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QUANTUM AND RELATIVISTIC COROLLARIES OF AN OPERATIVE DEFINITION OF SPACE TIME

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ABSTRACT

The paper introduces an "ab initio" theoretical model based on an operative definition of space time, regarded as a combination of the fundamental constants of the nature. The paper shows that significant concepts of quantum mechanics and relativity are straightforward consequence of the proposed definition of space time. Some cosmological implications of the model are also shown.

keywords

Space time; Quantum Uncertainty; Relativity; Cosmology.

Academic Disciplines And Sub-Disciplines

Astrophysics, Cosmology, Gravitation physics, Relativity, Quantum physics, Thermodynamics

1 INTRODUCTION

One of the most important outcomes of the general relativity is the disclosure of a 4-manifold, the space time, curved by the distribution of matter in it contained [1-5]. The fertile idea of gravity force equivalent to the local space time curvature, implies the necessity of merging space and time coordinates into a proper metrics to formulate the physical events in a covariant way [6-11]. This conceptual frame through which the physical phenomena are appropriately described, rises however a question: is the space time a simple arena where the events occur, or rather is it a real entity formulable and quantifiable itself via an operative physical definition? If the second chance is actual, then an appropriate mathematical formulation of the concept of space time could be introduced "ab initio" and implemented like any fundamental physical principle or law.

On the one hand, the sought definition must be consistent with all fundamental laws today known, possibly inferable as corollaries. On the other hand, any theoretical model based on this idea should provide a sensible answer to a further crucial question: if the birth of the universe is defined by the beginning of the space time and its inherent physics, does the sought definition help to outline even the evolution of the universe?

The paper proposes some answers to these questions by introducing a theoretical model based exclusively on the initial formulation of an operative definition of space time, regarded as a fundamental and productive principle of the nature rather than as a mere successful intuition. To this purpose the opening brainwaves of the paper are the Planck units, which by definition merge the fundamental constants of nature into single concepts only, e.g. length or time or energy and so on. In principle, however, nothing prevents combining arbitrarily these fundamental constants to obtain even more complex physical entities: the same idea underlying the Planck measure units is extended here to propose a new combination suitable to infer and exploit a composite physical concept, i.e. just that of space time. As the sought definition should expectedly include both space and time units, the proposed quantity to be tentatively implemented is

$$\frac{hG}{c^2} \qquad (1,1)$$

The physical dimensions of this ratio are the only hint and starting point of the theoretical model exposed below.

The challenge of the present paper is to extract all possible physical information from this seemingly innocuous position: the purpose is to demonstrate that actually this operative definition of space time makes inferable as corollaries several interesting features of the quantum mechanics and relativity.

By necessity, even well known topics are explicitly developed and exposed in the following. This is indispensable not only to make the paper as self-contained as possible, but mostly to show the effective chance of obtaining uniquely from the naive position (1,1) the physical laws that govern the space time, its evolution as a function of time and the related phenomena in it allowed to occur.

2 PHYSICAL BACKGROUND.

The proposed combination (1,1) of fundamental constants has physical dimensions $volume/time$, in principle sensibly consistent with the sought definition, and thus enables writing

$$\frac{hG}{c^2}=V\nu \quad \nu=\nu(\Delta t).(2,1)$$

This equation means that the definition of space time at left hand side concerns physical phenomena possibly occurring within the space volume V during the time range Δt, as a function of which is defined the frequency ν. In a sense, the right hand side is the aforementioned physical arena; the left hand side governs the dynamical variables that describe the physical events in this arena.

The suggested definition poses however some problems, now examined one by one.

-Expectedly the quantity defined by the eq (1,1) should allow regarding independently V and Δt; this requirement is in fact compatible with space and time coordinates separately definable, although intimately merged in describing the physical phenomena. The right hand side combines the space volume V and Δt through a function $\nu=\nu(\Delta t)$ having necessarily physical dimensions of a frequency. In general ν could be expressed as series expansion of an appropriate function; if in particular $\nu\equiv\Delta t^{-1}$, then V would be a monotonic increasing function of Δt. This agrees with the idea of dynamic space time, whose volume can however steadily increase only. Yet even steady or decreasing trends of V as a function of Δt seem reasonable and should be, at least in principle, possible.

In fact, the lack of hypotheses about V and ν suggests the feasibility of a more flexible definition of space time, where really the space and time coordinates can change independently each other like the single x, y, z themselves.

-The second problem concerns the time dependence of V in the proposed definition: whatever the time profile of V might be, a physical motivation is necessary to explain why V could in fact increase or decrease. Otherwise stated, as this motivation must be included itself in the position (1,1), a correct definition of space time requires inherently the chance of justifying and calculating an appropriate pressure acting against the internal or external boundaries of the space time volume.

-The third problem concerns just the pressure, which indeed requires itself admitting that the space time cannot be empty even in lack of matter; despite neither matter nor radiation energy appear explicitly in the eq (1,1), as previously assured this pressure must be inferable itself in the frame of the eqs (2,1) only.

-The fourth problem concerns the reference system R where are defined the lengths characterizing the size of V, expressed for example as Δx^3, and the time length Δt necessary to define ν. The quantities at left hand side are constant, thus invariant by definition in any reference system where are definable Δx^3 and Δt at right hand side. These latter must fulfill the physical dimensions of the position (1,1) and satisfy an appropriate property of invariance, necessary to make consistent both sides of the eq (2,1). In fact, however, no information is available "a priori" about the actual kind of transformation law compelled by the position (1,1) itself; if for instance the essential invariance would be that of Lorentz, then this condition could be satisfied in several ways: e.g. $V\nu=\left((c\Delta t)^2-\Delta x^2\right)^2/(\Delta x\Delta t)$ fulfills this requirement. More in general, any $V\nu=f(\Delta x\Delta t)$ via a suitable expansion of the function f in series of powers of the argument is appropriate too. Being arbitrary both time and space ranges here introduced and the function f itself linking them, any value of V can be in principle consistent with the condition $V\nu=const$ inherent the eq (2,1). Yet, once having fixed V the resulting ν is compatible with and corresponds to various values of Δt depending on the specific function $f(\Delta x\Delta t)$. Otherwise stated, the link between V and ν does not imply rigidly that between ν and Δt: in principle various values of V are therefore compatible with any given Δt via an appropriate $\nu(\Delta t)$.

Are thus interesting the definitions of volume $V=\left((c\Delta t)^2-\Delta x'^2\right)^2/\Delta x'=\Delta x^3$ and frequency $\nu=g/\Delta t$, being $g=g(\Delta x'\Delta t)$ an arbitrary dimensionless function of $\Delta x'$ and Δt. For sake of brevity and simplicity of notation, however, in the following the right hand side of the eq (2,1) will be shortly concerned as $\Delta x^3\nu$ only. Actually, being ν not necessarily coincident with Δt^{-1} but rather a more general function of Δt, these shortened forms subtend for instance

$$V=\Delta x^3 \quad \nu=\frac{g}{\Delta t} \quad V\nu=\frac{\left((c\Delta t)^2-\Delta x'^2\right)^2 g}{\Delta x'\Delta t} \quad g=g_0+\sum_j a_j\left(\frac{\Delta x'\Delta t}{x_0\tau}\right)^j (2,2)$$

where x_0, g_0 and τ are arbitrary constants. Note that apparently a more general definition of V should have the form $V'=\Delta x\Delta y\Delta z$; yet if Δy and Δz are arbitrary like Δx, then any value of V' is also allowed to and thus described by

V itself; so, without loss of generality, in the following will be shortly mentioned and implemented the given form of V only. The merit of the eqs (2,2) is that of having involved by necessity both time and space ranges; so both coordinates are merged together since the beginning in the model aimed to implement the eqs (2,1).

Having indicated, at least in principle, how to tackle the problem of the invariance, arises the further problem of the reference system R where are defined the ranges of the eq (2,2). Also in this respect, the eqs (2,1) do not provide any indication; moreover no information is available even about the properties of these ranges, e.g. their sizes. The only hint available is that the quantum uncertainty, expressed in its most agnostic form proposed in [12], disregards in fact the range sizes and the related reference system; this information is in effect inessential as concerns the calculation of the eigenvalues and thus the existence of physical observables [13,14]. This point is so important for its quantum and relativistic implications, that some concepts are preliminarily summarized now to clarify subsequently why the uncertainty is an essential corollary ensuring a coherent formulation of the model based on the eqs (2,1) only. This will be shortly shown in the section 9.2 as well.

Consider for instance $\Delta x = x_1 - x_0$. Knowing x_0 means having preliminarily introduced any kind of reference system, e.g. a coordinate axis in the simplest case or even a system of curvilinear coordinates, with respect to which the position of Δx in R can be determined; moreover, once knowing also x_1 the range size can be determined as well. Of course the role of x_0 and x_1 is identically interchangeable. Let however x_0 and x_1 be both undefined and indefinable in principle, i.e. conceptually and not as a sort of approximation to simplify some calculation; so, by fundamental assumption, neither the range size nor the range position in the reference system are in fact physically specifiable, whereas the concept of local x is disregarded itself in any R. Even in the case of the time coordinate, the current time t is replaced by $\Delta t = t_1 - t_0$: this means regarding t as a random variable ranging between the time boundaries t_0 and t_1 during which any physical event is allowed to occur; both boundaries are however arbitrary and unknowable by definition as well. Without such preliminary information, is not specifiable the origin of R and even what kind of reference system it actually symbolizes. Yet this agnostic standpoint is compatible with the chance of determining physical properties through the ranges of the dynamical variables only.

In the following, these ideas are better specified as necessary features of the present strategy: to gain all possible physical information from the definition of space time only. In previous papers the statistical formulation of the quantum uncertainty was assumed as a postulate to infer as corollaries the foundations of both quantum physics and relativity [15]. Here it is necessary to demonstrate that effectively the quantum uncertainty is inherent itself the definition (1,1).

This position has been introduced considering in general the Planck constant h. It is known however that several equations require the reduced constant $\hbar$. This depends on the kind of problem and related definition of ν: e.g. the Planck energy is given by $h\nu$, whereas the angular momentum is expressed as $(l \pm s)\hbar$. The reduced Planck constant is necessary in problems where the frequency ν is actually a circular frequency $\omega = 2\pi\nu$, e.g. to express the energy levels of hydrogenlike or many electron atoms where one or several electrons somehow move around the nucleus; in this case indeed $\hbar\omega = h\nu$. For simplicity of notation, in the present paper h symbolizes in general the Planck constant regardless of whether, case by case, it must be actually regarded as $\hbar$.

Investigating one by one all of the possible outcomes of the eqs (2,2) allows in fact a preliminary evaluation of the physical rationality of the eq (2,1). The next two sections aim to assess the self-consistency of these major points, invariance property and uncertainty, whose clarification legitimates the physical usefulness of the eq (2,1) and stimulates preliminary confidence on its physical rationality.

3 THE INVARIANT INTERVAL

Let us demonstrate first that effectively the eqs (2,1) imply just the Lorentz transformations. Rewrite the eq (2,1) replacing c with an arbitrary velocity $v < c$; one finds $hG/v^2 = v\delta x^3$, having put δx^3 instead of Δx^3 to introduce the new volume consistent with v. In fact, this is nothing else but dividing both sides by an arbitrary factor $0 < a < 1$ such that $c^2 a = v^2$ and $\Delta x^3/a = \delta x^3$. Having left unchanged v by definition, results $\delta x > \Delta x$. The ratio between the last equation and the eq (2,1) yields $c^2 = (\delta x/\Delta x)^3 v^2$. Multiply both sides by Δt^2; putting next $v\Delta t = \delta l$ and $(\delta x/\Delta x)^3 = 1 + q$, where $q > 0$ is an arbitrary real number, one finds then $c^2\Delta t^2 = \delta l^2 + \Delta l^2$ with $\Delta l^2 = q\delta l^2$. The conclusion is

$$\Delta l^2 = c^2\Delta t^2 - \delta l^2 \quad \text{or} \quad \delta l^2 = c^2\Delta t^2 - \Delta l^2 . (3,1)$$

Note that δl and Δl can be exchanged of place while leaving formally unchanged the result, i.e. their role is physically interchangeable with respect to $c\Delta t$. This symmetry implies that either δl^2 or Δl^2 must be an invariant with respect to the Lorentz transformations, as in fact it follows straightforwardly.

Multiply the former equation by an arbitrary coefficient ϑ such that by definition $\vartheta\Delta l^2 = \delta l^2$; then one obtains $\delta l^2 = c^2\Delta t'^2 - \delta l'^2$, being of course $\Delta t'^2 = \vartheta\Delta t^2$ and $\delta l'^2 = \vartheta\delta l^2$. The second eq (3,1) requires thus $c^2\Delta t'^2 - \delta l'^2 = c^2\Delta t^2 - \Delta l^2$, i.e.: whatever the changes of the single intervals at left hand side might be, the whole interval $c^2\Delta t^2 - \delta l^2$ is invariant. This holds in particular for time space uncertainty ranges in two inertial reference systems in reciprocal motion.

It is known that the invariancy rule is the basis of the special relativity [16].

All this has a conceptual cost. The classical intervals of the special relativity, exactly knowable, become now uncertainty ranges about which nothing is known; the interval rule and all its consequences hold identically, but become also compliant by definition with the Heisenberg principle. This way of deducing the invariant interval skips the existence of local coordinates and bypasses not only the introductory postulates of the special relativity but also the tensor calculus, which is in fact precluded in the present model that disregards "a priori" the local coordinates.

The next considerations will concern further the problem of the reference system and that of the covariancy.

4 THE SPACE TIME UNCERTAINTY

Rewrite identically the right hand side of the eq (2,1) as $(\Delta x/m)(gm\Delta x/\Delta t)\Delta x$ according to the definitions (2,2). Formally therefore one finds

$$V\nu = \frac{\Delta x}{m}\Delta x\Delta p_x \quad \Delta p_x = gmv_x = gp_x \quad v_x = \frac{\Delta x}{\Delta t};(4,1)$$

the subscript x symbolizes any space coordinate additional to the time. In the second equation the range Δp_x is consistent with the definition of g, as in general $gp_x = p_1 - p_0$ with $p_0 = -p_x\sum_j a_j(\Delta x\Delta t/x_0\tau)^j$ and $p_1 = g_0 p_x$; there is no constrain or restriction to define the boundaries of the uncertainty ranges, which in fact are completely arbitrary, unknown and unknowable.

Note that the physical dimension of momentum is defined without need of specific hypotheses, even without introducing explicitly the mass in the equation: m has been simply multiplied and divided the at the right hand side of the eq (2,1). Also, this step has contextually introduced the velocity component v_x via the time range Δt necessary for a hypothetical particle of mass m, possibly present in Δx, to travel throughout the given range size; otherwise stated, m is delocalized in Δx.

An analogous reasoning is carried out to rewrite $V\nu$ as $(\Delta x/m)gm(\Delta x/\Delta t)^2\Delta t$, which introduces the range $\Delta\varepsilon = gmv_x^2$ and yields now $V\nu = (\Delta x/m)\Delta t\Delta\varepsilon$; accordingly, the physical dimension of energy is introduced as well.

Considering that these outcomes share the quantity $V\nu m/\Delta x$, the conclusion is

$$\Delta x\Delta p_x = \frac{Vm\nu}{\Delta x} = \Delta\varepsilon\Delta t,(4,2)$$

which holds for any space time coordinate. Note that all terms have the physical dimensions of h and that $\Delta x\Delta p_x$ reads actually more in general $\Delta x \cdot \Delta p$, regardless of the actual number of space coordinates defining the scalar product. These equations thus do not conflict in principle with the existence of extra-dimensions additional to x, y, z, t; anyway, the merging of time and space coordinates, whatever their number might be, appears intrinsically inherent any approach based on the proposed definition of space time. Write then in general

$$\Delta x \cdot \Delta p = \mathrm{S} = \Delta\varepsilon\Delta t \quad \mathrm{S} = \frac{Vm\nu}{\Delta x} \quad \mathrm{S} = qh,(4,3)$$

which actually summarizes as many equations in principle compatible with quantum properties as the number of scalar components at left hand side; q is a dimensionless arbitrary factor to be defined, whereas the physical dimensions of S are that of the action.

To complete these remarks coherently with the lack of physical information about V and ν in the eq (2,1), it is enough to define conveniently S in the eq (4,3): let us put $S=nh$ with n arbitrary integer. To justify this point and explain its importance, let for instance $\Delta x\Delta p_x$ in R change to $\Delta x'\Delta p'_x$ in R'; it is clear that the respective n and n' are actually indistinguishable by definition, since both symbolize whole sets of unspecified and unspecifiable integers. In other words, whatever any particular $n=n_{xp}$ in R might be, its change in R' yields another particular $n'=n'_{x'p'}$, which however still reproduces an integer already contained in the whole set n. With this definition of n, therefore, S and S' are indistinguishable themselves, i.e. there is no specific or preferential correlation between any $S=nh$ and its distinctive R; otherwise stated, S is disconnected from a particular R, whereas this latter is indistinguishable from any other R' once admitting that neither x_0 and x_1 nor x'_0 and x'_1 are knowable.

This conclusion becomes even more evident noting that the eqs (4,3) can be expressed via Planck units; e.g. $\Delta x = n^*_x l_{Pl}$, being l_{Pl} the Planck length and n^*_x any real number arbitrary likewise the range size it describes. The main worth of expressing in this way the products of range sizes is that it highlights the true nature of the eqs (4,3), i.e. they actually concern products of real numbers times the corresponding Planck units linked by n only: so the eqs (4,3) take the weird form of a bare relationship between arbitrary and indeterminable numbers

$$n^*_x n^*_p = n = n^*_\varepsilon n^*_t . (4,4)$$

The asterisks indicate arbitrary real numbers, n only is an arbitrary integer; this notation evidences even more clearly the total lack of information about the range boundary coordinates and the reference systems defining them.

