\ ja})\\
&+\beta_I\vec{\nabla}\cdot(2\vec{\sigma}^a+\vec{\Lambda}^a)-\frac{g}{N}\beta_I f_{abc}\vec{D}^b\cdot(2\vec{\sigma}^c+\vec{\Lambda}^c)\\
&+2\frac{g}{N}\beta_I f_{abc}\phi^b(\sigma^c+\theta^c)+\frac{g}{N}\beta_I f_{abc}\phi^b(\tau^c+\Gamma_j^{\ jc})\\
&-2\rho_I\frac{\partial}{\partial t}(\sigma^a+\theta^a)+\rho_I\frac{\partial}{\partial t}(2\sigma_j^{\ ja}+2\theta_j^{\ ja}+\Gamma_j^{\ ja})\\
&+2\frac{g}{N}\rho_I f_{abc}\phi^b(\sigma^c+\theta^c)+\frac{g}{N}\rho_I f_{abc}\phi^b(2\sigma_j^{\ jc}+2\theta_j^{\ jc}+\Gamma_j^{\ jc})\\
&+iu_{[IJ]}f_{abc}\phi^{Jb}(\sigma^c+\theta^c+4\tau^c)-iu_{[IJ]}f_{abc}\phi^{Jb}(\sigma_j^{\ jc}+\theta_j^{\ jc}+2\Gamma_j^{\ jc})\\
&+v_{(IJ)}d_{abc}\phi^{Jb}(\sigma^c+\theta^c+4\tau^c)-v_{(IJ)}d_{abc}\phi^{Jb}(\sigma_j^{\ jc}+\theta_j^{\ jc}+2\Gamma_j^{\ jc})\\
&=0
\end{aligned}
\tag{90}
$$

Ampère law:

$$
\begin{aligned}
&-\alpha_I\vec{\nabla}\times(2d\vec{B}^a+2\alpha_J\vec{B}^{Ja}+\vec{b}^a)+\frac{g}{N}\alpha_I f_{abc}\vec{D}^b\times(2d\vec{B}^c+2\alpha_J\vec{B}^{Jc}+\vec{b}^c)\\
&-\frac{1}{2}i\gamma_{[IJ]}f_{abc}\vec{X}^{Jb}\times(d\vec{B}^c+\alpha_K\vec{B}^{Kc}+2\vec{b}^c)\\
&-ia_{[IJ]}f_{abc}\vec{X}^{Jb}\times(d\vec{B}^c+\alpha_K\vec{B}^{Kc}+2\vec{b}^c)-b_{[IJ]}d_{abc}\vec{X}^{Jb}\times(d\vec{B}^c+\alpha_K\vec{B}^{Kc}+2\vec{b}^c)\\
&+2ia_{(IJ)}f_{abc}\vec{X}^{Jb}\times\vec{b}^c-b_{(IJ)}d_{abc}\vec{X}^{Jb}\times(d\vec{B}^c+\alpha_K\vec{B}^{Kc}+2\vec{b}^c)\\
&+\alpha_I\frac{\partial}{\partial t}(2d\vec{E}^a+2\alpha_J\vec{E}^{Ja}+\vec{e}^a)-\frac{g}{N}\alpha_I f_{abc}\phi^b(2d\vec{E}^c+2\alpha_J\vec{E}^{Jc}+\vec{e}^c)\\
&+\frac{1}{2}i\gamma_{[IJ]}f_{abc}\phi^{Jb}(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)+ia_{[IJ]}f_{abc}\phi^{Jb}(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)\\
&+b_{[IJ]}d_{abc}\phi^{Jb}(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)+2ia_{(IJ)}f_{abc}\phi^{Jb}(\vec{e}^c)\\
&+b_{(IJ)}d_{abc}\phi^{Jb}(d\vec{E}^c+\alpha_K\vec{E}^{Kc}+2\vec{e}^c)+\beta_I\frac{\partial}{\partial t}(2\vec{\sigma}^a+\vec{\Lambda}^a)\\
&-\beta_I\vec{\nabla}(2\sigma^{ija}+2\theta^{ija}+\Gamma^{ija})+\frac{g}{N}\beta_I f_{abc}\vec{D}^b(2\sigma^{ijc}+2\theta^{ijc}+\Gamma^{ijc})\\
&-\frac{g}{N}\beta_I f_{abc}\phi^b(2\vec{\sigma}^c+\vec{\Lambda}^c)-2\rho_I\vec{\nabla}(\sigma^a+\theta^a)\\
&-2\rho_I\vec{\nabla}(2\sigma_j^{\ ja}+2\theta_j^{\ ja}+\Gamma_j^{\ ja})+2\frac{g}{N}\rho_I f_{abc}\vec{D}^b(\sigma^c+\theta^c)\\
&+\frac{g}{N}\rho_I f_{abc}\vec{D}^b(2\sigma_j^{\ jc}+2\theta_j^{\ jc}+\Gamma_j^{\ jc})+iu_{[IJ]}f_{abc}\vec{X}^{Jb}(\sigma^c+\theta^c+4\tau^c)\\
&-iu_{[IJ]}f_{abc}\vec{X}^{Jb}(\sigma_j^{\ jc}+\theta_j^{\ jc}+2\Gamma_j^{\ jc})+v_{(IJ)}d_{abc}\vec{X}^{Jb}(\sigma^c+\theta^c+4\tau^c)\\
&-v_{(IJ)}d_{abc}\vec{X}^{Jb}(\sigma_j^{\ jc}+\theta_j^{\ jc}+2\Gamma_j^{\ jc})\\
&=0
\end{aligned}
\tag{91}
$$

For $X_\mu^{\ ia}$, $\mu=0$:

$$
\begin{aligned}
& d\alpha_I d_{aek}\vec{\nabla}\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)-idf_{aek}\vec{\nabla}\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)\\
& -d\frac{g}{N}d_{ekl}f_{abl}\vec{D}^b\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)+id\frac{g}{N}f_{ekl}f_{abl}\vec{D}^b\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)\\
& -\frac{g}{N}\alpha_I d_{ekl}f_{abl}\vec{X}^{Ib}\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)+i\frac{g}{N}\alpha_I f_{ekl}f_{abl}\vec{X}^{Ib}\cdot(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e})\\
& -\frac{g}{N}\beta_I d_{ekl}f_{abl}\vec{X}^{Ib}\cdot(\vec{\sigma}^e+\frac{1}{2}\vec{\Lambda}^e)+i\frac{g}{N}\beta_I f_{ekl}f_{abl}\vec{X}^{Ib}\cdot(\vec{\sigma}^e+\frac{1}{2}\vec{\Lambda}^e)\\
& +\frac{g}{N}\beta_I d_{ekl}f_{abl}\phi^{Ib}(\sigma^e+\theta^e)-i\frac{g}{N}\beta_I f_{ekl}f_{abl}\phi^{Ib}(\sigma^e+\theta^e)\\
& +\frac{g}{2N}\beta_I d_{ekl}f_{abl}\phi^{Ib}(\tau^e+\Gamma_j^{\ je})-i\frac{g}{2N}\beta_I f_{ekl}f_{abl}\phi^{Ib}(\tau^e+\Gamma_j^{\ je})\\
& +\frac{g}{N}\rho_I d_{ekl}f_{abl}\phi^{Ib}(\sigma^e+\theta^e)-i\frac{g}{N}\rho_I f_{ekl}f_{abl}\phi^{Ib}(\sigma^e+\theta^e)\\
& +\frac{g}{N}\rho_I d_{ekl}f_{abl}\phi^{Ib}(\sigma_j^{\ je}+\theta_j^{\ je}+\frac{1}{2}\Gamma_j^{\ je})-i\frac{g}{N}\rho_I f_{ekl}f_{abl}\phi^{Ib}(\sigma_j^{\ je}+\theta_j^{\ je}+\frac{1}{2}\Gamma_j^{\ je})\\
& =0
\end{aligned}
\tag{92}
$$

and $\mu = i$:

$$
\begin{aligned}
& -d\frac{g}{N}d_{ekl}f_{abl}\vec{D}^b\times(d\vec{B}^e+\alpha_I\vec{B}^{Ie}+\frac{1}{2}\vec{b}^e)+id\frac{g}{N}f_{ekl}f_{abl}\vec{D}^b\times(d\vec{B}^e+\alpha_I\vec{B}^{Ie}+\frac{1}{2}\vec{b}^e)\\
& +dd_{aek}\vec{\nabla}\times(d\vec{B}^e+\alpha_I\vec{B}^{Ie}+\frac{1}{2}\vec{b}^e)-idf_{aek}\vec{\nabla}\times(d\vec{B}^e+\alpha_I\vec{B}^{Ie}+\frac{1}{2}\vec{b}^e)\\
& -\frac{g}{N}\alpha_I d_{ekl}f_{abl}\vec{X}^{Ib}\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\frac{1}{2}\vec{b}^e)+i\frac{g}{N}\alpha_I f_{ekl}f_{abl}\vec{X}^{Ib}\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\frac{1}{2}\vec{b}^e)\\
& -d\,d_{aek}\frac{\partial}{\partial t}(d\vec{E}^e+\alpha_I\vec{E}^{Ie}+\frac{1}{2}\vec{e}^e)+id\,f_{aek}\frac{\partial}{\partial t}(d\vec{E}^e+\alpha_I\vec{E}^{Ie}+\frac{1}{2}\vec{e}^e)\\
& +d\,\frac{g}{N}d_{ekl}f_{abl}\phi^b(d\vec{E}^e+\alpha_I\vec{E}^{Ie}+\frac{1}{2}\vec{e}^e)-id\,\frac{g}{N}f_{ekl}f_{abl}\phi^b(d\vec{E}^e+\alpha_I\vec{E}^{Ie}+\frac{1}{2}\vec{e}^e)\\
& +\frac{g}{N}\alpha_I d_{ekl}f_{abl}\phi^{Ib}(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)-i\frac{g}{N}\alpha_I f_{ekl}f_{abl}\phi^{Ib}(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)\\
& +\frac{g}{N}\beta_I d_{ekl}f_{abl}\vec{X}^{Ib}(\sigma^{ije}+\theta^{ije}+\frac{1}{2}\Gamma^{ije})-i\frac{g}{N}\beta_I f_{ekl}f_{abl}\vec{X}^{Ib}(\sigma^{ije}+\theta^{ije}+\frac{1}{2}\Gamma^{ije})\\
& -\frac{g}{N}\beta_I d_{ekl}f_{abl}\phi^{Ib}(\vec{\sigma}^e+\frac{1}{2}\vec{\Lambda}^e)+i\frac{g}{N}\beta_I f_{ekl}f_{abl}\phi^{Ib}(\vec{\sigma}^e+\frac{1}{2}\vec{\Lambda}^e)\\
& +\frac{g}{N}\rho_I d_{ekl}f_{abl}\vec{X}^{Ib}(\sigma^e+\theta^e)-i\frac{g}{N}\rho_I f_{ekl}f_{abl}\vec{X}^{Ib}(\sigma^e+\theta^e)\\
& +\frac{g}{N}\rho_I d_{ekl}f_{abl}\vec{X}^{Ib}(\sigma_j^{\ je}+\theta_j^{\ je}+\frac{1}{2}\Gamma_j^{\ je})-i\frac{g}{N}\rho_I f_{ekl}f_{abl}\vec{X}^{Ib}(\sigma_j^{\ je}+\theta_j^{\ je}+\frac{1}{2}\Gamma_j^{\ je})\\
& =0
\end{aligned}
\tag{93}
$$

Multiplying by t_k , one gets the corresponding X_μ^{ia} -Gauss law:

$$
\begin{aligned}
&- \alpha_I d_{aek} \vec{\nabla} \cdot (d\vec{E}^e + \alpha_J \vec{E}^{Je} + \frac{1}{2}\vec{e}^e) + i\alpha_I f_{aek} \vec{\nabla} \cdot (d\vec{E}^e + \alpha_J \vec{E}^{Je} + \frac{1}{2}\vec{e}^e) \\
&+ \frac{g}{N} d_{ekl} \alpha_I f_{abl} \vec{D}^b \cdot (d\vec{E}^e + \alpha_J \vec{E}^{Je} + \vec{e}^e) - i\frac{g}{N} \alpha_I f_{ekl} f_{abl} \vec{D}^b \cdot (d\vec{E}^e + \alpha_J \vec{E}^{Je} + \vec{e}^e) \\
&- \frac{1}{N} \gamma_{[IJ]} \vec{X}^{Ia} \cdot \vec{e}^k - \frac{1}{2} \gamma_{[IJ]} d_{abl} d_{ekl} \vec{X}^{Jb} \cdot \vec{e}^e + \frac{i}{2} \gamma_{[IJ]} d_{abl} f_{ekl} \vec{X}^{Jb} \cdot \vec{e}^e \\
&- f_{abl} f_{ekl} \vec{X}^{Jb} (a_{(IJ)} \vec{e}^e + a_{[IJ]} \vec{\Lambda}^e) - i f_{abl} d_{ekl} \vec{X}^{Jb} (a_{(IJ)} \vec{e}^e + a_{[IJ]} \vec{\Lambda}^e) \\
&- \frac{2}{N} \vec{X}^{Ja} \cdot (b_{[IJ]} \vec{e}^k + b_{(IJ)} \vec{\Lambda}^k) - d_{abc} d_{cek} \vec{X}^{Jb} (b_{[IJ]} \vec{e}^e + b_{(IJ)} \vec{\Lambda}^e) \\
&+ i d_{abc} f_{cek} \vec{X}^{Jb} (b_{[IJ]} \vec{e}^e + b_{(IJ)} \vec{\Lambda}^e) - 2\frac{d}{N} b_{[IJ]} \vec{X}^{Ja} \cdot (d\vec{E}^k + \alpha_K \vec{E}^{Kk}) \\
&- b_{(IJ)} d_{abl} d_{ekl} \vec{X}^{Jb} \cdot (d\vec{E}^k + \alpha_K \vec{E}^{Kk}) + i b_{[IJ]} d_{abl} f_{ekl} \vec{X}^{Jb} \cdot (d\vec{E}^k + \alpha_K \vec{E}^{Kk}) \\
&- \beta_I d_{aek} \partial_t (\sigma^e + \theta^e) + \frac{1}{2} \beta_I d_{aek} \partial_t (\tau^e + \Gamma_j{}^{je}) \\
&+ i\beta_I f_{aek} \partial_t (\sigma^e + \theta^e) - \frac{1}{2} i\beta_I f_{aek} \partial_t (\tau^e + \Gamma_j{}^{je}) \\
&+ \beta_I d_{aek} \vec{\nabla} \cdot (\vec{\sigma}^e + \frac{1}{2} \vec{\Lambda}^e) - i\beta_I f_{aek} \vec{\nabla} \cdot (\vec{\sigma}^e + \frac{1}{2} \vec{\Lambda}^e) \\
&- \frac{g}{2N} \beta_I d_{ekl} f_{abl} \vec{D}^b \cdot \vec{\sigma}^e + i\frac{g}{2N} \beta_I f_{ekl} f_{abl} \vec{D}^b \cdot \vec{\sigma}^e \\
&+ \frac{g}{2N} \beta_I d_{ekl} f_{abl} \phi^b (\sigma^e + \theta^e) - i\frac{g}{2N} \beta_I f_{ekl} f_{abl} \phi^b (\sigma^e + \theta^e) \\
&- \rho_I d_{aek} \partial_t (\sigma_j{}^{je} + \theta_j{}^{je} + \frac{1}{2} \Gamma_j{}^{je}) + i\rho_I f_{aek} \partial_t (\sigma_j{}^{je} + \theta_j{}^{je} + \frac{1}{2} \Gamma_j{}^{je}) \\
&- \rho_I d_{aek} \partial_t (\sigma^e + \theta^e) + i\rho_I f_{aek} \partial_t (\sigma^e + \theta^e) \\
&+ \frac{g}{N} \rho_I d_{ekl} f_{abl} \phi^b (\sigma^e + \theta^e) - i\frac{g}{N} \rho_I f_{ekl} f_{abl} \phi^b (\sigma^e + \theta^e) \\
&+ \frac{g}{N} \rho_I d_{ekl} f_{abl} \phi^b (\sigma_j{}^{je} + \theta_j{}^{je} - \frac{3}{4} \Gamma_j{}^{je}) - i\frac{g}{N} \rho_I f_{ekl} f_{abl} \phi^b (\sigma_j{}^{je} + \theta_j{}^{je} + \frac{3}{4} \Gamma_j{}^{je}) \\
&+ a_{[IJ]} f_{ekl} f_{abl} \phi^{Jb} (\tau^e + \Gamma_j{}^{je}) + i a_{[IJ]} d_{ekl} f_{abl} \phi^{Jb} (\tau^e + \Gamma_j{}^{je}) \\
&+ \frac{2}{N} b_{(IJ)} \phi^{Ja} (\tau^k + \Gamma_j{}^{jk}) + b_{(IJ)} d_{abc} d_{cek} \phi^{Jb} (\tau^e + \Gamma_j{}^{je}) - i b_{(IJ)} d_{abc} f_{cek} \phi^{Jb} (\tau^e + \Gamma_j{}^{je}) \\
&+ \frac{1}{2} u_{[IJ]} f_{abl} f_{ekl} \phi^{Jb} (\sigma^e + \theta^e) + \frac{i}{2} u_{[IJ]} f_{abl} d_{ekl} \phi^{Jb} (\sigma^{Ib} + \theta^e) \\
&- \frac{1}{2} u_{[IJ]} f_{abl} f_{ekl} \phi^{Ib} (\sigma_j{}^{je} + \theta_j{}^{je}) - \frac{i}{2} u_{[IJ]} f_{abl} d_{ekl} \phi^{Jb} (\sigma_j{}^{je} + \theta_j{}^{je}) \\
&+ u_{[IJ]} f_{ekl} f_{abl} \phi^{Jb} (2\tau^e + \Gamma_j{}^{je}) + i u_{[IJ]} d_{ekl} f_{abl} \phi^{Jb} (2\tau^e + \Gamma_j{}^{je}) \\
&+ \frac{1}{N} v_{(IJ)} \phi^{Ja} (\sigma^k + \theta^k) + \frac{1}{2} v_{(IJ)} d_{abl} d_{ekl} \phi^{Ib} (\sigma^e + \theta^e) - \frac{i}{2} v_{(IJ)} d_{abl} f_{ekl} \phi^{Ib} (\sigma^e + \theta^e) \\
&- \frac{1}{N} v_{(IJ)} \phi^{Ia} (\sigma_j{}^{jk} + \theta_j{}^{ke}) - \frac{1}{2} v_{(IJ)} d_{abl} d_{ekl} \phi^{Ib} (\sigma_j{}^{je} + \theta_j{}^{je}) + \frac{i}{2} v_{(IJ)} d_{abl} f_{ekl} \phi^{Ib} (\sigma_j{}^{je} + \theta_j{}^{je}) \\
&+ \frac{2}{N} v_{(IJ)} \phi^{Ja} (2\tau^k + \Gamma_j{}^{jk}) + v_{(IJ)} d_{abc} d_{cek} \phi^{Jb} (2\tau^e + \Gamma_j{}^{je}) - i v_{(IJ)} d_{abc} f_{cek} \phi^{Jb} (2\tau^e + \Gamma_j{}^{je}) \\
&= 0
\end{aligned}
\tag{94}
$$

and the X_μ^{ia} - Ampère law:

