The Deformation Structure of the Nuclei ^{32}S and ^{36}Ar

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Abstract

We applied two different approaches to investigate the deformation structures of the two nuclei ^{32}S and ^{36}Ar. In the first approach, we considered these nuclei as being deformed and have axes of symmetry. Accordingly, we calculated their moments of inertia by using the concept of the single-particle Schrödinger fluid as functions of the deformation parameter β. In this case, we calculated also the electric quadrupole moments of the two nuclei by applying Nilsson model as functions of β. In the second approach, we used a strongly deformed nonaxial single-particle potential, depending on β and the nonaxiality parameter γ, to obtain the single-particle energies and wave functions. Accordingly, we calculated the quadrupole moments of ^{32}S and ^{36}Ar by filling the single-particle states corresponding to the ground- and the first excited states of these nuclei. The moments of inertia of ^{32}S and ^{36}Ar are then calculated by applying the nuclear superfluidity model. The obtained results are in good agreement with the corresponding experimental data.

Keywords

Nuclear structure, deformed nuclei, single-particle Schrödinger fluid, Nilsson model, nuclear superfluidity model, moments of inertia, electric quadrupole moments.

PACS: 21.10Dr, 21.10.-K,21.10.Re.

1. Introduction

Many of the light nuclei are spherical. This is due to the success of the shell model, which is based on states in a field of spherical symmetry. According to the basic ideas of quantum mechanics the concept of rotation in a spherically symmetric system is meaningless. However, in an elongated nucleus the concept of rotation is meaningful, and the nucleus can rotate about an axis perpendicular to the axis of symmetry. The basic ideas concerning non spherical nuclei have been most completely described by A. Bohr [1]. A non-spherical nucleus is characterized by the moment of inertia about the axis perpendicular to the symmetry axis of the nucleus, its magnetic dipole moment and its electric quadrupole moment. The elongation of the nucleus is related to the interaction between the surface and the nucleons outside closed shells.

In treating the internal motion in a deformed nucleus, it is assumed that the individual nucleons move independently in a certain fixed non-spherical field of the nucleus. The Hamiltonian of the internal motion can then be represented, as in the ordinary model, in the form of a sum of one-particle Hamiltonians. One of the most successful models for generating realistic intrinsic single particle states of deformed nuclei is that first proposed by Nilsson [2]. According to Nilsson's model, the nucleons inside the nucleus are moving independently in an averaging field in the form of anisotropic oscillator, with $\omega_x = \omega_y \neq \omega_z$, added to it a spin-orbit term and a term proportional to the square of the orbital angular momentum of the nucleon. The nucleon energy eigenvalues and eigenfunctions are then obtained by solving the time-independent Schrödinger wave equation in spherical polar coordinates and applying the method of diagonalizing the matrices. This model was limited to nuclei with axially symmetric quadrupole deformations, where the deformation is measured by the deformation parameter β. Positive values of β correspond to prolate deformation and negative values to oblate deformation. The success of the description of many nuclei by means of deformed potential can be taken as an indication that by distorting a spherical potential in this manner we automatically obtain the right combination of spherical eigenfunctions which make the corresponding Slater determinant a better approximation to the real nuclear wave function. From this point of view, the deformed potential is a definite prescription for a convenient mixing of various configurations of the spherical potential. Considerable evidence has accumulated for the rotational structure of nuclei. The absolute values of the rotational energies or equivalently the moments of inertia require a knowledge of the fine details of the intrinsic nuclear structure. Consequently, the investigation of the nuclear moments of inertia is a sensitive check for the validity of the nuclear structure theories [3].

The study of the velocity fields for the rotational motion of the axially symmetric deformed nuclei led to the formulation of the so-called Schrödinger fluid [4,5]. Since the Schrödinger-fluid theory is an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory. For axially symmetric deformed nuclei, the best description of the moment of inertia can be carried out by applying the concept of the single-particle Schrödinger fluid [6-8]. For these nuclei one can apply Nilsson's model [2] to calculate the nuclear quadrupole moments.

For a nucleus which has not an axis of symmetry (usually called an asymmetric rotor), one can use a single-particle Hamiltonian for a nucleon moving in a nonaxial deformed potential and then solves the Schrödinger equation in this case to obtain the single-particle energy eigenvalues and eigenfunctions [9]. It is then possible to fill the ground state (or the excited state) of the considered nucleus by the resulting single-particle wave functions. As a consequence, the quadrupole moment can be obtained by calculating the expectation value of the well-known quadrupole-moment operator with respect to the ground (or the excited) state of the nucleus. The best description of the nuclear moments of inertia for a nucleus which has not an axis of symmetry can be obtained by applying the nuclear superfluidity model of Belyaev [10].

