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# Cosmological Consequences with Randers Conformal of Finsler space

Arunesh Pandey & R K Mishra
Department of Mathematics, SLIET Deemed University
Longowa-148106, Sangrur Punjab, India
Email : ravkmishra@yahoo.co.in
Email : ankpandey11@rediffmail.com

## ABSTRACT

In this paper we study an anisotropic model of space – time with Finslerian metric. The observed anisotropy of the microwave background radiation is incorporated in the Finslerian metric of space time.

**Key words:** Cosmology, Finsler metric, Randers conformal metric

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## 1-INTRODUCTION:

Recently in literature we have gone one paper published in physical letter respectively, B668(2008)453 and B676(2009)173 under which the aouther has considered that in Finslerian manifold there exists a unique linear connection known as Chern connection. It is torsion firness and metric compatibility, we are not agree with the result of the paper because there are well known result published by H. Rund that in Finsler geometry there exits and infinite number of linear connection defined by the same metric structure and that the Chern and Bernwald connection are not metric compatable.

cosmology and other reference of the same kind [ 1, 2 ,3 ].We are fully agreed with the theory expressed in the above referred publication regarding as it is based on tangent bundle on space time manifold are positively with local Lorentz violations which may be related with dark energy and dark matter models with variable $\Lambda$ in cosmology. Certainly those may be one of the most recent hidden connections between Finsler geometry and cosmology. Now in these days many researchers are constructing suitable cosmological models with variable Lambada term including our own research group [29-33]

Here in this paper we will study anisotropic model of General Theory of Relativity based on the frame work of Finsler geometry.

## 2. BASIC ASSUMPTIONS & CONSIDERATION:

In 1984, Shibata [7] introduce the concept of $\beta$ –change and defined as

$$\bar{L}(x, y) = L(x, y) + \beta(x, y)$$

Where, $\beta$ is a one-form metric and L is any Finsler metric, and obtained very interesting result in his paper. The conformal theory of Finsler spaces has been initiated by M. S. Knebelman [8] in 1929 and has been investigated in detail by many authors [9, 10, 11, 12, 13] etc. This change is defined as

$$L^{*} = e^{\sigma(x)} L(x,\ y)$$

Where $\sigma(x)$ is a function of position only and known as conformal factor.

## 3-Generalization of result with different assumption:

In the present paper we generalize the above changes and defined a Randers conformal metric as

$$L = e^{\sigma(x)}\alpha + \beta \tag{1}$$

Where, $\alpha = \sqrt{a_{ij}(x^i)y^i y^j}$ is a Riemannian metric, $\beta = b_i(x)y^i$ is a one-form and $\sigma(x)$ is the conformal factor. If $\sigma(x) = 0$, then this change is reduces to simple Randers metric and for this metric Stavrions and Diakogiannis [16] obtained the relationship between the anisotropic cosmological models of space time and Randers Finslerian metric. The purpose of the present study is to obtain the relationship between the anisotropic cosmological models of space time with above generalized Finslerian metric.

Let us consider an n- dimensional Finsler space ($M^n$, L) and an adaptable 1-form $\beta = b_i(x)dx^i$ on $M^n$. We shall use a Lagrangian function on $M^n$, given by the equation:

$$L = e^{\sigma(x)}\alpha + \varphi(x)\hat{b}_i y^i \tag{3}$$

Where $b_i(x) = \varphi(x)\hat{b}_i$ The vector $\hat{b}_i$ represents the observed anisotropic of the microwave background radiation.

A coordinate transformation on the total space TM is given by

$$\bar{x}^i = \bar{x}^i(x^0, \ldots\ldots x^3) \qquad \det\left\|\frac{\partial \bar{x}^i}{\partial x^j}\right\| \neq 0 \qquad \bar{y}^i = \frac{\partial \bar{x}^i}{\partial x^j} y^j \qquad y^j = \delta_i^j y^i \tag{4}$$

By definition [4, 15] a Finsler metric on M is a function $F : TM \to R$ having the properties:

1. The restriction of F to TM is positively homogeneous of degree 1 with respect to {$y^a$}.

$$F(x, ky) = kF(x, y), k \in R_+^{*}$$

2. The quadratic form on $R^n$ with the coefficients $g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$ (5)

Defined on TM, is non degenerate $\det\left\|\frac{\partial \bar{x}^i}{\partial x^j}\right\| \neq 0$, with rank $g_{ij} = 4$.

