



The Uncertainty Principle derived by the finite transmission speed of light and information

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ABSTRACT

This work shows that in the frame of the stochastic generalization of the quantum hydrodynamic analogy (QHA) the uncertainty principle is fully compatible with the postulate of finite transmission speed of light and information. The theory shows that the measurement process performed in the large scale classical limit in presence of background noise, cannot have a duration smaller than the time need to the light to travel the distance up to which the quantum non-local interaction extend itself. The product of the minimum measuring time multiplied by the variance of energy fluctuation due to presence of stochastic noise shows to lead to the minimum uncertainty principle. The paper also shows that the uncertainty relations can be also derived if applied to the indetermination of position and momentum of a particle of mass m in a quantum fluctuating environment.

Indexing terms/Keywords

Keywords: Quantum to classical transition; quantum decoherence; open quantum systems, quantum measure, quantum hydrodynamic analogy, non-local interaction and finite speed of light

Academic Discipline And Sub-Disciplines

Quantum Mechanics, Quantum decoherence, Quantum to classical transition, quantum non locality and light speed, EPR paradox, theory of measurement

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INTRODUCTION

How the classical behavior is achieved in passing from the quantum dynamics to the macroscopic scale is a problem of interest in many branches of physics [1]. The radical difference between the quantum world and the classical one makes this problem one of the most puzzling one of our century both on the philosophically and mathematically point of view [2]. For this reason a unified theory hosting both the quantum mechanics as well as the classical ones, where the transition between the two dynamics is gradual, coherent and systematic, is still underway [3-5].

The incompatibility between the quantum and classical mechanics comes mainly from the impossibility to manage the non-local interaction of the quantum mechanics that acts among the parts of a system whatever is their relative distance even if infinite. This infinite range of non-locality of the quantum mechanics makes it incompatible with the (local) classical mechanics that cannot be achieved on whatever large-scale limit of quantum theory.

The antagonism between the classical and the quantum mechanics leads also to the logical incompatibility between the quantum non-local interaction and the finite speed of transmission of light and information at the base of the relativistic (classical) theory.

Sometime the difficulty in passing from the quantum to classical description is tentatively overcome by simply posing the Plank's constant \hbar equal to zero.

Even if this procedure is intuitively understandable, it is not correct from the mathematical standpoint [6,7].

Even if the quantum to classical bridging is a great theoretical problem, from the empirical point of view, the solution appears quite natural. It has been shown by many authors that fluctuations may destroy quantum coherence and elicit the emergence of the classical behavior [9-11].

Recently, the author has shown that in the frame of the stochastic generalization of the QHA [8] (SQHA) [12-13], developed by Madelung contemporarily to Schrödinger, the quantum and the classical mechanics can be retrieved as asymptotical limiting dynamics in the frame of a unique theory: the quantum one in the limit of microscopic scale (respect to a theory self-defined length) and the classical one in the large scale limit. Obviously, the unification of the incompatible classical and quantum mechanics pays a price: the stochastic generalization of the QHA, following the method already used in obtaining the stochastic of Burger's equations [14], leads to a really cumbersome system of motion equation.

Large-scale dynamics and fluctuations clearly seem to be the "ingredients" that can lead to the classical behavior.

In this paper we show that starting by the SQHA approach, where it is possible to systematically pass from the quantum behavior to the classical (stochastic) one, also the great logical inconsistencies between the quantum and classical can be explained. Particularly, in this work we show that the finite transmission speed of light and information (the basic law of the classical relativistic world) is fully compatible with the concept of non-local interaction of quantum one (quantum entanglement). We show that all this is possible since the quantum non-local interaction (in presence of stochastic noise (i.e., in the SQHA theory)) has a finite distance of action. When a statistical measurement is performed (i.e., the measuring apparatus and system are not a unique quantum system and are not quantum entangled) such a process has a finite minimum time of realization since the transmission of information cannot be faster than the light speed.