The moments of inertia and the quadrupole moments of some deformed nuclei in the sd-shell have been investigated in frame work of different models. By applying Nilsson model, Bishop [11] calculated the moment of inertia and the quadrupole moment of the nucleus ^{27}Al. Also, Doma [6] applied the nuclear superfluidity model to calculate the moments of inertia of the nuclei ^{24}Mg and ^{26}Mg. Furthermore, Doma [12] applied the single-particle Schrödinger fluid to calculate the moments of inertia of the even-even nuclei in the sd-shell.

In the present paper, we investigated the deformation structure of the nuclei ^{32}S and ^{36}Ar. Accordingly, we calculated two characteristics for these nuclei by using different models which depend on the shape of the nucleus. In the case where the nucleus is assumed to be deformed and has an axis of symmetry, we applied the concept of the single-particle Schrödinger fluid for the calculation of the moments of inertia. Accordingly, the cranking-model moment of inertia and the rigid-body moment of inertia of the two nuclei ^{32}S and ^{36}Ar are calculated as functions of the deformation parameter β and the non-deformed oscillator parameter $\hbar\omega_0^0$. Furthermore, we calculated also the electric quadrupole moments of ^{32}S and ^{36}Ar by applying Nilsson model as function of β. We finally considered a single-particle deformed potential consisting of an anisotropic oscillator potential added to it a spin-orbit term and a term proportional to the square of the orbital-angular momentum of the nucleon to calculate the single-particle energy eigenvalues and eigenfunctions for a nucleon in a deformed non axial nucleus. As a consequence, the quadrupole moments of ^{32}S and ^{36}Ar are calculated by using the single-particle deformed wave functions. The moments of inertia of ^{32}S and ^{3}Ar are then calculated by applying the superfluidity nuclear model, as functions of β, the non-axiality parameter γ and the non-deformed oscillator parameter $\hbar\omega_0^0$.

2. Calculations Based on the Assumption that the Nucleus Is Deformed and Has an Axis of Symmetry

2.1 The Single-Particle Schrödinger Fluid

The detailed formulation of the concept of the single-particle Schrödinger fluid from the time dependent Schrödinger equation, by suitably chosen single-particle wave function, is given by Kane and Griffin [4,5]. The method of the application of this concept to the calculation of the nuclear moments of inertia is given by Doma [6-8]. The following expressions for the cranking-model and the rigid body-model moments of inertia can be easily obtained on the basis of the concept of the single-particle Schrödinger fluid [4,5]

$$\Im_{cr} = \frac{E}{w_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[\sigma^2(1+q)+\frac{1}{\sigma}(1-q)\right], \tag{2.1}$$

$$\Im_{rig} = \frac{E}{w_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}[(1+q)+\sigma(1-q)], \tag{2.2}$$

where q is the anisotropy of the configuration, which is defined by

$$q = \frac{\sum_{occ}(n_y+1)}{\sum_{occ}(n_z+1)}, \tag{2. 3}$$

and E is the total energy

$$E = \sum_{occ}\left[\hbar\omega_x\left(n_x+n_y+1\right)+\hbar\omega_z(n_z+1)\right]. \tag{2. 4}$$

In equations (2.3) and (2.4) n_x, n_y and n_z are the state quantum numbers of the oscillator. The summations in (2.3) and (2.4) are carried over all the occupied single-particle states. The method of filling these states is illustrated in [8]. Also, in (2.1) and (2.2) σ is a measure of the deformation of the potential and is defined by

$$\sigma = \frac{\omega_y-\omega_z}{\omega_y+\omega_z}\,. \tag{2. 5}$$

For the frequencies ω_x, ω_y and ω_z, Doma et al. [6-8] used Nilsson's frequencies [2], defined by

$$\omega_x^2 = \omega_y^2 = \omega_0^2(\delta)\left(1+\frac{2}{3}\delta\right), \tag{2.6}$$

$$\omega_z^2 = \omega_0^2(\delta)\left(1-\frac{4}{3}\delta\right), \tag{2.7}$$

$$\omega_0(\delta) = \omega_0^0\left(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right)^{-\frac{1}{6}}. \tag{2.8}$$

For the non-deformed frequency ω_0^0 we used the one which is given in terms of the mass number A, the number of neutrons N and the number of protons Z by [13]

$$\hbar\omega_0^0 = 38.6\,A^{-\frac{1}{3}} - 127.0\,A^{-\frac{4}{3}} + 14.75\,A^{-\frac{4}{3}}(\mathrm{N} - Z). \tag{2.9}$$

The well-known deformation parameter β is related to the parameter δ in equations (2.6), (2.7) and (2.8) by the following relation [2]

$$\beta = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\,\delta. \tag{2.10}$$

We note that the cranking-model and the rigid body-model moments of inertia are equal only when the harmonic oscillator is at the equilibrium deformation.

