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On approximate σ-homomorphisms and derivations in C^*-ternary algebras

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ABSTRACT

In this paper, we prove the generalized Hyers-Ulam stability of σ-homomorphisms and ternary derivations on C^*-ternary algebras associated wiith the generalized Cauchy-Jensen type additive functional equation

$$\sum_{i=1}^{n} f\left(x_i + \frac{1}{n-1}\sum_{j=1, j\neq i}^{n} x_j\right) = 2\sum_{i=1}^{n} f(x_i)$$

for all $x_i, x_j \in X$, where $n \in \mathbb{Z}^+$ is a fixed positive integer with $n \geq 2$.

KEYWORDS

C^*-ternary algebras; σ-homomorphisms; ternary derivation; generalized Hyers-Ulam stability

SUBJECT CLASSIFICATION

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1. INTRODUCTION

Ternary algebraic operations were considered in the 19th century by several mathematicians and physicists such as Cayley [3] who introduced the notion of a cubic matrix, which in turn was generalized by Kapranov et al. [10]. The simplest example of such nontrivial ternary operation is given by the following composition rule:

$$\{a,b,c\}_{ijk} = \sum_{1\le l,m,n\le N} a_{nil} b_{ljm} c_{mkn} \qquad (i,j,k\ldots = 1,2,\ldots,N)$$

Ternary structures and their generalization, the so-called n-ary structures, raise certain hopes in view of their applications in physics [11]. As it is extensively discussed in [25], the full description of a physical system implies the knowledge of three basic ingredients: the set of the observable, the set of the states, and the dynamics that describes the time evolution of the system by means of the time dependence of the expectation value of a given observable on a given statue. Originally the set of the observables were considered to be a C^*-algebra [8]. In many applications, however, this was shown not to be the most convenient choice, and so the C^*-algebra was replaced by a Von Neumann algebra. This is because the role of the representation turns out to be crucial, mainly when long range interactions are involved. Here we used a different algebraic structure.

A C^*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x,y,z) \mapsto [x,y,z]$ of A^3 into A, which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linear in the middle variable, and associative in the sense that $[x,y,[z,w,v]] = [x[w,z,y],v] = [[x,y,z],w,v]]$ and satisfies $\|[x,y,z]\| \le \|x\|\,\|y\|\,\|z\|$ and satisfies $\|[x,x,x]\| = \|x\|^3$ ([2], [27]). If a C^*-ternary algebra $(A, [\cdot,\cdot,\cdot])$ has an unit element $e \in A$ such that $x = [x,e,e] = [e,e,x]$ for all $x \in A$, then it is routine to verify that A, endowed with $x \circ y = [x,e,y]$ and $x^* = [e,x,e]$, is a unital C^*-algebra. Conversely, if $(A, \circ)$ is a unital C^*-algebra, then $[x,y,z] = x \circ y^* \circ z$ makes A into a C^*-ternary algebra.

Let A, B be C^*-algebras and σ be a permutation of $\{1,2,3\}$. A $\mathbb{C}$-linear mapping $H : A \to B$ is called a *σ-homomorphisms* if

$$H([x_1,x_2,x_3]) = [H(x_{\sigma(1)}), H(x_{\sigma(2)}), H(x_{\sigma(3)})]$$

for all $x_1, x_2, x_3 \in A$. A $\mathbb{C}$-linear mapping $D : A \to A$ is said to be a *ternary derivation* if

$$D([x,y,z]) = [D(x),y,z] + [x,D(y),z] + [x,y,D(z)]$$

for all $x, y, z \in A$.

The stability problem of functional equations originated from a question of Ulam [26] concerning the stability of group homomorphisms. In 1941, the famous Ulam stability problem was partially solved by Hyers [9] for linear functional equation of Banach spaces. In 1950, Aoki [1] was the second author to treat this problem for additive mappings. In 1978, Rassias [24] provided a generalization of Hyers' Theorem which allows the Cauchy difference to be unbounded. In 1982, Rassias [22] generalized the Hyers stability result by presenting a weak condition controlled by a product of different powers of norms. A generalization of the Rassias theorem was obtained by Găvruta [4] by replacing the unbounded Cauchy difference by a general control function in the sprite of the Rassias approach. Subsequently, various approaches to the stability problems have been extensively investigated by many mathematician. The interested readers for more information on such problems are referred to the works [5], [7], [12] – [17], [19] - [21], [23] and reference therein.

