ISSN 2347-3487

Formulation of Mathematical Modeling to Characterize the Aluminium Metals using Ultrasonic Non-Destructive Techniques.

A.R.Golhar[1] P.S.Sawadh[2] N.K.Choudhari [1]
[1]Bhagwati College of Engineering,
India[2]Bapurao Deshmukh college of Engineering India
E-mail: artigolhar@gmail.com, pratibha31@rediffmail.com,
drnitinchoudhari@gmail.com

Abstract

Predicting the type of aluminium metals and composition of elements present in the aluminium samples through Nondestructive testing (NDT) is a matter of very importance for aluminium Industry. The unique method to determine grade of the aluminium sample is required to characterize the aluminium metals. The Nondestructive Technique (NDT) and determination of characteristics and mechanical properties of aluminium metals are used to identify the grade of aluminium metals so that accordingly it can be used for the specific applications. Therefore a technique is required to predict the percentage of aluminium, Iron, Copper, Manganese of aluminium metals so as to categorize into different grades and applications. In Aluminium samples percentage of Aluminium plays very important role which may help to decide the grade of the aluminium metals hence its applications. The present work is focused on how the percentage of aluminium in aluminium samples can be calculated by adopting the mathematical modeling technique.

There are various parameters which generally affect the percentage of aluminium in aluminium samples, and play a very major role. Therefore through this investigation an attempt is being made to formulate an approximate mathematical model which will certainly predict the percentage of aluminium in aluminium samples. In advent of this a dimensionless pie terms of various prominent parameters or variables have been taken to form a mathematical model. Some of these variables used to formulate this model are given as follows (i) physical properties of the aluminium samples like hardness, density, modulus of elasticity etc (ii) Signal analysis properties like Peak amplitude of Time signal, FFT, PSD and (iii) both the properties. The data of such types of variables have been recorded and calculated and thus the formulation of model is being done by multiple regression analysis. The model is then optimized and the reliability of the model has also been estimated. In fact this type of model will be helpful to estimate the aluminium percentage.

Council for Innovative Research
Peer Review Research Publishing System
Journal of Advances in Physics
Vol 2, No.1
editor@cirworld.com
www.cirworld.com, member.cirworld.com

ISSN 2347-3487

INTRODUCTION

Ultrasonics techniques are the choice for inspecting the interior of material because ultrasound can penetrates deeply. For the purpose of detecting defects inside metals, it is the most practical alternative to heavy doses of x-rays or gamma rays. Ultrasonics can also be used to infer much information about the bulk microstructure of metals [1,2]and material properties[3]. Signal processing involves techniques that improve our understanding of information contained in the received ultrasonic data [4]. The formulation of mathematical model based on experimental model is a new approach to study the material. The theory of experimentation as suggested by Hilbert [5] is a good approach of representing the response of any phenomenon in terms of proper interaction of various inputs of the phenomenon. This approach finally establishes an experimental data based model for any phenomenon. As suggested in this article the experimentation has been carried out and the models are formulated. Aluminium percentage in the aluminium samples is determined to characterize the samples. This objective is only achievable by formulation of such models. Once models are formulated they are optimized using the optimization technique. The optimum conditions, which the independent variables should satisfy for maximum productivity, have been deduced.

EXPERIMENTAL APPROACH

In order to characterize the aluminium metals, different grades needs to be identified which is possible only if aluminium percentage and composition of other elements are known. It is observed that in aluminum percentage in the aluminium metals are varying with respect to other observed NDT parameters. The value of the parameters either increase or decrease with respect to other parameters. In order to develop the relationship between aluminium percentages with one of the observed NDT parameters is not possible. Hence the individual parameter is not sufficient to characterize the aluminium samples. However the combine effect of all these NDT observed parameters is utilized to characterize aluminium metals. Hence one is left with only alternative of formulating experimental data based models to be more specific field data based models. Normally the approach adopted for formulating generalized experimental model suggested by Hilbert for any complex physical phenomenon involves following steps. Hence, this aspect in general instigates to investigate a mathematical model, which can predict the percentage of aluminium. Indeed the model will be useful for researcher as well as industry to work on prominent variables by which they can estimate aluminium percentage.

AN APPROACH TO FORMULATE MATHEMATICAL MODEL:

The mathematical model can be established by an approach of experimental data based model formulation suggested by Schenck. H. Jr., in a modified form is adopted for this purpose. The modification is to the extent of covering only and it is done by

Identification of variables

Establishment of dimensionless pie terms

Formulating the model for prediction of aluminium percentage using other observed NDT parameters.