2.2 The Electric Quadrupole Moment

Assuming a charge distribution in accordance with the Thomas-Fermi statistical model applied to the oscillator potential one obtains, for the case of the axially symmetric nuclei, the intrinsic quadrupole moment, to the second-order in the deformation parameter δ [1]

$$Q_0 = 0.8ZeR^2\delta\left(1 + \frac{2\delta}{3}\right), \tag{2.11}$$

where Z is the number of protons and R is to be taken equal to the radius of charge of the nucleus. The relation between the measured quadrupole moment, denoted by Q_S, and Q_0 is given by

$$Q_S = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0, \tag{2.12}$$

where I is the total spin-quantum number of the specified nuclear state and K is its component along the body-fixed $z-axis$. Calculating the charge radius of the nucleus, the measured quadrupole moment for a nucleus with an axis of symmetry is then obtained as function of the deformation parameter δ.

3. Calculations Based on the Assumption that the Nucleus Is Deformed and Has Not an Axis of Symmetry

In the case where the nucleus is assumed to be deformed and has not, in principle, an axis of symmetry we proceed as illustrated in the next sections.

3.1 The Single-Particle Potential and the Method of Solution

Consider a nucleon which is moving in a deformed nuclear field whose Hamiltonian operator is given by [9]

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}\omega_0^2 r^2 + C\boldsymbol{l}.\boldsymbol{s} + Dl^2 - m\omega_0^2 r^2\beta cos\gamma Y_{2,0}(\theta,\varphi)$$
$$-\frac{\sqrt{2}}{2}m\omega_0^2 r^2\beta sin\gamma\{Y_{2,2}(\theta,\varphi) + Y_{2,-2}(\theta,\varphi)\}, \tag{3.1}$$

where $Y_{l,\Lambda}(\theta,\varphi)$ are the spherical harmonic functions, β is the deformation parameter and γ is the non-axiality parameter. The constants C and D in equation (3.1) are given by [2]

$$C = -2\chi\hbar\omega_0^0,\ D = -\mu\chi\hbar\omega_0^0, \tag{3.2}$$

where χ takes values in the interval $0.05 \leq \chi \leq 0.08$ and μ depends on the number of quanta of excitation N as given by Nilsson [2].

The Hamiltonian H, equation (3.1), can be rewritten in the form:

$$H = H^{(0)} + H^{(1)} + H^{(2)}, \tag{3.3}$$

where

$$H^{(0)} == -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}\omega_0^2 r^2 + C\boldsymbol{l}.\boldsymbol{s} + Dl^2, \tag{3.4}$$

$$H^{(1)} = -m\omega_0^2 r^2 \beta cos\gamma Y_{2,0}(\theta,\varphi), \tag{3.5}$$

$$H^{(2)} = -\frac{\sqrt{2}}{2} m\omega_0^2 r^2 \beta sin\gamma \{Y_{2,2}(\theta,\varphi) + Y_{2,-2}(\theta,\varphi)\}. \tag{3.6}$$

The solutions of the Schrödinger equation corresponding to the Hamiltonian (3.4) are straightforward [9] with eigenfunctions denoted by $|Nl\Lambda\Sigma\rangle$. Also, the solutions of the Schrödinger equation corresponding to the Hamiltonian $H^{(0)} + H^{(1)}$ are straightforward [9] by applying the variational method and as a result we obtain the eigenfunctions $|N\Omega^{\pi}\rangle$.

Finally, the Schrödinger equation representing the motion of a single nucleon in the non-axially deformed nuclear field, whose Hamiltonian operator is given by equation (3.1), can be solved by applying the stationary non-degenerate perturbation method, for the Hamiltonian $H^{(2)}$ as a perturbed term to $H^{(0)} + H^{(1)}$, with respect to the eigenfunctions $|N\Omega^{\pi}\rangle$. As a result, the single-particle energy eigenvalues and eigenfunctions, $|\Omega^{\pi}\rangle$, of a nucleon in a deformed nuclear field can be calculated for every level, with given value of the z-component of the total angular momentum Ω and parity π as functions of the potential parameters χ, and μ, the deformation parameter β, and the non-axiality parameter γ. In the above mentioned functions, N is the number of quanta of excitation, l and Λ are the nucleon orbital angular momentum quantum number and its z-component and Σ is the z-component of the nucleon spin ($= \pm\frac{1}{2}$).