Appear clearer now the previous considerations about the reference systems: having defined V and v via uncertainty ranges, all implications of the eqs (2,1) do not concern any reference system in particular. It it has been shown in [13,14] that n plays the role of the quantum numbers; so the quantization of the eigenvalues is a fingerprint itself of the proposed definition of space time. Concurrently, just this makes indistinguishable all reference systems as a corollary, not by assumption: while preventing any information about the local dynamical variables, the quantum uncertainty waives a reference system specifically related and thus excludes in fact privileged reference systems to describe the physical laws.

The requirement of relativistic covariancy is surrogated from the quantum standpoint by exploiting uniquely the aforesaid properties of the eqs (4,3), which hold identically regardless of how any physical law formulated in R transforms in R'; this simply requires that all laws be anyway formulated via uncertainty ranges of the dynamical variables only, not via their local values.

In the present model, therefore, the former only have physical meaning; the metric tensor defining lengths, distances, angles, geodesics and local curvature is unsuitable and thus worthless in the present context.

Are evident direct quantum and relativistic consequences of this way of thinking, e.g.: on the one hand is relevant the quantum indistinguishability of identical particles, due to the fact that the present approach moves the physical concern from the dynamical variables of the particles delocalized within uncertainty ranges to the way the sizes of these latter govern the quantum numbers; on the other hand is relevant the lack of privileged reference systems, which in fact result conceptually indistinguishable. It is not surprising that the eqs (4,3) written in the form

$$\Delta x \cdot \Delta p = nh = \Delta\varepsilon\Delta t . (4,5)$$

yield as corollaries the fundamental statements of both wave mechanics and relativity [15]. Note eventually that the classical physics simply follows putting $x_0=0$ and $t_0=0$, in which case Δx and Δt reduce to the local coordinates x and t; however the ranges are compliant with the Heisenberg principle by definition, the latter obviously do not.

Moreover the agnostic character of the present model agrees with the idea that the coordinates are a human artifact to carry out calculations: in effect the eigenvalues of the quantum mechanics, when calculable without need of numerical solutions, are expressed via the fundamental constants and quantum numbers, which however can take any value likewise the definition of n proposed here.

The fact that eqs (4,5) exclude the chance of identifying one particular R has expectedly further consequences, for instance the impossibility of calculating the local velocity. Determining the velocity requires indeed knowing the path between two fixed points traveled by a particle during an initial and final time, which however are both conceptually disregarded in the present model; in effect in the eqs (4,1) the component v_x has been formally expressed as ratio $\Delta x/\Delta t$ of uncertainty ranges. The only velocity specifiable is actually the light speed in the vacuum. Hence the eqs (4,5) leave out even the concept of accelerate or uniform reciprocal motion of different reference systems: it also implies removing the distinction between special and general relativity, as it will be shown for example in the next sections 5 and 9.2 and 13.2.

Even the postulate requiring that two coordinate frames in reciprocal uniform motion must fulfill the Lorentz transformations is in principle superfluous in this conceptual frame; the transformation rules, whatever they might be, are immaterial and skip any physical consequence once having disregarded both boundary coordinates of all ranges. Likewise, the same holds for the time reference system. Nevertheless, is just the eq (2,1) that requires by itself the Lorentz transformation.

The synthesis of this section is that the position (1,1) is physically sensible, given that anyway the right hand side of the eq (2,1) is compliant with the corollaries of quantum uncertainty and Lorentz transformations.

The aim of the present paper, to check all possible implications of the proposed definition of space time, is thus inherently based on the consequent statistical formulation of space time uncertainty; the eqs (4,5) are in fact the quantum equivalent of the relativistic covariancy.

5 THE EHERGY IN THE SPACE TIME

The physical dimensions of momentum and energy have been introduced in the previous section from the eq (2,1) contextually to the eqs (4,5) that require the quantization of the eigenvalues. Let us show now that also the explicit expressions of momentum, energy and energy density are in fact inherent the concept of space time.

Multiply both sides of the eq (2,1) by ν ; the resulting equation introduces the energy density η given by

$$\eta = \frac{(c\nu)^2}{G} \quad \eta = \frac{E}{V} \quad E = h\nu .(5,1)$$

Moreover, multiply and divide η by m ; one also obtains

$$\eta = \frac{h\nu}{V} = \frac{\varepsilon_0}{V_0} \quad V_0 = \frac{mG}{\nu^2} \quad \varepsilon_0 = mc^2 .(5,2)$$

The relativistic rest energy ε_0 and Planck energy E have been contextually inferred. Note that E is the energy of a wave entirely characterized by its frequency ν ; in lack of specific information, nothing hinders regarding ν as the frequency of electromagnetic radiation. So, whatever the size of V might be, η and E are defined regardless of the concept of mass. Instead ε_0 is the energy of a corpuscle uniquely characterized by its mass m , regardless of typical concepts of wave propagation. Nevertheless both energies have a common root in the initial eq (2,1), from which they have been obtained via trivial manipulations. Three conclusions are evident:

1) While introducing the mass in the energy field of the eq (5,1), the volume contextually changes from the initial value V to the new V_0 , the energy density being still η . The physical meaning of this statement, clearly due to the mass m , will be further concerned later; it is shortly anticipated here that the presence of mass deforms size, and expectedly geometry too, of the space time volume. Otherwise stated, the mass interacts with the space time and modifies its properties.

2) Despite the formal way to infer the eqs (5,2) and (5,1), ε_0 must be related to the corresponding $E = h\nu$: this conclusion introduces the corpuscle/wave dual behavior of matter.

It is immediate to show how are linked the wavelength λ of the wave and its propagation rate v . Divide side by side the third eqs (5,1) and (5,2); this yields

$$\frac{\varepsilon_0}{E} = \frac{\lambda_c}{\lambda_C} = \frac{c}{v} \quad \lambda_C = \frac{h}{mc} \quad \lambda_c = \frac{c}{\nu} ,(5,3)$$

having introduced the arbitrary velocity v by dimensional reasons. Moreover, rewriting identically $\varepsilon_0/E = mc^2 v/Ev$ and recalling that mv is momentum according to the eq (4,1), the position $mv = \vartheta p$, via the proportionality costant ϑ , one also finds

$$p = \varepsilon \frac{v}{c^2} \quad \varepsilon = \frac{\mathrm{n}E}{\vartheta} \quad \mathrm{n} = \frac{\varepsilon_0}{E} = \frac{c}{v} = \frac{V_0}{V} .(5,4)$$

These equations evidence the aforesaid link between the corpuscle and wave features of matter: writing explicitly the second equation as $\varepsilon v/c^2 = h\nu/\vartheta c$, the first equation reads

$$p = \frac{h}{\lambda} \quad \lambda = \frac{c\vartheta}{\nu} .(5,5)$$

The first equation is nothing else but the De Broglie momentum equation, inferred as a corollary together with the characteristic Compton length λ_C of m. This equation evidences the subtle link between corpuscular and wave formulation of momentum: p depends on m via ε_0; as this latter appears as ratio ε_0/E equal to c/v, which does not depend on m, the mass dependence turns into that of λ dependence.

3) The present approach merges typical quantum and relativistic concepts, i.e.: the dual behavior of matter, exemplified by the definitions of momentum of the eqs (4,1) and (5,5), and the space time/matter interaction together with the momentum eq ((5,4).

On the one hand, is crucial that the chance of defining an energy density η corresponding to E relies entirely in the frame of the proposed concept of space time only. On the other hand, the dual behavior of matter implies the position $\Delta x = \lambda$ i.e. λ is related to its corresponding delocalization range: the fact that h/λ is equivalent to $h/\Delta x$, i.e. the momentum expression is actually a way to rewrite the eqs (4,5), again emphasizes the link between De Broglie wavelength associated to the matter wave propagating throughout Δx and quantum delocalization of the corresponding corpuscle in the uncertainty range Δx.

Five remarks are useful on these conclusions:

(i) $\mathrm{n} > 1$, to be demonstrated in the next eq (7,3), requires $V_0 > V$; i.e. introducing the mass in V, a given η implies a greater volume V_0.

(ii) The eq (5,3) yields

$$\lambda_c = \mathrm{n}\lambda_C \quad \text{i.e.} \quad \lambda_c \geq \lambda_C;(5,6)$$

i.e. λ and thus Δx can be expressed as a function of the Compton length.

(iii) An interesting corollary of the eq (5,1) is shortly inferred noting that $\partial^2(c\nu)^2/\partial\nu^2 = v_0^2$; by dimensional reasons v_0 must be a constant velocity. Moreover put $\nu = v_0'/\lambda$, being $v_{0'}$ another constant velocity; one obtains in an analogous way $\partial^2(cv_0'/\lambda)^2/\partial(\lambda^{-1})^2 = v_0''^4$; hence, via a proportionality constant k such that $v_0^2 = v_0''^4 k$, one finds

$$\frac{\partial^2(c\nu)^2}{\partial\nu^2} = k\frac{\partial^2(cv_0'/\lambda)^2}{\partial(\lambda^{-1})^2} \quad k = \frac{v_0^2}{v_0''^4}.$$

Multiply now both sides by τ^4/τ'^4, being τ and τ' arbitrary constants having physical dimensions of $time$; then $c\nu\tau^2 = l_\tau$, being l_τ an appropriate length, whereas $\nu\tau'^2 = \Delta t$. In both cases the dynamical variables l_τ and Δt are simply introduced via the initial arbitrary variable ν times the dimensional constants $c\tau^2$ at numerator and τ'^2 at denominator. Operate in an analogous way at the right hand side; calling $cv'\tau^2/\lambda = l_v$ and $v'^2\tau'^2/\lambda = \Delta x$, the last equation reads

$$\frac{\partial^2 l_\tau^2}{\partial\Delta t^2} = kv'^4\frac{\partial^2 l_v^2}{\partial\Delta x^2} \quad l_\tau = c\nu\tau^2 \quad l_v = \frac{cv\tau^2}{\lambda}.$$

Hence follows the Dâ€™Alembert wave equation

$$\frac{\partial^2\psi}{\partial\Delta t^2} = v^2\frac{\partial^2\psi}{\partial\Delta x^2} \quad v^2 = \frac{v'^4 v_0^2}{qv_0''^4} \quad \psi = ql_v^2 = l_\tau^2.(5,7)$$

(iv) a further result can be inferred directly from the eqs (2,1) rewriting its right hand side as follows with the help of the eqs (5,3)

$$h\nu = \frac{h^2}{c^2\Delta x^2}\frac{G}{\Delta x} = m_1 m_2\frac{G}{k\lambda_C} \quad m^2 = k^2 m_1 m_2 \quad \Delta x = k\lambda_C;(5,8)$$

the last position requires that the factor k is arbitrary, as it relates Δx to the well defined value of λ_C. Owing to the eqs

(5,4) and (5,2), $\varepsilon_0 = Gm_1m_2\mathrm{n}/k\lambda_C$ reads introducing arbitrary numbers $q>0$ and $w>0$

$$\varepsilon = \mathrm{T}+U \quad \varepsilon=(w-q)\varepsilon_0 \quad \mathrm{T}=w\varepsilon_0 \qquad U=-G\frac{m_1m_2}{\Delta x'} \quad \Delta x'=\frac{k\lambda_C}{q\mathrm{n}};(5,9)$$

Now ε is in general any energy, independent of the initial ε_0 because of the factor $w-q$; the same holds for T because of w. Also, $\Delta x'$ is a new range still defined as a function of λ_C but in general different from Δx because of the factor $k/q\mathrm{n}$; indeed $\varepsilon \lessgtr 0$, i.e. the system is bound or not, depending on whether $w \lessgtr q$. This result shows that the initial $h\nu$ has been replaced by a new kind of energy governed by G.

The physical meaning of the function U will be clarified in the section 9.4. Note here the conceptual impossibility of determining whether m_1 or m_2 is the source of the gravitational field; clearly this means from a physical point of view that the concepts of inertial and gravitational mass are indistinguishable, i.e. they must necessarily coincide.

Also note that size and analytical form of $\Delta x'$ are arbitrary and unknown, whereas the definition of $\mathrm{n}=\varepsilon_0/h\nu$ prospects the possibility that $\mathrm{n}=\mathrm{n}(\Delta x')$ via ν; hence $\mathrm{n}\ge 1$ has in general the form of non-divergent series expansion like $\mathrm{n}=1+x_1/\Delta x'+x_2/\Delta x'^2+\dots$ via appropriate coefficients x_i. As $\mathrm{n}\to const$ for $\Delta x'\to\infty$, the Newton law holds in the particular case of small gravity fields at large distances only. Considering Δx or $(k/q\mathrm{n})\Delta x$ is physically irrelevant as concerns the ranges sizes themselves, which are in principle indefinable; it trivially means describing masses Δx apart or $\Delta x'$ apart. What is crucial instead is how U depends on the mutual distance between m_1 and m_2, schematically summarized by the following chances

$$U=U(x^{-1}) \quad U=U(\Delta x^{-1}) \quad U=U(\Delta x'^{-1}):(5,10)$$

in the Newtonian classical approximation holds the simple dependence $U=U(x^{-1})$, being x any local distance among that included in Δx, whereas in the quantum case the coordinate is replaced by its own uncertainty range. In the more general case, i.e. in a relativistic frame, this dependence is even more complex, as it appears from the series development of $\mathrm{n}(\Delta x')$: by necessity $U=U(\Delta x'^{-1})$ requires accounting for the space time deformation in the presence of m according to the eq (5,2). On the one hand this conclusion justifies the possibility of obtaining via the eqs (4,5) relevant results of the general relativity [17]; on the other hand this reasoning is further acknowledged considering the position $p=mv$ of the eq (4,1) here exploited. In the present context mv is not a classical definition of momentum, because the velocity is actually not defined itself for the reasons previously emphasized; so nothing hinders regarding $v=v'/\sqrt{1-(v'/c)^2}$, i.e. expressing the arbitrary and unknown v via another v' arbitrary and unknown as well. This more general form, actually required by the eq (3,1) [16], could be expressed as series expansion of v around $v'=0$, which shows that the classical approximation $v'\approx v$ holds for $v'<<c$ only and that the relativistic results correspond to and can be expressed by appropriate series expansions of non-relativistic outcomes.

In general it is possible to write $\Delta x'$ in a form that reduces to the plain classical range Δx for weak fields, i.e. for $\Delta x\to\infty$, and for $c\to\infty$; a typical form of series expansion fulfilling these requirements could be for example

$$\Delta x'=\frac{\Delta x}{1+\sum_{j=1}(a_j/c)(\Delta x+b)^{-j}}.(5,11)$$

v) Write the eq (2,1) as $V=hG/c^2\nu$ and consider that according to the eq (5,2) it turns into $V_0=hG/c^2\nu_0$ if mass is present in the space time. As the steady waves allowed in V and in V_0 must have different wavelengths, because anyway their nodes must must be at the boundaries of the volume containing them, one infers that the volume size change due to the mass implies $\lambda_0>\lambda$. This change involves both the wave frequency, because $V_0\nu_0=V\nu$ requires $\nu_0<\nu$, and the length of the time range Δt necessary to complete one wave cycle; indeed defining $\nu=\lambda/\Delta t$, the mass implies $\Delta t_0>\Delta t$. So, one infers qualitatively that the presence of mass implies red shift of a wave and time dilation; this can be due to nothing else but the gravity field created by the mass.

To get quantitative information on these statements, note that the eq (5,9) suggests defining

$$\varphi=\frac{U}{m_2}=-\frac{m_1G}{\Delta x'}$$

as the property of the gravitational mass of interest to describe the change $V\rightarrow V_0$. Note that this definition of φ holds regardless of the value of m_2; hence it holds even for a photon moving in the gravitational field of m_1. In particular, expressing m_2 according to the eq (5,4) as $m_2=\mathrm{n}h\nu/c^2$ and putting for simplicity $\mathrm{n}=1$, it possible to describe one photon moving in the vacuum in the gravitational field of m_1. Keeping the same notation, one finds

$$\frac{U}{h\nu}=-\frac{m_1G}{\Delta x'c^2}=\frac{\varphi}{c^2}\qquad \varphi=-\frac{m_1G}{\Delta x'}.$$

Since m_1G/c^2 is a length, put therefore

$$\frac{U}{h\nu}=-\frac{l^o}{\Delta x'}=\frac{\varphi}{c^2}\qquad l^o=\frac{m_1G}{c^2};$$

hence, differentiating φ with respect to $\Delta x\hat{a}$, this expression reads

$$\frac{\delta\varphi}{c^2}=\frac{l^o\delta\Delta x'}{\Delta x'^2}=-\frac{U\delta\Delta x'}{h\nu\Delta x'}.$$

Note that because of dimensional reasons $U\delta\Delta x'/h\Delta x'$ is to be regarded as a frequency change; so

$$\frac{\delta\varphi}{c^2}=-\frac{\delta\nu}{\nu},$$

i.e. $\delta\nu=U\delta\Delta x'/h\Delta x'$ is the sought frequency change related to $\delta\varphi/c^2$.

This is the well known red shift of a photon moving in a gravitational field.

In a completely analogous way is calculated also the time dilation $\Delta t_0>\Delta t$ related to $V_0>V$ in the presence of m_1.

