# An Explanation for the Equality between Inertia and Gravitational Mass Constantin SANDU ${ }^{1}$, Dan BRASOVEANU ${ }^{2}$ <br> ${ }^{1}$ Romanian Research and Development Institute for Gas Turbines-COMOTI, 220D, Iuliu Maniu Bvd., Sector 6, OP76, CP174, Bucharest, Romania constantin.sandu@comoti.ro <br> ${ }^{2}$ Systems Engineering Group Inc. (SEG), USA <br> 9861 Broken Land Pkwy, suite 350, Columbia, MD, 21046, USA <br> brasovdx@yahoo.com 


#### Abstract

The mechanism causing the equality of inertia and gravitational mass of a body, which was demonstrated experimentally by Eotvos in 1909 is still unexplained although this equality is the basis of Einstein's General Theory of Relativity (GTR).

Using consequences of GTR, this paper explains why the inertia mass is equally to the gravitational mass. The two masses are equal because the 'mechanism' that produces the inertia 'force' of a body is the same with the 'mechanism' that produces the gravitational 'force' of the same body.


## Indexing terms/Keywords

inertia mass, gravitational mass, equality of inertia mass and gravitational mass

## Academic Discipline And Sub-Disciplines

Physics, Relativity, General Theory of Relativity;

## SUBJECT CLASSIFICATION

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TYPE (METHOD/APPROACH)
Theoretical

## INTRODUCTION

Vladimir Fock has shown that the field equations of GTR (Eq.1) can be demonstrated starting from the equality between the gravitational and inertia mass and for this reason the 'Principle of Equivalence' is a derivative one [1]:
$R_{i k}-\frac{1}{2} \cdot g_{i k} \cdot R=-\frac{8 \pi G}{c^{4}} \cdot T_{i k}$
where the indices of coordinates in the 4D space-time continuum, $i$ and $k$ take values from 0 to 3 . The values 1,2 and 3 indicate space coordinates (for example $x_{1}=x, x_{2}=y, x_{3}=z$ when using Cartesian coordinates and $x_{1}=r$, i.e., radius, $x_{2}=\theta$, $x_{3}=\varphi$ when using spherical coordinates). The index value of zero is reserved for the temporal coordinate, which takes the form $x_{0}=c \cdot t$, where ' $c$ ' is the speed of light in vacuum and ' $t$ ' is time. $R_{i k}$, $g_{i k}$ and $T_{i k}$ are tensors; $R_{i k}$ is the curvature tensor of second order or Riemann's tensor, the tensor that defines the space metric, $g_{i k}$, is also called the metric or fundamental tensor, $R$ is Ricci's scalar curvature of space and $G$ is the universal constant from Newton's Law of gravitation. $\mathrm{T}_{\mathrm{ik}}$ is the energy-momentum tensor of matter (also known as the stress-energy tensor).

Fock's point of view generates a legitimate question : Could the cause of this equality be explained inside the General Theory of Relativity?

In this paper, the authors demonstrate that inertia mass is equal to gravitational mass because the 'mechanism' which generates the inertia of a body during acceleration is identical with the 'mechanism' of 'attraction 'of the same body in a gravitational field.

## DEMONSTRATION

Assume 2 photons are trapped inside a massless box having reflective walls (see Figure 1). If the box is accelerated for a time, $t$, with acceleration, a, by an observer, due to the Doppler effect, for a fixed observer, the photon moving to the right appears red-shifted [2]:
$v_{\text {right }}=v_{0}\left(1-\frac{v}{c}\right)=v_{0}\left(1-\frac{a \cdot t}{c}\right)=v_{0}\left(1-\frac{a \cdot \frac{L}{c}}{c}\right)=v_{0}\left(1-\frac{a \cdot L}{c^{2}}\right)$
and the photon moving to the left is blue-shifted:
$v_{\text {left }}=v_{0}\left(1+\frac{v}{c}\right)=v_{0}\left(1+\frac{a \cdot t}{c}\right)=v_{0}\left(1+\frac{a \cdot \frac{L}{c}}{c}\right)=v_{0}\left(1+\frac{a \cdot L}{c^{2}}\right)$
where $L$ is the length of box which is parallel to the direction of acceleration, $a$, and for simplicity it is assumed that the box speed, $\mathrm{v}=\mathrm{at} \ll \mathrm{c}$.