3.2 The Nuclear Superfluidity Model and the Moment of Inertia

The moment of inertia of a deformed nucleus which has not an axis of symmetry is then given by applying the nuclear superfluidity model [10], and as a result we obtain

$$\Im_{s.f.} = \hbar^2 \sum_{i,k} \frac{\langle i|J_x|k\rangle^2}{E_i + E_k}\left\{1 - \frac{(\zeta_i - \lambda)(\zeta_k - \lambda) + \Delta^2}{E_i E_k}\right\}, \tag{3.7}$$

where ζ_i are the eigenvalues of the self-consistent field, the eigenvalues of the Hamiltonian operator (3.1), λ is the chemical potential and the energy of elementary excitations of the nucleus, E_i, is given by

$$E_i = \sqrt{(\zeta_i - \lambda)^2 + \Delta^2}, \tag{3.8}$$

with Δ being the energy gap. The summation in equation (3.7) is taken over all states of the self-consistent field. The chemical potential λ is given by [10]

$$\sum_i \left\{1 - \frac{\zeta_i - \lambda}{\sqrt{(\zeta_i - \lambda)^2 + \Delta^2}}\right\} = \mathrm{N}_{p,n}, \tag{3.9}$$

where the summation, here, runs over all distinct neutron (or proton) energies and $\mathrm{N_{p,n}}$ is the number of protons or neutrons inside the nucleus.

3.3 The Quadrupole Moment

For the non-axial case, the intrinsic quadrupole moment, of a nucleus consisting of Z protons, is given by [1]

$$Q_0 = \sum_{i=1}^{Z} Q_i, \tag{3.10}$$

where the single-particle operator Q_i is given by

$$Q_i = e\sqrt{\frac{16\pi}{5}}\int\left(\Psi^i_{\Omega^\pi}\right)^2 r_i^2 Y_{2,0}(\theta_i,\phi_i)\,d\tau. \tag{3.11}$$

Carrying out the integration in equation (3.11) with respect to the wave functions $|\Omega^\pi\rangle$ which is evaluated in terms of the functions $|Nl\Lambda\Sigma\rangle$, one then obtains

$$Q_i = e\sqrt{\frac{16\pi}{5}}\sum_{\alpha,\beta} C^i_\alpha C^i_\beta \langle N_\alpha l_\alpha|r^2|N_\beta l_\beta\rangle\langle l_\alpha\Lambda_\alpha|Y_{2,0}|l_\beta\Lambda_\beta\rangle. \tag{3.12}$$

Filling the single-particle wave functions $|\Omega^\pi\rangle$ for the given nucleus in its ground- and excited- state (I^+), it is then possible to calculate the quadrupole moment by calculating the necessary matrix elements of equation (3.12) and evaluating the expansion coefficients of the functions $|\Omega^\pi\rangle$ in terms of the functions $|Nl\Lambda\Sigma\rangle$ as obtained from the variational and the perturbation methods.

4. Results for the Case of the Axial Symmetry

According to previous works [9], the parameters χ,μ and β are allowed to take on the values $\chi = 0.05, 0.06, 0.07,$ and 0.08, $\mu = 0$, for $N = 0,1$ and 2; and $\mu = 0.35$ for $N = 3$, β takes values in the interval $-0.50 \leq \beta \leq 0.50$ with a step 0.01.

In Tables-1 and 2 we present the variations of the values of the reciprocal moments of inertia of the nuclei ^{32}S and ^{36}Ar, by using the concept of the single-particle Schrödinger fluid for both of the cranking-and the rigid-body models, with respect to the deformation parameter β for the cases $\beta \leq 0$ and $\beta > 0$, respectively. The values of the non-deformed oscillator parameter $\hbar\omega_0^0$ are also given in Tables-1 and 2. Also, in Figures-1 and 2 we present the dependence of the reciprocal moments of inertia of ^{32}S and ^{36}Ar on the deformation parameter β.

