



## On mechanics of light propagation in free space with final temperature

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### ABSTRACT

Features of electromagnetic waves propagation of light range are considered in free space with final temperature 2.725K. The presence in space of temperature (and final density) allows justification to introduce the longitudinal component of electromagnetic field. A modified theory of electromagnetic waves propagation in free space is offered. Exact solutions of the nonlinear equations system in the presence of electric and gasdynamic interaction are obtained. Some of demonstrated exact solutions have a nature of continues and decretive spectrum.

**KEYWORDS:** light propagation mechanics, transverse-longitudinal waves, nonlinear theory, exact solution.



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## Introduction

Basing on Gauss's and Ampere's laws

$$\nabla \cdot \bar{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \times \bar{B} = \frac{\bar{j}}{\epsilon_0 c^2}$$

Maxwell using experimental data has calculated the constant  $c$  value, which coincides with the value of light speed in free space of vacuum equal to  $\sim 3 \cdot 10^8$  m/s [1-3]. His main conclusion sounded as follows: "We hardly can avoid an opinion that light is *transverse* wave movement of the same media which causes electric and magnetic phenomena". Statement about the transverse nature of electromagnetic waves followed, in particular, an effect of Faraday's law

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t},$$

which related electric field circulation with time-dependence of magnetic flow.

A principle background step by Maxwell became introduction in Ampere's law of unsteady component - displacement current

$$\nabla \times \bar{B} = \frac{\bar{j}}{\epsilon_0 c^2} + \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t}.$$

In his discourses Maxwell leaned on the necessity of real displacement onset of "charge-bearers" under action of force fields. Meanwhile, coming from the analogy with mechanical elastic deformation of continuum media only "charge-bearers" shift deformation were taken into account (without change of volume). In total, in the base of classical electrostatics the Maxwell's equations system was placed

$$\begin{aligned} \frac{\partial \bar{E}}{\partial t} - c^2 \nabla \times \bar{B} &= \frac{\bar{j}}{\epsilon_0}, \quad \nabla \cdot \bar{E} = \frac{\rho}{\epsilon_0}, \\ \frac{\partial \bar{B}}{\partial t} + \nabla \times \bar{E} &= 0, \quad \nabla \cdot \bar{B} = 0. \end{aligned} \quad (1)$$

In free space and in absence of concentrated charges and currents in (1) it is necessary to set  $\bar{j} = 0$  and  $\rho = 0$ . System (1) validity is repeatedly checked and causes no doubts in a linear case of weak electromagnetic perturbations propagation.

Note an important system (1) feature, namely, mathematical description of transverse electromagnetic waves propagation in an "incompressible" electromagnetic field case, when longitudinal compression-vacuum waves are absent (here it is possible to interpret propagation of the last with infinite speed). We used a hydrodynamic term "incompressible", following description of incompressible fluid movement, when speed vector field  $\bar{V}$  is set by equation  $\nabla \cdot \bar{V} = 0$ . The equation is valid both for stationary and unsteady incompressible fluid movement [4,5].

The system (1) property to describe propagation only for transverse waves leads to so-called "Lorentz's calibration" for vector  $\bar{A}$  and scalar  $\varphi$  potentials of electromagnetic field. Due to (1) we have

$$\bar{E} = \nabla \varphi - \frac{\partial \bar{A}}{\partial t}$$

and calibration requirement in form of

$$\nabla \cdot \bar{A} = -\frac{1}{c^2} \frac{\partial \varphi}{\partial t},$$

provides for realization of wave equations for  $\bar{A}$  and  $\varphi$

$$\begin{aligned} \frac{\partial^2 \bar{A}}{\partial t^2} - c^2 \Delta \bar{A} &= \frac{\bar{j}}{\epsilon_0}, \\ \frac{\partial^2 \varphi}{\partial t^2} - c^2 \Delta \varphi &= \frac{\rho}{\epsilon_0}. \end{aligned}$$



Making a graceful building of theoretical electrodynamics, as well as foundation of modern theoretical physics was completed in the first half of the XX century. However the second half of the XX century was marked by new basic experimental achievements of theoretical issues in the field of physics. First of all, note the finding of the final temperature of Cosmic Microwave Background Radiation (CMBR)  $T=2.725\text{K}$ . In 1955 Tigran Aramovich Shmaonov, young scientist of the Pulkovo Observatory, taking measurements of microwave radiation from space at wavelength 32 cm found the presence of an isotropic radiation with absolute efficient temperature value  $T=4\pm 3\text{K}$ . The results, obtained by Shmaonov T.A., he formulated in his dissertation and in article [6]. In 1965 two American radio astronomers Arno Penzias and Robert Wilson registered cosmic background radiation temperature related to the absolutely black body equilibrium radiation temperature with  $T\approx 3\text{K}$  [7].