Identification of Variables:

The first step in this process is identification of variables .The parameters of the phenomenon is called variables. These variables are of three types

(1) Independent variables,

(2) Dependent variable, and

(3) Extraneous variable.

The independent variables are those which can be changed without changing other variables of the phenomenon. Whereas, the dependent variables are those, that can only change if any change in the independent variables. The extraneous variables change in a random and uncontrolled manner in the phenomenon . This correlation is nothing but a mathematical model as a designed tool for such situation and this model is used to predict the aluminium percentage in aluminium metals which may help to identify its applications. As far as this phenomenon is concerned the dependent or response variable is Percentage of aluminium in the aluminium samples while the phenomenon is influenced by following variables.

S.N	Name of Variables	Representation	Dependent/ Independent
1	Percentage of aluminium in sample	Al% or Y	Dependent
2	Hardness	H	Independent
3	Density	P	Independent
4	Velocity	V	Independent
5	Attenuation	A	Independent
6	Modulus of elasticity	MOE	Independent
7	Peak amplitude of Time signal	TS(Y)	Independent
8	Peak ampl. of FFT	FFT(Y)	Independent
9	Peak Amp. of PSD	PSD(Y)	Independent
10	Peak amplitude time of time signal	TS(x)	Independent
11	Peak amplitude frequency of FFT	FFT(x)	Independent
12	Peak amplitude frequency of PSD	PSD(x)	Independent

Table1: List of Dependent and Independent Variables and their Symbols

Reduction of Independent Variables/Dimensional Analysis

Deducing the dimensional equation for a phenomenon reduces the number of independent variables in the experiments. The exact mathematical form of this dimensional equation is the targeted model. This has been achieved by applying Buckingham's π theorem (Hibert, 1961). When this theorem has been apply to a system involving n independent variables, (n minus number of primary dimensions viz. L, M, T, Q) i.e.(n-4) numbers of π terms are formed. When n is large, even by applying this theorem number of π terms will not be reduced significantly than number of all independent variables. Thus much reduction in number of variables is not achieved. It is evident that, if we take the product of the terms it will also be dimensionless number and hence a π term. This property has been used to achieve further reduction of the number of variables. Dimensional analysis has been used to reduce the variables.These Independent variables have been reduced into a group of pi terms is the second step of this process. The Equation 1 shows the dimension less pie terms for the phenomenon.

MODEL FORMULATION BY IDENTIFYING THE CONSTANT AND VARIOUS INDICES OF PI TERMS:

The multiple regression analysis helps to identify the indices of the different pi terms in the model aimed at, by considering three independent pi terms and one dependent pi term. Let model aimed at be of the form,

$$(\mathrm{Al}\ \%) = k \times \left[\frac{\mathrm{H} \times \mathrm{MOE}}{\rho^2 \times v^4}\right]^a \left[\frac{\mathrm{PSD\ Y} \times \mathrm{FFTX}^2 \times \alpha}{\rho \times v^4}\right]^b \left[\frac{\mathrm{TSX} \times \mathrm{TSY} \times \mathrm{PSDX}}{\mathrm{FFTY}}\right]^c \quad (1)$$

$$(Y) = K^*((\pi 1)a^*(\pi 2)\ b^*(\pi 3)\ c \quad (2)$$

The regression equations become as under.

$$\sum Y = n\ K1 + a\sum A + b\sum B + c\sum C$$

$$\sum YA = K1\sum A + a\sum A2 + b\sum AB + c\sum AC$$

$$\sum YB = K1\sum B + a\sum AB + b\sum B2 + c\sum BC$$

$$\sum YC = K1\sum C + a\sum AC + b\sum BC + c\sum C2 \quad (3)$$

In the above equations n is the number of sets of readings, A,B, and C represent the independent pi terms π1,π2, and π3. While, Y represents, dependent pi term. Next, calculate the values of independent pi terms for corresponding dependent pi term, which helps to form the equations in matrix form.

