



## Effect of uniform electromagnetic field on the trajectory of a free jet in a resistant medium proportional to the velocity.

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### ABSTRACT

The effect of electromagnetic field in a resistant medium of a free jet was carried out. The solutions of the governing equations showed that increase in electric field also increases the velocity profile of the fluid while increase in magnetic field decreases the velocity of the fluid particle. Increase in path angle beyond  $\pi/4$  radian, shows improved location of objects in a resistant medium proportional to the velocity.

### Keywords

electromagnetic field; Free jet; Resistant medium; Path angle

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## INTRODUCTION

The conceptual framework of two dimensional motions at the rudimentary stage cannot ignore the concept of projectile motion. [2] law of falling bodies led to an understanding of the motion of projectiles. Galeili studied the fall of a cannonball and described it as made up of two independent motions. The vertical component was uniformly accelerated and was in tandem to his law of falling bodies, the horizontal motion imparted to the body by a gunner was at constant speed [1]. When the horizontal and vertical components were combined, the resultant trajectory was a parabola. This seemingly abstract geometrical account had practical consequences for efficient gunnery. In military and defence, ballistic is concerned with projectiles fired from cannons or small arms, but it may be concerned also with the free flight of bombs and the powerful flight of rockets [1], The most important recent developments in ballistics is the use of computers. The calculus of exterior ballistics generally involves sets of second order partial differential equations to solve for the position of the projectile at various points along the path. In physics, projectile motion is a two dimensional motion as observed in the path of a base ball. The concept projectile motion which describes the trajectory of a particle has been studied under the effect of different parameters using different methods and configurations. [3] studied the effect of magnetic field on the trajectory of a free jet and deduced that the velocity of the particle was impeded.[4], [5], [6], [7],[8] and [9] all studied projectile motion using different methods and configurations but the present study is aimed at extending that of [3] and compare with other existing results.

### Mathematical Formulation of the Physical Problem

To investigate the effect of electromagnetic field on our problem, we employ the following equations

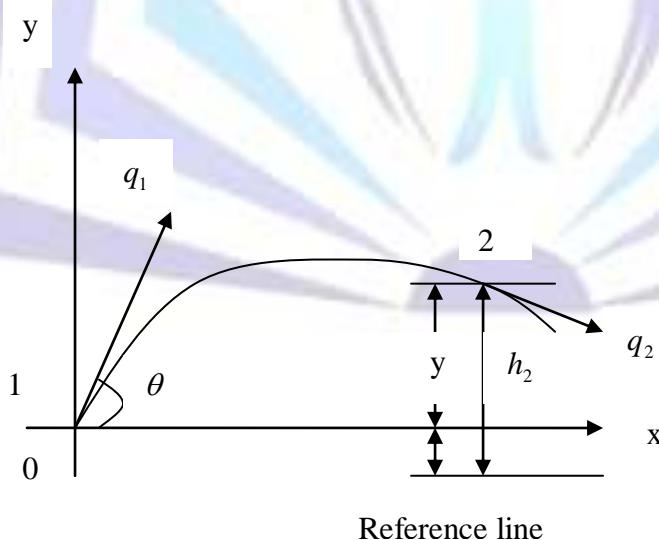
$$F = e(E + qx B) \tag{1}$$

$$E = -\nabla\phi - \frac{\partial A}{\partial t} \tag{2}$$

$$B = \nabla \times A \tag{3}$$

$$P_1 + \frac{1}{2} \rho q_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho q_2^2 + \rho g y_2 \tag{4}$$

where  $F$  is Lorentz force,  $e$  is particle charge,  $E$  is electric field intensity,  $q$  is particle velocity,  $\phi$  is scalar electric potential,  $A$  is vector magnetic potential,  $B$  is the magnetic induction vector,  $P$  is fluid pressure,  $\rho$  is fluid density,  $g$  is acceleration due to gravity and  $y$  is elevation. Further, we consider a liquid jet exiting from a small orifice of area of cross section  $S$ , with velocity  $q$  and making a path angle  $\theta$  with the horizontal as shown in figure 1 [10]



**Figure 1: The physical model and coordinate system of the problem**

We assume that the acceleration due to gravity is constant over the range of motion and directed downwards and the rotation of the earth does not affect the motion. Since the entire jet is in the atmosphere, we further assume that the pressure between the jet exit (regarded as point 1) and an arbitrary position (regarded as point 2) on the streamline are equal. The Bernoulli's equation as applied to a nonviscous incompressible fluid in steady flow reduces to



$$q_2^2 = q_1^2 - 2g(h_2 - h_1) \quad (5)$$

where  $q_1$  is velocity of influx,  $q_2$  is velocity of efflux

From figure 1 the physical model and the flow rate gives us the volume flow rate

$$Q = Sq_1 \quad (6)$$

where  $Q$  is the volume flow rate

Equation (5) reduces to

$$q_2^2 = \left(\frac{Q}{S}\right)^2 - 2gy \quad (7)$$

To determine the path of the jet, we consider the equation of motion for the jet along the horizontal component and the vertical component respectively as

$$\frac{dx}{dt} = q_1 \cos \theta - \frac{d}{dt} \left[ ex \left( -\frac{\partial \phi}{\partial x} + \frac{\partial(q.A)}{\partial x} - \frac{\partial A}{\partial t} \right) - kq \right] \quad (8)$$

$$\frac{dy}{dt} = q_1 \sin \theta - gt \quad (9)$$

Where  $(x, y)$  are coordinates of point 2 with point 1 as origin and  $k$  is proportional constant.