The Carton torsion coefficients $C_{ijk}$ are given by

$$C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij} \tag{6}$$

The torsions and curvatures which we use are given by [4, 5, 15]:

$$P_{ijk} = C_{ijk|l}\, y^l \tag{7}$$

$$S_{jikh} = C_{iks}C^s_{jh} - C_{ihs}C^s_{jk} \tag{8}$$

$$P_{ihkj} = C_{ijk|h} - C_{hjk|i} + C^r_{hj}C_{rik|l}\, y^l - C^r_{ij}C_{rkh|l}\, y^l \tag{9}$$

$$S^l_{ikh} = g^{lj} S_{jikh} \tag{10}$$

$$P^l_{ikh} = g^{lj} P_{jikh} \tag{11}$$

Differentiating equation (3) with respect to $y^i$, the normalized supporting elements $l_i = \dot{\partial}_i L$ is given by

$$l_i = \dot{\partial}_i L = e^{\sigma(x)}\left(\dot{\partial}_i \alpha + \varphi(x)\, b_i\right) \tag{12}$$

Where

$$\dot{\partial}_i \alpha = \dot{\partial}_i \sqrt{a_{ij} y^i y^j}$$

$$\dot{\partial}_i \alpha = \frac{1}{2\alpha}\left[a_{ij}(x)\, y^j + a_{ij}(x)\, y^j\right]$$

$$\dot{\partial}_i \alpha = \frac{2a_{ij} y^j}{2\alpha} = \frac{y_i}{\alpha} \quad \text{as} \quad y_i = a_{ij} y^j$$

$$\dot{\partial}_i \alpha = \frac{y_i}{\alpha} \tag{13}$$

From equation (12) and (13), we have

$$l_i = \dot{\partial}_i L = e^{\sigma(x)}\left(\frac{y_i}{\alpha} + \varphi(x)\, b_i\right) \tag{14}$$

Where $l_i = \dot{\partial}_i L$ is the normalized supporting element in the Finsler space $F^n$.

Differenting equation (14) with respect to $y^j$, the normalized supporting element

$$h_{ij} = L\dot{\partial}_i \dot{\partial}_j L = \frac{Le^{\sigma(x)}}{\alpha}\left[a_{ij} - \frac{y_i y_j}{\alpha^2}\right] \tag{15}$$

$$\dot{\partial}_i \dot{\partial}_j L = \frac{e^{\sigma(x)}}{\alpha}\left[a_{ij} - \frac{y_i y_j}{\alpha^2}\right] \tag{16}$$

Where $h_{ij} = L\dot{\partial}_i \dot{\partial}_j L$ is the angular metric tensor in the Finsler Space $F^n$.

**3- Anisotropic cosmological model with Randers conformal of Finsler space:**

The Lagrangian function on $M^n$, given by the equation:

$$L = e^{\sigma(x)}\left(\alpha + \varphi(x)\hat{b}_i y^i\right)$$

$$\alpha = \sqrt{a_{ij}(x^i)y^i y^j}$$

Where $a_{ij}$ is the Riemannian metric with signature (-, +, +. +).Because of the anisotropy, we must insert an additional term to the Riemannian line element α. This term fulfill the following requirements:

(i) It must give absolute maximum contribution for direction of movement parallel to the anisotropy axis.

(ii) The new line element must coincide with the Riemannian one for the direction vertical to the anisotropic axis.

(iii) It must not symmetric with respect to replacement $y^i \to -y^i$.

We see that a term which satisfies the above conditions is $\beta = b_i(x)y^i$

Where $b_i(x)$ expresses this anisotropic axis.

Let $b_i(x) = \varphi(x)\hat{b}_i$ , where $\hat{b}_i$ the unit vector in the direction is $b_i(x)$. Then $\varphi(x)$ plays the role of "length" of the vector $b_i(x)$, $\varphi(x) \in R$. β is the Finslerian line element and α is Riemannian one.