The theory shows that the minimum interval of time measurement multiplied by the energy variance due to the environment fluctuations is a constant (that can be set equal to the Plank's constant) and hence the uncertainty relations naturally appear.

The SQHA equation of motion

When space distributed noise is considered in the QHA, the motion equation for the particles density (PD) n (i.e., the wave function modulus squared (WFMS)), in the limit of small noise amplitude reads [13]

$$\partial_t n(q,t) = -\nabla_q \cdot (n(q,t) \dot{q}) + \eta(q,t,\Theta) \tag{1}$$

$$\lim_{\Theta \rightarrow 0} \langle \eta(q_\alpha,t), \eta(q_\beta + \lambda, t + \tau) \rangle = k\Theta \frac{\mu}{2\lambda_c^2} \exp[-(\frac{\lambda}{\lambda_c})^2] \delta(\tau) \delta_{\alpha\beta} \tag{2}$$

where $\dot{q}(t)$ is obtained by the solution of the differential equations

$$\dot{q} = \frac{p}{m}, \tag{4}$$

$$\dot{p} = -\nabla(V(q) + V_{qu}(n)), \tag{5}$$

where the noise correlation distance λ_c reads

$$\lim_{\Theta \rightarrow 0} \lambda_c = \pi \frac{\hbar}{(2mk\Theta)^{1/2}}, \tag{3}$$

where Θ is the amplitude of the spatially distributed noise η , μ is the PD mobility that depends by the specificity of the considered system [13], $V(q)$ represents the Hamiltonian potential and $V_{qu}(n)$ is the so-called (non-local) quantum potential [5-7] that reads

$$V_{qu} = -\left(\frac{\hbar^2}{2m}\right)n^{-1/2}\nabla \cdot \nabla n^{1/2}. \tag{6}$$

The Schrödinger equation is recovered for the complex variable [5]

$$\psi(q,t) = n^{1/2}(q,t)\exp\left[\frac{i}{\hbar}S(q,t)\right] \tag{7}$$

where $\nabla S(q,t) = p$.

The noise variance (2) is a direct consequence of the derivatives present into the quantum potential (i.e., $\nabla \cdot \nabla n^{1/2}$) that gives rise to a membrane elastic-like contribution to system energy [5] that reads

$$\overline{H}_{qu} = \int_{-\infty}^{\infty} n(q,t)V_{qu}(q,t)dq, \tag{8}$$

where higher curvature of the PD $n^{1/2}$ leads to higher energy (see figure 1).

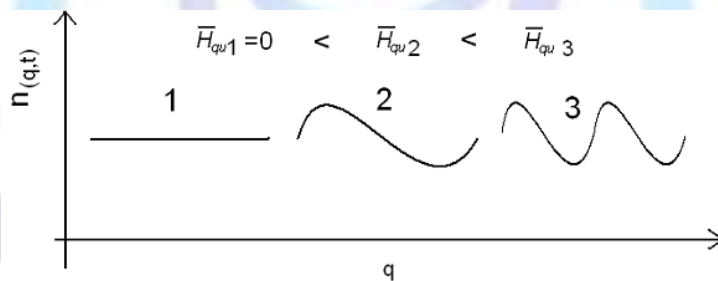


Figure 1 Quantum potential energy as a function of the “curvature” of the particle density n.

Therefore, independent fluctuations infinitesimally far apart and leading to very large curvature wrinkles of the PD and hence to an infinite quantum potential energy.

If we require that any physical system can attain configurations with finite energy, independent fluctuations of PD on shorter and shorter distance must have smaller and smaller amplitude. In the small noise limit, the energetic constraint

$$\lim_{\lambda_c \rightarrow 0} \overline{H}_{qu} = const < \infty$$

leads to the existence of a correlation distance (let's name it λ_c) for the noise variance [13] (see figure2) By imposing that the variance of the quantum potential energy fluctuations does not diverge in presence of a Gaussian noise it follows that the correlation distance must obey to the relation [13]

$$\lim_{\Theta \rightarrow 0} \lambda_c \propto \frac{\hbar}{(2mk\Theta)^{1/2}}. \tag{9}$$

The proportionality constant is defined by additional information that we discuss about further on in the paper.