### Solution Techniques

We take  $\frac{\partial A}{\partial t} = 0$  and integrate equations (8) and equations (9) respectively to obtain

$$x = (q_1 \cos \theta)t - ex(-\phi + q.A - kq)t + \varphi_1 \quad (10)$$

$$y = (q_1 \sin \theta)t - \frac{1}{2}gt^2 + \varphi_2 \quad (11)$$

where  $\varphi_1$  and  $\varphi_2$  are constants and having dropped the differential operators.

At start (point 1)  $t = 0, x = 0, y = 0$ . This renders  $\varphi_1 = \varphi_2 = 0$  and eliminating  $t$  from equations (10) and (11) give us

$$y = x[1 + e(-\phi + q.A - kq)] \tan \theta - \frac{1}{2}x^2 \left( \frac{g}{q_1^2} \right) [1 + e(-\phi + q.A - kq)]^2 \sec^2 \theta \quad (12)$$

If we put equations (6) and (12) into equation (7), the velocity vector of the fluid at any point of the jet is given by

$$q_2^2 = \left(\frac{Q}{S}\right)^2 - 2gx[1 + e(-\phi + q.A - kq)] \tan \theta + x^2 \left(\frac{gS}{Q}\right)^2 [1 + e(-\phi + q.A - kq)]^2 \sec^2 \theta \quad (13)$$

### Results

For numerical validation of the problem the following values have been chosen



$$Q = 0,4m^3 / s^2$$

$$S = 0,2m^2$$

$$g = 9.8m / s^2$$

$$e = 1.6 \times 10^{-19} C$$

$$k = 0.34$$

$$q = 2m / s$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{0.19}, \frac{\pi}{0.22}, \frac{\pi}{0.28}$$

$$\phi = 0.3, 0.6, 0.9, 1.2, 1.5$$

$$A = 0.2, 0.4, 0.6, 0.8, 1.0$$

The dependence of velocity squared on space coordinate with electric field varying

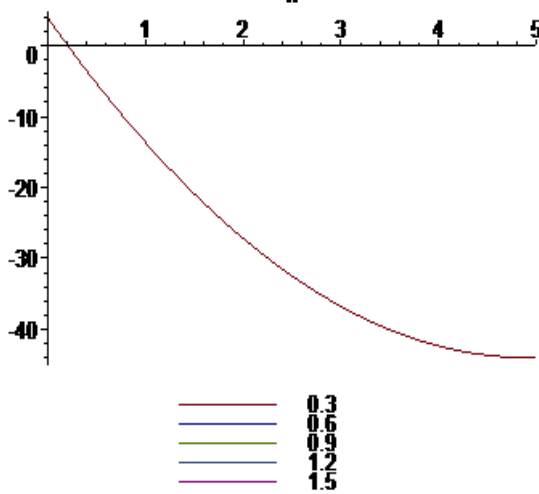


Figure 2: Plot of velocity squared against horizontal distance with  $\phi$  varying

The dependence of velocity squared on space coordinate with magnetic field varying

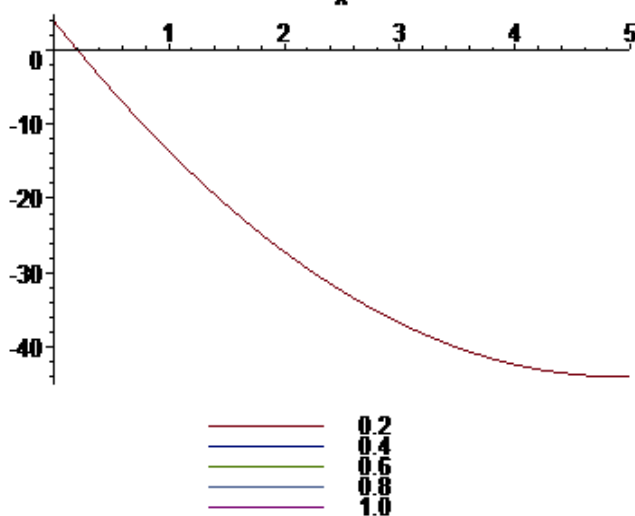
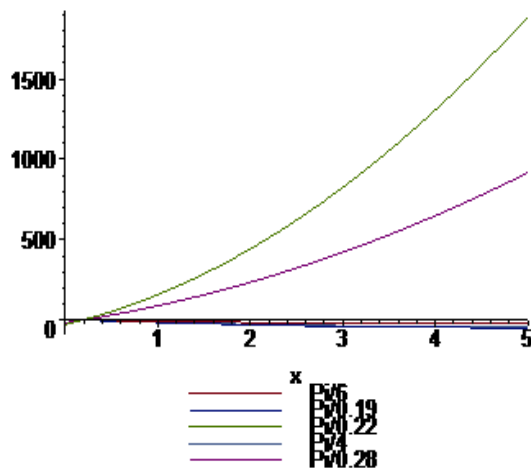


Figure 3: Plot of velocity squared against horizontal distance with  $A$  varying



The dependence of velocity squared on space coordinate with path angle varying

Figure 4: Plot of velocity squared against horizontal distance with  $\theta$  varying

## Discussions

Increase in the electric field increases the velocity of the projectile motion as depicted from figure 1 but the change is not very noticeable due to the presence of particle charge ( $e$ ) which is very small in magnitude. Figure 3 shows that increase in magnetic field decreases the velocity profile of the trajectory which is in accord with all earlier studies but again, the presence of particle charge made it appear not too visible. Finally, increase in path angle less than  $\pi/4$  radian may not be good enough for any appreciable effect for application in the initial positioning and aiming at target but greater than or equal to  $\pi/4$  radian is better as shown in figure 4 particularly in a resistant medium proportional to velocity.

## Conclusions

The application of projectile motion in effective gunnery and location of objects or positions is very important and knowledge of appropriately determining the path angle is of utmost necessity for accurate timing.

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