6 THE ENERGY STATES

Multiply both sides of the eq (2,1) by h/m; one finds $h^2G/mc^2=h\nu/\rho$, being of course $\rho=m/V$. Recalling the eqs (5,1), (5,2) and (5,5), this result reads

$$\rho Gh^2=(mc^2)(h\nu)=\mathrm{n}(h\nu)^2=\frac{(mc^2)^2}{\mathrm{n}}\qquad(6,1)$$

and shows that $h\nu$ and mc^2 are in principle both compatible with negative values of m and ν, i.e. with the existence of negative energy states $h\overline{\nu}$ and $\overline{m}c^2$. In fact this conclusion does not conflict with the eqs (2,1), (2,2) and (4,5), provided that some specifications are made about m and ν. As concerns the eqs (2,1), is evident the requirement $V\nu>0$. Therefore $V\nu=\Delta x^3\nu$ shows that the inequality is fulfilled simply changing $\Delta x\rightarrow-\Delta x$ when $\nu\rightarrow\overline{\nu}$. On the other hand, according to the eqs (2,2) the second position requires simply $\Delta t\rightarrow-\Delta t$: indeed the sign of $\Delta x\Delta t$ defining g remains unchanged. More shortly, the first eq (2,2) is clearly unaffected changing the signs of both time and space ranges including them. As concerns the sign change inherent $m\rightarrow\overline{m}$, it appears that the uncertainty equation $\Delta x\Delta p_x$ is unaffected itself because $\overline{m}$ implies negative local values of p_x; these values require being included in a negative uncertainty range $-\Delta p_x$, so that $\Delta x\Delta p_x\equiv(-\Delta x)(-\Delta p_x)$. An analogous reasoning holds for the energy: negative energy states are included in a negative range $-\Delta\varepsilon$, which however yields the identity $(-\Delta\varepsilon)(-\Delta t)\equiv\Delta\varepsilon\Delta t$. In conclusion the negative energy states consistent with the eq (6,1) require simply a backwards running time coordinate and a mirror space coordinate, which in practice merely exchanges left and right. Before considering a third condition, also necessary, multiply both sides of the eq (2,1) by $h/\Delta x^2$; being $V=\Delta x^3$, are admissible both chances $(hG/c\Delta x)^2/G = h\nu\Delta x$ and $(hG/c\Delta x)^2/G = h\overline{\nu}(-\Delta x)$. Multiply again these equations by $|\Delta x|^{-1}$: summing up first and subtracting next side by side the resulting equations, one finds respectively $0 = h\nu + h\overline{\nu}$ and $2(hG/c\Delta x)^2/G\Delta x = h\nu - h\overline{\nu}$. Since $(hG/c\Delta x)^2/G\Delta x$ is in fact an energy, call it ε , the conclusion is $h\nu - h\overline{\nu} = 2\varepsilon$ and $h\nu + h\overline{\nu} = 0$.

The implications of this result concern of course both mc^2 and $h\nu$.

As regards the former, the chance of writing

$$mc^2 - \overline{m}c^2 = 2\varepsilon_0 \quad (m+\overline{m})c^2 = 0 \quad mc^2 + |\overline{m}|c^2 = 2mc^2 \, (6,2)$$

means that the energy gap between positive and negative states ε and $\overline{\varepsilon}$ is 2ε and that particles and antiparticles allowed to interact annihilate their mass releasing the energy 2ε. The mass balance of the second equation is closely related to the energy balance of the third equation, the notations have an analogous physical meaning: the former emphasizes the annihilation of the masses, but hides the energy contextually released; the latter emphasizes energies and equivalent masses of particle and antiparticle explicitly introduced and individually regarded, but skips mentioning their mutual annihilation when interacting. According to the third equation, the antimatter is described by a separate class of antiparticles here symbolized by $\overline{m}$.

As regards $h\nu$, rewrite the second eq (5,4) noting that $\mathrm{n} > 1$ can be certainly expressed via the fine structure constant α as $\mathrm{n} = \mathrm{n_o}\alpha$, being $\mathrm{n_o}$ an arbitrary number subjected to the condition $\mathrm{n_o} \ge \alpha^{-1}$ only. In this way $\mathrm{n}h\nu = \mathrm{n_o}e^2/\lambda$, of course with $\lambda = c/\nu$, implies also $\mathrm{n}h\overline{\nu} = -\mathrm{n_o}e^2/\lambda$. The minus sign is interpretable in two ways. It can be due to $-\mathrm{n_o}$ i.e. to a negative refraction index n, which in fact has been experimentally observed in particular metamaterials at selected light frequencies. Moreover it is also compatible with $\mathrm{n_o}\overline{e}^2/(-\lambda)$, in agreement with the fact that the negative energy states require $\Delta x \to -\Delta x$. Eventually the sign even regards $-e^2 = e\overline{e}$. The last result is interesting because it suggests that the antimatter requires the position $\overline{e} \to -e$ for the charge; in this way, writing e^2 or $-e\overline{e}$ does not change the absolute value or the sign of α . More shortly, all this appears considering $\alpha hG/c^2 = e^2G/c^3$; since $G/c^3 = l_{Pl}^2/h$ by definition, one finds thus $\alpha hG/c^2 = (l_{Pl}e)^2/h = (l_{Pl}\overline{e})^2/h$. Again the position (1,1) yields a positive value corresponding to a square quantity, where now appears the charge. Writing thus in summary

$$\alpha\frac{hG}{c^2} = \frac{(l_{Pl}e)^2}{h} = \frac{(-l_{Pl}\overline{e})^2}{h} = \alpha V\nu \quad mc^2 = \frac{e^2}{\lambda'} = -\frac{e\overline{e}}{\lambda'} = \mathrm{n}h\nu \quad \overline{m}c^2 = \frac{e\overline{e}}{\lambda'} = \mathrm{n}h\overline{\nu} \quad \lambda' = \frac{\lambda}{\mathrm{n_o}}, (6,3)$$

on the one hand one infers the Coulomb law with repulsive or attractive energies $e \leftrightarrow e$ or $e \leftrightarrow \overline{e}$; on the other hand either choice describes a charged or neutral system, because the total charge in V corresponding to e^2 is $2e$ whereas $-e\overline{e}$ implies a neutral system. To confirm this conclusion, implement the fact that the forbidden gap 2ε can be overcome by interaction with photons of appropriate energy: the charge conservation during the annihilation of a possible couple of charged particles and antiparticles, requires opposite charges for these latter in agreement with the null total charge before and after their mutual annihilation. Of course has been waived here for brevity the fact that a third particle, e.g. a nucleus, must be also present to fulfill the conservation laws during the transition from negative to positive energy states: the opposite charges of particles and antiparticles are the crucial point to be emphasized. These considerations, based uniquely on the eqs (2,1) and (4,5), will be further implemented in this paper.

The conclusion is that matter and antimatter can be regarded in an equivalent way, and thus their physics is indistinguishable under these three concurring changes, in agreement with the well known CPT theorem. The present reasoning, although well known, has been carried out merely to stress that even the negative energy states and the CPT theorem are compatible with and inferable from the eq (2,1).

7 QUANTUM UNCERTAINTY AND DIFFERENTIAL CALCULUS

So far preliminary results have been obtained via elementary manipulations of the eq (2,1) only, without need of differential calculus. As the mathematical approach requires taking into account the corollary eqs (4,5), i.e. that delocalization ranges of the dynamical variables systematically replace the respective local dynamical variables, this section concerns the way to regard the differential calculus in agreement with the agnostic concept of quantum uncertainty governing the space time: in a theoretical model disregarding the point coordinates, the concept of local increment of a dynamical variable is useless because the new derived function would be undefinable itself in the incremental point likewise the original function.

Nevertheless the definition (1,1) of space time was adequate to infer the gravity potential in the presence of mass even without introducing the local space time curvature, though with the conceptual limitation that $U(\Delta x)$ of the eqs (5,9) is actually not definable as a local value: the definition of potential energy at a given distance Δx from the source, wherever it might be, is unavoidably replaced by that of potential energy field within an uncertainty range Δx including all possible distances. Moreover appeared in the eq (5,1) the necessity of introducing the volume $V_0 \neq V$ when calculating in (5,2) the energy density η as a function of the corpuscle energy mc^2 instead of the wave energy $h\nu$: so even in this agnostic context still emerges the idea that the mass modifies the geometry of the space time.

This is in effect the leading edge of the present approach: the physics of the events is essential, not its mathematical formalism.

While in the present context the tensor calculus is in fact ineffective, the most intuitive way of defining differentials with the same features of the uncertainty ranges is to implement the ranges themselves: likewise as the eqs (4,5) waive specifying any particular reference system, even their ratios are disconnected from the choice of a particular R. So the concept of uncertainty in its most agnostic form introduces in general the derivatives as ratios of uncertainty ranges and thus in the frame of the eq (2,1) only.

An example is v_x of the eq (2,2), which is a mere average quantity related to the time range Δt necessary for any particle to travel the space range Δx. The consequence already emphasized is that v_x defined in this way is conceptually unknown and unknowable; details about its local value at any x within Δx and t during Δt are unaccessible. Nevertheless, valuable information is obtained even via this agnostic standpoint. Consider the eq (5,4); replacing $v = \Delta x/\Delta t$ one finds $p\Delta t = \varepsilon\Delta x/c^2$. This result is particularly significant in the case of a system consisting of several particles, for simplicity assumed non-interacting. With vector notation this result reads $p_j\Delta t = \varepsilon_j\Delta r_j/c^2$ for the j-th particle. Summing over j, one finds thus $c^2\Delta t\sum p_j = \sum\varepsilon_j\Delta r_j$. Normalizing both sides with respect to the total energy $\sum\varepsilon_j$, this equation defines

$$v = \frac{c^2\sum\Delta p_j}{\sum\varepsilon_j} \qquad R = \frac{\sum\varepsilon_j\Delta r_j}{\sum\varepsilon_j};$$

these are the relativistic equations of the center of mass of the system of particles.

It has been highlighted that V of the eq (2,1) is subjected to change as a function of time; this means that the size Δx introduced in the eq (2,2) to define V at an arbitrary time t_0 in fact changes at a later time t_1 by $\delta\Delta x$. Of course the eqs (4,5) require that the conjugate range Δp_x changes by $\delta\Delta p_x$ too; it must be still true that $(\Delta x + \delta\Delta x)(\Delta p_x + \delta\Delta p_x) = nh$. As in fact $\delta\Delta p_x$ and $\delta\Delta x$ are correlated, the connection between force field and space time deformation is immediately acknowledged: altering the extent of space delocalization of a hypothetical particle or wave during the time range increment $\delta\Delta t$ means affecting all allowed local values p_x and thus the rising of a force field $\delta\Delta p_x/\delta\Delta t$ throughout $\Delta x + \delta\Delta x$. The corresponding classical local force would be of course $\delta p_x/\delta t$.

Note that $\delta\Delta x$ is identically definable as $\delta\Delta x = (x + \delta x) - x_0$ or $\delta\Delta x = x - (x_0 - \delta x)$, which however imply a subtle but crucial physical difference. Suppose that x_0 is the coordinate defining the position of Δx in R at $t = t_0$: the former way to define $\delta\Delta x$ with x_0 fixed means that Δx is at rest in R, i.e. its deformation is obtained stretching or shrinking its upper boundary only; the latter way with mobile x_0 and fixed x implies instead that actually Δx deforms and displaces in R. These are the points of view of two observers sitting on x_0 and x. Since anyway $\delta\Delta x$ during $\delta\Delta t$ implies $\delta\Delta p_x$, and thus the change of the conjugate momenta allowed to a particle delocalized in Δx, this means that are indistinguishable the situations where the momenta change because of a force interacting on the particle/wave or because Δx merely accelerates in R. The full reasoning, described in [15] and omitted here for brevity, shows that one of the fundamental postulates of the general relativity is actually a corollary of the quantum uncertainty. It is of interest here that, owing to the arbitrariness of Δx and $\delta\Delta x$, in general the incremental range size is independent on the initial range size. Nevertheless it is also possible to write $\delta\Delta x = k\Delta x$ and $\delta\Delta t = k'\Delta t$ via the arbitrary and independent factors k and k'. So

$$\frac{\delta\Delta x}{\delta\Delta t} = k''\frac{\Delta x}{\Delta t} \qquad k'' = \frac{k}{k'}(7,1)$$

If in particular $k = k'$, then $\delta\Delta x/\delta\Delta t = \Delta x/\Delta t$; thus the definition of velocity in the eqs (4,1) is a particular case of a more general definition $v = \delta\Delta x/\delta\Delta t$, while being anyway not specifiable v and its pertinent R. By consequence integrating an equation means that the range sizes Δx or $\delta\Delta x$, both arbitrary by definition, are so small with respect to x_0 to be treated as a usual differential dx. However even this latter is actually a small sized uncertainty range, for which hold therefore all of the previous considerations. So appears more appropriate the notation $\delta x = x - x_0$ for an uncertainty range with $x - x_0 << x$ or $x - x_0 << x_0$.

For instance the eqs (4,5) yield $\Delta\dot{p}_x = -nh\Delta\dot{x}/\Delta x^2$, being by definition

$$\Delta\dot{p}_x = \frac{\delta\Delta p_x}{\delta\Delta t} \qquad \Delta\dot{x} = \frac{\delta\Delta x}{\delta\Delta t}.(7,2)$$

The eqs (7,1) have two interesting consequences, related to the physical dimensions of S of the eqs (4,3) and to the eqs (7,2).

(i) Calculate the change $\delta\mathrm{S}$ of S during the time range change $\delta\Delta t$; putting $V/\Delta x = A > 0$ and reasoning on the physical dimensions of S, one finds $\delta\mathrm{S} = m\delta A/\Delta t - mA\delta\Delta t/\Delta t^2$. Hence

$$\frac{\delta\mathrm{S}}{\delta\Delta t} = 2\frac{m\delta A}{\delta\Delta t^2} - \frac{mA}{\Delta t^2} \qquad A = A(\Delta t).$$

Regard A as a dynamical variable. If it is constant or corresponds to a maximum/minimum as a function of Δt, the energy $\delta\mathrm{S}/\delta\Delta t$ is negative because $\dot{A} = \delta A/\delta\Delta t = 0$ vanishes; moreover, is expectable in general that $\delta A/\delta\Delta t \neq 0$ even at values of $\Delta t^{\S}$ where $A(\Delta t^{\S}) = 0$. This suggests that the function $\delta\mathrm{S}/\delta\Delta t = -\mathrm{H}$ describes a bound system where the energy $2m\delta A/\delta\Delta t^2 = \mathrm{T}$ governed by the dynamical variable $\dot{A}$ represents the kinetic term, whereas the energy $mA/\Delta t^2 = U$ governed by the dynamical variable A represents the potential term. So the conclusion is

$$\frac{\delta\mathrm{S}}{\delta\Delta t} = -\mathrm{H} = \mathrm{T} - U(7,3)$$

(ii) Owing to the eqs (4,5), the first eq (7,2) yields $\Delta F = -nh\Delta\dot{x}/\Delta x^2$. If in particular $\Delta\dot{x} = \pm const$, then the last result reads $\Delta F = \pm const/\Delta x^2$, as the deformation of Δx can be due in principle to its shrinking or stretching. For example $const = e_1 e_2$ means that ΔF is related to repulsive or attractive force field between charges, as already found in the eq (6,3): ΔF at left hand side accounts for all possible local values of F corresponding to each x included in Δx. Only if Δx is small sized, then ΔF takes the limit meaning of "quasi-local" force. As $\Delta\dot{x}$ is in general not constant, it can be certainly written as series expansion of Δx; e.g.

$$\Delta\dot{x} = \frac{c}{const + \sum_j a_j(\Delta x + b)^{-j}},(7,4)$$

being a_j and b appropriate coefficients. In this way $\Delta\dot{x}$ reduces to a constant for $\Delta x \to \infty$, whereas ΔF takes in this particular case of weak field the standard form of the Coulomb force. Clearly hold also now all considerations made about the eqs (5,11) and (5,10), where the series expansion has an analogous physical meaning of relativistic generalization.

From now on, the symbol δ indicates the change of the concerned quantity; for brevity however in the following the derivative will be sometime indicated with the usual notation ∂. The question that arises now is: are these positions physically verifiable and productive? The next two sections examine just this question.

8 CLASSICAL COROLLARY OF THE SPACE TIME UNCERTAINTY

An interesting corollary of the eqs (4,5) is easily inferred for $n \to \infty$; in this case $\delta n = n - n_1$, even though still arbitrary integer positive by definition, can be regarded as a differential for n tending to infinity. Differentiating the right hand side of the eqs (4,5), one finds $\Delta\varepsilon\delta\Delta t + \Delta t\delta\Delta\varepsilon = h\delta n$; moreover, replacing in this result $\Delta\varepsilon$ and Δt via nh and squaring both sides, trivial manipulations yield $(\delta\Delta t/\Delta t + \delta\Delta\varepsilon/\Delta\varepsilon)^2 = \delta n^2/n^2$. Since $\delta n \ge 1$ is independent on the current n, i.e. any given δn can be obtained simply determining appropriately n_1, write $(\delta\Delta t/\Delta t + \delta\Delta\varepsilon/\Delta\varepsilon)^2 \ge 1/n^2$ and thus $\delta\Delta t/\Delta t + \delta\Delta\varepsilon/\Delta\varepsilon = \beta$ for $n \to \infty$: here $\beta >: 0$ is a value close to zero for large n and tending to zero for $n \to \infty$. Hence the last result reads $\delta\log(\Delta t/\Delta t_0) + \delta\log(\Delta\varepsilon/\Delta\varepsilon_0) = \beta$ and thus $\delta\log(\Delta t\Delta\varepsilon/\Delta t_0\Delta\varepsilon_0) = \beta$, which requires $\Delta t\Delta\varepsilon >: \Delta t_0\Delta\varepsilon_0$. If $\Delta t_0\Delta\varepsilon_0 = h$, i.e. if it represents the minimum uncertainty consistent with the eqs (4,3), then $\beta \to 0_+$ requires

$$\Delta t\Delta\varepsilon >: h \qquad \Delta x\Delta p_x >: h;(8,1)$$

indeed an identical reasoning holds considering Δx and the conjugate Δp_x.

An interesting consequence of the first eq (8,1) is obtained multiplying both sides of the inequality by ν; being $\nu\Delta t = q$ an arbitrary dimensionless number, one infers $q\Delta\varepsilon >: h\nu$. As the eq (2,1) implies two kinds of energy, $h\nu$ and mc^2 of the eqs (5,1) and (5,2), it is sensible to regard this inequality as $q\Delta mc^2 >: h\nu$; calling $q\Delta m = m' - m_{0'}$, the result $m'c^2 - m_{0'}c^2 >: h\nu$ reads thus

$$m'c^2 > h\nu,(8,2)$$

valid for any m', unless $m'_0 << m'$ in which case still holds $>:$. So for matter holds $\mathrm{n} > 1$, i.e. necessarily the matter particles move at rate $v < c$, as in effect it has been done in the eqs (5,2) and (5,5). Write in general the eq (8,2) as

$$mc^2 = h\nu + h\nu' \qquad \nu' > 0,(8,3)$$

being ν' an arbitrary frequency for which hold of course the same considerations introduced in the eq (2,2) for ν.