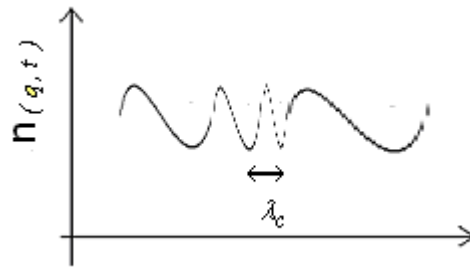


Figure 2. The particle density $n(q, t)$ owing Gaussian Fluctuations

It is worth mentioning that the stochastic equations (1-5) deriving by the following system of differential equations

$$\partial_t S(q, t) = -V(q) + \frac{\hbar^2}{2m} \frac{\nabla^2 A(q, t)}{A(q, t)} - \frac{1}{2m} (\nabla S(q, t))^2 \quad (10)$$

$$\partial_t A(q, t) = -\frac{1}{m} \nabla A(q, t) \cdot \nabla S(q, t) - \frac{1}{2m} A \nabla^2 S(q, t) + A^{-1} \eta(q, t, \Theta) \quad (11)$$

for the complex variable (7), are equivalent to the stochastic Schrödinger equation [11]

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(q) \psi + i \frac{\psi}{|\psi|^2} \eta(q, t, \Theta). \quad (12)$$

Local large-scale dynamics

In addition to the noise correlation function (2), in the large-distance limit, it is also important to know the behavior of the

quantum force $\dot{p}_{qu} = -\nabla_q V_{qu}$.

The relevance of the force generated by the quantum potential at large distance can be evaluated by the convergence of the integral [2]

$$\int_0^{\infty} |q|^{-1} |\nabla V_{qu}| dq \quad (13)$$

If the quantum potential force $-\nabla_q V_{qu}$ goes like $\lim_{|q| \rightarrow \infty} |\nabla V_{qu}| \propto |q|^{-\varepsilon}$, where $\varepsilon > 0$, at large distance so

that $\lim_{|q| \rightarrow \infty} |q|^{-1} |\nabla V_{qu}| \propto |q|^{-(1+\varepsilon)}$, the integral (13) converges. In this case, the mean weighted distance

$$\lambda_q = 2 \frac{\int_0^{\infty} |q|^{-1} \frac{dV_{qu}}{dq} dq}{\lambda_c^{-1} \left| \frac{dV_{qu}}{dq} \right|_{(q=\lambda_c)}} \quad (14)$$

can be used to evaluate the quantum potential range of interaction.

For analyzing the interplay between the Hamiltonian potential and the quantum one in a general non-linear case, we can consider the linear case. For the linear interaction, the Gaussian-type eigenstates leads to a quadratic quantum potential

and, hence, to a linear quantum force, so that $\lim_{|q| \rightarrow \infty} |q|^{-1} |\nabla V_{qu}| \propto \text{constant}$ and λ_q diverges.

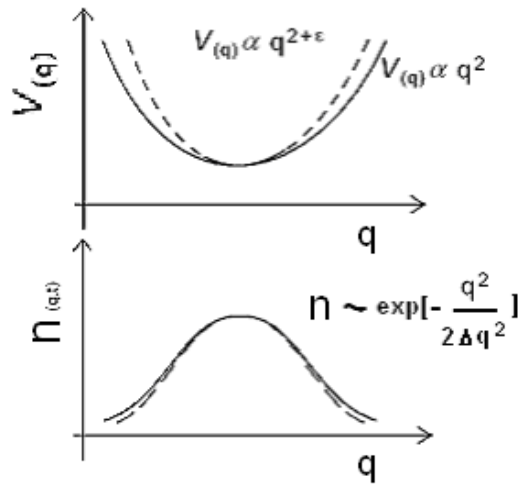


Figure 3. Particle density $n(q,t)$ for near-linear Hamiltonian potential

Moreover, since faster the Hamiltonian potential grows, more localized is the WFMS and hence stronger is the quantum potential, in order to have λ_q finite (so that the large-scale classical limit can be achieved) we have to deal with a system of particles interacting by a