In 1969 a large-scale dipole anisotropy of background radiation related with the Solar system movement around background at a speed of about 370 km/s aside the Leo Constellation and with motion of the entire local group of galaxies, including the Milky Way, for background radiation with a speed of approximately 630 km/s. The circumstance allows entrance of the chosen spatial coordinates system rigidly tied with CMBR [8].

Important for additional experimental validation of our research results is registration in the Universe of Dark Matter (DM), which forms beside 96% of our Universe whole matter [9,10]. At present the problem of DM existence is the most intriguing undecided one of the modern science.

Here we also note rather interesting experimental fact of free space polarization. The polarization effect is exemplified by the presence of polarized space near electron (so named "fur coat" of electron), as well as similar polarized space near proton and positive charged atom nucleus. A limiting case of polarization effect manifestation might be the birth of positron-electron pair at collision of two rather strong electromagnetic pulses [11]. Incorporated by Maxwell the displacement current in free space characterizes also polarization of this space [12].

Our article attempts to agree the classical electrodynamics theory with outer space final temperature, presence of dedicated coordinate system in vicinities of the Earth (and Solar system), and discovered rather significant volume (96%) of our Universe matters. Main attention is paid for consideration of electromagnetic waves of light range, in which the mentioned effects can be most greatly shown. Herewith the proposed mathematical description comprises the traditional classical electrodynamics as a private case.

## 1. Hidden Mass Bozon

Return from positions of modern experimental data to physics origins of XX century beginning (to basic works by M. Plank, A. Einstein, Louis de Broglie, and others) allows building rather justified both linear and nonlinear theory of light propagation in free space with final temperature. Reference theory positions follow from the correlations

$$E = mc^2 = h\nu \approx kT, \quad (2)$$

allowing definition of particle mass  $m \approx kT / c^2$ . Experimental registration of physical vacuum temperature (CMBR)  $T=2.725\text{K}$  under the known perturbations propagation speed in vacuum  $c = 2.998 \cdot 10^8$  m/s and Boltzmann constant  $k$  defines characteristic value of vacuum particle mass (Hidden Mass Boson - HMB), which under correct calculation forms  $m = 5,6 \cdot 10^{-40}$  kg  $= 3,4 \cdot 10^{-4}$  eV [12-14].

In our study, coming from final (not zero) obtained mass  $m$ , we postulate the presence in free space (the vacuum) of gaseous HMB medium identified with DM. Under a set value to density and temperature on the classic equation of ideal gas state one may define the pressure value in free space [13-15]. So, we have at medium density value equal to critical Universe density  $\rho_* = 10^{-26}$  kg/m<sup>3</sup> and temperature  $T=2.725\text{K}$  the pressure value

$$p = \rho_* \frac{k}{m} T \approx 10^{-9} \text{ Pa.}$$

Estimation of pressure value in Solar system vicinity at setting  $\rho = 10^{-23}$  kg/m<sup>3</sup> gives us

$$p = \rho \frac{k}{m} T = 10^{-6} \text{ Pa.}$$

In papers [13-18] an agreed thermodynamic system of conservation laws is drawn describing a joint dynamic of gaseous two-phase medium of common baryon matter (the traditional gas) and gaseous HMB medium (DM), and some important practical conclusions are made for a case of external and internal aerodynamics problems.

The presence of final mass value  $m$  for HMB and usage of hydrodynamic models for description of HMB dynamics allows us to write in the considered phenomenological approach the total derivative (derivative in a particle moving at speed  $\bar{V}$ )



$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla$$

and so provide hereunder invariance of our description relative to Galilean transformation groups [19].

## 2. HMB Structure

We shall go now to an important question of possible structure clarification of the massive particle - HMB determined by us. Here first of all we should rest on experimental registered phenomena of vacuum polarization, electromagnetic waves propagation in vacuum (in particular, shift current), and possible onset of positron-electron pairs from vacuum. Similar and some other observable effects can be explained by postulating HMB structures in the form of classic dipole [13-15]. Coming from estimation of the mass, charge, and sizes of electron and proton, we get linear dipole size  $l = 7 \cdot 10^{-20}$  m and its electric charge value  $q \approx 10^{-28}$  C. HMB dipole electric moment value is  $p = ql \approx 7 \cdot 10^{-48}$  C·m. In spite of calculated tiny sizes, we consider that all the known electric dipoles characteristics are saved and in our case. Hereunder we can say automatically perform the above mentioned characteristics of vacuum polarization and others. Herewith in the considered model of physical vacuum by natural way we model the birth in vacuum of positron-electron pairs [11]. In our case such a pair origins from HMB material medium rather than from emptiness complying with all the laws of conservation of mass, charge, impulse, and energy.