$$\begin{aligned}
&-\alpha_I d_{aek}\vec{\nabla}\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\frac{1}{2}\vec{b}^e)+i\alpha_I f_{aek}\vec{\nabla}\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\frac{1}{2}\vec{b}^e)\\
&+\frac{g}{N}\alpha_I d_{ekl}f_{abl}\vec{D}^b\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\vec{b}^e)-i\frac{g}{N}\alpha_I f_{ekl}f_{abl}\vec{D}^b\times(d\vec{B}^e+\alpha_J\vec{B}^{Je}+\vec{b}^e)\\
&+\frac{1}{N}\gamma_{[IJ]}\times\vec{b}^k+\frac{1}{2}d_{abl}d_{ekl}\gamma_{[IJ]}\times\vec{b}^e-\frac{i}{2}d_{abl}f_{ekl}\gamma_{[IJ]}\times\vec{b}^e+a_{(IJ)}f_{abl}f_{ekl}\vec{X}^{Jb}\times\vec{b}^e\\
&+ia_{(IJ)}f_{abl}d_{ekl}\vec{X}^{Jb}\times\vec{b}^e+\frac{2}{N}b_{[IJ]}\vec{X}^{Jb}\times\vec{b}^k+b_{[IJ]}d_{abc}d_{cek}\vec{X}^{Jb}\times\vec{b}^e\\
&-ib_{[IJ]}d_{abc}f_{cek}\vec{X}^{Jb}\times\vec{b}^e-\frac{2}{N}b_{(IJ)}\vec{X}^{Ja}\times(d\vec{B}^k+\alpha_J\vec{B}^{Jk})\\
&-b_{(IJ)}d_{abl}d_{ekl}\vec{X}^{Jb}\times(d\vec{B}^e+\alpha_K\vec{B}^{Ke})+ib_{(IJ)}d_{abl}f_{ekl}\vec{X}^{Jb}\times(d\vec{B}^e+\alpha_K\vec{B}^{Ke})\\
&+\alpha_I d_{aek}\partial_t(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)-i\alpha_I f_{aek}\partial_t(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\frac{1}{2}\vec{e}^e)\\
&-\frac{g}{N}\alpha_I f_{abl}d_{ekl}\phi^b(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\vec{e}^e)+i\frac{g}{N}\alpha_I f_{abl}f_{ekl}\phi^b(d\vec{E}^e+\alpha_J\vec{E}^{Je}+\vec{e}^e)\\
&+\frac{2}{N}\gamma_{[IJ]}\phi^{Ja}(\vec{e}^k)+\frac{1}{2}\gamma_{[IJ]}d_{abl}d_{ekl}\phi^{Jb}(\vec{e}^e)-\frac{i}{2}\gamma_{[IJ]}d_{abl}f_{ekl}\phi^{Jb}(\vec{e}^e)\\
&+a_{(IJ)}f_{abl}d_{ekl}\phi^{Jb}(\vec{e}^e)+ia_{(IJ)}f_{abl}d_{ekl}\phi^{Jb}(\vec{e}^e)+\frac{2}{N}b_{[IJ]}\phi^{Ja}(\vec{e}^k)
\end{aligned}$$

$$
\begin{aligned}
&+ b_{[IJ]}d_{abc}d_{cek}\phi^{Jb}(\vec{e}^e) - ib_{[IJ]}d_{abc}f_{cek}\phi^{Jb}(\vec{e}^e) \\
&+ \frac{2}{N}b_{(IJ)}\phi^{Ja}(d\vec{E}^k + \alpha_K\vec{E}^{Kk} + \vec{e}^k) + b_{(IJ)}d_{abl}d_{ekl}\phi^{Ja}(d\vec{E}^k + \alpha_K\vec{E}^{Kk}) \\
&+ ib_{(IJ)}d_{abl}f_{ekl}\phi^{Ja}(d\vec{E}^k + \alpha_K\vec{E}^{Kk}) \\
&+ \beta_I d_{aek}\partial_t(\vec{\sigma}^e + \frac{1}{2}\vec{\Lambda}^e) - i\beta_I f_{aek}\partial_t(\vec{\sigma}^e + \frac{1}{2}\vec{\Lambda}^e) \\
&- \beta_I d_{aek}\vec{\nabla}(\sigma^{ije} + \theta^{ije} + \frac{1}{2}\Gamma^{ije}) + i\beta_I f_{aek}\vec{\nabla}(\sigma^{ije} + \theta^{ije} + \frac{1}{2}\Gamma^{ije}) \\
&+ \frac{g}{2N}\beta_I d_{ekl}f_{abl}\vec{D}^b(\sigma^{ije} + \theta^{ije}) - i\frac{g}{2N}\beta_I f_{ekl}f_{abl}\vec{D}^b(\sigma^{ije} + \theta^{ije}) \\
&- \frac{g}{2N}\beta_I f_{abl}d_{ekl}\phi^b(\sigma^e) + i\frac{g}{2N}\beta_I f_{abl}f_{ekl}\phi^b(\sigma^e) \\
&- \rho_I d_{aek}\vec{\nabla}(\sigma^e + \theta^e) + i\rho_I f_{aek}\vec{\nabla}(\sigma^e + \theta^e) \\
&- \rho_I d_{aek}\vec{\nabla}(\sigma_j{}^{je} + \theta_j{}^{je} + \frac{1}{2}\Gamma_j{}^{je}) + i\rho_I f_{aek}\vec{\nabla}(\sigma_j{}^{je} + \theta_j{}^{je} + \frac{1}{2}\Gamma_j{}^{je}) \\
&+ \frac{g}{N}\rho_I d_{ekl}f_{abl}\vec{D}^b(\sigma^e + \theta^e) - i\frac{g}{N}\rho_I f_{ekl}f_{abl}\vec{D}^b(\sigma^e + \theta^e) \\
&+ \frac{g}{N}\rho_I d_{ekl}f_{abl}\vec{D}^b(\sigma_j{}^{je} + \theta_j{}^{je} + \frac{3}{4}\Gamma_j{}^{je}) - i\frac{g}{N}\rho_I f_{ekl}f_{abl}\vec{D}^b(\sigma_j{}^{je} + \theta_j{}^{je} + \frac{3}{4}\Gamma_j{}^{je}) \\
&+ a_{[IJ]}f_{abl}f_{ekl}\vec{X}^{Jb}(\Gamma^{ije}) + ia_{[IJ]}f_{abl}d_{ekl}\vec{X}^{Jb}(\Gamma^{ije}) \\
&- a_{[IJ]}f_{abl}f_{ekl}\phi^{Jb}(\vec{\Lambda}^e) - ia_{[IJ]}f_{abl}d_{ekl}\phi^{Jb}(\vec{\Lambda}^e) \\
&+ \frac{2}{N}b_{(IJ)}\vec{X}^{Ja}(\Gamma^{ijk}) + b_{(IJ)}d_{abc}d_{cek}\vec{X}^{Jb}(\Gamma^{ije}) - ib_{(IJ)}d_{abc}f_{cek}\vec{X}^{Jb}(\Gamma^{ije}) \\
&- b_{(IJ)}d_{abc}d_{cek}\phi^{Jb}(\vec{\Lambda}^e) + ib_{(IJ)}d_{abc}f_{cek}\phi^{Jb}(\vec{\Lambda}^e) \\
&+ \frac{1}{2}u_{[IJ]}f_{abl}f_{ekl}\vec{X}^{Jb}(\sigma^e + \theta^e) + \frac{i}{2}u_{[IJ]}f_{abl}d_{ekl}\vec{X}^{Ib}(\sigma^e + \theta^e) \\
&- \frac{1}{2}u_{[IJ]}f_{abl}f_{ekl}\vec{X}^{Ib}(\sigma_j{}^{je} + \theta_j{}^{je}) - \frac{i}{2}u_{[IJ]}f_{abl}d_{ekl}\vec{X}^{Ib}(\sigma_j{}^{je} + \theta_j{}^{je}) \\
&+ u_{[IJ]}f_{abl}f_{ekl}\vec{X}^{Jb}(2\tau^e + \Gamma_j{}^{je}) + iu_{[IJ]}f_{abl}d_{ekl}\vec{X}^{Jb}(2\tau^e + \Gamma_j{}^{je}) \\
&+ \frac{1}{N}v_{(IJ)}\vec{X}^{Ja}(\sigma^k + \theta^k) + \frac{1}{2}v_{(IJ)}d_{abl}d_{ekl}\vec{X}^{Ib}(\sigma^e + \theta^e) \\
&- \frac{i}{2}v_{(IJ)}d_{abl}f_{ekl}\vec{X}^{Ib}(\sigma^e + \theta^e) - \frac{1}{N}v_{(IJ)}\vec{X}^{Ia}(\sigma_j{}^{jk} + \theta_j{}^{jk}) \\
&- \frac{1}{2}v_{(IJ)}d_{abl}d_{ekl}\vec{X}^{Ib}(\sigma_j{}^{je} + \theta_j{}^{je}) + \frac{i}{2}v_{(IJ)}d_{abl}f_{ekl}\vec{X}^{Ib}(\sigma_j{}^{je} + \theta_j{}^{je}) \\
&+ \frac{2}{N}v_{(IJ)}\vec{X}^{Ja}(\tau^k + 2\Gamma_j{}^{jk}) + v_{(IJ)}d_{abc}d_{cek}\vec{X}^{Jb}(2\tau^e + \Gamma_j{}^{je}) \\
&- iv_{(IJ)}d_{abc}f_{cek}\vec{X}^{Jb}(2\tau^e + \Gamma_j{}^{je}) \\
&= 0
\end{aligned} \tag{95}
$$