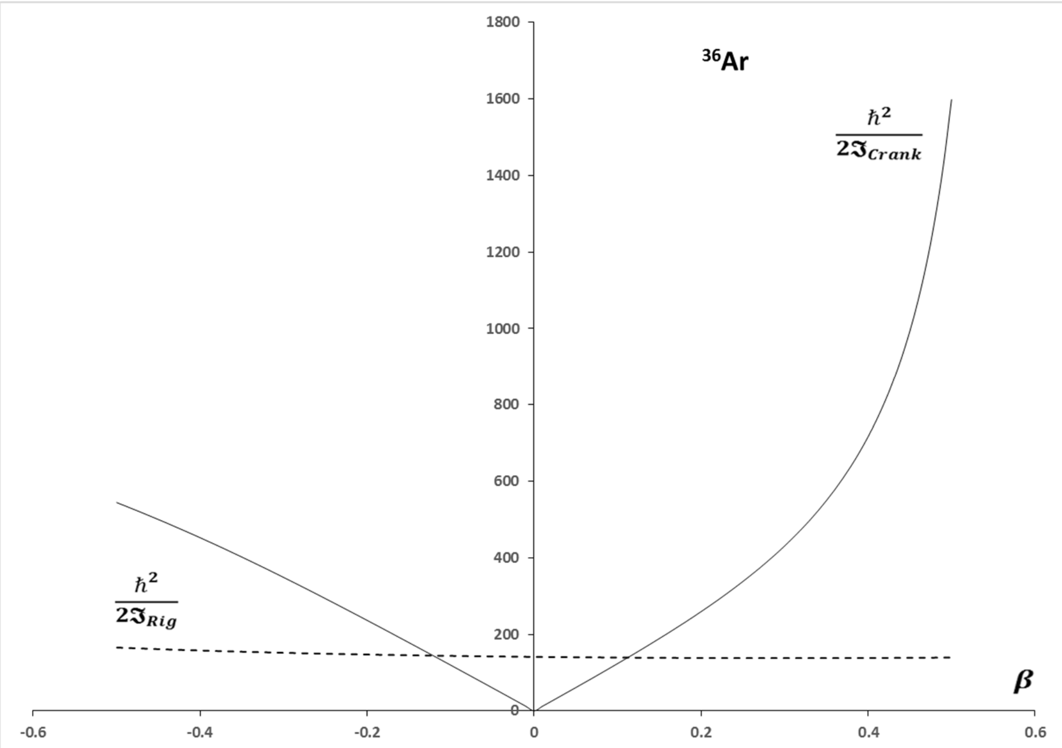

Fig.1 Reciprocal moments of inertia of the nucleus ^{32}S. Solid line for cranking model and dashed line for rigid body model.

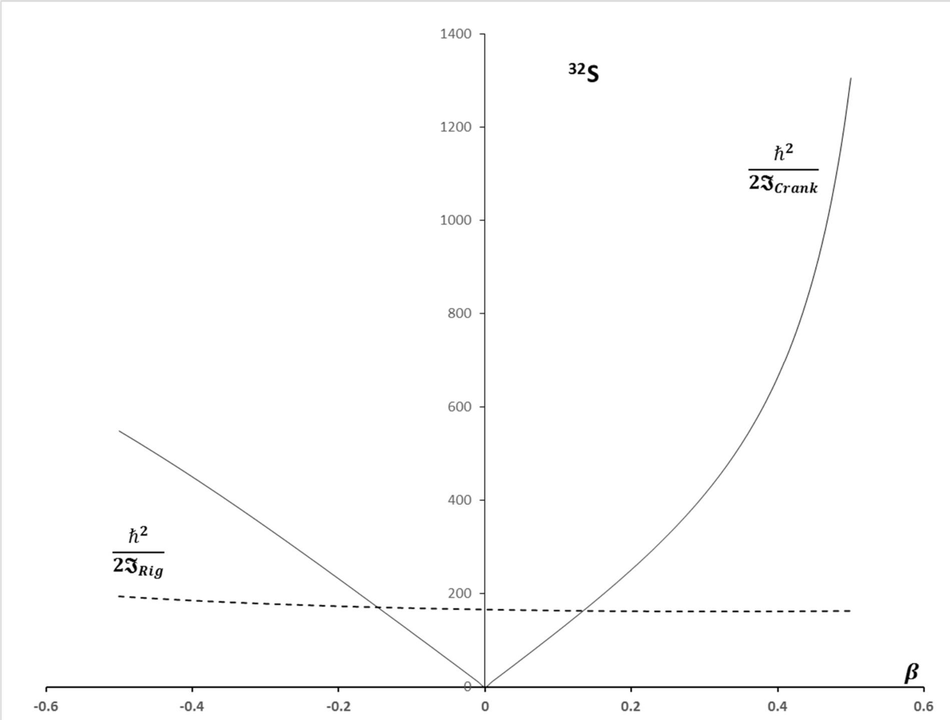

Fig.2 Reciprocal moments of inertia of the nucleus ^{36}Ar. Solid line for cranking model and dashed line for rigid body model.