The presence of characteristic charge  $q \approx 10^{-28}$  C allows us to introduce a characteristic linear size of polarized space (Debye screening radius D) and characteristic frequency  $\omega$  (analogue of Langmuir plasmon frequency) [18]. Write down the known expressions for these two typical parameters at preset dielectric vacuum penetrability  $\epsilon_0$  and dipole concentrations  $n$ . We have

$$D = \sqrt{\frac{\epsilon_0 kT}{nq^2}}, \quad \omega = \sqrt{\frac{nq^2}{\epsilon_0 m}}.$$

.The product of these parameters gives us the same value of characteristic velocity, which follows from (2),

$$c = D\omega = \sqrt{\frac{kT}{m}}$$

.The stated modeling of physical vacuum (and DM) as gaseous compressible HMB medium and dipoles allows us to consider propagation in vacuum of longitudinal (potential) perturbations (waves of rare-compression), alongside with transverse (solenoidal) perturbations (shift waves).

## 3. Extended system of Maxwell electrodynamics equations

The following natural step in modeling physical vacuum at the presence of final temperature of HMB compressible medium in the form of classical dipoles is iteration for the case considered of the Maxwell's method of getting linear electrodynamics equations (1). Here as a matter of discourses convenience we write down equations (1) for free space without charges and currents ( $\rho=0$  and  $\vec{j} = 0$ ) for intensity of electric field  $\vec{E}$  and magnetic field  $\vec{H}$  in the manner of (Gauss system of units)

$$\begin{aligned} \frac{\partial \vec{E}}{\partial t} - c \nabla \times \vec{H} &= 0, \quad \nabla \cdot \vec{E} = 0, \\ \frac{\partial \vec{H}}{\partial t} + c \nabla \times \vec{E} &= 0, \quad \nabla \cdot \vec{H} = 0. \end{aligned} \quad (3)$$

The Maxwell background idea consisted of introduction to system (3) of displacement current (first component in the left-hand part of the first equation). In our model the shift current should be explained by charges shift, which we characterize by changing power lines density of electric and magnetic fields (accordingly  $p$  and  $q$ ). Here we follow the analogies with dynamics of compressible fluid and instead of "incompressible" relationships ( $\nabla \vec{E} = 0, \nabla \vec{H} = 0$ ) write down

$$\frac{\partial p}{\partial t} + c_0 \nabla \vec{E} = 0, \quad \frac{\partial q}{\partial t} + c_0 \nabla \vec{H} = 0 \quad (4)$$

Taking into account the presence of longitudinal components of electric and magnetic fields intensity requires also extended record of the two other Maxwell equations in the manner of



$$\begin{aligned}\frac{\partial \bar{E}}{\partial t} - c_1 \nabla \times \bar{H} + c_0 \nabla p &= 0 \\ \frac{\partial \bar{H}}{\partial t} + c_1 \nabla \times \bar{E} + c_0 \nabla q &= 0\end{aligned}\quad (5)$$

Underline, that in system of equations (4) and (5) velocity value  $c_0$  defines spreading longitudinal potential perturbations (waves of rare-compression), and velocity value  $c_1$  – common propagation speed of transverse solenoidal perturbations (shift waves) of electromagnetic field.

#### 4. Potential and solenoidal waves

We show for our considered linear case the presence of two wave types of in elastic space of physical vacuum. Here we use the classical Poisson approach for elastic media (see, for instance, [20]) and rather simple but defining many topological particularities a theorem about presentation of arbitrary vector field in the manner of superposition of potential and vortex [21]. We present vector fields  $\bar{E}$  and  $\bar{H}$  as amounts of potential  $\bar{E}_p$  and  $\bar{H}_p$  and solenoidal  $\bar{E}_s$  and  $\bar{H}_s$  vector components

$$\begin{cases} \bar{E} = \bar{E}_p + \bar{E}_s = \text{grad}\varphi + \text{rot}\bar{A}, \\ \bar{H} = \bar{H}_p + \bar{H}_s = \text{grad}\psi + \text{rot}\bar{B}, \end{cases}\quad (6)$$

where  $\varphi, \psi$ , and  $\bar{A}, \bar{B}$  - scalar and vector potentials of electric and magnetic fields. Without limiting generalities, one can suppose [21]

$$\text{div}\bar{A} = 0, \quad \text{div}\bar{B} = 0. \quad (7)$$

Substituting (6) in (4) and (5) with provision for (7) get the D'Alembert equations for all the considered scalar and vector fields. So, using operation *div* to system (5) equations we have

$$\frac{\partial}{\partial t} \text{div}\bar{E} + c_0 \Delta p = 0, \quad \frac{\partial}{\partial t} \text{div}\bar{H} + c_0 \Delta q = 0 \quad (8)$$

and, expressing  $\text{div}\bar{E}$  and  $\text{div}\bar{H}$  through system (4) equations come to D'Alembert equations for scalar fields  $p$  and  $q$

$$\frac{\partial^2 p}{\partial t^2} - c_0^2 \Delta p = 0, \quad \frac{\partial^2 q}{\partial t^2} - c_0^2 \Delta q = 0, \quad (9)$$

where  $\Delta$  means Laplace operator  $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ .

Using operation *div* to expressions (6), come to correlations

$$\text{div}\bar{E} = \Delta\varphi, \quad \text{div}\bar{H} = \Delta\psi \quad (10)$$

and exclusive of derivatives of values  $p$  and  $q$  from (4), (5) with provision for (10) get

$$\Delta\left(\frac{\partial^2 \varphi}{\partial t^2} - c_0^2 \Delta\varphi\right) = 0, \quad \Delta\left(\frac{\partial^2 \psi}{\partial t^2} - c_0^2 \Delta\psi\right) = 0,$$

which are automatically satisfied, if  $\varphi$  and  $\psi$  meet D'Alembert equations

$$\frac{\partial^2 \varphi}{\partial t^2} - c_0^2 \Delta\varphi = 0, \quad \frac{\partial^2 \psi}{\partial t^2} - c_0^2 \Delta\psi = 0 \quad (11)$$

The executed transformations and equations (9) and (11) demonstrate that scalar fields  $p$  and  $q$  and scalar potentials  $\varphi$  and  $\psi$  of electric and magnetic fields have a characteristic velocity of spreading longitudinal vacuum-compression waves  $c_0$ .



Let's analyze spreading velocity of vector potentials  $\bar{A}$  and  $\bar{B}$  of electric and magnetic fields. Using operation *rot* to system (5) first equation and noticing that due to (6)

$$\text{rot}\bar{E} = -\Delta\bar{A}, \quad \text{rot}\bar{H} = -\Delta\bar{B},$$

we come to equations

$$\Delta\left(\frac{\partial^2\bar{A}}{\partial t^2} - c_1^2\Delta\bar{A}\right) = 0, \quad \Delta\left(\frac{\partial^2\bar{B}}{\partial t^2} - c_1^2\Delta\bar{B}\right) = 0,$$

which are also automatically satisfied with solutions of D'Alembert equations

$$\frac{\partial^2\bar{A}}{\partial t^2} - c_1^2\Delta\bar{A} = 0, \quad \frac{\partial^2\bar{B}}{\partial t^2} - c_1^2\Delta\bar{B} = 0 \quad (12)$$

From (12) follows the conclusion that vector solenoidal (vortex) potentials  $\bar{A}$  and  $\bar{B}$  of electric and magnetic fields spread at shift waves speed  $c_1$ .

In conclusion of this section we once again underline that we recorded an extended linear system of Maxwell electrodynamics equations, which at existence of material elastic carrier (DM, classical ether, or "mass" photon gas) aside from traditional transverse electromagnetic waves includes also description of longitudinal waves (rare-compression waves) at speed  $c_0$ . Moreover, as already noted, this extended system is reduced to a traditional system of Maxwell electrodynamics equations under negligible quantity of longitudinal components of electromagnetic field.

## 5. Particularities of spreading interactions

For our further interpretation we remind of important issue of spreading interactions and rather weak perturbations for well known three cases: acoustic (gasdynamic one), electromagnetic waves propagation, and perturbations propagation when describing hydrodynamic quasi-neutral plasma under simultaneous action of gasdynamic and electrodynamic interactions [22-28].

In the first case we have acoustic longitudinal waves (rare-compression) [5,19], which at absence of dispersion are described by D'Alembert equation (11) with constant velocity  $c_0$ . Here, the main role is played by gasdynamic interaction defined by disturbed pressure gradients ( $\nabla p$ ). When spreading classic transverse electromagnetic waves in the second case of electromagnetic interactions we have also description by means of D'Alembert equations for electric and magnetic fields intensity and scalar and vector potentials (see previous section).

An important feature of the D'Alembert wave equation is invariance for Lorentz transform. However, in gasdynamics the wave equation also is broadly used for getting solutions in a coordinate system moving at constant speed  $U$  [5,19] (including study of stationary solutions). We bring here a stationary form of transonic equations for velocity potential [29] in coordinate system moving with constant velocity  $U$  (i.e. when using Galilean transformation for D'Alembert equation)

$$(1 - M^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0. \quad (13)$$

Here -  $M = U/c$  Mach number. Under  $M > 1$  equation (13) has a hyperbolic type, under  $M \leq 1$  - elliptical type. Thereby, in gasdynamics D'Alembert equation is used and converted both with the help of Lorentz transform (then we come to Prandtl-Glauert and Ackerett singularity rule [5,26,30]) and Galilean transformation [19].