The corresponding vectorial expressions for the Bianchi identities are

$$
\begin{aligned}
&\vec{D} \times (d\vec{E} + \alpha_J\vec{E}^J) = -\frac{D}{Dt}(d\vec{B} + \alpha_J\vec{B}^J) \\
&\vec{D} \cdot (d\vec{B} + \alpha_J\vec{B}^J) = 0
\end{aligned} \tag{96}
$$

and

$$\begin{aligned} \vec{D} \times \vec{e} &= -\frac{D}{Dt}\vec{b} \\ \vec{D} \cdot \vec{b} &= 0 \end{aligned} \tag{97}$$

8 Lagrangian Building Blocks

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_A + \mathcal{L}_S \\ \mathcal{L}_A &= \lambda_1 Z_{[\mu\nu]} Z^{[\mu\nu]} + \lambda_2 z_{[\mu\nu]} z^{[\mu\nu]} + \lambda_3 Z_{[\mu\nu]} z^{[\mu\nu]} \\ \mathcal{L}_S &= \xi_1 Z_{[\mu\nu]} Z^{[\mu\nu]} + \xi_2 z_{[\mu\nu]} z^{[\mu\nu]} + \xi_3 Z_{[\mu\nu]} z^{[\mu\nu]} \end{aligned} \tag{98}$$

$(1)\ \ \mathcal{L} = \operatorname{tr} Z_{[\mu\nu]} Z^{[\mu\nu]}$

$$\begin{aligned} \mathcal{L}_K &= 2a_I a_J [(\partial_\mu G_\nu^I)(\partial^\mu G^{\nu J}) - (\partial_\mu G_\nu^I)(\partial^\nu G^{\mu J})] \\ \mathcal{L}_{\text{int}}^{(3)} &= -4iga_K a_{(IJ)} (\partial_\mu G_\nu^K)[G^{\mu I}, G^{\nu J}] \\ \mathcal{L}_{\text{int}}^{(4)} &= -g^2 a_{(IJ)} a_{(KL)} [G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \end{aligned} \tag{99}$$

Equation of motion

$$\operatorname{tr} \lambda_1 (4a_I \partial_\nu Z^{[\mu\nu]} t_a - 4iga_{(IJ)} G_\nu^{Jb} Z^{[\mu\nu]} [t_a, t_b]) = 0 \tag{100}$$

Taking the above relations' trace

$$2a_I \partial_\nu Z^{[\mu\nu]a} + 2ga_{(IJ)} f_{abc} G_\nu^{Jb} Z^{[\mu\nu]c} \tag{101}$$

Multiplying by t_k and taking the trace:

$$a_I (d_{aek} - if_{aek}) \partial_\nu Z^{[\mu\nu]e} + ga_{(IJ)} f_{abc} (d_{ekc} - if_{ekc}) G_\nu^{Jb} Z^{[\mu\nu]e} \tag{102}$$

$(2)\ \ \mathcal{L} = \operatorname{tr} z_{[\mu\nu]} z^{[\mu\nu]}$

$$\begin{aligned} \mathcal{L}_{\text{int}}^{(4)} = & \; b_{(IJ)} b_{(KL)} [G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \\ & + 2b_{(IJ)} c_{(KL)} [G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\} \\ & + 2\gamma_{[IJ]} b_{(KL)} G_\mu^I G_\nu^J [G^{\mu K}, G^{\nu L}] \\ & + c_{[IJ]} c_{[KL]} \{G_\mu^I, G_\nu^J\}\{G^{\mu K}, G^{\nu L}\} \\ & + 2\gamma_{[IJ]} c_{[KL]} G_\mu^I G_\nu^J \{G^{\mu K}, G^{\nu L}\} \\ & + \gamma_{[IJ]} \gamma_{[KL]} G_\mu^I G_\nu^J G^{\mu K} G^{\nu L} \end{aligned} \tag{103}$$