Table-1 Schrödinger fluid reciprocal moments of inertia of ^{32}S and ^{36}Ar as functions of β, $\beta \leq 0$.

Case	^{32}S $\hbar\omega_0^0 = 10.908$		^{36}Ar $\hbar\omega_0^0 = 10.622$	
β	$\frac{\hbar^2}{2\mathfrak{I}_{Crank}}$	$\frac{\hbar^2}{2\mathfrak{I}_{Rig}}$	$\frac{\hbar^2}{2\mathfrak{I}_{Crank}}$	$\frac{\hbar^2}{2\mathfrak{I}_{Rig}}$
-.500	548.020300	193.265300	544.238000	164.409900
-.490	538.614000	192.287800	535.632800	163.596600
-.480	529.116500	191.338600	526.897800	162.807400
-.470	519.527600	190.416700	518.031900	162.041100
-.460	509.847100	189.521000	509.035600	161.296800
-.450	500.075600	188.650300	499.908800	160.573600
-.440	490.213300	187.803700	490.652600	159.870700
-.430	480.261500	186.980300	481.268200	159.187200
-.420	470.221200	186.179100	471.757000	158.522500
-.410	460.093800	185.399300	462.120700	157.875800
-.400	449.881400	184.640300	452.361800	157.246600
-.390	439.585700	183.901200	442.482500	156.634200
-.380	429.208600	183.181400	432.485300	156.037900
-.370	418.752900	182.480200	422.373500	155.457400
-.360	408.220500	181.797100	412.150300	154.892200
-.350	397.614400	181.131500	401.818600	154.341500
-.340	386.937000	180.482700	391.382200	153.805100
-.330	376.191400	179.850400	380.844700	153.282600
-.320	365.380300	179.234000	370.210100	152.773600
-.310	354.506700	178.633100	359.482000	152.277500

-.300	343.573400	178.047200	348.664400	151.794100
-.290	332.583700	177.476000	337.761400	151.323100
-.280	321.540300	176.919000	326.777300	150.864000
-.270	310.446500	176.375900	315.715900	150.416700
-.260	299.305000	175.846200	304.581400	149.980700
-.250	288.119000	175.329900	293.377900	149.555800
-.240	276.891300	174.826300	282.109600	149.141900
-.230	265.624600	174.335300	270.780300	148.738500
-.220	254.321800	173.856600	259.394000	148.345500
-.210	242.985500	173.389900	247.954300	147.962700
-.200	231.618100	172.934900	236.465100	147.589700
-.190	220.222300	172.491400	224.929600	147.226400
-.180	208.799800	172.059200	213.351700	146.872700
-.170	197.353200	171.638000	201.734100	146.528300
-.160	185.884000	171.227600	190.079900	146.193000
-.150	174.394300	170.827900	178.392000	145.866800
-.140	162.885500	170.438600	166.672800	145.549300
-.130	151.358800	170.059400	154.924600	145.240500
-.120	139.815400	169.690400	143.149300	144.940200
-.110	128.255900	169.331300	131.348700	144.648300
-.100	116.681200	168.981900	119.524000	144.364600
-.090	105.091300	168.642100	107.676500	144.089100
-.080	93.486510	168.311700	95.806760	143.821600
-.070	81.866290	167.990600	83.915240	143.562000
-.060	70.230280	167.678800	72.001590	143.310100
-.050	58.577240	167.375900	60.065510	143.065900
-.040	46.906120	167.082000	48.106110	142.829300
-.030	35.215160	166.797000	36.121970	142.600300
-.020	23.502260	166.520600	24.111300	142.378600
-.010	11.764880	166.252900	12.071650	142.164300
.000	0.000000	165.993700	0.000000	141.957300

Table-2 Schrödinger fluid reciprocal moments of inertia of ^{32}S and ^{36}Ar as functions of β, $\beta > 0$.