In conclusion, this well known formulation of the uncertainty is nothing else but the classical limit of the more general statistical quantum formulation eq (4,5). There is however a crucial difference between the eqs (4,5) and (8,1): the former enable finding quantum eigenvalues and relativistic equations, the latter are useful as boundary conditions to help solving specific problems [18].

9 UNCERTAINTY AND GENERALIZED COORDINATES

This section highlights that in the present theoretical context: (i) the uncertainty ranges play the same role of generalized coordinates, (ii) the space time is governed by quantum and relativistic laws, (iii) the concept of derivative is successfully regarded and expressed as mere ratio of uncertainty range sizes.

The following four examples highlight shortly that these hints and their physical implications are direct consequences of the position (1,1).

9.1 LAGRANGE EQUATIONS

Implement the eqs (4,5) considering the chance that the range sizes change during a short time lapse $\delta\Delta t$. Owing to the eqs (7,1) and (7,2) the uncertainty provides the following equivalent equations

$$\Delta\varepsilon = \Delta p_x \frac{\Delta x}{\Delta t} = \Delta p_x \Delta\dot{x} \qquad \Delta\varepsilon = \Delta x \frac{\Delta p_x}{\Delta t} = \Delta x \Delta\dot{p}_x.(9,1)$$

The possibility of defining $\Delta\dot{x}$ independently of Δx and $\Delta\dot{p}_x$ independently of Δp_x allows inferring from the eqs (9,1) $\delta\Delta\varepsilon/\delta\Delta\dot{x} = \Delta p_x$ and $\delta\Delta\varepsilon/\delta\Delta x = \Delta\dot{p}_x$: differentiating both sides, the former equation relates $\delta(\delta\Delta\varepsilon/\delta\Delta\dot{x})$ to $\delta\Delta p_x$

, the latter relates $\delta(\delta\Delta\varepsilon/\delta\Delta x)$ to $\delta\Delta\dot{p}_x$. Then dividing these changes by $\delta\Delta t$, one finds

$$\frac{\delta}{\delta\Delta t}\frac{\delta\Delta\varepsilon}{\delta\Delta\dot{x}}=\frac{\delta\Delta p_x}{\delta\Delta t}.$$

Despite its notation, $\delta\Delta t = t_1' - t_0'$ is not mere mathematical differential, but a small alteration of Δt; it is thus a short time lapse, but still an uncertainty range, during which the quantity $\delta\Delta\varepsilon/\delta\Delta\dot{x}$ is allowed to change by $\delta(\delta\Delta\varepsilon/\delta\Delta\dot{x})$; hold therefore for t_1' and t_0' all previous considerations about the eqs (4,5). Moreover, the second equation (9,1) yields

$$\frac{\delta\Delta\varepsilon}{\delta\Delta x}=\frac{\Delta p_x}{\Delta t}.$$

Owing to the eq (7,1), put $\Delta p_x/\Delta t = \delta\Delta p_x/\delta\Delta t$ without loss of generality because of the arbitrariness of both Δp_x and its change $\delta\Delta p_x$. As an identical reasoning holds for $\delta\Delta t$ and Δt too, combining these equations the result is

$$\frac{\delta}{\delta\Delta t}\frac{\delta\Delta\varepsilon}{\delta\Delta\dot{x}}=\frac{\delta\Delta\varepsilon}{\delta\Delta x}.(9,2)$$

Thinking the symbol δ likewise as the usual ∂, this result is nothing else but the Lagrange equation as a function of the generalized coordinate ranges Δx and $\Delta\dot{x}$; it is well known that this result is fulfilled specifying in particular $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1 = \mathrm{T} - U$, where of course T and U are now functions of Δx and $\Delta\dot{x}$ replacing local generalized coordinates and velocities. This point has been concerned in the eq (7,3).

The chance of obtaining the Lagrange equations as straightforward corollary of the eqs (4,5) shows that the uncertainty ranges surrogate successfully the generalized dynamical variables of the classical mechanics: however the physical worth of this result is that the the former fulfill the Heisenberg principle by definition, the latter of course do not. For this reason the present conceptual frame moves the physical interest from the local dynamical variables, unknown and unknowable, to their uncertainty ranges, which are in fact related to the eigenvalues of quantum systems.

This explains how to make consistent classical and quantum mechanics simply replacing the local dynamical variables with the respective uncertainty ranges.

Of course nothing compels just this way only of defining $\Delta\varepsilon$, which actually is not specifiable by definition; in agreement with the concept of uncertainty, $\Delta\varepsilon$ is a mere range in principle referable to and thus including any kind of local energy. Showing this point is just the purpose of the next example, where $\Delta\varepsilon$ is regarded and implemented in a different way; the aim is to highlight how an apparently different result can be once more obtained via the eq (2,1) and the corollary eqs (4,5) only.

9.2 HYDROGENLIKE ATOMS AND LIGHT BEAM BENDING

Consider a quantum system governed by a central potential having the attractive form $-\vartheta/\Delta r$ with $\vartheta > 0$, where Δr is the radial distance between the interacting partners at any assigned time. The classical Hamiltonian inferable from the eqs (9,2) reads $\Delta\varepsilon = \Delta p^2/2m - \vartheta/\Delta r$, being now m the reduced mass of the system; Δp is the momentum change with respect to the state where the particles do not interact, e.g. when $\Delta r \to \infty$. Write thus $\Delta\varepsilon = \Delta p_r^2/2m + \Delta \mathrm{M}^2/2m\Delta r^2 - \vartheta/\Delta r$, which putting $\Delta p_r = nh/\Delta r$ reads identically

$$\Delta\varepsilon=\left(\frac{\Delta p_r}{\sqrt{2m}}-\frac{\sqrt{2m}\vartheta}{2nh}\right)^2+\frac{\Delta \mathrm{M}^2}{2m\Delta r^2}-\frac{\vartheta^2 m}{2(nh)^2};(9,3)$$

$\Delta\varepsilon$ results as the difference of two parts: the sum of the first two addends is certainly positive, the only negative energy term is the third that therefore represents the energy gain ensuring the stability of the system. So it is natural to regard $\Delta\varepsilon = \varepsilon_{nb} - \varepsilon_b$, where ε_{nb} includes all positive terms whereas ε_b is just the binding energy. Minimize the non-bonding part of $\Delta\varepsilon$ putting equal to zero the term in parenthesis; this means minimizing ε_{nb} with respect to all of the possible r included in Δr to ensure the maximum stability of the bound state. The eqs (4,5) yield then

$$\Delta p_r = \frac{m\vartheta}{nh} \quad \Delta r = \frac{(nh)^2}{m\vartheta} \quad \varepsilon_b = -\frac{\vartheta^2 m}{2(nh)^2}. (9,4)$$

It is possible in particular that

$$\vartheta = Z_1 Z_2 e^2 \quad \text{or} \quad \vartheta = Gm_g m = Gm_g E/c^2. (9,5)$$

(i) If the potential has the Coulomb form, one recognizes in the last term (9,4) the energy $-Z^2e^4m/2(nh)^2$ of the Bohr hydrogenlike atom with a nucleus of charge Ze, whereas $\Delta r = (nh)^2/Ze^2m$ is just the Bohr radius; this confirms that effectively the number n of allowed quantum states surrogates the hydrogenlike quantum number, which is indeed by definition an arbitrary and unspecifiable integer. As previously stated, the physical interest does not concern details of the motion of m, but the ranges allowed to its dynamical variables. It would be really trivial to prove that the residual positive term of the eq (9,3) leads to steady orbital radii if $\Delta \mathrm{M}^2 < (nh)^2$ and thus $l < n$ in order to have a negative energy balance [13].

(ii) Let ϑ represent now the gravitational interaction between a central mass m_g and a moving mass m, which of course can be introduced via its equivalent energy according the the eq (5,4), to describe a gravitational quantum system still implementing the eqs (9,4) and (9,5). Write

$$\Delta r = \frac{(nh)^2}{(Gm_g/c^2)(mc)^2} = \frac{(n\lambda_C)^2}{Gm_g/c^2}$$

recalling the definition of Compton length. Put $n^2 = n_\varphi n_r$; despite n is arbitrary integer, n_φ and n_r are in general any real numbers. The last equation reads identically

$$\frac{\lambda_\varphi}{\Delta r} = \frac{Gm_g}{\lambda_r c^2} \quad \lambda_r = n_r \lambda_C \quad \lambda_\varphi = n_\varphi \lambda_C.$$

Likewise as in the eq (5,6), n_φ and n_r define the lengths λ_φ and λ_r. Here m does no longer appear explicitly; appear instead the central gravitational mass m_g and the characteristic length Gm_g/c^2 only. So, owing to the physical meaning of Δr defined in the previous example, this equation can describe even the behavior of a light beam under the gravitational field of m_g: likewise as the electron is delocalized around the nucleus, let the photon be delocalized around m_g.

Consider an arbitrary point r_0 on a sphere of radius Δr centered on m_g, and let λ_φ be the distance traveled by the photon from r_0 along an arc $\delta s = \Delta r \delta\varphi$ of circumference. If so, then it is necessary to put $\delta s = 2\lambda_\varphi = (r_0 + \lambda_\varphi) - (r_0 - \lambda_\varphi)$: the former addend corresponds to a clockwise displacement of the photon from r_0, the latter to a counterclockwise displacement. These displacements are indistinguishable and equiprobable, thus both concurring to δs compatibly with a unique travel distance λ_φ. So $\lambda_\varphi = \Delta r \delta\varphi/2$ implies scaling appropriately λ_r as well. For this latter to be consistent with both symmetric displacements λ_φ, put $\lambda_r = \Delta r/2$ in order that $\delta\varphi$ be uniquely defined as $\delta\varphi = \delta s/\Delta r$ and as $\delta\varphi = \lambda_\varphi/\lambda_r$ too; both read indeed $2\lambda_\varphi/\Delta r$. Hence the previous equation becomes

$$\delta\varphi = 4\frac{Gm_g}{c^2 \Delta r}. (9,6)$$

Is clear now the notation: λ_φ concerns the angular displacement corresponding to $\delta\varphi$, whereas λ_r can be nothing else but the radial distance of the photon from m_g. Indeed $\delta\varphi \to 0$ for $\lambda_r \to \infty$, as it must be.

It is worth noticing that is immaterial the size of Δr, which is an uncertainty range about which nothing in knowable; in effect it symbolizes arbitrary distances of the photon from m_g depending on its size. Instead the factor 4 is physically essential: it introduces physical information about the indistinguishability of the situations where the photon coming from minus infinity goes to infinity after being deflected or comes from infinity and points to minus infinity. In this sense, writing Δr or $\Delta r/4$ is profoundly different.

Since the angular deviation $\delta\varphi$ is equal to the angle between the tangents to the circle at the boundaries $r_0 - \lambda$ and $r_0 + \lambda$ of δs, this result yields the deflection angle of a light beam passing at distance λ_r from m_g. This result is in principle approximate, as the trajectory of a photon in proximity of m_g is actually different from a circular section; in practice this approximation is the same as that characterizing the Einstein result, as it is evident since the final formula is identical. Yet the result is satisfactory, as it is well known, because any curve can be approximated by the osculating circle. This is especially true for small deflection angles.

This well known result and the chance of inferring the equations of the gravitational waves too [19] confirm the validity of the positions in the eqs (9,4) and (9,5), suitable to calculate both the hydrogenlike energy levels and the light beam bending.

9.3 MOMENTUM AND ENERGY OF A FREE PARTICLE

Consider now the second eq (9,1) and implement the arbitrariness of all range sizes to consider in particular $\Delta x = x_1 - x_0$ in the case where $\Delta x << x_0$; so Δx can be regarded mathematically as a differential. Rewriting thus this equation with the usual notation $d\varepsilon = dx dp_x/dt = v_x dp_x$ and integrating this expression, one finds

$$\varepsilon = \int v_x dp_x = \int v_x (dp_x/dv_x) dv_x \quad \varepsilon = \varepsilon(v_x) \quad p_x = p_x(v_x).(9,7)$$

Define without loss of generality $\varepsilon = f_x p_x$, being $f_x = f(v_x)$ a suitable function of v_x to be found. Without preliminary information on this function, it could be $f_x = \sum_j a'_j v_x^j$ or $f_x = \sum_j a_j v_x^{-j}$, being $a'_j > 0$ and $a_j > 0$ appropriate coefficients of the series. Both expansions are examined considering for simplicity the first order terms only, i.e. $v_x^{\pm 1}$. Hence one obtains the chances

$$\varepsilon = p_x a' v_x \quad \text{or} \quad \varepsilon = p_x a/v_x.$$

These positions are easily acknowledged: the former corresponds to the classical $\varepsilon \propto v_x^2$ found to infer the eq (4,2), the latter to the relativistic eq (5,4). However it is instructive ignoring this information and proceed uniquely on the basis of either mathematical chance, i.e. simply guessing how in principle p_x might depend on v_x by integration of the eq (9,7).

Replacing the former chance in the eq (9,7), one finds

$$a' p_x v_x = \int v_x (dp_x/dv_x) dv_x \quad \varepsilon = \int v_x (d(\varepsilon a'^{-1} v_x^{-1})/dv_x) dv_x;$$

these equations are integrable in closed form and yield

$$p_x = C'_p v_x^{a'/(1-a')} \quad \varepsilon = C'_\varepsilon v_x^{1/(1-a')},(9,8)$$

being C'_p and C'_ε the integration constants.

Replacing next the second chance in the eq (9,7), one finds

$$p_x a v_x^{-1} = \int v_x (dp_x/dv_x) dv_x \quad \varepsilon = \int v_x (d(\varepsilon a^{-1} v_x)/dv_x) dv_x;$$

also these expressions are easily integrable in closed form, the solutions are

$$p_x = \frac{C_p^* v_x}{\sqrt{v_x^2 - a}} \quad \varepsilon = \frac{C_\varepsilon^*}{\sqrt{v_x^2 - a}}.$$

with C_p^* and C_ε^* new integration constants. To hold even in a reference system where $v_x = 0$, let these constants be imaginary because $a > 0$. Putting thus $C_p^* = im\sqrt{a}$ and $C_\varepsilon^* = imc^2\sqrt{a}$, these solutions read

$$p_x = \frac{mv_x}{\sqrt{1 - v_x^2/a}} \qquad \varepsilon = \frac{mc^2}{\sqrt{1 - v_x^2/a}};(9,9)$$

this also shows that v_x is upper bound.

It would be trivial to demonstrate that the eqs (9,9) are the relativistic generalizations of the respective eqs (9,8) putting $a' = 0.5$; also, with the given choice of the integration constants, mc^2 of the eq (5,2) is the expected limit of ε for $v_x \to 0$. All this would also be inferable directly from the eq (3,1) [16]; for brevity these elementary considerations are here omitted, while emphasizing however once more the physical analogy between uncertainty ranges and generalized coordinates.

9.4 THE GRAVITY FORCE

The eqs (5,1) and (5,2) have shown that h and c of the eq (2,1) are linked to the energies $h\nu$ and mc^2, thus introducing the uncertainty eqs (4,5) and the Lagrange eqs (9,2) as well. It is sensible to expect that even G could be related to an appropriate form of energy. This point is elucidated in the next example, aimed to show how the gravity force is introduced contextually to the previous results and to the eqs (5,9); this example is significant, as it confirms that the way of explaining the result $V \neq V_0$ of the eq (5,2) is effectively correct.

The connection between force field and space time deformation has been already acknowledged in the section 7, i.e. considering the size changes $\delta\Delta p_x$ and $\delta\Delta x$ of the momentum and coordinate uncertainty ranges during the time range change $\delta\Delta t$. It is easy to confirm that $\delta\Delta x$ implies a force field by differentiating the eqs (4,5) and writing $\Delta F = -nh\Delta\dot{x}/\Delta x^2$, defined by $\Delta F = \delta\Delta p_x/\delta\Delta t$ and $\Delta\dot{x} = \delta\Delta x/\delta\Delta t$. Note that $\Delta\dot{x}$ has physical dimensions of velocity, i.e. it is the rate v with which displace the boundaries of Δx during $\delta\Delta t$; moreover, it is possible to replace h via the Planck mass $m_{Pl}^2 = hc/G$. Hence, let us write this result as $\Delta F = -nm_{Pl}^2 Gv/c\Delta x^2$, which reads more expressively as $\Delta F = -G(m_{Pl}\sqrt{nv_1/c})(m_{Pl}\sqrt{nv_2/c})\Delta x^{-2}$ with $\sqrt{v_1 v_2} = v$. Regard first $v = const$. Since the number n of states and the change rate $\Delta\dot{x}$ of the space time uncertainty range Δx are both arbitrary and of course independent each other, it is possible to write $m_{Pl}\sqrt{nv_1/c} = m_1$ and $m_{Pl}\sqrt{nv_2/c} = m_2$, being m_1 and m_2 two arbitrary masses. The result is thus $\Delta F = -Gm_1 m_2/\Delta x^2$, where ΔF is a force field including all local values $F = -Gm_1 m_2/x^2$ implied by the mutual distances x possible between m_1 and m_2 delocalized within Δx. This result shows that the gravity force is related to the deformation rate $\Delta\dot{x}$ of the space time uncertainty range Δx, which necessarily involves $\Delta\dot{p}_x$ as well; yet, since nothing is known about Δx, it is impossible to relate the local force F to the local curvature of the range Δx or to any kind of its actual variation. Nevertheless, despite the lack of information about Δx and its deformation, it is possible to guess intuitively that $\Delta\dot{x}$ represents the stretching rate of a line initially on a plane to a geodesic on a surface curved by the mass. Two interesting consequences are:

-Hold again for Δx the considerations carried out to explain that the Newton law of the eq (5,9) is actually the approximate form of a more complex potential fulfilling the requirements of the general relativity. Indeed nothing is known about $v_x = \delta\Delta x/\delta\Delta t$, so that it is reasonable to expect $v = v(\Delta x)$; since however $v \leq c$, is reasonable a non-divergent form of series expansion like $v = c/(x_0 + x_1\Delta x^{-1} + x_2\Delta x^{-2} + ...)$ tending to the constant value c/x_0 for $\Delta x \to \infty$. Clearly the series expansions affects ΔF, which now results multiplied by $x_0 + x_1\Delta x^{-1} + x_2\Delta x^{-2} + ...$ replacing the zero order constant term; as already found in the eqs (7,4) and (5,11), also here appears explicit the relativistic correction to the Newton law that becomes negligible for $\Delta x \to \infty$ only.