Let's consider in detail the third case on an example of hydrodynamic approximation of quasi-neutral plasma at presence simultaneously of gasdynamic and electrodynamic interactions. The following detailed analysis is a key momentum of our article and is brought for the reason of illustrative demonstration of influence on spreading nature of nonlinear interaction effects and own electrogasdynamic fields.

### Statement of nonlinear task of plasmodynamic interaction

In this section are analyzed exact solutions demonstrating a rich palette in the form of periodic (cnoidal) and isolated waves (solitons). Considered is a hydrodynamic "twin-fluid" model of cold plasma in absence of external fields. It is expected that ion and electronic plasma components weakly interact with each other and in general have different temperatures  $T_i$  and  $T_e$  (i.e. plasma is rather rarefied).

Main equations of plasma ion and electronic components motion in this approach take the form [22-28]

$$\frac{\partial n_{i,e}}{\partial t} + \text{div}(n_{i,e}\bar{v}_{i,e}) = 0, \quad (14)$$



$$\frac{\partial \bar{v}_{i,e}}{\partial t} + (\bar{v}_{i,e} \cdot \nabla) \bar{v}_{i,e} = -\frac{1}{n_{i,e} m_{i,e}} \nabla p_{i,e} - \frac{e_{i,e}}{m_{i,e}} \nabla \Phi, \tag{15}$$

$$\Delta \Phi = -4\pi e(n_i - n_e). \tag{16}$$

In (14) - (16) indexes  $i, e$  indicate a sort of particles (ions, electrons, moreover  $e_i = e = -e_e$ ),  $n$  - particles concentration,  $p$  - pressure,  $\bar{v}$  - velocity vector,  $\Phi$  - electric field potential. The equations provided (14) and (15) are invariant for the Galilean transformation groups [19].

Consider in detail the case of rather slow (ion) plasma fluctuations with negligible small time to electron relaxations. In this case electrons concentration is described with good approximation by Boltzmann distribution. Under  $T_e = \text{const}$  and  $p_e = n_e T_e$  we have

$$n_e = n_0 \exp(e\Phi / T_e) \tag{17}$$

In (17) value  $n_0$  presents itself unperturbed concentration of charged particles (unperturbed plasma is considered electrically neutral).

Providing suggestions made, the ion gas movement equation is drawn like

$$\begin{aligned} \frac{\partial n_i}{\partial t} + \text{div}(n_i \bar{v}_i) &= 0, \\ \frac{\partial \bar{v}_i}{\partial t} + (\bar{v}_i \cdot \nabla) \bar{v}_i &= -\frac{1}{n_i m_i} \nabla p_i - \frac{e}{m_i} \nabla \Phi, \\ \Delta \Phi &= 4\pi e(n_0 \exp(e\Phi / T_e) - n_i). \end{aligned} \tag{18}$$

Equations system (18) is closed by the ion gas state equation. For isothermic motion we have

$$p_i = n_i T_i, \tag{19}$$

As a matter of convenience we enter for further interpretation the following positive parameters

$$\begin{aligned} D^2 &= T_e / 4\pi n_0 e^2, \quad c_i^2 = T_i / m_i, \quad c_e^2 = T_e / m_e, \\ c_0^2 &= T_e / m_i = m_e c_e^2 / m_i. \end{aligned} \tag{20}$$

Here  $D$  – Debye radius,  $c_i$  and  $c_0$  - characteristic velocities calculated by ion and electron temperature.

Using (19) and (20), system (14)-(16) takes the shape (index  $i$  here and hereinafter we omit)

$$\begin{aligned} \frac{\partial n}{\partial t} + \text{div}(n \bar{v}) &= 0, \\ \frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \bar{v} &= -\frac{c_i^2}{n} \nabla n - c_0^2 \nabla \Phi, \\ D^2 \Delta \Phi &= \exp(\Phi) - n. \end{aligned} \tag{21}$$

In (21) and below all the values are considered non-dimensional, moreover ions concentration is referred to its unperturbed value  $n_0$ , potential - to value  $\Phi_0 = T_e / e$ , velocity and coordinate - to their characteristic values  $v_0$  and  $l_0$ , time - to  $t = l_0 / v_0$ .