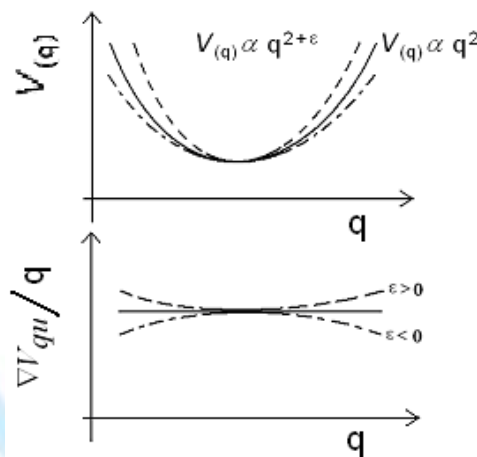


Figure4. The scaled quantum force $q^{-1}\nabla V_{qu}$ for near-linear Hamiltonian potential

weaker than the linear interaction.

For Hamiltonian potential weaker than the quadratic one ($\epsilon < 0$ in figure 4), the quantum potential force (QPF) ∇V_{qu} at large distance grows less than linearly. Given the case of a pseudo-Gaussian WFM such as $\lim_{|q| \rightarrow \infty} n \propto \exp[-P^k(q)]$,

for $k < \frac{2}{3}$ the integral $\int_0^\infty q^{-1} \nabla V_{qu} / dq$ converges [13].

When the length resolution $\Delta\Omega$ of our physical problem is larger than λ_c and λ_q so that we have $\lambda_c \cup \lambda_q \ll \Delta\Omega \ll \Delta L$, where ΔL is the physical length of the system, equation (11) reads [13]



$$\begin{aligned}
 \lim_{\Theta \rightarrow 0} \dot{q} &= \frac{p}{m} = \lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} \lim_{\Theta \rightarrow 0} \frac{\nabla S}{m} \\
 &= \nabla \left\{ \lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} \lim_{\Theta \rightarrow 0} \frac{1}{m} \int_{t_0}^t dt \left(\frac{p \cdot p}{2m} - V_{(q)} - V_{qu(n)} \right) \right\} \\
 &= \nabla \left\{ \lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} \frac{1}{m} \int_{t_0}^t dt \left(\frac{p \cdot p}{2m} - V_{(q)} - V_{qu(n_0)} - (V_{qu(n)} - V_{qu(n_0)}) \right) \right\} \\
 &= \lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} \frac{1}{m} \nabla_q \left\{ \int_{t_0}^t dt \left(\frac{p \cdot p}{2m} - V_{(q)} - \lim_{\Theta \rightarrow 0} (V_{qu(n)} - V_{qu(n_0)}) \right) \right\} = \frac{p_{cl}}{m} + \frac{\delta p}{m} \cong \frac{p_{cl}}{m}
 \end{aligned} \tag{15}$$

where , given that $\lim_{q \rightarrow \infty} -\nabla_q V_{qu} = 0$, it has been posed $\lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} V_{qu(n_0)} = 0$, where

$$\dot{p}_{cl} = \lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} -\nabla_q V_{(q)}, \tag{16}$$

is the classical force and where δp is a small fluctuation of the momentum since the variance of the quantum potential fluctuations (i.e., $V_{qu(n)} - V_{qu(n_0)}$) tends to zero for vanishing Θ (for the convergence to the deterministic limit warranted by (2)).