-The sign of F is due to that of ΔF: this range includes in fact negative values of force. This result, obtained and better explained in a previous paper [20], implicitly assumes $\Delta\dot{x} > 0$; yet there is no reason that actually this is the only chance. The initial size of Δx, whatever it might be, can change by stretching or shrinking; so that $\delta\Delta x$, and thus F, can take in principle both signs during $\delta\Delta t$. In effect the concept of anti-gravity has been already introduced in that paper depending on either sign allowed by the kind of deformation rate of Δx. In conclusion the last equality reveals that, as shown in the eq (5,8), $h\nu$ of the eq (5,2) is equivalent to a different form of energy governed by G.

-The Newton law in its standard attractive form implies the potential energy $-U$; this is the reason why in the eq (5,9) it was preliminarily introduced with the negative sign. However the last conclusion on the existence of anti-gravity force, if experimentally verified, is compatible even with positive potential energy.

Despite very shortly introduced, these examples emphasize some relevant evidences, i.e. there are no wave equations to be solved to find quantum results; moreover even relativistic results are contextually obtainable together with general equations of the particle mechanics, like the Lagrange equations. Eventually, the gravity enters in a natural way in the frame of quantum and relativistic results; yet these results introduce even the anti gravity in a natural way. The fact that specific problems of different character are compatible with typical outcomes of relativistic and quantum mechanics involving generalized coordinates, is not surprising; the paper [15] shows specifically that both theories are conceptually rooted in the eqs (4,5). In this context, both theories appear compatible with the initial assumption (1,1) even more fundamental than the space time uncertainty itself.

10 ENERGY DENSITY AND PRESSURE

The link between pressure and energy density is well known. Yet the next considerations are necessary not only to allow a self-contained exposition of the present model, but mostly to demonstrate that even this topic fits the properties of the space time still in the frame of the eqs (2,1). This is significant: the conclusions inferred below will elucidate in the next sections 14.3 and 15 further crucial topics, like the temperature of the cosmic background microwave radiation and the nature of dark matter and energy.

The starting points are the eqs (5,1). As η has the same physical dimensions of a pressure, it is possible to write $P=\xi\eta$ via a dimensionless constant ξ ; this also yields $\delta P=\xi\delta\eta$. So the existence of an internal pressure P related to the time dynamics of V , whatever its actual nature might be, is in principle self-consistent with the mere definition (2,1) of space time.

Note that η depends intuitively on Δx^{-3} in the case where E is the energy of matter particles. In the case of a radiation field propagating at rate v with wavelength $\lambda = v/\nu$ one expects $\eta = h\nu/\lambda\Delta x^3$; the existence of stationary waves having $\lambda_n = n_\lambda \Delta x$ within V , i.e. the arbitrary number n_λ of higher harmonics with nodes at the boundaries of Δx , implies volume dependence of the radiation field energy density that follows the $\eta \propto \Delta x^{-4}$ law.

Recalling the eq (5,1), $\nu = c/\lambda$ implies

$$V=\left(\frac{c}{\nu}\right)^3 \qquad \frac{\delta V}{V}=-3\frac{\delta\nu}{\nu}=-3\frac{\delta E}{E};(10,1)$$

in this result, ν has been assumed uniquely defined in V .

Let be in general $\eta = \varepsilon/V$ due to the energy ε in the volume V ; whatever ξ might be, to describe the expansion from the initial $V=\Delta x^3$ to $V+\delta V$ write then

$$\frac{\delta P}{P}=\frac{\delta\varepsilon}{\varepsilon}-\frac{\delta V}{V}.(10,2)$$

To find the link between P and η, consider now $V=\Delta x^3$ according to the eq (2,1) to calculate δV . Introducing explicitly Δx to express δV , however, requires to account for the uncertainty: any given δV is compatible with the change of one size only, i.e. $V=\Delta x_0^2\Delta x$, or two sizes, i.e. $V=\Delta x_0\Delta x^2$, or all three sizes, i.e. Δx^3. These ways of describing δV are identically possible and obviously indistinguishable; so they are concurrently admissible. It means that $\delta V/V=\xi\delta\Delta x/\Delta x$, with $\xi=1,2,3$ respectively.

Both ways to express $\delta V/V$ must of course coincide: clearly this last result and the eq (10,1) concern the dual corpuscle/wave behavior of matter. Hence, taking their ratio side by side, one finds

$$\frac{\xi}{3}\frac{E}{\Delta x}=-\frac{\delta E}{\delta\Delta x} \qquad \xi=1,2,3.$$

The right hand side defines the average force $\langle F\rangle$ in V ; if indeed $E=\langle F\rangle\Delta x$, then $\delta E=\langle F\rangle\delta\Delta x$. As expected, $\langle F\rangle$ is actually an average value because it represents the force field in the whole Δx^3. Dividing both sides by the surface

Δx^2, one finds

$$\frac{k}{3}\frac{E}{\Delta x^3} = -\frac{\langle F\rangle}{\Delta x^2} \quad \text{i.e.} \quad P = -\frac{\xi}{3}\eta \quad \eta = \frac{E}{\Delta x^3} \quad \xi = 1,2,3.$$

The negative sign of P is related to that of $\delta\Delta x$; having tacitly assumed $\delta\Delta x > 0$ it means that $P < 0$ generated within V pushes the boundary of V outwards. A positive sign consistent with $\delta\Delta x < 0$, identically possible, would mean of course that an external pressure tends to shrink V. Anyway, regardless of the sign, the possible relationships between pressure and energy density corresponding to the values of ξ are $P = \eta$, $P = 2\eta/3$ and $P = \eta/3$, to which are expectedly related three ways of regarding E.

It is known that $P = \eta/3$ holds for light completely absorbed by the internal walls that delimit V, whereas $P = 2\eta/3$ holds for particles or even for photons that are reflected by the walls and bounce back elastically inside V; in this case the kinetic momentum transferred against the walls is obviously twice than before. Eventually the relationship $PV = 2\varepsilon/3$ introduces itself the third chance $PV = \varepsilon''$ as a function of the energy ε'' defined by $\varepsilon = 3\varepsilon''/2$; this in effect concerns a known result of the elementary kinetic theory of the ideal gases once identifying $\varepsilon'' = k_B T$. The concept of temperature will be introduced in a more fundamental way in the section 13.4.

The present result deserves three comments.

1) The coefficient ξ does not depend on the spin of the particles.

2) Another significant case where $P = \eta$ is the following. Put without loss of generality in the eq (5,1) $g = g'\alpha/3$, where g' is an arbitrary function of Δt. Recalling the eq (2,2) and replacing g in the eq (5,1), one finds via the eq (2,1)

$$\eta' = \frac{c^2}{h^2 G}\left(\frac{e^2}{3c\Delta t}\right)^2 = \frac{1}{Vh\nu}\left(\frac{e^2}{3c\Delta t}\right)^2 \quad \eta' = \frac{\eta}{g'^2} \quad g' = g'(\Delta x\Delta t).(10,3)$$

Next, putting $e^2/c\Delta t = \varepsilon'$, the result $\eta' = (\varepsilon'/3V)^2 V/h\nu$ yields

$$\eta'\eta_\nu = P^2 \quad \eta_\nu = h\nu/V \quad P = \pm\varepsilon'/3V.$$

This result introduces thus P as radiation pressure with both signs; it is the geometric average of the energy densities η' and η_ν and adds a further chance of calculating explicitly the negative pressure acting inside V as a function of time. Indeed the eq (10,3) yields

$$\eta' = \frac{\alpha^2}{9G}\left(\frac{c}{\Delta t}\right)^2.(10,4)$$

3) An interesting consequence of the eq (10,2) is expressed as follows:

$$\delta P = \left(\frac{PV}{\varepsilon}\frac{\delta\varepsilon}{\delta V} - P\right)\frac{\delta V}{V} = \left(\frac{P}{\eta}\frac{\delta\varepsilon}{\delta V} - P\right)\frac{\delta V}{V} = \left(\xi\frac{\delta\varepsilon}{\delta V} - P\right)\frac{\delta V}{V},$$

whence, since $\delta P = \xi\delta\eta$,

$$\delta\eta = \left(\frac{\delta\varepsilon}{\delta V} - \frac{P}{\xi}\right)\frac{\delta V}{V}.(10,5)$$

To handle in general $\delta\varepsilon/\delta V$, consider both cases of extensive and intensive energy fields in principle possible inside V.

-The field fills uniformly the region V of space time; so the total ε available in V increases proportionally to the extent of this latter, i.e. $\varepsilon = wV$ with w proportionality constant. This typically occurs when the field is an intrinsic property of the space time. So $\delta\varepsilon/\delta V = w = \varepsilon/V = \eta$, whence $\xi\delta\eta = \delta(\xi\eta) = (\xi\eta - P)\delta V/V$.

-The field does not fill uniformly the region V. For instance, the average ε decreases as a function of V allowed to particles interacting with strength proportional to their reciprocal square distance; if so, then the average ε due to this kind of interaction decreases when larger and larger distances are allowed to the particles. Thus $\varepsilon = w/V$ yields $\delta\varepsilon/\delta V = -w/V^2 = -\varepsilon/V = -\eta$, whence $\delta(\xi\eta) = -(\xi\eta + P)\delta V/V$.

These results merge into the unique equation

$$\delta\eta' = -(P \pm \eta')\frac{\delta V}{V} \quad \eta' = \xi\eta;(10,6)$$

regardless of the value of ξ, is crucial the fact that at both sides appears $\xi\eta$, which can be regarded more in general as an arbitrary energy density η'. It is also interesting to rewrite this equation dividing both sides by an arbitrary time range $\delta\Delta t$, during which $\delta\eta'$ is allowed by the change $\delta\Delta x$. This yields with the help of the eq (10,2) and (10,1) calculated with c/ν

$$\dot{\rho}c^2 = -3H(P \pm \rho c^2) \quad H = \frac{\Delta\dot{x}}{\Delta x} = -\frac{1}{\nu}\frac{\delta\nu}{\delta\Delta t} \quad \eta = \rho c^2 \quad \dot{\rho} = \frac{\delta\rho}{\delta\Delta t} \quad \Delta\dot{x} = \frac{\delta\Delta x}{\delta\Delta t}.(10,7)$$

If ν is a decreasing function of Δt, i.e. the space time volume expands, then $H > 0$. The conclusion of this paragraph is that even the existence of an internal pressure that accounts for the time dependence of V is found as a corollary of the eq (5,1). These outcomes imply three remarks:

(i)once more the electric charge is introduced into the concept of space time via α, i.e. through a further combination of fundamental constants;

(ii) as no hypothesis has been made about in the eq (10,2), it is reasonable to expect that even the vacuum, in addition to the real matter or radiation field, can originate a pressure;

(iii) the equations

$$P = \eta'' \quad \eta'' = \pm\sqrt{\eta_\nu \eta'}(10,8)$$

still represent a possible link between energy density and pressure; in this case P merges two different kinds of energy densities via their geometric average, e.g. in the case of the quantum vacuum through which propagates a radiation field. Also now both signs appear possible for P.

Till now no explicit reference has been made to the existence of matter, despite appears self-consistently justified the concept of energy density in the eq (5,2). The next sections concern just this point, especially in connection with another energy density field, i.e.that due to the quantum vacuum.

11 THE FIRST LAW

This section is shortened as much as possible, being mostly aimed to clarify the meaning of negative pressure; the considerations are reduced to the minimum necessary to show that even the first law is inferable in the conceptual context of the eq (2,1), i.e. as a corollary of the eq (10,2).As $\delta\varepsilon = (\varepsilon/PV)(V\delta P + P\delta V)$ reads $\delta\varepsilon = (\varepsilon/PV)\delta PV$ whereas $\xi\varepsilon = PV$, one finds

$$\xi\delta\varepsilon_i = \delta(PV) \quad P = P_i \quad \delta\varepsilon_i = \delta\varepsilon_i(P = P_i).(11,1)$$

The subscript i emphasizes that actually P and δP concern the internal pressure and its change inside the space time region; indeed $\delta\varepsilon_i$ is by definition the energy change of the matter or radiation field within V. This equation, rewritten as $\xi\delta\varepsilon_i = \delta(U + PV) - \delta U$, yields

$$\delta U = \delta\mathrm{H} - \delta(PV) \quad \mathrm{H} = U + PV$$

If $P = const$, the first equation reads $\delta U = \delta Q - P\delta V$, whereas the enthalpy change $\delta\mathrm{H}$ reduces to δQ. With an internal pressure $P < 0$ the second addend is positive, which suggests that an external pressure $P > 0$ makes negative the second addend; this is in fact the first principle, which is usually written as a function of the external pressure acting on a thermodynamic system.

It is also interesting to rewrite explicitly the eq (10,2) as a function of a possible external pressure P_e acting from the outside of V against its external surface; in this case any change δP_e reverts the action on V with respect to that described by δP_i in the eq (10,2). Whatever the physical reason of δP_e might be, this means anyway that such an effect is described replacing δV with $-\delta V$ in the eq (10,2). Hence the eq (10,2) yields now $\xi\delta\varepsilon = V\delta P - P\delta V$, so that

$$\xi\delta\varepsilon_e = \delta(PV) - 2P\delta V \quad P = P_e \quad \delta\varepsilon_e = \delta\varepsilon_e(P = P_e, \delta V).$$

Since $\xi\delta\varepsilon_e = \delta(U + PV) - \delta U - 2P\delta V$ reads $\xi\delta\varepsilon_e = \delta\mathrm{H} - \delta Q - P\delta V = \delta(\mathrm{H} - Q) - P\delta V$, once having put $\delta Q = \delta U + P\delta V$, it is possible to write

$$\xi\delta\varepsilon_e = \delta G - P\delta V \quad G = \mathrm{H} - Q = U + PV - TS \quad Q = TS;(11,2)$$

in effect it is known that $V\delta P = \delta G$ at $T = const$, so that $\xi\delta\varepsilon_e = \delta G - P\delta V$. The concept of entropy S will be introduced in the next section 13.4. These results are the first law.

Consider eventually $\delta\varepsilon_e - \delta\varepsilon_i = -\eta\delta V$ subtracting side by side the eqs (11,1) and (11,2): it is clear that the limit $\delta\varepsilon_e - \delta\varepsilon_i \to 0$ holds if $P_e \to P_i$, in which case $\delta V \to 0$ too. This is the well known concept of reversible process, which implies an infinitely slow change of the system under a near equilibrium difference between the internal and external pressures leading to a steady V as a limit case.

12 THE QUANTUM VACUUM

The eq (6,3) has related e^2/λ to mc^2 and to $\mathrm{n}h\nu$, both inferred as direct consequences of the eq (2,1); so it is reasonable to think that e^2/λ and $-e\bar{e}/\lambda$, although both formally introduced via α, must have their own physical meaning. This section concerns just this point. In the section 5 both sides of the eq (2,1) have been multiplied by the frequency ν already inherent the definition of space time; the same operation is now repeated by implementing an arbitrary frequency $\nu_o \neq \nu$, which yields

$$\Lambda_t = \nu_o\nu = \frac{h\nu_o}{V}\frac{G}{c^2}.(12,1)$$

This position introduces the new quantity Λ_t and energy $E_o = h\nu_o$ formally similar to that of the eq (5,1), which reads

$$h\nu_o = \frac{\Lambda_t c^2 V}{G}.(12,2)$$

The eq (12,1) also defines the square energy $\varepsilon_\Lambda^2 = h^2\Lambda_t$ and the time range Δt_Λ during which $\varepsilon_\Lambda \Delta t_\Lambda >: h$. Hence $\varepsilon_\Lambda^2 \Delta t_\Lambda^2 >: h^2$ yields

$$\Lambda_t >: \Delta t_\Lambda^{-2} \quad \Lambda_t = \Lambda_t(\Delta t).(12,3)$$

The eq (12,2) introduces the related energy density η_Λ and density ρ_Λ as follows

$$\eta_\Lambda = \frac{h\nu_o}{V} = \frac{\Lambda_t c^2}{G} \quad \rho_\Lambda = \frac{\Lambda_t}{G}.(12,4)$$

The physical meaning of ρ_Λ is uniquely due to the physical dimensions of G; the time dependence of V upon $\nu(\Delta t)$ in the eqs (2,1) is now merged with that of Λ_t, whose physical meaning is further elucidated defining the volume

$$V_\Lambda = \frac{c^2 l_\Lambda}{\Lambda_t},(12,5)$$

being l_Λ an appropriate length to be determined. To highlight the physical meaning of l_Λ, note that it defines energy and mass without introducing any particle, but simply implementing the physical dimensions of G

$$\varepsilon_\Lambda = \frac{c^4}{G} l_\Lambda \quad m_\Lambda = \frac{c^2 l_\Lambda}{G}.(12{,}6)$$

The eqs (12,4) depend explicitly on Λ_t only, whereas the corresponding quantities of the eqs (12,6) depend on the length l_Λ only. Since even these quantities are not related to V, which indeed does not appear explicitly, one infers the existence of an energy field pervading all volume of space time, whose amount per unit volume is just η_Λ. This field is due to charged virtual particles, assumed to exist in the space time in agreement with the eq (6,3) and with ε_Λ of the eq (12,6) itself.