Let's exclude from two last equations (21) electric field potential value  $\Phi$  and obtain a quasilinear system of third order differential equations [22-26]



$$\begin{aligned} \frac{\partial n}{\partial t} + \frac{\partial nu}{\partial x} &= 0, \\ \frac{\partial nu}{\partial t} + \frac{\partial nu^2}{\partial x} &= -(c_i^2 + c_0^2) \frac{\partial n}{\partial x} + \\ &+ D^2 \frac{\partial^2}{\partial x^2} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{c_i^2}{n} \frac{\partial n}{\partial x} \right) + \\ &+ \frac{D^2}{2c_0^2} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{c_i^2}{n} \frac{\partial n}{\partial x} \right)^2, \end{aligned} \tag{22}$$

where  $u$  - ion gas velocity projection on axis  $x$ ,  $c^2 = c_i^2 + c_0^2$  - square of so named ion sound velocity. This total value  $c$  defines the perturbations propagation velocity in our problem.

### 6. Qualitative analysis of exact solutions

Let equations system (22) possesses stationary solutions in coordinate system  $(X, t)$ , where  $t$  - former,  $X = x - Ut$ , and  $U$  - constant motion velocity of the new coordinate system relatively the origin one. (Galilean transformation). After turning to variable  $(X, t)$  and single integration on  $X$  from (22) we get

$$\begin{aligned} n(u - U) &= C_1, \\ n(u - U)^2 &= -c^2 n + D^2 \frac{d}{dX} \left[ (u - U) \frac{du}{dX} + \frac{c_i^2}{n} \frac{dn}{dx} \right] + \\ &+ \frac{D^2}{2c_0^2} \left( (u - U) \frac{du}{dX} + \frac{c_i^2}{n} \frac{dn}{dx} \right)^2 + C_2. \end{aligned} \tag{23}$$

Suppose that at  $X \rightarrow \infty$  value  $n$  strives to its unperturbed value 1, and velocity  $u$  goes to zero along with its own derivatives. Then  $C_1 = -U$  and  $C_2 = U^2 + c^2$ . From system (23),  $n$  exclusive, we find a common second order differential equation

$$\begin{aligned} D^2 \frac{\left( -U^2 - c_i^2 \frac{d^2 u}{dX^2} - \frac{u \left( U^2 + U(u - U) \right)}{u - U} \right)}{u - U} &= \\ - \frac{D^2}{\left( -U^2 - c_i^2 \right)} \left( \left( -U^2 + c_i^2 + \frac{\left( -U^2 - c_i^2 \right)^2}{2c_0^2} \right) \left( \frac{du}{dX} \right)^2 \right), \end{aligned} \tag{24}$$

describing ion gas velocity variation.

Carrying in as independent variables  $v = u - U$  and dependent variables  $p = dv/dX$ , we get a single first order equation

$$\frac{dp}{dv} = - \frac{(c^2 + Uv)(v + U)v + D^2 p^2 (v^2 + c_i^2 + (v^2 - c_i^2)^2 / 2c_0^2)}{D^2 (v^2 - c_i^2) pv}, \tag{25}$$

from which one can easily obtain a linear equation for function  $w(v) = D^2 p^2(v)$ .

The picture of equation (25) integral curves on phase plane  $(v, p)$  is symmetrical relative to horizontal axis  $p = 0$  and contains on this axis three special points with coordinates

$$v_1 = 0, \quad p_1 = 0; \tag{26}$$

$$v_2 = -U, \quad p_2 = 0; \tag{27}$$



$$v_3 = -\frac{c^2}{U}, \quad p_3 = 0. \tag{28}$$

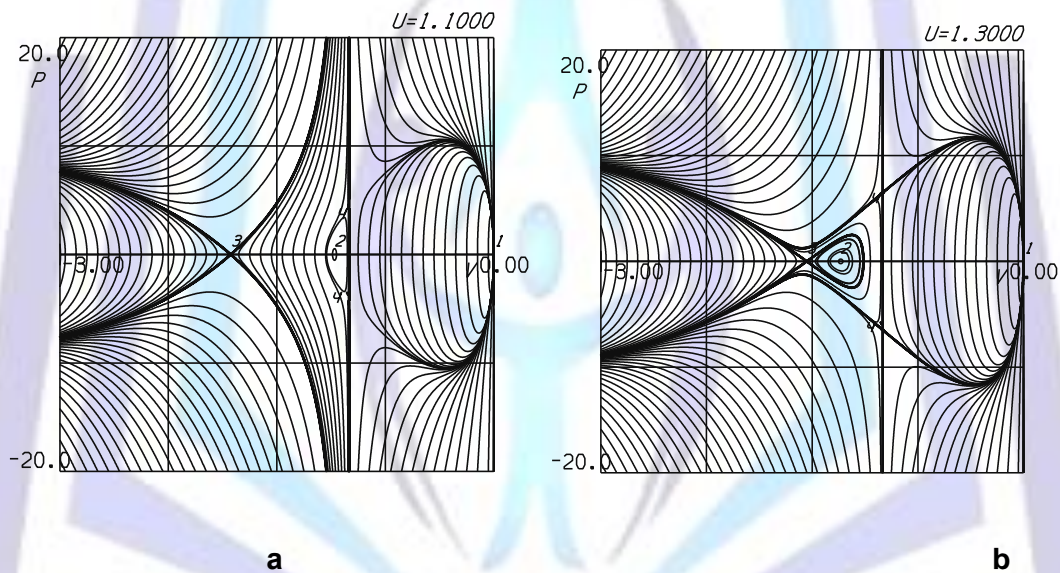
Analysis shows that the first special point - degenerate node. Integral curves in this point touch the vertical axis at  $v \leq 0$ . The second special point - a centre under and saddle under or. The third special point - a centre under  $c_i^2 < U^2 < c^2$  and saddle under  $U^2 < c_i^2$  or  $U > \tilde{n}^2 / c_i$ .