Case	^{32}S $\hbar\omega_0^0 = 10.908$		^{36}Ar $\hbar\omega_0^0 = 10.622$	
β	$\frac{\hbar^2}{2\Im_{Crank}}$	$\frac{\hbar^2}{2\Im_{Rig}}$	$\frac{\hbar^2}{2\Im_{Crank}}$	$\frac{\hbar^2}{2\Im_{Rig}}$
.010	11.795180	165.743000	12.106490	141.757400
.020	23.625170	165.500600	24.252580	141.564700
.030	35.494220	165.266600	36.442800	141.379000
.040	47.407260	165.040800	48.682530	141.200300
.050	59.369950	164.823100	60.977860	141.028600
.060	71.388450	164.613500	73.335590	140.863900
.070	83.469570	164.412000	85.763710	140.706000
.080	95.621090	164.218400	98.270660	140.554800
.090	107.851400	164.032800	110.866100	140.410600
.100	120.169700	163.855100	123.560800	140.273000
.110	132.586300	163.685300	136.366500	140.142200
.120	145.112600	163.523300	149.296300	140.018100
.130	157.760600	163.369100	162.364700	139.900700
.140	170.544300	163.222700	175.587500	139.789900
.150	183.478300	163.084000	188.982400	139.685900
.160	196.578900	162.953100	202.569000	139.588400
.170	209.864500	162.829900	216.368600	139.497600
.180	223.354700	162.714400	230.405000	139.413400
.190	237.071400	162.606700	244.704700	139.335900
.200	251.038700	162.506600	259.297000	139.265000
.210	265.283100	162.414300	274.214100	139.200800
.220	279.834300	162.329600	289.492300	139.143200
.230	294.725000	162.252700	305.171900	139.092200
.240	309.991400	162.183600	321.298400	139.048000
.250	325.674200	162.122300	337.922300	139.010500
.260	341.818600	162.068600	355.100400	138.979600
.270	358.475700	162.022800	372.897600	138.955600
.280	375.702200	161.984800	391.386700	138.938200
.290	393.563200	161.954700	410.651300	138.927700
.300	412.131100	161.932400	430.786100	138.924000
.310	431.489500	161.918000	451.900700	138.927200
.320	451.733200	161.911600	474.121300	138.937300
.330	472.971500	161.913200	497.594200	138.954300
.340	495.330000	161.922800	522.490700	138.978300
.350	518.954300	161.940500	549.011800	139.009400
.360	544.014800	161.966200	577.395400	139.047500
.370	570.711900	162.000000	607.925700	139.092600
.380	599.281300	162.042000	640.943400	139.144900

.390	630.004700	162.092200	676.863400	139.204300
.400	663.220800	162.150600	716.193300	139.270900
.410	699.339400	162.217200	759.563200	139.344700
.420	738.862500	162.292000	807.763000	139.425800
.430	782.411600	162.375100	861.798700	139.514100
.440	830.765300	162.466500	922.971600	139.609700
.450	884.913100	162.566000	992.994500	139.712400
.460	946.131000	162.673700	1074.168000	139.822400
.470	1016.096000	162.789500	1169.661000	139.939600
.480	1097.054000	162.913300	1283.958000	140.063900
.490	1192.087000	163.045000	1423.624000	140.195100
.500	1305.541000	163.184400	1598.674000	140.333200

In Table-3 we present the best values of the calculated reciprocal cranking-model moments of inertia for the two nuclei ^{32}S and ^{36}Ar. The values of the corresponding deformation parameter β and the experimental moments of inertia of the two nuclei are also given in this table. Concerning the values of the rigid-body moments, they are not presented in this table since they are not in good agreement with the corresponding experimental values, as shown from Tables-1 and 2, as expected.

Table-3 Best values of the calculated reciprocal moments of inertia by using the cranking-model for ^{32}S and ^{36}Ar

Nucleus	β	$\frac{\hbar^2}{2\Im_{Crank}}$ (KeV)	$\frac{\hbar^2}{2\Im_{exp}}$ (KeV) [14]
^{32}S	-0.335	371.533	371.72
	0.278	371.621	
^{36}Ar	-0.325	374.544	374.55
	0.272	374.442	

We considered the equilibrium moment of inertia, for the two nuclei, as the value where both of the cranking model and the rigid-body model moments of inertia are equal. In Table-4 we present the equilibrium reciprocal moments of inertia of the two nuclei together with the values of the deformation parameter β and the corresponding experimental reciprocal moments of inertia of the two nuclei.