Equation of motion:

$$\operatorname{tr} \lambda_2 \left(4b_{(IJ)} G_\nu^{Jb} z^{[\mu\nu]} [t_a, t_b] + 4c_{[IJ]} G_\nu^{Jb} z^{[\mu\nu]} \{t_a, t_b\} + 2\gamma_{[IJ]} G_\nu^{Jb} z^{[\mu\nu]} [t_a, t_b] \right) = 0 \tag{104}$$

Taking the trace from above equation

$$2ib_{(IJ)} f_{abc} G_\nu^{Jb} z^{[\mu\nu]c} + 2c_{[IJ]} d_{abc} G_\nu^{Jb} z^{[\mu\nu]c} + \gamma_{[IJ]} d_{abc} G_\nu^{Jb} z^{[\mu\nu]c} \tag{105}$$

Multiplying by t_k and taking the trace

$$
\begin{aligned}
& b_{(IJ)} f_{abc}(id_{eck} - f_{eck}) G_\nu^{Jb} z^{[\mu\nu]e} + \frac{2}{N} c_{[IJ]} G_\nu^{Ja} z^{[\mu\nu]k} \\
& + c_{[IJ]} d_{abc}(d_{eck} + i f_{eck}) G_\nu^{Jb} z^{[\mu\nu]e} + \frac{1}{N} \gamma_{[IJ]} G_\nu^{Ja} z^{[\mu\nu]k} \\
& + \frac{1}{2} \gamma_{[IJ]} d_{abc}(d_{eck} + i f_{eck}) G_\nu^{Jb} z^{[\mu\nu]e}
\end{aligned} \tag{106}
$$

(3) $\mathcal{L} = \operatorname{tr} Z_{[\mu\nu]} z^{[\mu\nu]}$

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(3)} &= 2a_I b_{(JK)} \partial_\mu G_\nu^I [G^{\mu J}, G^{\nu K}] \\
& + 2a_I c_{[JK]} \partial_\mu G_\nu^I \{G^{\mu J}, G^{\nu K}\} \\
& + 2a_I \gamma_{[JK]} \partial_\mu G_\nu^I G^{\mu J} G^{\nu K}
\end{aligned} \tag{107}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(4)} &= -ig a_{(IJ)} b_{(KL)} [G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \\
& - ig a_{(IJ)} c_{[KL]} [G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\} \\
& - ig a_{(IJ)} \gamma_{[KL]} [G_\mu^I, G_\nu^J] G^{\mu K} G^{\nu L}
\end{aligned} \tag{108}
$$

Equations of motion:

$$
\operatorname{tr} \lambda_3 \begin{pmatrix} 2a_I \partial_\nu z^{[\mu\nu]} t_a - 2ig a_{(IJ)} G_\nu^{Jb} z^{[\mu\nu]} [t_a, t_b] + 2b_{(IJ)} G_\nu^{Jb} Z^{[\mu\nu]} [t_a, t_b] \\ +2c_{[IJ]} G_\nu^{Jb} Z^{[\mu\nu]} \{t_a, t_b\} + 2\gamma_{[IJ]} G_\nu^{Jb} Z^{[\mu\nu]} \{t_a, t_b\} \end{pmatrix} = 0 \tag{109}
$$

Multiplying by t_k and taking the trace

$$
\begin{aligned}
& a_I \partial_\nu z^{[\mu\nu]a} + g a_{(IJ)} f_{abc} G_\nu^{Jb} z^{[\mu\nu]c} + i b_{(IJ)} f_{abc} G_\nu^{Jb} Z^{[\mu\nu]c} \\
& + c_{[IJ]} d_{abc} G_\nu^{Jb} Z^{[\mu\nu]c} + \gamma_{[IJ]} d_{abc} G_\nu^{Jb} Z^{[\mu\nu]c}
\end{aligned} \tag{110}
$$

Multiplying by t_k and taking the trace

$$
\begin{aligned}
& \frac{1}{2} a_I (d_{aek} - i f_{aek} \partial_\nu z^{[\mu\nu]e} + \frac{g}{2} a_{(IJ)} f_{abc}(d_{ekc} - i f_{ekc}) G_\nu^{Jb} z^{[\mu\nu]e} \\
& + \frac{1}{2} b_{(IJ)} f_{abc}(id_{eck} - f_{eck}) G_\nu^{Jb} Z^{[\mu\nu]e} + \frac{1}{N} c_{[IJ]} G_\nu^{Ja} Z^{[\mu\nu]k} \\
& + \frac{1}{2} c_{[IJ]} d_{abc}(d_{eck} + i f_{eck}) G_\nu^{Jb} Z^{[\mu\nu]e} + \frac{1}{N} \gamma_{[IJ]} G_\nu^{Ja} Z^{[\mu\nu]k} \\
& + \frac{1}{2} \gamma_{[IJ]} d_{abc}(d_{eck} + i f_{eck}) G_\nu^{Jb} Z^{[\mu\nu]e}
\end{aligned} \tag{111}
$$

(4) $\mathcal{L} = \operatorname{tr} Z_{(\mu\nu)} Z^{(\mu\nu)}$

$$
\begin{aligned}
\mathcal{L}_K &= 2\beta_I \beta_J [(\partial_\mu G_\nu^I)(\partial^\mu G^{\nu J}) + (\partial_\mu G_\nu^I)(\partial^\nu G^{\mu J})] \\
& + 8\rho_J g_{\mu\nu} (\partial^\mu G^{\nu I})(\partial_\alpha G^{\alpha J}) + 4\rho_I \rho_K g_{\mu\nu} g^{\mu\nu} (\partial_\alpha G^{\alpha I})(\partial_\alpha G^{\alpha J})]
\end{aligned} \tag{112}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(3)} &= 4i(g_I \beta_J \beta_K - g_J \beta_I \beta_K) \partial_\mu G_\nu^K [G^{\mu I}, G^{\nu J}] \\
& - 8ig_J \beta_I \rho_K g_{\mu\nu} \partial_\alpha G^{\alpha K} [G^{\mu I}, G^{\nu J}] - 8ig_K \beta_I \rho_J g_{\mu\nu} \partial^\mu G^{\nu I} [G_\alpha^J, G^{\alpha K}] \\
& - 8ig_J \rho_I \rho_K g_{\mu\nu} g^{\mu\nu} \partial_\alpha G^{\alpha K} [G_\alpha^I, G^{\alpha J}]
\end{aligned} \tag{113}
$$

$$\begin{aligned}\mathcal{L}_{\text{int}}^{(4)} &= 2(g_J g_K \beta_I \beta_L - g_J g_L \beta_I \beta_K)[G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \\ &\quad - 8 g_J g_L \beta_I \beta_K g_{\mu\nu}[G^{\mu I}, G^{\nu J}][G_\alpha^K, G^{\alpha L}] \\ &\quad - 4 g_J g_L \rho_I \rho_K g_{\mu\nu} g^{\mu\nu}[G_\alpha^I, G^{\alpha J}][G_\alpha^K, G^{\alpha L}]\end{aligned} \tag{114}$$