On the phase plane can exist else two special points with coordinates

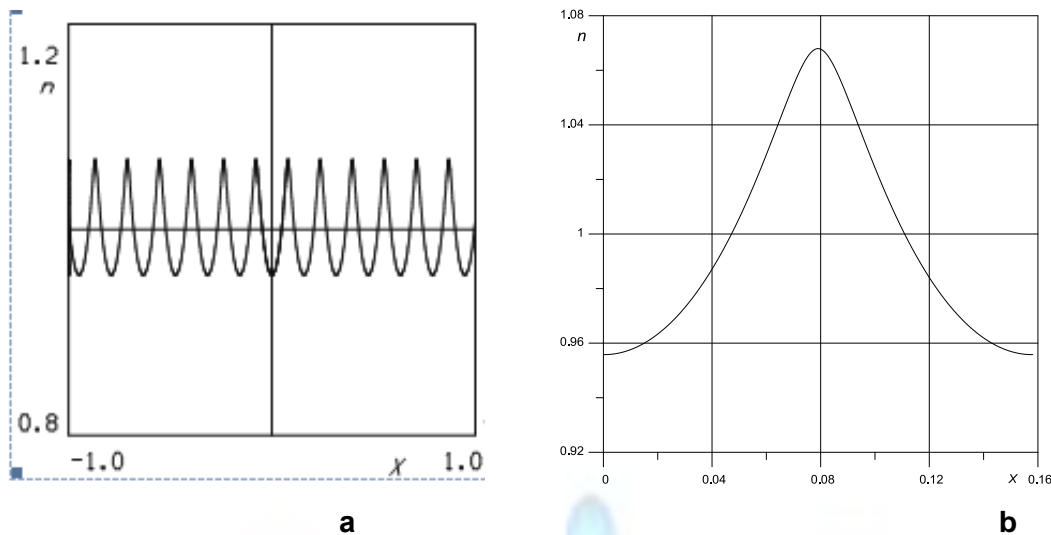
$$v_{4,4'} = -c_i, \quad p_{4,4'} = \pm \left( \frac{(\tilde{n}^2 - Uc_i)(U - c_i)}{2D^2c_i} \right)^{1/2}. \tag{29}$$

Under  $c_i < U < \tilde{n}^2 / c_i$  these are special saddle points. In rest cases they have imaginary ordinate values. Besides, equation (25) has special points with abscissa  $v = c_i$  and imaginary ordinate, which we shall not consider.

Typical examples of integral curves phase portraits are presented in Fig.1 for the supercritical case ( $U > c_i$ ), at  $U=1.1$  and  $U=1.3$ . ( $U > c_i$ ), under  $U=1.1$  and  $U=1.3$ . The special points (26)-(29) are shown by points with corresponding numbers.



**Fig.1. Phase portraits of exact solutions in supercritical velocity range at values  $U=1.1$  (a) and  $U=1.3$  (b).**



**Fig. 2. Periodic wave: concentrations variation in wave (a) and individual wave (b).**

Consider some interesting for us solutions of system (23) - the type of periodic waves and solitary waves – solitons spreading with supercritical ( $U > c_i$ ) velocity. The periodic wave deals with motion along closed integral curve around special point 2 (the type of centre). A solution example in the form of periodic wave is brought in Fig.2. It shows wave concentration variations (Fig.2, a) and concentrations separate wave (Fig.2, b). The separatrix coming from special point 3 (Fig.1, b) represents the solution in the manner of solitons (soliton's shape is similar submitted in Fig.2,b). Coordinate  $X$  variation from  $-\infty$  to  $+\infty$  corresponds to motion on separatrix around special point 2 (the type of centre). Cnoidal waves (Fig.1, b) are appropriate with movement on closed integral curve around special point 2 in internal area limited by separatrix (special point 3).