In order for the Finslerian metric to be physically consistent with General theory of relativity, it must have the same signature with the Riemannian metric (-, +, +, +). We have

$$\alpha = cd\tau = c\mu dt = \mu d(ct) = \mu dx^0 \tag{17}$$

Where $\mu = \sqrt{1 - \frac{v^2}{c^2}}$ and v: 3- velocity in Riemannian space- time. One possible explanation of the anisotropy axis could be that it represents the resultant of spin densities of the angular momenta of galaxies in a restricted area of space ( $b_i(x)$ space like). It is known that the mass is anisotropically distributed for regions of space with radius ≤ $10^8$ light years [4].

The Finsler metric tensor $g_{ij}$ is

$$g_{ij} = h_{ij} + l_i l_j$$

$$g_{ij} = \frac{Le^{\sigma(x)}}{\alpha}\left[a_{ij} - \frac{y_i y_j}{\alpha^2}\right] + \left[\frac{e^{\sigma(x)}y_i}{\alpha} + \varphi(x)\hat{b}_i\right]\left[\frac{e^{\sigma(x)}y_j}{\alpha} + \varphi(x)\hat{b}_j\right]$$

$$g_{ij} = \frac{Le^{\sigma(x)}}{\alpha}a_{ij} + \left(\varphi(x)\right)^2\hat{b}_i\hat{b}_j + \frac{e^{\sigma(x)}\varphi(x)}{\alpha}\left(\hat{b}_i y_j + \hat{b}_j y_i\right) - \frac{\beta e^{\sigma(x)}}{\alpha^3}y_i y_j$$

$$g_{ij} = \gamma a_{ij} + \{\varphi(x)\}^2 \hat{b}_i \hat{b}_j + \frac{e^{\sigma(x)}\varphi(x)}{\alpha}(\hat{b}_i y_j + \hat{b}_j y_i) - \frac{\beta e^{\sigma(x)}}{\alpha^3} y_i y_j \tag{18}$$

Where $$\gamma = \frac{Le^{\sigma(x)}}{\alpha} \tag{19}$$

Where we put $y_i = a_{ij} y^j$ and $a_{ij}$ is the fundamental tensor for the Finsler space $F^n$. It will be easy to see that the det. $\|g_{ij}\|$ does not vanish, and the reciprocal tensor with components $g^{ij}$ is given by

$$g^{ij} = \gamma^{-1} a^{ij} + y^i y^j - \alpha^{-1}\gamma^{-2}(y^i \hat{b}^j + y^j \hat{b}^i) \tag{20}$$

$$\varphi = e^{-2\sigma(x)}(\alpha e^{\sigma(x)} b^2 + \beta L)^{-3} \tag{21}$$

Where $g^{ij}$ is the reciprocal tensor of $g_{ij}$ and $b^2 = \hat{b}_i \hat{b}^i$, $\hat{b}^i = a^{ij}\hat{b}_j$ And $a^{ij}$ is the inverse matrix of $a_{ij}$. As it may be verified by direct calculation, where $m = \hat{b}_i \hat{b}^i = 0, \pm 1$ according whether $\hat{b}_i$ is null, space like or time like. It must be noted, however, that if $y^i$ represents the velocity of a particle (time like) then $\hat{b}^i$ is bound to be space like. this follows from the fact that one possible value of $\hat{b}_i y^i$ is zero.

The Carton covariant tensor C with the components $C_{ijk}$ is obtained as follows:

$$C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij}$$

$$C_{ijk} = \frac{1}{2}\left[\begin{array}{l} \dot{\partial}_k \left\{\frac{Le^{\sigma(x)}}{\alpha}\right\} a_{ij} + \dot{\partial}_k \left\{\frac{e^{\sigma(x)}\varphi(x)}{\alpha}\right\}(\hat{b}_i y_j + y_i \hat{b}_j) \\ + \frac{e^{\sigma(x)}\varphi(x)}{\alpha}(\hat{b}_i \dot{\partial}_k y_j + \dot{\partial}_k y_i \hat{b}_j) - \dot{\partial}_k\left(\frac{\beta e^{\sigma(x)}}{\alpha^3}\right) y_i y_j \\ - \frac{\beta e^{\sigma(x)}}{\alpha^3}\left((\dot{\partial}_k y_i) y_j + (\dot{\partial}_k y_j) y_i\right) \end{array}\right]$$