By (15) we can see that in the large scale limit the quantum action S that reads

$$\lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} S = \int_{t_0}^t dt \left(\frac{p \cdot p}{2m} - V_{(q)} \right) + \delta S_{(V_{qu(n)} - V_{qu(n_0)})} = S_{cl} + \delta S \tag{17}$$

converges to the classical value S_{cl} plus a small fluctuation and that the noise variance reads

$$\lim_{\Delta L / \lambda_c \rightarrow 0} \lim_{\Theta \rightarrow 0} \langle \eta(q_\alpha, t), \eta(q_\alpha + \lambda, t + \tau) \rangle = \underline{\mu} \delta_{\alpha\beta} \frac{2k\Theta}{\lambda_c} \delta(\lambda) \delta(\tau) \tag{18}$$

The features of the large-scale classical limit

The stochastic classical dynamics defined by (15-16) is obtained by neglecting the quantum potential interaction that at large distance reads

$$\lim_{\Delta L / \lambda_c \rightarrow \infty} \lim_{\Delta L / \lambda_q \rightarrow \infty} V_{qu(n_0)} = 0 \ll V_{qu(n)} - V_{qu(n_0)}$$

This operation that in absence of noise is not mathematically correct, in presence of noise does not alter the outputs of the equations since it is much smaller than the fluctuations of the quantum potential itself $V_{qu(n)} - V_{qu(n_0)}$.

Obviously, the effect of the quantum potential fluctuations $V_{qu(n)} - V_{qu(n_0)}$ on the dynamics of the system is not equal to the effect of their mean value to which $V_{qu(n_0)}$ may be close. The stochastic sequence of inputs of quantum potential do not lead to any coherent reconstruction of superposition of state whether or not we take into account the very small deterministic quantum potential $V_{qu(n_0)}$.

As a consequence of this selective efficacy of quantum potential interaction that acts on small-scale dynamics while it becomes ineffective on large ones, the stochastic large-scale mechanics in the SQHA maintains its local behavior until the resolution size $\Delta\Omega$ is larger than the quantum coherence length λ_c .

Moreover, higher is the amplitude of the noise Θ , higher is the attainable degree of spatial precision within the classical (local) limit. On the other hand, higher is the amplitude of noise, higher are the fluctuations of the observable such as the variance of energy measurements.

This mutual conflicting effect is basically the same of that one of the conjugated variables leading to the Heisenberg's principle of uncertainty. To verify this, let's evaluate the uncertainty relation between the time interval Δt and the variance in the energy measurement on a punctual particle of mass m in the classical mechanical limit of the SQHA.

If on distances smaller than λ_c any system behave as a wave so that any its subparts cannot be perturbed without disturbing all the system, it follows that the independence between the measuring apparatus and the measured system requires that they must be far apart more than λ_c and hence for the finite speed of propagation of interactions and information the measure process must last longer than the time $\tau = \frac{\lambda_c}{c}$ (see figure 5).

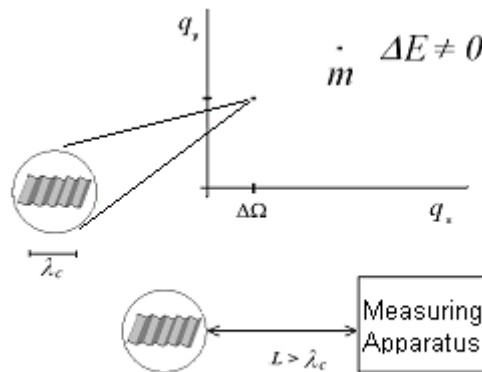


Figure 5. The coarse-grained classical view in a fluctuating quantum environment

Moreover, given that the noise $\eta(q,t,\Theta)$ in (1) in the small noise limit (i.e. Θ tends to zero) is Gaussian [13] we have that the mean value of the energy fluctuation for one particle is $\Delta E_{(\Theta)} = \left(\frac{3}{2}\right)^{1/2} k\Theta \propto k\Theta$ [15].