Now, we consider a mapping $f : X \to Y$ satisfying the following functional equation:

$$\sum_{i=1}^{n} f\left(x_i + \frac{1}{n-1}\sum_{j=1, j\ne i}^{n} x_j\right) = 2\sum_{i=1}^{n} f(x_i) \qquad (1.1)$$

for all $x_i, x_j \in X$, where $n \in \mathbb{Z}^+$ is a fixed positive integer with $n \ge 2$. Note that in the case $n = 2$, the functional equation (1.1) yields the Cauchy additive equation $f(x_1 + x_2) = f(x_1) + f(x_2)$. Also, for $n = 3$ in (1.1), we have

$$f\left(x_1 + \frac{x_2 + x_3}{2}\right) + f\left(x_1 + \frac{x_2 + x_3}{2}\right) + f\left(x_1 + \frac{x_2 + x_3}{2}\right) = 2(f(x_1) + f(x_2) + f(x_3)). \qquad (1.2)$$

The functional equation (1.2) yields the Cauchy-Jensen additive equation

$$f\left(\frac{x_1 + x_2}{2}\right) + f\left(x_1 + \frac{x_2}{2}\right) + f\left(x_2 + \frac{x_1}{2}\right) = 2(f(x_1) + f(x_2))$$

If $x_3 = 0$. Therefore, the functional equation (1.1) is called the *generalized Cauchy-Jensen type additive functional equation*.

In this paper, we investigate the generalized Hyers-Ulam stability of σ-homomorphisms and ternary derivations on C^*-ternary algebras associated with the generalized Cauchy-Jensen type additive functional equation (1.1) in the sprite of Hyers, Ulam and Rassias.

2. Main Results

Throughout this section, let A, B be C^*-ternary algebras. Assume that $n \in \mathbb{Z}^+$ is a fixed positive integer with $n \geq 2$ and $T^1 = \{\mu \in C : |\mu| = 1\}$. For a given mapping $f : A \to B$, we define

$$D_\mu f(x_1, \dots, x_n) = \sum_{i=1}^{n} f\left(\mu x_i + \frac{1}{n-1} \sum_{j=1, j\neq i}^{n} \mu x_j\right) - 2\mu \sum_{i=1}^{n} f(x_i)$$

for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$.

We prove the generalized Hyers-Ulam stability of σ-homomorphisms on C^*-ternary algebras for the functional equation $D_\mu f(x_1, \dots, x_n) = 0$. We need the following lemma in the main theorems.

Lemma 2.1 [5]. Let X, Y be linear spaces and $n \in \mathbb{Z}^+$ (≥ 2) be a fixed positive integer. A mapping $f : X \to Y$ satisfies (1.1) if and only if f is additive.

Theorem 2.2. Let $p < 1$, $s < 3$ and θ be positive real numbers. If a mapping $f : A \to B$ satisfies

$$\|D_\mu f(x_1, \dots, x_n)\| \leq \theta \textstyle\sum_{j=1}^{n} \|x_j\|^p \tag{2.2}$$

and

$$\|f([x_1, x_2, x_3]) - [f(x_1), f(x_2), f(x_3)]\| \leq \theta\left(\|x_1\|^s + \|x_2\|^s + \|x_3\|^s\right) \tag{2.3}$$

for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$, then there exists a unique σ-homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \leq \frac{\theta\|x\|^p}{2 - 2^p} \tag{2.4}$$

for all $x \in A$.

Proof. Substituting $x_1 = \cdots = x_n = x$ and $\mu = 1$ in (2.2), we have

$$\|nf(2x) - 2n f(x)\| \leq n\,\theta\|x\|^p, \tag{2.5}$$

which gives

$$\left\|f(x) - \frac{f(2x)}{2}\right\| \leq \frac{\theta\|x\|^p}{2} \tag{2.6}$$

for all $x \in A$. If we replace x by $2^j x$ and divide 2^j both sides of (2.6), then we have

$$\left\|\frac{f(2^j x)}{2^j} - \frac{f(2^{j+1}x)}{2^{j+1}}\right\| \leq \frac{\theta\|x\|^p}{2}(2^{p-1})^j$$

for all $x \in A$ and all $j = 0, 1, 2, \dots$. Therefore, we obtain

$$\left\|\frac{f(2^k x)}{2^k} - \frac{f(2^m x)}{2^m}\right\| \leq \frac{\theta\|x\|^p}{2} \sum_{j=k}^{m-1} (2^{p-1})^j \tag{2.7}$$