The qualitative analysis executed shows that demonstrated solutions can model continues spectrums (in the form of periodic and cnoidal waves) and discrete spectrums (as solitons). Herewith the considered case of nonlinear modeling under simultaneous presence of power fields of gasdynamic and electrodynamic nature greatly increases the classes of realized solutions. The next step of this study will be presentation of qualitative analysis for longitudinal components of light range waves.

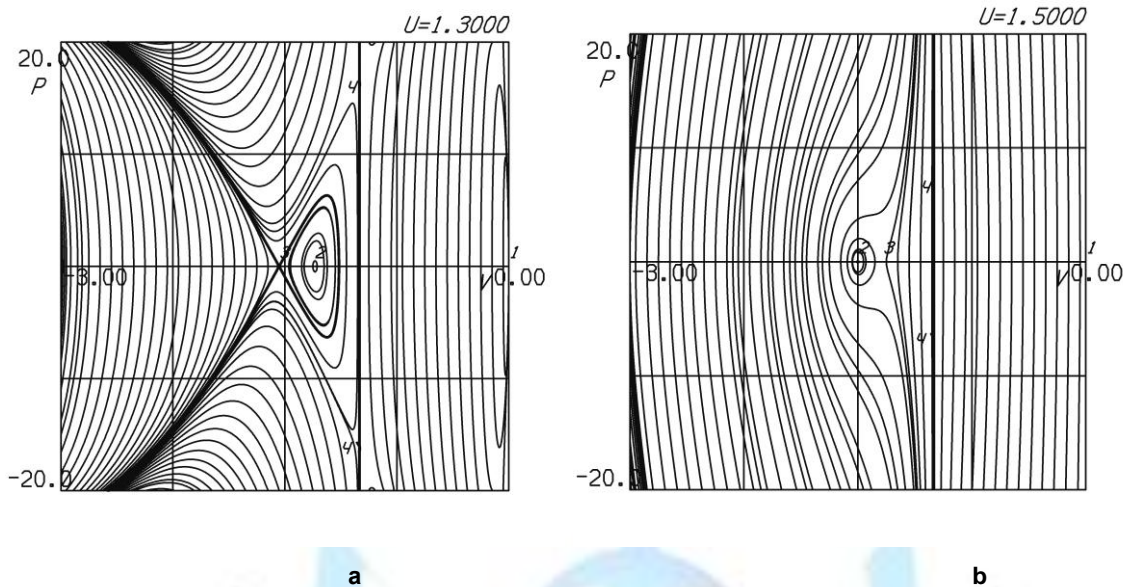
## 7. Qualitative analysis of exact solutions for light waves longitudinal components

First of all notice the main admissions, under which we shall conduct this analysis, similar to the analysis of previous section of the article. Physical vacuum is considered at CMBR temperature  $T=2.725$  K. Coming from seminal ideas by M. Planck, A. Einstein, and Louis de Broglie with the use of correlation (2), we consider the HMB gaseous medium (section 1). The HMB structure, due to the physical vacuum polarization phenomena, is postulated in the form of classic dipole (section 2). Thereby, the considered model of free space (physical vacuum) presents itself a quasi-neutral medium, for which one may apply twin-fluid hydrodynamic description (14) - (15), (19), completely repeating the known hydrodynamic description of quasi-neutral plasma [22-28]. However, a principle moment stays the equation for electric potential  $\varphi$  (16), which should be modified so that it became an effect of the D'Alembert equation when turning in a coordinate system moving with constant velocity  $U$ . Executing gasdynamic transformations and similar transformations obtaining correlation (13), we come to the equation

$$\left(1 - \frac{U^2}{c^2}\right) \varphi_{xx} + \varphi_{yy} + \varphi_{zz} = -4\pi e(n_i - n_e) \quad (30)$$

Equation (30) in our case replaces equation (16), and it is easy to repeat the whole analysis of previous section of the article. The given analysis complies with already executed one saving for the second special point remains a centre under  $U^2 > c^2$ .

The corresponding phase portraits of these solutions are brought in Fig. 3, a and b for  $U=1.3$  and  $U=1.5$ . The phase curves picture in Fig. 3, a is similar to the picture in Fig. 1, b. In this case are modeled also solid (in the manner of periodic waves) and linear (as isolated solitons) spectrums of spreading interactions in the considered models of physical vacuum.



**Fig. 3. Phase portraits of exact solutions for light waves longitudinal components at velocity values  $U=1.3$  (a) and  $U= 1.5$  (b).**

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