$$C_{ijk} = \frac{\gamma}{2L}(h_{ij} m_k + h_{ji} m_i + h_{ki} m_j) \tag{22}$$

Where $$m_i = \hat{b}_i - \beta\alpha^{-2} y_i \tag{23}$$

The Covariant indices j is replaced by k and k by s, we have

$$C_{iks} = \frac{\gamma}{2L}(h_{ik} m_s + h_{ks} m_i + h_{si} m_k) \tag{24}$$

$$C_{ijk} g^{jh} = C^h_{ik} = \frac{1}{2L}(h^h_i m_k + h^h_k m_i + h_{ik} m^h) - \frac{\gamma^{-1}}{2\alpha L}(2 m_i m_k + m^2 h_{ik}) y^h \tag{25}$$

Where $$m^2 = m_i m^i \tag{26}$$

Replacing the covariant indices h by s, i by j and k by h in equation (25), we have

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$$C^{s}_{jh} = \frac{1}{2L}\left(h^{s}_{j} m_h + h^{s}_{h} m_j + h_{jh} m^{s}\right) + \frac{\gamma^{-1}}{2\alpha L}\left(2 m_j m_h + m^2 h_{jh}\right) y^{s} \tag{27}$$

Now,

$$S_{jikh} = C_{iks} C^{s}_{jh} - C_{ihs} C^{s}_{jk} \tag{28}$$

$$C_{iks} C^{s}_{jh} = \frac{\gamma}{2L}\left(h_{ik} m_s + h_{ks} m_i + h_{si} m_k\right)$$

$$\left[\frac{1}{2L}\left(h^{s}_{j} m_h + h^{s}_{h} m_j + h_{jh} m^{s}\right) + \frac{\gamma^{-1}}{2\alpha L}\left(2 m_j m_h + m^2 h_{jh}\right) y^{s}\right]$$

$$C_{iks} C^{s}_{jh} = \frac{\gamma}{4L^2}\left\{\begin{aligned} &2\gamma h_{ik} m_h m_j + m^2 h_{ik} h_{jh} + h_{kj} m_i m_h + h_{kh} m_i m_j \\ &+ h_{ij} m_k m_h + h_{ih} m_k m_j + 2\gamma h_{jh} m_i m_k \end{aligned}\right\} \tag{29}$$

Replacing k by h and h by k in equation (29), we get

$$C_{ihs} C^{s}_{jk} = \frac{\gamma}{4L^2}\left\{\begin{aligned} &2\gamma h_{ih} m_k m_j + m^2 h_{ih} h_{jk} + h_{hj} m_i m_h + h_{kh} m_i m_j \\ &+ h_{ij} m_k m_h + h_{ik} m_h m_j + 2\gamma h_{jk} m_i m_h \end{aligned}\right\} \tag{30}$$

Thus, equation (28), Equation (29) and equation (30) yields

$$S_{jikh} = \frac{\gamma}{4L^2}\left[\begin{aligned} &m^2\left(h_{ik} h_{jh} - h_{ih} h_{jk}\right) + 2\gamma m_j\left(h_{ik} m_h - h_{ih} m_k\right) + m_i\left(h_{kj} m_h - h_{hj} m_k\right) \\ &+ m_j\left(h_{ih} m_k - h_{ik} m_h\right) + 2\gamma m_i\left(h_{jh} m_k - h_{jk} m_h\right) \end{aligned}\right]$$

$$S_{jikh} = \frac{\gamma}{4L^2}\left[m^2\left(h_{ik} h_{jh} - h_{ih} h_{jk}\right) + (2\gamma - 1) m_j m_h h_{ik} - (2\gamma - 1) m_j m_k h_{ih} + (2\gamma - 1) m_k m_i h_{jh} + (2\gamma - 1) m_i m_h h_{jk}\right]$$

$$S_{jikh} = \frac{\gamma}{4L^2}\left[m^2\left(h_{ik} h_{jh} - h_{ih} h_{jk}\right) + (2\gamma - 1) m_j\left(h_{ik} m_h - h_{ih} m_k\right) + (2\gamma - 1) m_i\left(h_{jh} m_k - h_{jk} m_h\right)\right] \tag{31}$$

Thus, S- curvature represents the measure of anisotropy of matter.