for all $x \in A$ and all integers k, m with $m > k \geq 0$. Then, the sequence $\left\{\frac{f(2^n x)}{2^n}\right\}$ is a Cauchy sequence for all $x \in A$. Since B is complete, the sequence $\left\{\frac{f(2^n x)}{2^n}\right\}$ converges. So, we can define a mapping $H : A \to B$ by

$$H(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

for all $x \in A$. Moreover, letting $n \to \infty$ in (2.7) with $k = 0$, we obtain the desired inequality (2.4). It follows from (2.2) that

$$\|D_\mu H(x_1, \dots, x_n)\| \leq \lim_{n\to\infty} \frac{1}{2^n} \|D_\mu f(2^n x_1, \dots, 2^n x_n)\| \leq \theta \lim_{n\to\infty} (2^{p-1})^n\left(\|x_1\|^p + \cdots + \|x_n\|^p\right) = 0,$$

which gives $D_\mu H(x_1, \dots, x_n) = 0$ for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$. If we put $\mu = 1$ in $D_\mu H(x_1, \dots, x_n) = 0$, then by Lemma 2.1, the mapping H is additive. Letting $x_1 = x$ and $x_2 = \cdots = x_n = 0$ in the last equality, then $H(\mu x) = \mu H(x)$. By the same reasoning as that the proof of Theorem 2.1 of [18], the mapping H is $\mathbb{C}$-linear. Also, it follows from linearity of H and (2.3) that

$$\left\|H([x_1,x_2,x_3]) - [H(x_{\sigma(1)}), H(x_{\sigma(2)}), H(x_{\sigma(3)})]\right\|$$
$$\leq \lim_{n\to\infty} \frac{1}{2^{3n}} \|f([2^n x_1, 2^n x_2, 2^n x_3]) - [f(2^n x_{\sigma(1)}), f(2^n x_{\sigma(2)}), f(2^n x_{\sigma(3)})]\|$$
$$\leq \theta \lim_{n\to\infty} (2^{s-3})^n \left(\|x_1\|^s + \|x_2\|^s + \|x_3\|^s\right) = 0$$

for all $x_1, x_2, x_3 \in A$. Thus, the mapping $H : A \to B$ is a σ-homomorphism. Now, let $H' : A \to B$ be another additive mapping satisfying (2.4). Then we have

$$\|H(x) - H'(x)\| \leq \frac{1}{2^n}\|H(2^n x) - H'(2^n x)\| \leq \frac{2\theta\|x\|^p}{2-2^p}(2^{p-1})^n,$$

which tends to zero as $n \to \infty$ for all $x \in A$. So, we can conclude that $H(x) = H'(x)$ for all $x \in A$. This proves the uniqueness of H. This completes the proof.

Theorem 2.3. Let $p > 1$, $s > 3$ and θ be positive real numbers, and let $f : A \to B$ be a mapping such that (2.2) and (2.3). Then there exists a unique σ-homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \leq \frac{\theta\|x\|^p}{2^p - 2} \tag{2.8}$$

for all $x \in A$.

Proof. It follows from (2.5) that

$$\left\|2^k f\left(\frac{x}{2^k}\right) - 2^m f\left(\frac{x}{2^m}\right)\right\| \leq \frac{\theta\|x\|^p}{2^p} \sum_{j=k}^{m-1} (2^{1-p})^j \tag{2.9}$$

for all $x \in A$ and all integers k, m with $m > k \geq 0$. It follows that the sequence $\{2^n f\left(\frac{x}{2^n}\right)\}$ is a Cauchy sequence for all $x \in A$ and it converges. Thus, we can define a mapping $H : A \to B$ by

$$H(x) = \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$$

for all $x \in A$. Letting $n \to \infty$ in (2.9) with $k = 0$, we get (2.8). It follows from (2.2) that

$$\|D_\mu H(x_1, \dots, x_n)\| \leq \lim_{n\to\infty} 2^n \left\|D_\mu f\left(\frac{x_1}{2^n}, \dots, \frac{x_n}{2^n}\right)\right\| \leq \theta \lim_{n\to\infty} \frac{2^n}{2^{np}} \left(\|x_1\|^p + \dots + \|x_n\|^p\right) = 0$$

for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$. So, $D_\mu H(x_1, \dots, x_n) = 0$ for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$. By the same reasoning as that the proof of Theorem 2.1 of [18], the mapping H is C-linear. Also, it follows from linearity of H and (2.3) that