Equation of motion

$$\operatorname{tr} \xi_1 \begin{pmatrix} -4\beta_I \partial_\nu Z^{(\mu\nu)} t_a - 4\rho_I g_{\rho\sigma} \partial^\mu Z^{(\rho\sigma)} t_a + \\ +4i(g_I \beta_J - g_J \beta_I) G_\nu^{Jb} Z^{(\mu\nu)}[t_a, t_b] + \\ +4i(g_I \rho_J - g_J \rho_I) g_{\rho\sigma} G^{\mu Jb} Z^{(\rho\sigma)}[t_a, t_b] \end{pmatrix} = 0 \tag{115}$$

Taking the trace

$$\begin{aligned}&- \beta_I \partial_\nu Z^{(\mu\nu)a} - 2\rho_I \partial^\mu Z_{(\nu}{}^{\nu)a} - 2(g_I \beta_J + g_J \beta_I) f_{abc} G_\nu^{Jb} Z^{(\mu\nu)c} \\ &- 2(g_I \rho_J + g_J \rho_I) f_{abc} G^{\mu Jb} Z_{(\nu}{}^{\nu)c}\end{aligned} \tag{116}$$

Multiplying by t_k and taking the trace

$$\begin{aligned}&\beta_I (i f_{aek} - d_{aek}) \partial_\nu Z^{(\mu\nu)e} + \rho_I (i f_{aek} - d_{aek}) \partial^\mu Z_{(\nu}{}^{\nu)e} \\ &+ (g_I \beta_J + g_J \beta_I) f_{abc} (i f_{aek} - d_{aek}) G_\nu^{Jb} Z^{(\mu\nu)e} \\ &+ (g_I \rho_J + g_J \rho_I) f_{abc} (i f_{aek} - d_{aek}) G^{\mu Jb} Z_{(\mu}{}^{\mu)e}\end{aligned} \tag{117}$$

(5) $\mathcal{L} = \operatorname{tr} z_{(\mu\nu)} z^{(\mu\nu)}$

$$\begin{aligned}\mathcal{L}_{\text{int}}^{(4)} &= b_{[IJ]} b_{[KL]}[G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \\ &\quad + 2 b_{[IJ]} u_{[KL]} g_{\mu\nu}[G^{\mu I}, G^{\nu J}][G_\alpha^K, G^{\alpha L}] \\ &\quad + 2 b_{[IJ]} v_{(KL)} g_{\mu\nu}[G^{\mu I}, G^{\nu J}]\{G_\alpha^K, G^{\alpha L}\} \\ &\quad + 2 b_{[IJ]} c_{(KL)} g_{\mu\nu}[G^{\mu I}, G^{\nu J}]\{G^{\mu K}, G^{\nu L}\} \\ &\quad + u_{[IJ]} u_{[KL]} g_{\mu\nu} g^{\mu\nu}[G_\alpha^I, G^{\alpha J}][G_\alpha^K, G^{\alpha L}] \\ &\quad + 2 u_{[IJ]} v_{(KL)} g_{\mu\nu} g^{\mu\nu}[G_\alpha^I, G^{\alpha J}]\{G_\alpha^K, G^{\alpha L}\} \\ &\quad + 2 u_{[IJ]} c_{(KL)} g_{\mu\nu}[G_\alpha^I, G^{\alpha J}]\{G^{\mu K}, G^{\nu L}\} \\ &\quad + v_{(IJ)} v_{(KL)} g_{\mu\nu} g^{\mu\nu}\{G_\alpha^I, G^{\alpha J}\}\{G^{\mu K}, G^{\nu L}\} \\ &\quad + 2 v_{(IJ)} c_{(KL)} g_{\mu\nu}\{G_\alpha^I, G^{\alpha J}\}\{G^{\mu K}, G^{\nu L}\} \\ &\quad + c_{(IJ)} c_{(KL)}\{G_\mu^I, G_\nu^J\}\{G^{\mu K}, G^{\nu L}\}\end{aligned} \tag{118}$$

Equation of motion

$$\operatorname{tr} \xi_2 \begin{pmatrix} +4 b_{[IJ]} G_\nu^{Jb} z^{(\mu\nu)}[t_a, t_b] + 4 c_{(IJ)} G_\nu^{Jb} z^{(\mu\nu)}\{t_a, t_b\} \\ +4 u_{[IJ]} G^{\mu Jb} g_{\rho\sigma} z^{(\rho\sigma)}[t_a, t_b] + 4 u_{(IJ])} G^{\mu Jb} g_{\rho\sigma} z^{(\rho\sigma)}\{t_a, t_b\} \end{pmatrix} = 0 \tag{119}$$

Taking the trace

$$\begin{aligned}&2i b_{[IJ]} f_{abc} G_\nu^{Jb} z^{(\mu\nu)c} + 2 c_{(IJ)} d_{abc} G_\nu^{Jb} z^{(\mu\nu)c} \\ &+ 2i u_{[IJ]} f_{abc} G^{\mu Jb} z_{(\nu}{}^{\nu)c} + 2 u_{(IJ)} d_{abc} G^{\mu Jb} z_{(\nu}{}^{\nu)c}\end{aligned} \tag{120}$$

Multiplying by t_k and taking the trace

$$
\begin{aligned}
& b_{[IJ]} f_{abc}(id_{ekc}+f_{ekc}) G_\nu^{Jb} z^{(\mu\nu)e} + \frac{2}{N} c_{(IJ)} G_\nu^{Ja} z^{(\mu\nu)k} \\
& + c_{(IJ)} d_{abc}(d_{ekc} - i f_{ekc}) G_\nu^{Jb} z^{(\mu\nu)e} \\
& + u_{[IJ]} f_{abc}(id_{ekc}+f_{ekc}) G^{\mu Jb} z_{(\nu}{}^{\nu)e} + \frac{2}{N} v_{(IJ)} G^{\mu Ja} z_{(\nu}{}^{\nu)k} \\
& + v_{(IJ)} d_{abc}(d_{ekc} - i f_{ekc}) G^{\mu Jb} z_{(\nu}{}^{\nu)e}
\end{aligned} \tag{121}
$$