To support this conclusion, note that replacing G and V in ε_Λ via the eqs (2,1) one finds $\varepsilon_\Lambda = (c/\nu)^2 l_\Lambda h\nu/\Delta x^3$. As $l_\Lambda \nu$ has physical dimensions of a velocity, it is possible to write $l_\Lambda \nu = qc$ via an appropriate numerical coefficient q; moreover, being c/ν a length, it is also possible to regard $(c/\nu)^2$ as a surface A. Hence this straightforward elaboration of ε_Λ reveals the form

$$\frac{\varepsilon_\Lambda}{A} = q\frac{hc}{\Delta x^3}.$$

Is attracting the chance of expressing $q = q'\alpha$, being q' a proportionality factor; in this way the last equation reads $\varepsilon_\Lambda/A = q'e^2/\Delta x^3$. So $(e^2/\Delta x)/\Delta x^2$ at the right hand side shows that ε_Λ/A is proportional to the energy/surface ratio of charged virtual particles. To get from this expression the force per unit surface, it is enough that $q' = 1/3$: dividing both sides of $\varepsilon_\Lambda/A = e^2/\Delta x^3/3$ by Δx one finds the expected form $\eta = P/3$, being indeed $\eta = \varepsilon_\Lambda/V$ and $P = e^2/\Delta x^4$. In conclusion with $q = \alpha/3$ the previous equation reads

$$\frac{\varepsilon_\Lambda}{A} = \frac{\alpha}{3}\frac{hc}{\Delta x^3} = \frac{1}{3}\frac{e^2}{\Delta x^3},$$

where the factor 1/3 specifies that the energy density corresponds to wave-like pressure. According to the eq (6,3), however, an analogous result can be obtained from $(e\bar{e}/\Delta x)/V$ i.e. multiplying by $-\alpha = e\bar{e}/hc$: this implies $F_\Lambda = -\varepsilon_\Lambda/\Delta x$: in the former case the force is repulsive, in the latter case attractive while the vacuum results neutral according to the considerations of the section 6. In this latter case the pressure is due to couples of virtual charges and anti-charges that form and annihilate releasing the concerned energy, which is thus is identifiable with the Casimir force. In effect, the numerical value of the coefficient $hc\alpha/3$ fits well that $\hbar c\pi^2/(3\times 240)$ reported in the literature, whereas the physical meaning of this force is easily explainable in terms of selected virtual charges existing in the quantum vacuum between two plates of surface A put Δx apart.

The distance Δx between the surfaces A selects the vacuum waves allowed inside Δx^3, i.e. steady waves with nodes at both surfaces A: the number of vacuum frequencies allowed between these surfaces is smaller than that allowed in the free vacuum. Just the reduced number residual waves causes the attractive force corresponding to ε_Λ, as it is inferable from the coefficient 1/3: the attractive effect is nothing else but the "richer" vacuum that squeezes the "poorer" vacuum.

This confirms that the quantities with the subscript Λ are due to virtual particles and antiparticles that characterize the quantum vacuum.

After having introduced density and volume together with energy and energy density of a virtual world inherent the space time, whose physical properties in fact do not involve real masses or radiation, is deducible their connection with the real world previously introduced.

Multiply both sides of the eqs (2,1) by c^3 and note that c^3V has the same physical dimensions of $(h/m_0)^3$, being m_0 an arbitrary mass; hence, assuming c^3V proportional to $(h/m_0)^3$, the eq (2,1) yields $hcG = \xi_0(h/m_0)^3\nu$, where ξ_0 is a dimensionless proportionality constant. Owing to the initial definition of $\nu = hG/(c^2\Delta x^3)$, one finds

$$m_0 = h\left(\frac{\xi_0 \nu}{hcG}\right)^{1/3} = \frac{h}{c\Delta x_0} \quad \Delta x_0 = \xi_0^{-1/3}\Delta x \quad V_0 = \Delta x_0^3 = \frac{V}{\xi_0}(12,7)$$

The first equation is the Compton length Δx_0 of the mass m_0, already introduced in the eqs (5,3), which results here proportional to $(V/\xi_0)^{1/3}$ via the factor ξ_0. Hence ξ_0, expectedly >1, is the scale factor between the macroscopic volume V of space time of the eqs (2,1) and the local size V_0 around the mass m_0. Also,

$$m_0 c^2 = h\left(\frac{\xi_0 \nu}{t_{Pl}^2}\right)^{1/3} \quad hcG = \left(\frac{V\zeta\Lambda_t}{\xi_0}\right)^2 \quad \frac{1}{t_{Pl}^2} = \frac{c^5}{hG}:(12,8)$$

the second equation is inferred from $hcG = (V'\nu''^2)^2$, guessed by dimensional reasons, replacing $V' = V/\xi_0$ in accordance with the eq (12,7) and $\nu''^2 = \zeta\Lambda_t$ via the further proportionality constant ζ. Formally ξ_0 and ζ could be merged into one multiplicative factor only; yet the proposed notation better emphasizes that the former refer to the space scale, the latter to the time scale. Note that now m_0 is not a mere multiplicative factor introduced at both sizes of an equation, as done for instance in inferring the eq (4,1) of the section 4; here the mass is an actual physical entity explicitly introduced and defined by the Compton length and the energy m_0c^2 via the Planck time t_{Pl}, as it appears in the eq (12,8). Just these results link hcG and the early definition hG/c^2 of space time; i.e., since V is still that of the eq (2,1), the second eq (12,8) reads

$$hcG = \left(\frac{hG}{c^2}\frac{\zeta\Lambda_t}{\xi_0 \nu}\right)^2.(12,9)$$

The eq (12,9) highlights the sought form of hcG and provides the expected link between ξ_0 and ζ

$$\zeta\Lambda_t = \frac{\xi_0 \nu}{t_{Pl}} \quad \text{i.e.} \quad \nu_o = \frac{\xi_0}{t_{Pl}\zeta};(12,10)$$

then

$$\varepsilon_\Lambda = h\sqrt{\Lambda_t} = h\sqrt{\frac{\xi_0 \nu}{t_{Pl}\zeta}}.(12,11)$$

On the one hand the eqs (12,10) and (12,8) yield

$$hcG = \left(\frac{m_0 G}{\xi_0}\right)^2 \quad m_0 = \frac{V\zeta\Lambda_t}{G} \quad \xi_0 = \frac{m_0c^2}{h/t_{Pl}} = \frac{m_0}{m_{Pl}};(12,12)$$

on the other hand, for Δt_Λ to fulfill the condition (12,3), the second eq (12,12) requires

$$\zeta <: \frac{m_0 \Delta t_\Lambda^2}{V}G.(12,13)$$

The second eq (12,12) equals via the factor ζ the mass densities m_0/V and Λ_t/G, having clearly different physical meaning. Whatever the values of m_0 and V might be, the former is the usual density defined by a real mass and a real volume; the latter is a virtual density defined by the physical dimensions of Λ_t and G only, thus not attributable to any volume or amount of matter actually existing in the space time. As m_Λ and ε_Λ are proportional via the length l_Λ to the characteristic fundamental units that define the space time as a whole, this remark confirms that to the properties of the space time concur also virtual particles, whose mass and energy and respective densities are nonetheless still definable likewise that of the ordinary matter. Thus, despite their different physical properties, it is reasonable to expect that their energy density concurs to the total pressure acting inside V. Is important the chance of defining the total energy density in the space time volume due to the virtual particles plus that due to the possible presence of ordinary matter; according to the second eq (12,12), it is given by

$$\rho_{tot} = \frac{\Lambda_t}{G} + \frac{m_0}{\zeta V} = 2\frac{\Lambda_t}{G} = 2\frac{m_0}{\zeta V}.(12,14)$$

Here is the sought result: via the proportionality factor ζ, vacuum and matter are at the equilibrium.

A further property of Λ_t of interest is inferred from the inequality (12,3). Introduce an appropriate length Δr_Λ such that $c/\Delta r_\Lambda <: \Delta t_\Lambda^{-1}$; multiplying side by side this inequality and (12,3), Δr_Λ is defined in order that

$$\Delta t_\Lambda^3 = \frac{\Delta r_\Lambda}{c\Lambda_t};(12,15)$$

The conclusions of this simple reasoning are

$$\frac{c}{\Delta r_\Lambda} <: H \qquad \Delta t_\Lambda <: \frac{1}{H}.(12,16)$$

The next sections highlight the implications of the groundwork hitherto exposed.

13 SOME PHYSICAL COROLLARIES

After having concerned physical properties directly implied by the eq (2,1), this section elaborates some among the results so far introduced to highlight additional features of the space time.

13.1 THE FREE PARTICLE

Let us show that the Compton length λ_C of m given by the eq (5,3) can be also inferred via n of the eq (5,4) noting that this equation yields $v = h\nu/mc$, so that

$$c\frac{\partial \mathrm{n}^{-1}}{\partial \nu} = \frac{\partial c/\mathrm{n}}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{h}{mc}.(13,1)$$

To implement further n via the eqs (4,5), define next $\Delta t_\mathrm{n} = nh/\Delta(\mathrm{n}E)$ and $\Delta t = nh/\Delta E$, with the same number n of allowed states; then $\Delta(\mathrm{n}E) > \Delta E$, due to $\mathrm{n} > 1$ according to the eq (8,2), compels $\Delta t_\mathrm{n} < \Delta t$. Introduce thus a velocity v such that $c\Delta t_\mathrm{n} = v\Delta t$; so, $v\Delta(\mathrm{n}E) = c\Delta E$ yields

$$\frac{v}{c} = \frac{\Delta t_\mathrm{n}}{\Delta t} = \frac{\Delta E}{\Delta(\mathrm{n}E)} = \frac{\Delta \nu}{\Delta(\mathrm{n}\nu)}$$

and thus

$$v = \frac{c}{\Delta(\mathrm{n}\nu)/\Delta\nu}.(13,2)$$

This result, written in differential form in the limit $\Delta \to \partial$, i.e. for very small range sizes, takes the familiar form

$$v = \frac{c}{\partial(\mathrm{n}\nu)/\partial\nu}.$$

The mass does not appear explicitly in this result, which therefore holds for $m \neq 0$ and even for $m = 0$; the former case implies that m moves at the rate we call group velocity of the particle, in the second case v describes a packet of light waves moving at the slower group rate in a dispersive medium with refraction index $\mathrm{n} > 1$. An important consequence follows from this result for n constant, $\mathrm{n} = \mathrm{n}_0$: the eq (13,2) reads $c\delta\nu/\delta(\mathrm{n}c/\lambda)$ and then thanks to the eq (5,5)

$$v^* = \frac{c}{\mathrm{n}_0} = \frac{\partial \nu}{\partial(1/\lambda)},(13,3)$$

which is the fundamental equation of the quantum mechanics together with the De Broglie momentum.

Consider now the eq (8,3); squaring both sides, one obtains $(h\nu)^2=(mc^2)^2+(h\nu')^2-2h\nu' mc^2$. Next, putting by definition $\nu'=c/\lambda'$, so that $h\nu'=p'c$, this equation reads $(h\nu)^2+2mc^2p'c=(mc^2)^2+(p'c)^2$ according to the eq (5,5); the prime symbol emphasizes the link with ν'. Note that with ν' defined in this way via c, the right hand side is an invariant; so at the left hand side appears an invariant square energy. Put now $h\nu'=qmc^2$ via the factor $q<1$; this position is reasonable because $h\nu=mc^2(1-q)$ of the eq (8,2) agrees with $h\nu<mc^2$ of the eq (5,4). Hence $2mc^2p'c=2(p'c)^2/q$. Consider a boundary condition for the eq (8,3): holds the limit $mc^2\rightarrow h\nu'$ for $h\nu\rightarrow 0$, in which case $q\rightarrow 1$; this means that the value of q is related to that of $h\nu$. Put therefore $q=m_oc^2/h\nu$, being m_o an arbitrary constant mass introduced by dimensional reasons: in this way $m_o\rightarrow 0$ for $h\nu\rightarrow 0$ is consistent with the finite limit of q and thus with $\nu'\neq 0$. So the result is $(h\nu)^2=(mc^2)^2+(p'c)^2-2(h\nu)p'^2/m_o$. Since ν and ν' are arbitrary, this equation reads in general

$$\varepsilon^2=(mc^2)^2+(pc)^2-(2/m_o)\varepsilon p^2.(13,4)$$

This equation, which compels modifying the rest term in the standard relativistic energy equation, is a well known feature of the quantum gravity; it solves the three cosmological paradoxes emphasized in [21].

13.2 THE BLACK HOLE LENGTH

Rewrite the eq (2,1) as $mV\nu^2/h\nu=mG/c^2$; the right hand side describes a length characterized by m only, which therefore represents a physical property of m. Replacing $h\nu$ with $p_\nu c^2/v_\nu$ via the eqs (A09) and (9,9) and expressing without loss of generality $V=(c/\nu)^2\lambda$ via an appropriate length λ, the result is

$$\lambda=\frac{p_\nu}{mv_\nu}\frac{mG}{c^2}\qquad\frac{p_\nu}{mv_\nu}=\left(1-(v_\nu/c)^2\right)^{-1/2}\geq 1,(13,5)$$

where the equality holds in the limit $v_\nu\rightarrow 0$. The dimensionless ratio $p_\nu/\mathrm{m}v_\nu$, in principle arbitrary, implies important consequences for $v_\nu=0$ or for particular values of $v_\nu\neq 0$. If $v_\nu\neq 0$, then in general the eq (13,5) is simply a way to express λ in mG/c^2 units via arbitrary values of the ratio p_ν/mv_ν. Yet, two particular values of this ratio deserve attention. These cases are separately concerned.

1) The limit $v_\nu\rightarrow 0$ concerns in particular $\lambda_0=mG/c^2$. Multiplying both sides by an arbitrary mass m_1, according to the eq (5,9) this equation describes a gravitational system of two masses m and m_1 a distance λ_0 apart:

$$-m_1c^2=-G\frac{m_1m}{\lambda_0}$$

It is easy to find the Newtonian limit of the virial theorem for a gravitational system of two masses, since this result suggests a possible connection with the eqs (5,9) according which $m_1mG/\lambda_0=-U$; it implies a rest mass m_1, e.g. fixed at the origin of an arbitrary system of coordinates, and a mass m orbiting circularly λ_0 apart, whose potential energy <0 corresponds to the attractive gravitational field of m_1. Hence, $-m_1c^2$ must account for the binding energy $\varepsilon_b<0$ of the whole system and for the kinetic energy ε_{kin} of m. Clearly ε_{kin} is lower and upper bound by the condition $0\leq\varepsilon_{kin}\leq m_1c^2$, without which m would escape from the gravitational field of m_1. On average therefore $\langle\varepsilon_{kin}\rangle=m_1c^2/2$, whereas $\langle U\rangle=U$ yields $\langle\varepsilon_{kin}\rangle=-U/2$; also, $\langle\varepsilon_{kin}\rangle+U=U/2$ yields $-m_1c^2/2=\langle\varepsilon_b\rangle$, where at the right hand side appears of course the average binding energy. In conclusion

$$\langle\varepsilon_{kin}\rangle=-\langle U\rangle/2=-\langle\varepsilon_b\rangle.(13,6)$$

The Newtonian approximation is clearly due to the way of defining ε_{kin} via the rest term of the eq (9,9) only, which holds for $v << c$ only, instead of implementing the series expansion of the eq (5,9) to generalize the eq (13,6). This generalization, requiring a longer discussion, is omitted for sake of brevity; it is far beyond the purposes of the present paper, merely aimed to show that sensible results are obtainable from the eq (2,1).

2) Rewrite identically the eqs (13,5) as follows

$$1-\left(\frac{v_\nu}{c}\right)^2=\left(\frac{m_\nu mG}{\lambda m_\nu c^2}\right)^2;$$

owing to the chance of writing $c=\mathrm{n}v_\nu$, eq (5,4), this equation reads

$$1=\left(\frac{v_\nu}{c}\right)^2+\left(\frac{m_\nu mG}{|M|c}\right)^2 \qquad |M|=\mathrm{n}\lambda m_\nu v_\nu, (13,7)$$

being by dimensional reasons $|M|$ the modulus of the angular momentum of m_ν orbiting at the distance $\mathrm{n}\lambda$ from m . Multiplying both sides by $(c/v_\nu)^2$, one finds

$$\left(\frac{c}{v_\nu}\right)^2=1+\left(\frac{c}{v_\nu}\right)^2\left(\frac{m_\nu mG}{|M|c}\right)^2:$$

this result confirms once again that necessarily $c>v$. Moreover note that the terms at right hand sides of the eq (13,7) can be regarded as probabilities, whose sum yields the certainty; trusting therefore on the probabilistic character of this equation, $v_\nu<c$ implies by necessity $m_\nu mG/\Delta r<|M|c/\Delta r$, where Δr is the average orbiting distance of m_ν from m. It means that the second addend yields the probability of finding m_ν in an arc $\Delta r\delta\phi$ of its orbit. So this probability reads $\Delta r\delta\phi/2\pi\Delta r=\delta\phi/2\pi$. Note that v_ν is the modulus of a vector having space components that point towards three orthogonal directions, one of which only is of interest here: the one pointing along the tangent to the arc of orbit defined by $\Delta r\delta\phi$. So, since nothing is known about v_ν for the reasons previously introduced, the probability of displacement of m_ν just along $\pm\delta\phi$ is actually 1/3 of $\delta\phi/2\pi$. Hence

$$\frac{\delta\phi}{6\pi}=\left(\frac{m_\nu mG}{|M|c}\right)^2.$$

The eqs (5,10) and (5,11) explain and motivate this conclusion, which concerns the well known perihelion displacement of orbiting bodies simply regarding Δr as average distance of the elliptic orbit of m_ν around m. It is known that the classical form of gravitational potential does not account correctly for the perihelion precession of orbiting bodies, an additional term $U^*=\beta/r^2$ would be necessary [22]; unfortunately, however, the classical physics does not justify such an ancillary term. Yet here the Einstein result has been obtained via a probabilistic reasoning correspondingly to the necessity of considering $U(\Delta x'^{-1})$ instead of the plain Newtonian $U(x^{-1})$, as emphasized in the positions (5,10); this term appears in a natural way and justifies the relativistic effect found here.