$$\left\|H([x_1,x_2,x_3]) - [H(x_{\sigma(1)}), H(x_{\sigma(2)}), H(x_{\sigma(3)})]\right\|$$
$$= \lim_{n\to\infty} 2^{3n} \left\| f\left(\frac{[x_1,x_2,x_3]}{2^n\cdot 2^n\cdot 2^n}\right) - \left[f\left(\frac{x_{\sigma(1)}}{2^n}\right), f\left(\frac{x_{\sigma(2)}}{2^n}\right), f\left(\frac{x_{\sigma(3)}}{2^n}\right)\right]\right\|$$
$$\leq \theta \lim_{n\to\infty} (2^{3-s})^n \left(\|x_1\|^s + \|x_2\|^s + \|x_3\|^s\right) = 0$$

for all $x_1, x_2, x_3 \in A$. Now, let $H' : A \to B$ be another additive mapping satisfying (2.8). Then we have

$$\|H(x) - H'(x)\| \leq 2^n \left\|H\left(\frac{x}{2^n}\right) - f\left(\frac{x}{2^n}\right)\right\| + \left\|H'\left(\frac{x}{2^n}\right) - f\left(\frac{x}{2^n}\right)\right\| \leq \frac{2\cdot 2^n\theta\|x\|^p}{2^{np}(2^p-2)}$$

which tends to zero as $n \to \infty$ for all $x \in A$. So, we can conclude that $H(x) = H'(x)$ for all $x \in A$. This proves the uniqueness of H. This completes the proof.

Theorem 2.4. Let $p = \sum_{j=1}^{n} |p_j| < 1$, $s < 1$ and θ be positive real numbers, and let $f : A \to B$ satisfies

$$\|D_\mu H(x_1, \dots, x_n)\| \leq \theta \prod_{j=1}^{n} \|x_j\|^{p_j} \tag{2.10}$$

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and

$$\left\|f([x_1,x_2,x_3]) - [f(x_1), f(x_2), f(x_3)]\right\| \le \theta\left(\|x_1\|^s \ \|x_2\|^s \ \|x_3\|^s\right) \qquad (2.11)$$

for all $x_1, \dots, x_n \in A$ and $\mu \in T^1$. Then there exists a unique σ-homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \le \frac{\theta\|x\|^p}{n(2-2^p)} \qquad (2.12)$$

for all $x \in A$.

Proof. Let us assume $x_1 = \cdots = x_n = x$ and $\mu = 1$ in (2.10). Then we have

$$\left\|f(x) - \frac{f(2x)}{2}\right\| \le \frac{\theta\|x\|^p}{2n} \qquad (2.13)$$

and so, we have

$$\left\|\frac{f(2^k x)}{2^k} - \frac{f(2^m x)}{2^m}\right\| \le \frac{2\theta\|x\|^p}{2n} \sum_{j=k}^{m-1} (2^{p-1})^j \qquad (2.14)$$

for all $x \in A$ and all integers k, m with $m > k \ge 0$ It follows that the sequence $\left\{\frac{f(2^n x)}{2^n}\right\}$ is a Cauchy sequence for all $x \in A$, and so it converges. By the same reasoning as the proof of Theorem 2.2, we can define the $\mathbb{C}$-linear additive mapping $H : A \to B$ by

$$H(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

for all $x \in A$. Letting $n \to \infty$ in (2.14) with $k = 0$, we obtain the desired inequality (2.12). It follows from (2.11) that

$$\left\|H([x_1, x_2, x_3]) - \left[H\left(x_{\sigma(1)}\right), H\left(x_{\sigma(2)}\right), H\left(x_{\sigma(3)}\right)\right]\right\|$$

$$\le \lim_{n\to\infty} \frac{1}{2^{3n}} \left\|f([2^n x_1, 2^n x_2, 2^n x_3]) - \left[f\left(2^n x_{\sigma(1)}\right), f\left(2^n x_{\sigma(2)}\right), f\left(2^n x_{\sigma(3)}\right)\right]\right\|$$

$$\le \theta \lim_{n\to\infty} \frac{2^{3ns}}{2^{3n}} \left(\|x_1\|^s \ \|x_2\|^s \ \|x_3\|^s\right) = 0$$

For all $x_1, x_2, x_3 \in A$. The rest of proof is similar method to the proof of Theorem 2.2. Thus, the mapping $H : A \to B$ is a unique σ-homomorphism satisfying (2.12). This completes the proof.

Corollary 2.5. Let $p = \sum_{j=1}^{n} |p_j| > 1, s > 1$ and θ be positive real numbers. If $f : A \to B$ satisfies (2.10) and (2.11), then there exists a unique σ-homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \le \frac{\theta\|x\|^p}{n(2^p - 2)} \qquad (2.15)$$

for all $x \in A$.