(6) $\mathcal{L} = \operatorname{tr} Z_{(\mu\nu)} z^{(\mu\nu)}$

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(3)} = {} & 2\beta_I b_{[JK]} \partial_\mu G_\nu^I [G^{\mu J}, G^{\nu K}] \\
& + 2\rho_I b_{[JK]} g_{\mu\nu} \partial_\alpha G^{\alpha I} [G^{\mu J}, G^{\nu K}] \\
& + 2\beta_I u_{[JK]} g_{\mu\nu} \partial^\mu G^{\nu I} [G_\alpha^J, G^{\alpha K}] \\
& + 2\rho_I u_{[JK]} g_{\mu\nu} g^{\mu\nu} \partial_\alpha G^{\alpha I} [G_\alpha^J, G^{\alpha K}] \\
& + 2\beta_I v_{(JK)} g_{\mu\nu} \partial^\mu G^{\nu I} \{G_\alpha^J, G^{\alpha K}\} \\
& + 2\rho_I v_{(JK)} g_{\mu\nu} g^{\mu\nu} \partial_\alpha G^{\alpha I} \{G_\alpha^J, G^{\alpha K}\} \\
& + 2\rho_I c_{(JK)} g_{\mu\nu} \partial_\alpha G^{\alpha I} \{G^{\mu J}, G^{\nu K}\}
\end{aligned} \tag{122}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(4)} = {} & i(g_I \beta_J b_{[KL]} - g_J \beta_I b_{[KL]}) [G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}] \\
& - 2i g_J \rho_I b_{[KL]} g_{\mu\nu} [G_\alpha^I, G^{\alpha J}][G^{\mu K}, G^{\nu L}] \\
& - 2i g_J \beta_I u_{[KL]} g_{\mu\nu} [G^{\mu I}, G^{\nu J}][G_\alpha^K, G^{\alpha L}] \\
& - 2i g_J \rho_I u_{[KL]} g_{\mu\nu} g^{\mu\nu} [G_\alpha^I, G^{\alpha J}][G_\alpha^K, G^{\alpha L}] \\
& - 2i g_J \beta_I v_{(JK)} g_{\mu\nu} [G^{\mu I}, G^{\nu J}]\{G_\alpha^K, G^{\alpha L}\} \\
& - 2i g_J \rho_I v_{(JK)} g_{\mu\nu} g^{\mu\nu} [G_\alpha^I, G^{\alpha J}]\{G_\alpha^K, G^{\alpha L}\} \\
& + i(g_I \beta_J c_{(KL)} - g_J \beta_I c_{(KL)}) [G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\} \\
& - 2i g_J \rho_I c_{(JK)} g_{\mu\nu} [G_\alpha^I, G^{\alpha J}]\{G^{\mu K}, G^{\nu L}\}
\end{aligned} \tag{123}
$$

Equation of motion

$$
\operatorname{tr} \xi_3 \begin{pmatrix} -2\beta_I \partial_\nu z^{(\mu\nu)} t_a - 2\rho_I \partial^\mu g_{\rho\sigma} z^{(\rho\sigma)} t_a + 2b_{[IJ]} G_\nu^{Jb} Z^{(\mu\nu)}[t_a, t_b] + 2c_{(IJ)} G_\nu^{Jb} Z^{(\mu\nu)}\{t_a, t_b\} + \\ + 2u_{[IJ]} G^{\mu Jb} g_{\rho\sigma} Z^{(\rho\sigma)}[t_a, t_b] + 2v_{(IJ)} G^{\mu Jb} g_{\rho\sigma} Z^{(\rho\sigma)}\{t_a, t_b\} + \\ + 2i(g_I \beta_J - g_J \beta_I) G_\nu^{Jb} z^{(\mu\nu)}[t_a, t_b] + 2i(g_I \rho_J - g_J \rho_I) G^{\mu Jb} g_{\rho\sigma} z^{(\rho\sigma)}[t_a, t_b] \end{pmatrix}
$$
$$
= 0 \tag{124}
$$

Taking the trace

$$
\begin{aligned}
& - \beta_I \partial_\nu z^{(\mu\nu)a} - (g_I \beta_J - g_J \beta_I) f_{abc} G_\nu^{Jb} z^{(\mu\nu)c} \\
& - \rho_I \partial^\mu z_{(\nu}{}^{\nu)a} - (g_I \rho_J - g_J \rho_I) f_{abc} G^{\mu Jb} z_{(\nu}{}^{\nu)c} \\
& + i b_{[IJ]} f_{abc} G_\nu^{Jb} Z^{(\mu\nu)c} + c_{(IJ)} d_{abc} G_\nu^{Jb} Z^{(\mu\nu)c} \\
& + i u_{[IJ]} f_{abc} G^{\mu Jb} Z_{(\nu}{}^{\nu)c} + v_{(IJ)} d_{abc} G^{\mu Jb} Z_{(\nu}{}^{\nu)c}
\end{aligned} \tag{125}
$$

Multiplying by t_k and taking the trace

$$
\begin{aligned}
&\frac{1}{2}\beta_I(if_{aek}-d_{aek})\partial_\nu z^{(\mu\nu)e}+\frac{1}{2}\rho_I(if_{aek}-d_{aek})\partial^\mu z_{(\nu}{}^{\nu)e}\\
&+\frac{1}{2}(g_I\beta_J+g_J\beta_I)f_{abc}(if_{aek}-d_{aek})G^{Jb}_\nu z^{(\mu\nu)e}\\
&+\frac{1}{2}(g_I\rho_J+g_J\rho_I)f_{abc}(if_{aek}-d_{aek})G^{\mu Jb}z_{(\mu}{}^{\mu)e}\\
&+\frac{1}{2}b_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{Jb}_\nu Z^{(\mu\nu)e}+\frac{1}{N}c_{(IJ)}G^{Jb}_\nu Z^{(\mu\nu)k}\\
&+\frac{1}{2}c_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{Jb}_\nu Z^{(\mu\nu)e}\\
&+\frac{1}{2}u_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{\mu Jb}Z_{(\nu}{}^{\nu)e}+\frac{1}{N}v_{(IJ)}G^{\mu Jb}Z_{(\nu}{}^{\nu)k}\\
&+\frac{1}{2}v_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{\mu Jb}Z_{(\nu}{}^{\nu)e}
\end{aligned}
\tag{126}
$$

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