From equation (9), we have

$$P_{ihkj} = C_{ijk|h} - C_{hjk|i} + C^{r}_{hj} C_{rik|0} - C^{r}_{ij} C_{rkh|0} \tag{32}$$

Differenting equation (22) with respect to covariant h, we get

$$C_{ijk/h} = \left(\frac{\gamma}{2L}\right)_{/h}\left(h_{ij} m_k + h_{jk} m_i + h_{ki} m_j\right) + \frac{\gamma}{2L}\left\{\begin{aligned} &h_{ij|h} m_k + h_{ij} m_{k|h} + h_{jk|h} m_i \\ &+ h_{jk} m_{i|h} + h_{ki} m_{j|h} \end{aligned}\right\}$$

$$h_{ij|h} = 0 \qquad L_{|h} = 0 \qquad l_{|h} = 0 \qquad \alpha_{|h} = 0$$

Since

$$C_{ijk|h} = \frac{1}{2}\frac{e^{\sigma}_{|h}}{\alpha}\left(h_{ij} m_k + h_{jk} m_i + h_{ki} m_j\right) + \frac{\gamma}{2L}\left(h_{ij} m_{k|h} + h_{jk} m_{i|h} + h_{ki} m_{j|h}\right) \tag{33}$$

Where $\left(\frac{\gamma}{2L}\right)_{|h} = \left\{\frac{Le^{\sigma}}{2L\sigma}\right\}_{|h} = \left\{\frac{e^{\sigma}}{2\sigma}\right\}_{|h} = \frac{1}{2}\left[\frac{\alpha e^{\sigma}_{|h} - e^{\sigma}_{|h}\alpha_{/h}}{\alpha^2}\right]$

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$$\left(\frac{\gamma}{2L}\right)_{|h} = \frac{e^{\sigma}_{|h}}{\alpha} \quad , \qquad \because \alpha_{|h} = 0$$

Replacing i by h and h by i in equation (33), we get

$$C_{hjk/i} = \frac{1}{2}\frac{e^{\sigma}_{|h}}{\alpha}\left(h_{hj}m_k + h_{jk}m_h + h_{kh}m_j\right) + \frac{\gamma}{2L}\left(h_{hj}m_{k|i} + h_{jk}m_{h|i} + h_{kh}m_{j|i}\right) \tag{34}$$

Equation (33) and Equation (34) gives

$$C_{ijk|h} - C_{hjk|i} = \theta_{(h)}\left\{\left[\frac{1}{2}\frac{e^{\sigma}_{|h}}{\sigma}\left(h_{ij}m_k + h_{jk}m_i + h_{ki}m_j\right) + \frac{\gamma}{2L}\left(h_{ij}m_{k|h} + h_{jk}m_{i|h} + h_{ki}m_{j|h}\right)\right]\right\} \tag{35}$$

Where $\theta_{(h)}$ is interchanging the indices and substitution.

From equation (33), we have

$$C_{ijk|0} = C_{ijk|h}y^h = \frac{1}{2}\frac{e^{\sigma}_{|0}}{\alpha}\left(h_{ij}m_k + h_{jk}m_i + h_{ki}m_j\right) + \frac{\gamma}{2L}\left(h_{ij}m_{k|0} + h_{jk}m_{i|0} + h_{ki}m_{j|0}\right) \tag{36}$$