[y]=[X] x[a] (4)

Solving, the above matrix by using MATLAB software, the values of constant and different indices of the proposed model have been found out

$$Y = 89.742894 * (\pi_1)^{-0.0042} * (\pi_2)^{-0.0018} * (\pi_3)^{0.0005}$$

INTERPRETATION OF MODEL:

Interpretation of model is being reported in terms of several aspects viz (1) order of influence of various inputs (causes) on outputs (effects) (2) Interpretation of curve fitting constant K (3) Sensitivity of causes (4) optimization (5) reliability

(A) Order of influence of various observed NDT parameters

The above equation is established using the various observed NDT parameters. It indicates that π_3 which is related to the signal analysis parameters of aluminium metals has highest influence as 0.0005 on the measurement of aluminium percentage in aluminium samples. The least influence is seen for $\pi_1$as -0.0042, which is related to physical properties of aluminium metals. The negative sign indicate the inverse relation. The π_2, which is related to properties and signal analysis parameters has moderate influence as -0.0018 on the measurement of aluminium percentage in the samples.

(B)Interpretation of curve fitting constant (K)

The value of curve fitting constant in this model is 89.742894. This collectively represents the combined effect of all, which are influenced on the dependent terms. Further, as it is positive, this indicates that there are good numbers of NDT parameters which have influence on estimating the aluminium percentage in the samples.

(C) Sensitivity Analysis

Sensitivity analysis is done to evaluate the sensitiveness of each pi term involved in the model. This analyze by introducing percentage change in each independent pi term, one can see the change in effect on the dependent pi term. Hence, in the present work change of ± 10% is introduced in individual pi terms independently (one at time). Therefore total rate of introduced change is 20%. After introducing the change in independent pi term, one may have obtained the change in the dependent pi term. The percentage of change in dependent pi term by introducing 20% change in each independent pi terms is shown graphically in Figure (1). The sensitivity is evaluated and discussed below.

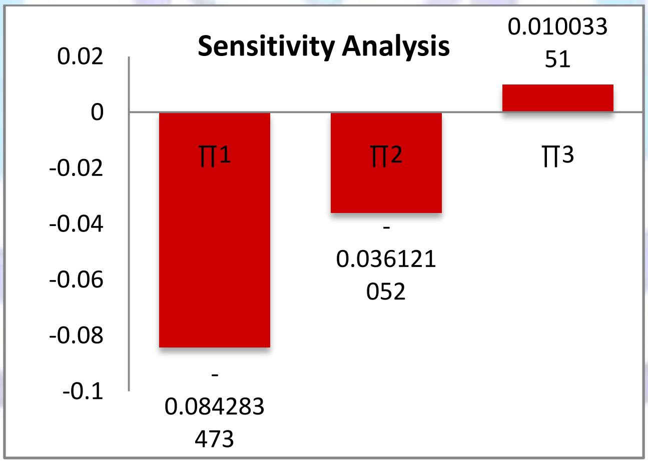

Figure (1): Sensitivity Chart of independent pi terms

(D)Effect of introduced change on the dependent pie term

In this model (Equation), when the total 20% of change is introduced in the independent term π3, then the change of 0.010033% has occurred in the value of dependent pi term (Y) (computed from model). Whereas, the least change in the value of dependent pi term (Y) is -0.08428%. This change is due to pi term π1. Similarly, the change of about -0.03612%, take place because of change in the values of π2 respectively.

It is noticed that, the highest change in the dependent pi term is due to the independent term π3. While the pi term π1 is responsible for least change in the dependent pi term. Hence, π3 is the most sensitive pi term and π1 is the least sensitive pi term. The sequence of the various pi terms in descending order of sensitivity is π3, π2, π1.

(E) Optimization of Models

The basic aim of establishing the mathematical model is not only to simulate the phenomenon but also to find out the best set of independent variables. This will result in maximization or minimization of objective function. In the present work the ultimate objective is to obtain the percentage of aluminium. In fact, the models are in non linear form. Hence, for the purpose of optimization of model, this is to be converted into linear form. This is done by taking Log on both the sides. To maximize linear function one can use linear programming technique as shown below.

For the dependent π term, we have

(π) = K*((π1) a*(π2) b*(π3) c*

Taking Log on both the side of this equation,

Log (Y) =Log K + a Log (π1)+b Log (π2) + c Log (π3)

Let, Log (Y) =Z; Log K=K; Log (π1)=X1 ; Log (π2)=X2 ; Log (π3)=X3;

Then the linear model in the form of first degree of polynomial can be written as,

Z=k+ a *X1+b*X2+c*X3

Thus the percentage of aluminium in the aluminium samples will be the objective function for the optimization with specific goal of maximization and minimization for the purpose of linear programming problem. The next step would be to apply the constraints to the problem. During observation of various NDT parameters of certain range of independent pi term are achieved. In fact this range has a minimum and maximum value. Thus, it can be said that the range is constraint for the problem. Therefore, there are two constraints for each independent variable.