Proof. It follows from (2.10) that

$$\left\|2^k f\left(\frac{x}{2^k}\right) - 2^m f\left(\frac{x}{2^m}\right)\right\| \le \frac{\theta\|x\|^p}{2n} \sum_{j=k}^{m-1} (2^{1-p})^j \qquad (2.16)$$

for all $x \in A$ and all integers k, m with $m > k \ge 0$. The sequence $\left\{2^n f\left(\frac{x}{2^n}\right)\right\}$ is a Cauchy sequence for all $x \in A$ and it converges. So, we can define a mapping $H : A \to B$ by $H(x) = \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$ for all $x \in A$. Letting $n \to \infty$ in (2.16) with $k = 0$, we obtain (2.15). The rest of proof is similar method to the proof of Theorem 2.3. This completes the proof.

2.2. Stability of ternary derivations

In this subsection, we investigate the generalized Hyers-Ulam stability of ternary derivations on C^*-ternary algebra A for the functional equation $D_\mu f(x_1, \dots, x_n) = 0$.

Theorem 2.6. Let $p < 1$, $s < 3$ and θ be positive real numbers, and $f : A \to A$ be a mapping such that (2.2) and

$$\|f([x,y,z]) - [f(x),y,z] - [x,f(y),z] - [x,y,f(z)]\| \leq \theta\left(\|x\|^s + \|y\|^s + \|z\|^s\right) \tag{2.17}$$

for all $x,y,z \in A$ and $\mu \in T^1$. Then there exists a unique ternary derivation $D : A \to A$ such that

$$\|f(x) - D(x)\| \leq \frac{\theta\|x\|^p}{2 - 2^p} \tag{2.18}$$

for all $x \in A$.

Proof. By the same method as in the proof of Theorem 2.2, there exists a unique $\mathbb{C}$-linear mapping $D : A \to A$ satisfying (2.18). The mapping is given by $D(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ for all $x \in A$. It follows from (2.17) that

$$\| D([x,y,z]) - [D(x),y,z] - [x,D(y),z] - [x,y,D(z)] \| \leq \lim_{n\to\infty} \frac{\theta 2^{np}}{2^{3n}} \left(\|x\|^s + \|y\|^s + \|z\|^s\right) = 0$$

for all $x,y,z \in A$. Then, we have

$$D([x,y,z]) = [D(x),y,z] + [x,D(y),z] + [x,y,D(z)]$$

for all $x,y,z \in A$. Thus, the mapping $D : A \to A$ is a unique ternary derivation satisfying (2.18). This completes the proof.

Corollary 2.7. Let $p > 1$, $s > 3$ and θ be positive real numbers. Let $f : A \to A$ be a mapping such that (2.2) and

$$\| f([x,y,z]) - [f(x),y,z] - [x,f(y),z] - [x,y,f(z)]\| \leq \theta\left(\|x\|^s + \|y\|^s + \|z\|^s\right)$$

for all $x,y,z \in A$ and $\mu \in T^1$. Then there exists a unique ternary derivation $D : A \to A$ such that

$$\|f(x) - D(x)\| \leq \frac{\theta\|x\|^p}{2^p - 2} \tag{2.19}$$

for all $x \in A$.

Proof. By the same method as in the proof of Theorem 2.3, there exists a unique $\mathbb{C}$-linear mapping $D : A \to A$ satisfying (2.19). The mapping is given by $D(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in A$. The rest of proof is similar method to the proof of Theorem 2.6. This completes the proof.

Theorem 2.8. Let $p = \sum_{j=1}^{n} |p_j| \neq 1$, $s \neq 1$ and θ be positive real numbers. If $f : A \to A$ satisfies (2.10) and

$$\| f([x,y,z]) - [f(x),y,z] - [x,f(y),z] - [x,y,f(z)]\| \leq \theta\left(\|x\|^s \|y\|^s \|z\|^s\right)$$

for all $x,y,z \in A$ and $\mu \in T^1$, then there exists a unique ternary derivation $D : A \to A$ such that

$$\|f(x) - D(x)\| \leq \begin{cases} \dfrac{\theta\|x\|^p}{n(2 - 2^p)}, & p < 1, \quad s < 1 \\ \dfrac{\theta\|x\|^p}{n(2^p - 2)}, & p > 1, \quad s > 1 \end{cases}$$

for all $x \in A$.

Proof. The proof is similar to the proofs Theorem 2.4, 2.6 and Corollary 2.5.

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