With the help of equation (27) and equation (36), we write

$$C^r_{hj}C_{rik|0} = \left[\frac{1}{2L}\left(h^r_j m_h + h^r_h m_j + h_{jh}m^r\right) + \frac{\gamma^{-1}}{2\alpha L}\left(2m_j m_h + m^2 h_{jh}\right)y^r\right].$$

$$\left[\frac{1}{2}\frac{e^{\sigma}_{|0}}{\alpha}\left(h_{ir}m_k + h_{rk}m_i + h_{ki}m_r\right) + \frac{\gamma}{2L}\left(h_{ir}m_{k|0} + h_{rk}m_{i|0} + h_{ki}m_{r|0}\right)\right]$$

$$C^r_{hj}C_{rik|0} = \frac{e^{\sigma}_{|0}}{4\alpha L}\left(h_{ij}m_h m_k + h_{ih}m_j m_k + 2h_{jh}m_k m_i + h_{jk}m_i m_h + h_{hk}m_i m_j + 2h_{ki}m_h m_j + m^2 h_{jh}h_{ki}\right)$$

$$+\frac{\gamma}{4L^2}\left\{\begin{array}{l} h_{ij}m_h m_{k|0} + h_{hi}m_j m_{k|0} + h_{jh}m_i m_{k|0} + h_{jk}m_h m_{i|0} + h_{hk}m_j m_{i|0} \\ + h_{jh}m_k m_{i|0} + h_{ki}m_h m_{j|0} + h_{ki}m_j m_{h|0} + h_{jh}h_{ki}\overline{m} \end{array}\right\} \tag{37}$$

Where $\overline{m} = m^r m_{r|0}$

$$C^r_{hj}C_{rik|0} - C^r_{ij}C_{rhk|0} = \frac{e^{\sigma}_{|0}}{4\alpha L}\theta_{(h)}\left(h_{ij}m_h m_k + h_{jh}m_k m_i + h_{hk}m_i m_j + h_{ki}m_h m_j + m^2 h_{jh}h_{ki}\right)$$

$$+\frac{\gamma}{4L^2}\theta_{(h)}\left(h_{jk}m_h m_{i|0} + h_{jh}m_k m_{i|0} + h_{ki}m_h m_{j|0} + h_{jh}h_{ki}\overline{m}\right)$$

$$C^r_{hj}C_{rik|0} - C^r_{ij}C_{rhk|0} = \frac{e^{\sigma}_{|0}}{4\alpha L}\theta_{(h)}\left(h_{jh}m_k m_i + h_{ki}m_h m_j + m^2 h_{jh}h_{ki}\right)$$

$$+\frac{\gamma}{4L^2}\theta_{(h)}\left(h_{jk}m_h m_{i|0} + h_{jh}m_k m_{i|0} + h_{ki}m_h m_{j|0} + h_{jh}h_{ki}\overline{m}\right) \tag{38}$$

The equation (32), equation (35) and equation (38) yields

$$P_{ihkj} = \theta(h)\left[\frac{1}{2}\frac{e^{\sigma}_{|h}}{\sigma}\left(h_{ij}m_k + h_{jk}m_i + h_{ki}m_j\right) + \frac{\gamma}{2L}\left(h_{ij}m_{k|h} + h_{jk}m_{i|h} + h_{ki}m_{j|h}\right)\right]$$

$$+ \frac{e^{\sigma}_{|0}}{4\alpha L}\theta(h)\left(h_{jh}m_k m_i + h_{ki}m_h m_j + m^2 h_{jh}h_{ki}\right) + \frac{\gamma}{4L^2}\theta(h)\left(h_{jk}m_h m_{i|0} + h_{jh}m_k m_{i|0} + h_{ki}m_h m_{j|0} + h_{jh}h_{ki}\overline{m}\right) \quad (39)$$

## 4- Concluding Remarks:

Particular attention over the last decade has been paid on the so-called Finsler-Randers cosmological model [17].In general metrical extensions of Riemann geometry can provide a Finslerian geometrical structure in a manifold which leads to generalized gravitational field theories. During the last decade there is a rapid development of applications of Finsler geometry in its FR context, mainly in the topics of general relativity, astrophysics and cosmology [18–28]. Keeping this in mind in this paper we have considered the randers conformal metric to the general relativity. From the above section it is clear that If $\sigma(x) = 0$, then this change is reduces to simple Randers metric and GTR is well connected with Randers metric. For this metric, we have also obtained the relationship between the anisotropic cosmological models of space time with above generalized Finslerian metric which is shown in the equations derived in section 3

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