Thence, for the classical case ($mc^2 \gg k\Theta$) for a particle of mass m leads to an energy variance ΔE that reads

$$\begin{aligned} \Delta E &\approx \left(\langle (mc^2 + \Delta E_{(\Theta)})^2 \rangle - (mc^2)^2 \right)^{1/2} \cong \left(\langle (mc^2)^2 \rangle + 2\Delta E_{(\Theta)} - (mc^2)^2 \right)^{1/2} \\ &\cong (2mc^2 \langle \Delta E_{(\Theta)} \rangle)^{1/2} \cong (2mc^2 k\Theta)^{1/2} \end{aligned} \quad (19)$$

from which it follows that

$$\Delta E \Delta t > \Delta E \Delta \tau = \frac{(2mc^2 k\Theta)^{1/2} \lambda_c}{c} = \infty \hbar, \quad (20)$$

therefore if we use the freedom to choose the proportionality constant to introduce into the SQHA model to the physical information of the uncertainty principle $\Delta E \Delta t > \frac{\hbar}{2}$, we obtain that $\infty = \pi$. It is worth noting that the product $\Delta E \Delta \tau$ is

constant since the growing of the energy variance with the square root of Θ $\Delta E \approx (2mc^2 k\Theta)^{1/2}$ is exactly compensated the decrease of the minimum time of measurement

$$\tau \propto \frac{\hbar}{(2mc^2 k\Theta)^{1/2}} \quad (21)$$



The same result is achieved if we derive the uncertainty relations between the position and momentum of a particle of mass m . If we measure the spatial position of a particle with a precision of $\Delta L > \lambda_c$ so that we do not perturb its quantum wave function (that extends its on a smaller spatial domain) the variance Δp of the modulus of its relativistic momentum $(p^\mu p_\mu)^{1/2} = mc$ due to the vacuum fluctuations reads

$$\begin{aligned} \Delta p &\approx (\langle (mc + \frac{\Delta E_{(\Theta)}}{c})^2 - (mc)^2 \rangle)^{1/2} \cong (\langle (mc)^2 + 2m\Delta E_{(\Theta)} - (mc)^2 \rangle)^{1/2} \\ &\cong (2m \langle \Delta E_{(\Theta)} \rangle)^{1/2} \cong (2mk\Theta)^{1/2} \end{aligned} \tag{22}$$

Leading to the uncertainty relation\

$$\Delta L \Delta p > \lambda_c (2mk\Theta)^{1/2} = \frac{\hbar}{2} \tag{23}$$

If we require measuring the spatial position with a precision $\Delta L < \lambda_c$, we have to perturb the particle quantum state.

Due to the spatial confinement of the wave function, an increase of the quantum potential energy is generated so that the final particle momentum gets a variance Δp higher than (22) and still satisfying the uncertainty relations steaming by the property of the Fourier transform relations.

(22) holds as far as we remain in the classical approach and we do not explore the space into domains whose length is smaller than λ_c so that momentum variance is given just by the environmental fluctuations.

DISCUSSION

For sake of completeness we have to say that a basically equivalent result to equation (19) has been previously obtained by Olszewski et al. [16-20]. In this paper we additionally show that the energy variance ΔE is linked (i.e., proportional) to the amplitude of the background noise Θ and hence to the quantum correlation distance λ_c that, on its turn, determines the minimum time interval to perform the measurement (leading to a self-consistent model). Actually, the present approach gives a physical reason behind the uncertainty relations of conjugate pair of observables.

When we act in the (local) classical world and go down to look to a smaller and smaller scale phenomena, the non-local (quantum wave nature) character of particles emerges through the uncertainty relations linking conjugated variables. If we try to increase the localization of quantum state (e.g., by increasing the background noise) the momentum of such a system will necessarily be perturbed with the increase of its variance. Behind the uncertainty relations there is the physics of quantum decoherence and the transition to the classical mechanics.

Particularly, the SQHA model shows that for Θ that goes to zero (i.e., $\lambda_c \rightarrow \infty$ and the standard quantum mechanics is realized) the measuring time goes to infinity and the energy fluctuation ΔE goes to zero (we have a perfect overall quantum system with exactly defined energy levels) complying with the uncertainty relations.