3) Is also interesting one specific value of the ratio p_ν/v_ν, such that λ takes the particular physical meaning of space range where is confined m. Being $p=h/\lambda$, this condition requires that even the longest steady wavelength of the momentum wave be entirely included in mG/c^2, otherwise m could not be actually confined therein. Let λ_{bh} be the sought wavelength; as any steady wave has nodes at the boundaries of mG/c^2, the corresponding λ_{bh} must be twice the size of this latter. Otherwise stated, λ_{bh} consists of two half-wavelengths each one of which is equal to mG/c^2. This identifies the particular value $p_\nu/mv_n=2$ of interest to describe the confinement of m, i.e.

$$\lambda_{bh} = 2\frac{mG}{c^2} \quad p_{bh} = \frac{h}{\lambda_{bh}} = \frac{1}{2}\frac{hc^2}{mG} \quad \varepsilon_{bh} = cp_{bh} = \frac{1}{2}\frac{\text{n}hc^3}{mG}.(13,8)$$

This first equation is confirmed considering directly the corpuscle velocity v_x. Putting $\Delta p_x = mv_x - (-mv_x)$, the identity $\Delta x \equiv \Delta x^2 \Delta p_x / nh$ also reads according to the eqs (4,1) $\Delta x \equiv 2\Delta x^2 mv_x / nh$: it means that regarding m in an arbitrary point x_0 of the space time, the momentum range corresponding to its total displacement Δx around x_0 is in fact compatible with both components v_x and $-v_x$ of velocity along the x axis. The reasoning is identical to that already introduced to infer the eq (9,6). Put then $|v_x| = \nu\Delta x$, where ν describes the frequency with which the corpuscle bounces back and forth throughout Δx; the last identity reads $\Delta x \equiv 2\Delta x^3 m\nu / nh$ and thus, according to the eqs (2,1), $\Delta x \equiv 2mG/nc^2$. Since $n = 1$ identifies the maximum range size consistent with this condition, $\Delta x_{n=1} = \lambda_{bh}$ is the maximum length crossable by the particle whatever its velocity might be; as the velocity does not appear explicitly in defining Δx, the result holds even for the light speed. The reasoning carried out for m holds identically for any particle moving inside Δx, which is a physical property of m. This allows concluding that no particle can escape beyond the boundary defined by the range size $\Delta x_{n=1}$.

The size of λ_{bh} containing all momentum steady wavelengths of m, is thus the diameter of a "no escape hypersphere" centered on m. This is in fact the meaning of the factor 2, which doubles the half longest momentum wavelength and corresponds to the total range accessible to delocalize m. Note that the orientation of the range $2mG/c^2$ in the space time, along which run the momentum wavelengths, is not definable if the space time is homogeneous and isotropic; so, thinking fixed one boundary of λ_{bh}, the mobile boundary describes a larger hypersphere centered on the fixed boundary. The hyperspherical volume of radius $2mG/c^2$, i.e. defined by all orientations of λ_{bh} in the space time, is the event horizon of m. So, event horizon of m and confinement range outside which m does not escape are here different concepts.

The factor 2 is interesting for at least three reasons.

(i) Replace $\lambda' = \lambda_{bh}$ to define $h\nu'$ of the eq (8,3), as done to infer the eq (13,4); one finds in agreement with the eq (13,8)

$$h\nu' = p'c = \frac{1}{2}\frac{hc^3}{mG},(13,9)$$

which yields

$$mc^2 = h\nu + \frac{1}{2}h\nu \quad \nu = \frac{c^3}{mG}.(13,10)$$

This result further clarifies the meaning of the inequality (8,2), which together with the eq (8,3) implies $\text{n} > 1$ and thus $c > v$ in the eq (5,4) simply because of the uncertainty: the reason is the existence of the zero point energy, which in effect is a quantum property hidden into the inequality $mc^2 > h\nu$.

(ii) Replacing $l_\Lambda = 2m_H G/c^2$ in the eq (12,5), one finds $V_\Lambda = 2m_H G/\Lambda_t$; moreover the eq (12,2) reads $h\nu_o = 2m_H c^2$, i.e.

$$m_H c^2 = \frac{1}{2}h\nu_o.(13,11)$$

The physical meaning of V_H and l_H leading to this result will be described next below. It is anticipated here that the virtual charges behave as oscillators with their own zero point energy.

(iii) To highlight the physical meaning of ξ_0, note that the eq (12,12) yields with the help of the eq (13,8)

$$\xi_0^2=\frac{m_0^2G^2}{hcG}=\frac{c^3}{4hG}\left(\frac{2m_0G}{c^2}\right)^2,$$

which can be regarded in two ways.

On the one hand, assuming in particular an hyper-spherical geometry of radius λ_{bh}, one finds

$$\xi_0^2=\frac{A}{16\pi l_{Pl}^2}\qquad A=4\pi\lambda_{bh}^2.(13,12)$$

On the other hand, owing to the eq (12,6), ξ_0^2 is proportional to the ratio of two energies

$$\xi_0^2=\frac{m_0^2G/\Delta r}{hc/\Delta r}=\frac{\alpha m_0^2G}{e^2/\Delta r}.(13,13)$$

The eq (13,12) concerns the boundary of a stationary hyper-spherical black hole having surface A; the fact that it is also proportional to the ratio of two energies, recalls the classical concept of dimensionless entropy compatible with the energy ratio $\delta Q/k_BT$. In effect ξ_0^2 is proportional to the Hawking-Beckenstein entropy $A/4l_{Pl}^2$ [23] of a stationary black hole via a constant factor $(4\pi)^{-1}$; note however that the present result would coincide with that of HB waiving the hyper-spherical geometry, i.e. putting simply $A=\lambda_{bh}^2$.

As a final remark, the eqs (13,5) and (5,6) show that the arbitrariness of the respective values of v and n allow the chance of describing any length l via the mG/c^2 and h/mc, whence the positions

$$l=n_{bh}^*\frac{mG}{c^2}\qquad l=n_C^*\frac{h}{mc}=c\frac{\partial(n_C^*/\mathrm{n})}{\partial\nu};(13,14)$$

of course n_{bh}^* and n_C^* are arbitrary real numbers, whose subscripts emphasize their reference to the respective fundamental lengths, the last equality implements the eq (13,1). This remark is not trivial, because of the different mass dependence of either reference length: the former increases with m, whereas the latter decreases. Although in principle both ways to describe any length via the respective numerical coefficients are equivalent, the next section 14.1 will show that this formal equivalence is not so obvious from a physical standpoint in describing the size evolution of the space time.

13.3 RED SHIFT AND GRAVITATIONAL BINDING ENERGY

A further corollary of the eq (13,8) is inferred implementing either eq (3,1). Put for instance in the second eq (3,1) $\delta l=2mG/c^2$, which is possible in principle because the quantities at both sides are invariant; of course this replacement requires specifying accordingly Δl too. Since Δl is not required to be invariant itself, let be $\Delta l^2=q\Delta r^2$, with $\Delta r=c\Delta t$ and $q<1$ arbitrary constant; this yields then $2mG/(c^2\Delta r)=\sqrt{1-q}$. The left hand side must have the form $(v/c)^2$ because $2mG/\Delta r$ has physical dimensions of square velocity; so q must be such that $\sqrt{1-q}=(v/c)^2$, i.e. $q=1-(v/c)^4$, in agreement with the fact that $\Delta l^2<(c\Delta t)^2$. Hence $2mG/(c^2\Delta r)=(v/c)^2$ yields $\left(1-2mG/(c^2\Delta r)\right)^{-1/2}=\left(1-(v/c)^2\right)^{-1/2}$. The right hand side represents the Lorentz contraction factor between two lengths in reference systems in reciprocal motion, as it is easily inferable from the interval invariance itself [16]; so the left hand side yields in particular the red shift of a proper wavelength λ_e emitted by a light source moving at rate v with respect to an observer along its sight line. This latter records instead a wavelength λ_{obs} given by

$$\frac{\lambda_{obs}}{\lambda_e}=\frac{1}{\sqrt{1-\dfrac{2mG}{c^2\Delta r}}}=1+z(13,15)$$

which defines the red shift z.

Note that this expression reads identically

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$$\frac{\varepsilon_e}{\varepsilon_{obs}}=\frac{1}{\sqrt{1-\varepsilon'_{be}/\varepsilon_0}}\qquad\frac{\varepsilon'_{be}}{\varepsilon_0}=\frac{(2/3)m^2G/\Delta r}{mc^2/3}\qquad\frac{\varepsilon_e}{\varepsilon_{obs}}=\frac{\lambda_{obs}}{hc}\frac{hc}{\lambda_e},(13,16)$$

where the subscript be stands for binding energy. The interest of these results is due to the ratio $\varepsilon'_{be}/\varepsilon_0$. It is known that the gravitational binding energy of a homogeneous spherical body of matter of radius Δr is obtained classically considering the gravitational interaction between a core mass $4\pi r^3\rho/3$ and an external shell mass $4\pi r^2\rho dr$ of the body; integrating their product times r^{-1} over $0\le dr\le\Delta r$ according to the eq (5,9), the result is $\varepsilon_{be}^{cl}=-0.6m^2G/\Delta r$. Regard here

$$\varepsilon_{be}=-\varepsilon'_{be}=-\frac{2}{3}\frac{m^2G}{\Delta r}$$

as the classical gravitational binding energy of a spherical body formed by a system of particles: 2/3 fits well the coefficient 0.6 affected by the approximation inherent the classical Newtonian potential r^{-1}, whereas the eq (13,16) to infer ε_{be} waives instead any reference to the analytical form of gravitational potential.

Also, mc^2 is the energy of the total mass of the system, whose particles are initially supposed at the infinity and thus non-interacting.

The physical meaning of the ratios $\varepsilon_{be}/\varepsilon_0$ and $\varepsilon_e/\varepsilon_{obs}$ is then inferred as follows.

(i) Write the former as $\varepsilon_{be}/\varepsilon_0=\eta_{be}/\eta_0$: to obtain this result ε_{be} and ε_0 have been ideally divided by an arbitrary reference volume V_{ref}, which can even coincide with that of the whole homogeneous body, to calculate the respective energy densities. Since $mc^2=\mathrm{n}h\nu$ according to the eq (5,4), the second eq (13,16) reads

$$\frac{\varepsilon_{be}}{\varepsilon_0}=-\frac{(2/3)\left(m^2G/\Delta r\right)/V_{ref}}{(1/3)\mathrm{n}h\nu/V_{ref}}\qquad\varepsilon_{be}=-\varepsilon'_{be}.$$

Hence, putting $\mathrm{n}h\nu/V_{ref}=\chi^{-2}(c\nu)^2/G$ whatever V_{ref} might be via the proportionality factor χ that includes n, one finds

$$\frac{\varepsilon_{be}}{\varepsilon_0}=\chi^2\frac{P_{be}^{mat}}{P^{rad}}\qquad P^{rad}=-\frac{1}{3}\frac{(c\nu)^2}{G}\qquad P_{be}^{mat}=\frac{2}{3}\frac{m^2G}{V_{ref}\Delta r}:(13,17)$$

The positive P_{be}^{mat} is related to the attractive gravitational energy density of the matter, which tends to shrink the system enclosed by V_{ref}; the negative P^{rad} is related to a form of pressure necessarily negative also concurring in the reference volume, which tends to swell the system. The notation emphasizes the assumption of radiation pressure with reference to the eq (5,1) and to the coefficient 1/3, i.e. due to the whole radiation field surely existing within $V=V_{ref}$ in agreement with ν and acting against the internal surface of V that is thus pushed outwards.

(ii) Since $\varepsilon_e\to\varepsilon_{obs}$ is the non relativistic limit of ε_e for $c\to\infty$, regard reasonably $\varepsilon_e/\varepsilon_{obs}$ as the ratio of $\varepsilon_{be}^{rel}=\phi\varepsilon_e$ and $\varepsilon_{be}^{cl}=\phi\varepsilon_{obs}$ via an appropriate proportionality factor ϕ: the aim is to define $\phi\nu_{obs}$ in order that $\phi h\nu_{obs}$ represents ε_{be}^{cl} and $\phi\nu_e$ in order that $\phi h\nu_e$ represents ε_{be}^{rel}. Then the first eq (13,16) reads

$$\varepsilon_{be}^{rel}=\frac{\varepsilon_{be}^{cl}}{\sqrt{1-2mG/c^2\Delta r}}\qquad\varepsilon_{be}^{cl}=-\frac{2}{3}\frac{m^2G}{\Delta r}.(13,18)$$

For $y=mG/c^2\Delta r<<1$ the first equation reduces to $0.66y/(1-y)$, whereas an analogous literature result is $0.6y/(1-y/2)$ with $y=mG/c^2r$ [24]: despite the literature result concerns a hot dense supernova star of mass m, whereas here ε_{be}^{rel} considers the total mass m of several corpuscles regarded as a whole gravitational system, for

$y << 1$ both expressions are reasonably comparable.

A corollary of the eq (13,18) is the link between the relativistic gravitational energy gain due to the mass m of a system of particles and the energy necessary to create a higher amount of mass $M > m$. Note preliminarily that the physical conclusions of the following reasoning would be in principle expectable also with the quoted literature equation; regardless of a possible small difference of numerical values of calculations carried out with the eq (13,18) or with the literature formula, the essential point is to use the relativistic binding energy and not the classical one. Implementing the eq (13,18) as follows

$$-\varepsilon_{be}^{rel} = Mc^2 = \frac{(2/3)ymc^2}{\sqrt{1-2y}} \quad y = \frac{mG}{c^2\Delta r} \quad M = m + m', (13,19)$$

the gravitational binding energy at the right hand side appears adequate to account not only for its own mass m but also for the equivalent mass of any energy field possibly associated to the outwards pushing or inwards shrinking effects; this additional field is related to and symbolized by the equivalent mass m'. Note that the eq (13,19) admits a solution: for $y \to 1/2$, i.e. $m \to c^2\Delta r/2G$, the right hand sides diverges, so in principle the equation can be solved whatever the additional m' might be. Hence even a small value of $m << M$ is in principle compatible with the occurring of a large extra-amount of mass/energy generically denoted here as m'. Eventually let us emphasize a final remark about the eqs (13,15) and (13,16). In the particular case of hyper-spherical geometry of space time where is distributed the gravitational mass m, i.e. $V = 4\pi\Delta r^3/3$, the eq (13,17) reads

$$\frac{P_{be}^{mat}}{|P^{rad}|} = \frac{1}{\chi^2}\frac{2mG}{c^2\Delta r} = \frac{8\pi mG}{3V}\left(\frac{\Delta r}{\chi c}\right)^2 = \frac{8\pi\rho G}{3H^2} \quad H = \chi\frac{c}{\Delta r} \quad \rho = \frac{m}{V}, (13,20)$$

being H a new function with physical dimensions $time^{-1}$. It is clear that V expands or shrinks or is at rest depending on the force balance of the gravity driven contraction pressure P_{be}^{mat} and the internal radiation driven pushing effect P^{rad} acting on its boundary: in this way the inwards or outwards effect depends on whether $P_{be}^{mat}/|P^{rad}| \lessgtr 1$. In effect the last term of the chain of equations is a well known result, reported in the literature as Ω, obtained solving the Friedman equations.

13.4 THERMODYNAMIC SYSTEMS OF PARTICLES

Let us multiply the eq (2,1) by ∇m in order to introduce the vector

$$L = \frac{hG}{c^2}\nabla m,$$

whose modulus has the same physical dimensions as h. This means considering a complex system of masses m_j such that $m = \sum_j m_j$, whose mutual positions are compatible with the existence of mass density gradients. The global gradient $\nabla m \neq 0$ introduces the concept of non-equilibrium configuration possible for the mass existing in the space time. Let $\varepsilon_{0j} = m_j c^2$ be the particular value of energy related to the j-th mass of the system. Writing identically $L = \varepsilon_0(\nabla\varepsilon_0/\varepsilon_0)(hG/c^4)$ and specifying this expression for each particular mass element, $L_j = \varepsilon_{0j}(\nabla \log \varepsilon_{0j})hG/c^4$ yields thus

$$L_j = \frac{UhG}{c^4}\frac{\varepsilon_{0j}}{U}\nabla \log\left(\frac{\varepsilon_{0j}}{U}\right) \quad U = U(\Delta t) \quad \varepsilon_{0j} = m_j c^2 (13,21)$$

being U a time dependent energy to be defined; so U is compatible with the possible time dependence of ∇m_j. Rewrite the right hand side of the first equation as

$$\Pi_j \nabla \log \Pi_j = \nabla(\Pi_j \log \Pi_j) - (\nabla \Pi_j)\log \Pi_j \quad \Pi_j = \Pi_j(\Delta t) = \frac{\varepsilon_{0j}}{U}$$

and define via the first eq (13,21) the scalar

$$\mathrm{S}_j = L_j \cdot u = \frac{UhG}{c^4} u \cdot \nabla m_j = \frac{UhG}{c^4}\left[u \cdot \nabla\left(\Pi_j \log \Pi_j\right) - \left(u \cdot \nabla \Pi_j\right)\log \Pi_j\right] (13,22)$$

that introduces the function S having the physical dimensions of h through a dimensionless arbitrary unit vector u. As this result holds for any j-th mass of the system, let U, not yet specified, be such that Π_j takes the meaning of a probability; e.g. this interpretation could be satisfied putting $\sum_j \Pi_j = 1$ by definition and $U = f\sum_j \varepsilon_{0j}$, where $f = f(\Delta t)$ is an appropriate function accounting for the time dependence of U. Summing both sides over j, this result takes the form

$$\frac{\mathrm{S}}{h} = \frac{UG}{c^4} u \cdot \sum_j \nabla\left(\Pi_j \log \Pi_j\right) - \frac{UG}{c^4} u \cdot \sum_j \log \Pi_j \nabla \Pi_j \qquad \mathrm{S} = \frac{hGU}{c^4} u \cdot \sum_j \nabla m_j = \sum_j \mathrm{S}_j (13,23)$$

Note that $UGu/c^4 = r$ has physical dimensions of a vector length and that the summation of the first addend can be rewritten as $r \cdot \nabla \sum_j \left(\Pi_j \log \Pi_j\right)$; hence it is possible to rewrite the last equation as

$$\frac{\mathrm{S}}{h} = r \cdot \nabla \sum_j \left(\Pi_j \log \Pi_j\right) - r \cdot \sum_j \log \Pi_j \nabla \Pi_j \qquad r = r(\Delta t) = \frac{UG}{c^4} u. (13,24)$$

As S has the physical dimensions of h, the second addend of the eq (13,24) can be obtained by nothing else but the scalar product of r and an arbitrary momentum p/h. This equation reads thus more conveniently

$$\frac{\mathrm{S}}{h} = -r \cdot \nabla \sigma + \frac{r \cdot p}{h} \qquad \sigma = -\sum_j \Pi_j \log \Pi_j \qquad \frac{p}{h} = -\sum_j \log \Pi_j \nabla \Pi_j . (13,25)$$

The reason of this notation is that $\log \Pi_j < 0$ because by definition $0 < \Pi_j < 1$; in this way σ and p are thus positive. Are evident three relevant results.