Table-4 Equilibrium reciprocal moments of inertia of ^{32}S and ^{36}Ar

Nucleus	β	$\frac{\hbar^2}{2\Im_{equ}}$ (KeV)	$\frac{\hbar^2}{2\Im_{exp}}$ (KeV) [14]
^{32}S	-0.146	170.6241	371.72
	0.135	163.3611	
^{36}Ar	-0.121	144.8934	374.55
	0.114	140.0742	

As seen from Table-4, the values of the equilibrium reciprocal moments of inertia of ^{32}S and ^{36}Ar are not in good agreement with the corresponding data since they are closely related to the rigid-body values.

In Table-5 we present the values of the electric quadrupole moments of ^{32}S and ^{36}Ar for the axially-symmetric case together with the corresponding values of the total spin of the considered nuclear state and its parity, I^{π}, the deformation parameter β, the root mean-square radius, R, and the experimental data.

Table-5 Electric quadrupole moments of ^{32}S and ^{36}Ar for the axially-symmetric case

Nucleus	I^{π}	β	R (in fm)	Q_S (efm^2)	Q_{exp} (efm^2)
^{32}S	2^+	0.278	4.92	-14.88	-14.9
^{36}Ar	2^+	-0.11	5.12	10.96	11.0

5. Results for the Case where the Nucleus Has Not an Axis of Symmetry

In Table-6 we present the deformation parameters and the parameters of the single-particle potential for the two nuclei ^{32}S and ^{36}Ar.

Table-6 Model and potential parameters for the ^{32}S and ^{36}Ar nuclei

Nucleus	β	γ	χ	$\hbar\omega_0^0$ (MeV)	λ (MeV)	Δ (MeV)
^{32}S	0.395	25^0	0.06	10.908	3.24	2.07
^{36}Ar	-0.376	20^0	0.06	10.622	3.25	1.93

In Table-7 we present the superfluidity reciprocal moments of inertia of ^{32}S and ^{36}Ar together with the corresponding moments and the deformation parameters.

Table-7 Superfluidity reciprocal moments of inertia of ^{32}S and ^{36}Ar

Nucleus	β	γ	$\frac{\hbar^2}{2\Im_{S.F.}}$ (KeV)	$\frac{\hbar^2}{2\Im_{exp}}$ (KeV)
^{32}S	0.395	25^0	371.782	371.72
^{36}Ar	-0.376	20^0	374.665	374.55

Finally, in Table-8 we present the calculated values of the electric quadrupole moments of ^{32}S and ^{36}Ar by using the single-particle wave functions of the nonaxial potential. The values of the deformation parameters. the total spin and parity are also given.

Table-8 The electric quadruple moments of ^{43}S and ^{36}Ar in the non-axial case

nucleus	I^{π}	β	γ	Q_{cal} (efm^2)	Q_{exp} (efm^2)
^{32}S	2^+	0.45	25^0	-14.92	-14.9
^{36}Ar	2^+	-0.38	20^0	-10.9	11

6. Conclusion

It is seen from Tables-1 and 2 that the calculated values of the moments of inertia of the considered nuclei by using the cranking model of the concept of the single-particle Schrödinger fluid are in good agreement with the corresponding experimental values, a result which shows that the concept of this fluid is reliable and can be applied successfully to deformed nuclei in the $s-d$ shell. It is seen, also, from Tables-1 and 2 that the two nuclei ^{32}S and ^{36}Ar have nearly equal values of the deformation parameter $0.27 < \beta < 0.28$ (or $-0.33 < \beta < -0.32$). The disagreement between the value of the rigid-body reciprocal moment of inertia and the corresponding experimental data is due to the fact that the pairing correlation is not taken in concern in this model [3]. Furthermore, according to the results of the moments of inertia by using the concept of the single-particle Schrödinger fluid, the two nuclei ^{32}S and ^{36}Ar may have prolate deformation shape (positive value of β) as well as oblate deformation shape (negative value of β).

It is well-known that the quantity that characterizes the deviation from spherical symmetry of the electrical charge distribution in a nucleus is its quadrupole moment Q. If a nucleus is extended along the axis of symmetry, then Q is a positive quantity, but if the nucleus is flattened along the axis, it is negative. On the other hand, according to the results of the electric quadrupole moments of the two nuclei in the axially-symmetric case, the nucleus ^{32}S has prolate deformation shape while the nucleus ^{36}Ar has oblate deformation shape. In the case where the nucleus is assumed to be deformed and has not an axis of symmetry, the results obtained from the calculations of the superfluidity moments of inertia and the electric quadrupole moments of the two nuclei show also that the nucleus ^{32}S has prolate deformation shape while ^{36}Ar has oblate deformation shape. Accordingly, the two models, although different in their structures, agree in the assumption that the ^{36}Ar nucleus has an oblate deformation shape.

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