This makes clear that in a perfect quantum universe the measuring process is endless (i.e., not possible) confirming that the classical behavior is needed to perform a measurement and hence to the definition of the quantum mechanics (based on the measuring process).

The SQHA approach shows that the logical contradiction between the non-local quantum interaction and the finite transmission speed of interactions comes by two singularities of the standard approach: the infinite velocity of light in the non-relativistic limit and the infinite value of the quantum coherence length of the standard quantum mechanics.

This conceptual incongruity ends in the frame of the SQHA model where quantum coherence length is finite and the systematic passage to the classical mechanics is allowed.

It is worth mentioning that the Heisenberg indetermination principle in the SQHA is achieved by the fact that the minimum duration of the measurement process multiplied by variance of the energy fluctuation leads to a constant. This can happen

since the energy variance going like $\Delta E \approx (2mc^2 k\Theta)^{1/2}$ exactly compensates the Θ -dependence of the minimum

time of measurement $\tau \propto \frac{\hbar}{(2mc^2 k\Theta)^{1/2}}$. The same compensation happens for the position-momentum uncertainty

relation of a particle of mass m .



For sake of precision, we observe that the indetermination relations (20,23) corresponds in the Ozawa formulation, [20-22] to the product of the standard deviation of the probe observable (e.g., position) by the disturbance of the conjugated one (e.g., momentum). Moreover, the present derivation is slightly more general since it is derived without the requirement of instantaneous measuring process.

In the frame of the SQHA model, the minimum uncertainty principle is fully compatible with the relativistic requirement of finite speed of light and information transmission. This is a very important achievement in order to obtain a reliable and satisfying quantum description compatible with the large scale classical and relativistic behavior.

Moreover, if we consider the SQHA model as the correct approach, the uncertainty postulate can be re-visited as deriving by the requirement of finite transmission speed of interactions to travel the distance over which quantum non-local interaction holds.

CONCLUSION

The stochastic generalization of the QHA shows that the local classical description, in which it is possible to divide a system into independent sub-parts (so that we can improve the precision of local variables) has a physical limit given by the distance on which the non-local quantum interaction takes place.

Given that below the length on which the quantum non-locality sets-in, any subpart of a system cannot collect and extract statistically independent information on the remaining one, the measuring process cannot be performed in a time shorter than that one needed to the interactions and information to be transmitted beyond such a distance.

The localization of the quantum states in the SQHA model (achieved as a consequence of fluctuations) obeys to the uncertainty relations.

The achievement of the indetermination principle into the SQHA dynamics can be derived by the fact that the minimum duration of the measurement process multiplied by variance of the energy fluctuation results a constant.

In the frame of this approach the minimum uncertainty postulate derives by requiring that the finite transmission speed of information is compatible with the non-local character of the quantum mechanics.

NOMENCLATURE

n : squared wave function modulus	l^{-3}
S : action of the system	$m^{-1} l^{-2} t$
m : mass of structureless particles	m
\hbar : Plank's constant	$m l^2 t^{-1}$
c : light speed	$l t^{-1}$
k : Boltzmann's constant	$m l^2 t^{-2} \text{ } ^\circ\text{K}^{-1}$
Θ : Noise amplitude	$^\circ\text{K}$
H : Hamiltonian of the system	$m l^2 t^{-2}$
V : potential energy	$m l^2 t^{-2}$
V_{qu} : quantum potential energy	$m l^2 t^{-2}$
η : Gaussian noise of WFMS	$l^{-3} t^{-1}$
λ_C : correlation length of squared wave function modulus fluctuations	l
λ_L : range of interaction of non-local quantum interaction	l
$G(\lambda)$: dimensionless correlation function (shape) of WFMS fluctuations	pure number
$\underline{\mu}$: WFMS mobility form factor	$m^{-1} t l^{-6}$
μ = WFMS mobility constant	$m^{-1} t$



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