Fig.1: The 'mechanism' of inertia force-The man feels the inertia force with his hand
Obviously, the inertia force felt by the observer on his hand will be the difference of the forces exerted by the two photons on the right and left wall of box. Considering that the box is a prism having area of each base equal to S , the volume of box is ( $\mathrm{L} \cdot \mathrm{S}$ ) and the energy density of photons near the left side base will be:
$w=\frac{h \cdot v_{\text {left }}}{L \cdot S}=\frac{h \cdot v_{0}\left(1+\frac{a \cdot L}{c^{2}}\right)}{L \cdot S}$
The pressure of light on the left base of box is [3]:
$\mathrm{p}=\mathrm{w}$
and the force on the left base of box is correspondingly:
$F_{1}=p \cdot S=w \cdot S=\frac{h \cdot v_{0}\left(1+\frac{a \cdot L}{c^{2}}\right)}{L \cdot S} \cdot S=\frac{h \cdot v_{0}\left(1+\frac{a \cdot L}{c^{2}}\right.}{L}$
Using the same methodology, the force on the right base of box is:
$F_{r}=p \cdot S=w \cdot S=\frac{h \cdot v_{0}\left(1-\frac{a \cdot L}{c^{2}}\right)}{L \cdot S} \cdot S=\frac{h \cdot v_{0}\left(1-\frac{a \cdot L}{c^{2}}\right.}{L}$
And the inertia force felt by observer on his hand is:
$F_{i}=F_{I}-F_{r}=\frac{2 \cdot h \cdot v_{0} \cdot \frac{a \cdot L}{c^{2}}}{L}=\frac{2 \cdot h \cdot v_{0}}{c^{2}} \cdot a=m \cdot a$
Consider the same box containing the two photons is placed in a constant gravitational field (see Figure 2) having the potential:
$\Phi=-g \cdot x$
where $\Phi$ is the gravitational potential, $g$ is the local intensity of gravitational field ( $\mathrm{g}=\mathrm{a}$ ) and x is the distance. Such a gravitational field is like that of a large planet when the box length $L$ is small in comparison with the distance from that planet.

The law of energy conservation for the photon moving on the distance $L$ to the right base of box can be written:
$E_{\text {photon }}=h \cdot v_{0}-\frac{h \cdot v_{0}}{\mathrm{c}^{2}} \cdot \mathrm{~g} \cdot \mathrm{~L}=\mathrm{h} \cdot v_{0}\left(\cdot 1-\frac{\mathrm{g} \cdot \mathrm{L}}{\mathrm{c}^{2}}\right)$
According to Eq. (9), the frequency of the photon moving to the right base of box, $v_{r}$, is:
$v_{r}=v_{0}\left(-1-\frac{g \cdot L}{c^{2}}\right)$


Figure 2- The 'mechanism' of gravitational force: The man feels the gravitational force in his hand. This force is equal to the inertia force from Figure 1

Obviously, the photon suffers a 'red-shift'. Similarly, the photon moving to the left base of box suffers a 'blue-shift 'because its energy increases in the gravitational field:
$v_{\mathrm{i}}=v_{0}\left(1+\frac{\mathrm{g} \cdot \mathrm{L}}{\mathrm{c}^{2}}\right)$
Using the same procedure as in the case of inertia force,
$F_{g}=F_{l}-F_{r}=\frac{2 \cdot h \cdot v_{0} \cdot \frac{g \cdot L}{c^{2}}}{L}=\frac{2 \cdot h \cdot v_{0}}{c^{2}} \cdot g=m \cdot g=m \cdot a=F_{i}$
The conclusion is simple: the inertia and gravitational mass are identical because the 'mechanism' which produces inertia 'force' is identical with the mechanism 'which produces the so called gravitational 'force'.

Of course, these 'mechanism' can be explained only according to General Theory of Relativity which stipulated for the first time the red-shift of light in gravitational fields of stars.

## CONCLUSIONS

This paper demonstrated using a simplified model why the inertial and gravitational masses are equal.
For an observer, the photons trapped in a box having mirror walls suffer the same red and blue shift if the box is accelerated by the hand of an observer with acceleration, a , or when attracted by a gravitational field having intensity $\mathrm{g}=\mathrm{a}$. Thus, the photons exert different pressures/forces on box's walls. As a result, the observer perceives with his hand that inertia and gravitational force are the same. While the acceleration, a, and the intensity of the constant gravitational field, g , are identical, the result is that the inertial and gravitational mass must be identical.
Similarly, for a body composed of condensed matter, the bosons and fermions which form the body matrix can be considered as propagating waves. Due to the same Doppler effect, the gravitational and inertia 'forces' exerted on the observer's hand are the same when the body is placed in a gravitational field having local intensity, g , or when accelerated with acceleration $\mathrm{a}=\mathrm{g}$.
These 'mechanisms' can be explained only based on General Theory of Relativity which stipulated for the first time the red shift of light in gravitational fields of stars.

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This represents original research belonging to authors

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## Author' biography with Photo


, Romanian scientist. Scientific researcher at Romanian Research and Development Institute for Gas Bucharest, Romania. PhD in power transmission through mechanical vibrations. Over 35 papers in the field of physics of flight and aerospace propulsion. Project manager in 5 national and European research projects, 8 inventions.


Dr. Dan Brasoveanu is an expert in theoretical and applied physics including physics of propulsion, guidance navigation and control (GNC). He is also an expert in satellite sensors and nuclear detection systems. He worked for NASA, USA Department of Defense and Department of Homeland Security. He is the author or co-author of more than 40 science papers on propulsion, satellite GNC and theoretical physics published in science journals and refereed conference publications and of a book on quantum and relativistic mechanics included in the library of Conseil Européen pour la Recherche Nucléaire (CERN).