Let, the maximum and minimum value of independent pi term is π1 max and π1 min, then first two constraints for the problem will be obtained by taking Log of these quantities and by substituting the value of multipliers of all other variables except the one under consideration equal to zero. Let the log limits be defined as C1 and C2 (i.e. C1= Log π1 max), (i.e. C2= Log π1 min).

Hence the equation of constraints becomes.

1*X1 + 0*X2 +0*X3 ≤ C1

1*X1 + 0*X2 +0*X3 ≥ C2

The other constraints are also found to be.

0*X1 + 1*X2 +0*X3 ≤ C3

0*X1 + 1*X2 +0*X3 ≥ C4

0*X1 + 0*X2 +1*X3 ≤ C5

0*X1 + 0*X2 +1*X3 ≥ C6

After solving, the linear programming problem, one would get the maximum value of Z and set of values of the independent variables to achieve this maximum value. The value of independent pi term and maximum value of dependent pi term can be obtained by taking antilog of Z, X1, X2, and X3. The linear programming problem can be solved with the help of MS solver, which is available in MS excel of Microsoft office computer program. By solving, the above problem with MS solver, we obtained, Z=2.000646811; X1= -11.0107; X2=-1.93779;X3=-4.17252;Hence, Zmax=(Y)max=antilog(2.000646811)=100.149 and corresponding values of independent pi terms are obtained by taking the antilog of X1,X2,and X3, These values are 9.76 x10-12, 0.01154, 6.72 x 10-05 respectively.

Similarly for minimum optimization , we get Z=1.992124; X1=-10.0149; X2=-0.05342; X3=-6.06933; Hence Zmin = (Y)min = antilog(1.992124)= 98.20283and corresponding values of independent pi terms are obtained by taking the antilog of X1,X2,and X3, These values are 9.66 x 10-11, 0.88425, 8.52 x 10-07

(F) RELIABILITY OF MODELS:

In general reliability is a term associated with the chance of failure. Hence reliability also finds value, which is used to show the performance of the model. The reliability of model is evaluated as follows.

For available mathematical model, the known value of independent variables has substituted in the mathematical model. So that, one will obtain, the required values of dependent variable, which is known as calculated value of dependent variable. Now, one can find error in the calculated value of dependent variable and observed value of dependent variable. For this, it is necessary to subtract calculated value from the observed value of dependent variable. Once the error is calculated, then one can calculate the reliability of model by calculating the mean error.

This can be done by using following formula,

Reliability= 1- Mean error

Where, Mean error= ΣXIFI/ ΣFI

Where, ΣXIFI= Summation of the product for percentage of error and frequency of error occurrence and ΣFI= Summation of frequency of error occurrence.

Hence for the present model reliability is obtained as 99.40351%

This model may help to identify the aluminium percentage present in the aluminium sample.

CONCLUSION

This mathematical model provides and helps to make the NDT monitoring system for the analysis of characterization of aluminium metals to predict the percentage of aluminium, in the aluminium metals. This may help to identify the types of aluminium and its applications

REFERENCES

[1]. S. Djordjevic, 'The effect of ultrasound solidification on aluminum' J.Min.Met. , 39(3-4)B , (2007) , p.p. 527-532.

[2]. Richard weaver, 'Ultrasonics is in an aluminum foam' ultrasonics international, Volume 36, Issue 1-5, (1998) , p.p. 435-442.

[3]. D. M. Profunser, J. Vollmann and J. Dual, 'Determination of the material properties of microstructure by laser based ultrasound' Proceedings of ultrasounics International, Volume 42 , Issue 1-9, (April 2004), 641.

[4. S. Femmam, N.K.Sirdi and A Ouahabi, ' Perception and characterization of materials using signal processing techniques' IEEE trans. on Insrumentation and measurements, Vol. 50,No. 5, October 2001, p.p. 1203-1211.

[5]. H.Schenck Jr, ' Reduction of variablesDimensional analysis' , Theories of Engineering Experimentation, Mc Graw Hill Book Co, New York, p.p 60-81.

[6]. Schenck H. Jr., 1967 "Theories of Engineering Experimentation", First Edition McGraw Hill Inc.

[7]. Best J W; Kahn J V. 2006. Research in Education, New Delhi: Prentice-Hall of India Pvt. Ltd.