(i) The chance of defining a function S proportional to σ via a constant k_B, i.e.

$$S = k_B \sigma = -k_B \sum_j \left(\Pi_j \log \Pi_j\right) (13,26).$$

(ii) According to the Euler homogeneous function theorem, $r \cdot \nabla \sigma = \sigma$ if the function σ is a first degree homogeneous function of x, y, z; so the eq (13,25) reads $\mathrm{S} = -\sigma h + r \cdot p$. Eventually, note that the eq (Y3Y) allows to find $\Delta S = R\log(V_2/V_1)$ for the isothermal expansion of an ideal gas and that this result is identically inferable also directly from the eq (13,26). The conclusion is that σ is a dimensionless entropy and thus it should be the ratio of two arbitrary energies E/ε. In effect, being $h = \varepsilon t$ according to the eqs (4,5), one finds $\sigma h = (\mathrm{E}/\varepsilon)h = \mathrm{E}t$. So one finds

$$\mathrm{S} = -\mathrm{E}t + r \cdot p. (13,27)$$

Note that the physical dimensions of S of the eq (13,23) are equal to that of the eqs (4,3); hence holds also now the eq (7,3).

(iii) Noting that $-d\sigma/d\Pi_j = 1 + \log \Pi_j$, one finds

$$-\Pi_j \frac{\delta \sigma}{\delta \Pi_j} = \Pi_j + \Pi_j \log \Pi_j;$$

summing over j, the second equation yields by definition the average value of $\delta\sigma/\delta\Pi_j$

$$-\left\langle \frac{\delta \sigma}{\delta \Pi} \right\rangle = -\sum_j \Pi_j \frac{\delta \sigma}{\delta \Pi_j} = 1 - \sigma \qquad \sum_j \Pi_j = 1,$$

which yields eventually

$$-\left\langle \frac{\delta\sigma}{\delta\Pi} \right\rangle + \sigma = 1.(13,28)$$

The sum of two terms equal to 1 means that these terms regard two probabilities whose sum yields the certainty. This suggests that

$$order + disorder = 1 \quad order = -\left\langle \frac{\delta\sigma}{\delta\Pi} \right\rangle \quad disorder = \sigma(13,29)$$

The entropic term measures the degree of disorder; since any thermodynamic system is characterized by its internal degrees of order and disorder, the second addend is reasonably related to the degree of order. If the former addend increases for an isolated term, the second term must be a decreasing function. A system with one particle only is an ordered system, as there is one accessible configuration only. Note that these results have been inferred via the eq (2,1) simply implementing the idea of non-equilibrium configuration inherent $\nabla m \neq 0$.

13.5 THE SECOND AND THIRD LAW

Start from the eq (13,26) and the concurrent boundary condition $\sum_j \Pi_j = 1$ to calculate the quantity $\sigma + \sigma_a$, where the subscript stands for additive; the notation emphasizes that $\sigma_a > 0$ is a dimensionless entropy additive to σ. Hence σ_a is not necessarily constant; like σ, it is however required not to depend on the dummy index j by definition. As such, σ_a represents the evolution of the whole system, not that of some allowed states. As $\sigma + \sigma_a = -\sum_j \Pi_j \log \Pi_j + \sum_j \sigma_a \Pi_j$, trivial algebraic steps yield

$$\sigma + \sigma_a = -\exp(\sigma_a) \sum_j \Pi'_j \log \Pi'_j \quad \Pi'_j = \Pi_j \exp(-\sigma_a).(13,30)$$

Since $\Pi'_j < \Pi_j$, in principle Π'_j is still consistent with the meaning of probability; note that the entropy increase implies a number j of Π'_j states necessarily greater than that initially introduced for Π. The entropy increase is expressed as a function of the value of a new entropy σ'

$$\sigma + \sigma_a = \exp(\sigma_a)\sigma' \quad \sigma' = -\sum_j \Pi'_j \log \Pi'_j.(13,31)$$

Calculate now the entropy change $\delta\sigma$ due to an ideal modification of the state of the system from an initial configuration σ_0 to a final configuration σ_{fin}. Let $\delta\sigma = \sigma_{fin} - \sigma_0$ be expressed without loss of generality as $\delta\sigma = (\sigma_{fin} - \sigma_a) - (\sigma_0 - \sigma_a)$, i.e. as a function of σ' only. So $\delta\sigma = \exp(\sigma_a)\sigma' \pm \vartheta^{-1}\sigma'$: even the amount $\sigma_0 - \sigma_a$ of entropy added or subtracted has been still expressed as a function of σ' via an arbitrary coefficient $\pm\vartheta^{-1}$, correspondingly to either chance $\sigma_0 < \sigma_a$ or $\sigma_0 > \sigma_a$ in principle possible. The linear combination emphasizes that the transition from σ_0 to σ_{fin}, whatever they might be, is consistent in general with an increase or decrease of the initial entropy. Write then

$$\delta\sigma = \Lambda_\sigma \sigma' \quad \Lambda_\sigma = \exp(\sigma_a) \pm \vartheta^{-1}.(13,32)$$

Given that $\sigma_a > 0$ and being $\sigma > 0$ and $\sigma' > 0$ by definition, either sign of $\delta\sigma$ depends on ϑ only: e.g. if this latter is positive and tending to zero, then it is possible that $\delta\sigma < 0$. Consider thus separately the two cases.

i) $\Lambda_\sigma < 0$. In this case the eq (13,32) reads

$$\delta\sigma = -|\Lambda_\sigma|\sigma' \quad \delta\sigma < 0.(13,33)$$

Clearly $\delta\sigma$ governed by $\Lambda_\sigma = \exp(\sigma_a) - \vartheta^{-1}$ concerns the case where an appropriate thermodynamic process subtracts the amount of entropy $\vartheta^{-1}\sigma'$ from a system initially described by $\exp(\sigma_a)\sigma'$.

ii) $\Lambda_\sigma > 0$. Let us write first the eq (13,32) as follows

$$\delta\sigma = \frac{\sigma'}{n_{FB}} \qquad n_{FB} = \frac{1}{\Lambda_\sigma} = \frac{\vartheta}{\exp(\Delta\sigma_a)\pm 1} \qquad \Delta\sigma_a = \sigma_a + \log\vartheta.(13,34)$$

So $n_{FB} > 0$ regardless of the $\pm$ sign via an appropriate choice of σ_a and ϑ; now

$$\delta\sigma > 0.(13,35)$$

The crucial difference between the eqs (13,33) and (13,35) is that $\delta\sigma < 0$ can be obtained only subtracting a suitable amount of entropy controlled by the parameter ϑ from the former term, whereas instead $\delta\sigma > 0$ is naturally obtained even without subtracting or adding anything. Hence the latter is an isolated system characterized by its natural tendency to increase the entropy, the former does not: the necessity of subtracting $\vartheta^{-1}\sigma'$ to get $\delta\sigma < 0$ indicates that part of its previous entropy was removed by interaction with another system. This is the second law.

A possible understanding of the eq (13,34) is that n_{FB} represents the number of particles possible in a given state of the system, in which case the change $\delta\sigma$ is proportional to the initial entropy of each one of them; moreover the sign $\pm$ suggests that exist two different statistical distribution of particles between the allowed quantum states. At least in principle this is acceptable: being $\delta\sigma$ still an entropy, it simply indicates its extensive property, and agrees with the eq (13,32): the positive coefficient n_{FB}^{-1} allows $-\sum_j \Pi'_j \log\Pi'_j$ to represent entropy change of all particles. Three remarks are necessary at this point.

-The eq (13,31) shows that actually the entropy is defined an additive arbitrary constant apart; indeed the reasoning about σ_a surely holds even for a constant term σ_o. Moreover in this case it is possible that $\sigma_o < 0$; being σ_o arbitrary, in principle it can be defined in agreement with the condition $\Pi_j^{\max}\exp(|\sigma_o|) < 1$ even for the largest one among the various Π_j, in order to ensure according to the eq (13,30) that all Π'_j are still compliant with the meaning of probability. Under this constrain $\sigma - \sigma_o$ is still an entropy change, whereas the first eq (13,31) reads

$$\sigma = \exp(|\sigma_o|)\sigma' + \sigma_o.$$

-The eq (13,30) shows that $\Pi'_{j'} < \Pi_j$; being anyway $\sum_{j'}\Pi'_{j'} = \sum_j \Pi_j = 1$, then $j' > j$ ensures that the entropy increases along with the number of quantum states accessible to the concerned thermodynamic system. Hence any configuration of allowed quantum states is the evolution of a previous one with a lower number of quantum states and thus with a smaller entropy. Going back towards simpler and simpler configurations, one infers that the most fundamental configuration is that with $j = 1$, with one quantum state only, whose entropy is zero or equal to the constant σ_o. This is the third law.

-Since σ_a has been introduced with the physical meaning of entropy it must have a form related to $\delta Q_{rev}/k_B T$, as it has been shown in the previous section. Hence it most general form is reasonably that of linear combination $a_1 dQ_{rev}/k_B T + a_2$, which can be also rewritten as $(\varepsilon_2 - \varepsilon_1)/k_B T$ merging the coefficient a_2 with $\log(\vartheta)$. So

$$n_{FB} = \frac{\vartheta}{\exp(\Delta\varepsilon/k_B T)\pm 1}.$$

This is the well known form of the BE and FD statistical distributions: the ways to occupy the quantum states result examining the trend of the respective statistical distributions of particles as a function of T. Examining the trend of n_{BF}, one infers "a posteriori" the different character of either statistical distribution. In this way, coherently with the purposes of the present model, the existence of fermions and bosons comes from the eq (2,1), from which has been inferred the eq (13,26) here implemented, instead of being purposely hypothesized via either kind of occupancy of quantum states to explain specific topics. The spin of particles, however, has not yet been introduced in the frame of the eq (2,1). This point is examined in the next section.

13.6 THE SPIN

This section aims to show that even the spin of quantum particles and the related statistics are inferable in the present conceptual frame based on the eq (2,1). Start to this purpose from the eqs (4,5) and write

$$\Delta x\Delta p = nh = (x_1p_1 + x_2p_2) - (x_2p_1 + x_1p_2) \quad \Delta x = x_2 - x_1 \quad \Delta p = p_2 - p_1 (13,36)$$

to describe a particle delocalized in Δx whose kinetic momentum p falls in the range Δp of local values; x_1 and x_2 are any boundary coordinates, arbitrary likewise as p_1 and p_2. Regard the quantities at the right hand side as x components of a new vector, call it M in agreement with the eq (13,7) by dimensional reasons. Being both signs possible for the components of any vector along an arbitrary direction, the eq (13,36) reads

$$|\mathrm{M}_x| - |\mathrm{M}_{x12} + \mathrm{M}_{x21}| = nh \quad \mathrm{M}_x = x_1p_1 + x_2p_2 \quad \mathrm{M}_{x12} = x_1p_2 \quad \mathrm{M}_{x21} = x_2p_1 . (13,37)$$

Both M_{x12} and M_{x21} are mere quantum properties: they vanish if the uncertainty ranges reduce to the respective classical dynamical variables. E.g. putting $x_2 = 0$ and $p_2 = 0$ results instead $\mathrm{M}_x = x_1p_1 \neq 0$; the same holds of course putting $x_1 = 0$ and $p_1 = 0$, which imply $\mathrm{M}_x = x_2p_2 \neq 0$. Moreover it can also be $\mathrm{M}_x = 0$ for $x_1p_1 = -x_2p_2$ while being $\mathrm{M}_{x12} + \mathrm{M}_{x21} = x_2(p_1^2 - p_2^2)/p_1 \neq 0$ since $p_1 \neq p_2$ by definition. Hence: (i) M_{x12} and M_{x21} have a physical meaning analogous, but different from that of M_x; (ii) all of them are nevertheless components of angular momentum. Thus not only $|\mathrm{M}_x| \neq 0$ and $|M_{x12} + \mathrm{M}_{x21}| = 0$ but also $|\mathrm{M}_x| = 0$ and $|M_{x12} + \mathrm{M}_{x21}| \neq 0$ must be satisfiable together with the chance of both terms equal to zero or different from zero, because there is no physical reason to exclude anyone of them; for this reason the moduli $|\mathrm{M}_x|$ and $|\mathrm{M}_{x12} + \mathrm{M}_{x21}|$ have been considered separately. Moreover the point (i) also suggests that $|\pm\mathrm{M}_{x12}| = |\pm\mathrm{M}_{x21}|$: this position makes indeed $\mathrm{M}_{x12} + \mathrm{M}_{x21}$ compatible with both $\pm(\mathrm{M}_{x12} + \mathrm{M}_{x21}) = \pm 2\mathrm{M}_{x12}$ and $\pm(\mathrm{M}_{x12} - \mathrm{M}_{x21}) = 0$, so that the addends at the left hand side of the eq (13,37) take the expected forms $|\pm\mathrm{M}_x| = n_{or}h$ and $2|\pm\mathrm{M}_{x12}| = n_{sp}h$. The notation expresses that n_{or} and n_{sp} are in principle independent whole sets of arbitrary integers likewise n, in agreement with the concept of uncertainty. Split then the first eq (13,37) as

$$|\pm\mathrm{M}_x| = (n_{or} - 1)h \quad 2|\pm\mathrm{M}_{x12}| = (n_{sp} - 1)h \quad n_{or} - n_{sp} = n$$

The reason of these positions is that $n \geq 1$, likewise $n_{or} \geq 1$ and $n_{sp} \geq 1$ even simultaneously, whereas instead it is possible that $|\pm\mathrm{M}_x| = 0$ and $|\pm\mathrm{M}_{x12}| = 0$ too. The chance that $|\pm\mathrm{M}_{x12}| = 0$ compels writing $n_{sp} - 1$ and thus $n_{or} - 1$ as well, which in turn is compatible with $|\mathrm{M}_x| = 0$.

As concerns the given component of M, it is worth summarizing the possible chances

$$\mathrm{M}_x = \pm n_{orbit}h \quad \mathrm{M}_{x12} = \pm n_{spin}\frac{h}{2} \quad n_{orbit} \geq 0 \quad n_{spin} \geq 0.$$

The quantum properties of M have been inferred considering one of its components only, actually without specifying anything; just this entails the integer and half integer quantizations that appear merged in $j_x = n_{orbit} \pm n_{spin}/2$ according to the usual notation in h units. Clearly n_{orbit} and n_{spin} are independent each other: the fact that the latter is a mere quantum property that vanishes considering the local coordinates only, indicates that the former is the quantum property corresponding to the classical angular momentum. The spin is thus an intrinsic and distinctive property of the quantum particles.

It is evident that if n_{spin} is even, then the total angular momentum component of N particles $N_{tot} = Nn_{orbit} + Nn_{spin}/2$ is an integer number of h units; hence, being n_{orbit} arbitrary, the angular momentum a quantum system with N particles is indistinguishable from that with a different number $N + N'$ of particles. Instead for n_{spin} odd, the system jumps from half integer to integer values after the addition of a new particle, i.e. the quantum states of the system are distinguishable upon addition of each particle. It is clear that this is nothing else but a different formulation of the Pauli principle. This result has been also found in [12]. The reasoning here carried out for one component of M cannot be repeated for another component: this would trivially mean changing the notation of the component with a unique physical information only. Otherwise stated this means that in lack of a new physical information, actually non existing, one component only of M is knowable. So in a homogeneous and isotropic space it is reasonable to expect $<\mathrm{M}_x^2>=<\mathrm{M}_y^2>=<\mathrm{M}_z^2>$: the square averages waive the signs of each component. Summing j_i^2 from arbitrary $-J$ to J yields $\sum_{-J}^{J} j_i^2 = J(J+1)(2J+1)/3$. Hence one finds $\mathrm{M}^2 = \sum_{i=1}^{3} <\mathrm{M}_i^2> = J(J+1)h^2$ as average value of $2J+1$ states including 0, with one i_{th} component given by $j_i = \pm n_{orbit} \pm n_{spin}/2$.

13.7 THE BLACK BODY

In the section 10 the change of the energy density $\eta = \varepsilon/V$ was assumed due to δV and to $\delta\varepsilon$ inside V; the frequency ν of the eq (2,1) was implicitly assumed changing only because of transformations occurring inside V, e.g. because of quantum fluctuations of the space time itself, but not dissipated outside V. In lack of specific hypotheses, however, losses of energy outside V cannot be excluded. Admit therefore the chance that $\delta\nu$ is due just to the irradiation of energy $h\nu$ around the space time volume V, now assumed constant, and consider the eq (5,1) to infer that $\delta\eta/\delta\nu = 2\nu c^2/G$; replacing c^2/G via the eq (2,1) one finds $\delta\eta/\delta\nu = 2h/V$. Assuming that the energy irradiated is very small with respect to $h\nu$, let us express the volume as $V = (c/\nu)^3$, i.e. as a function of ν itself, to find how the energy density of steady radiation waves still inside V changes because of the radiation loss; one finds thus $\delta\eta/\delta\nu = 2h(\nu/c)^3$. Suppose also that the small amount of energy irradiated does not perturb appreciably the equilibrium conditions inside V: then the uniform distribution of radiation present in V produces a homogeneous front of energy irradiated in any element of solid angle $d\Omega$ around the surface of V. Hence, integrating the uniform energy irradiation all around V, means regarding $\delta\eta/\delta\nu$ as a constant; so one finds $\delta\eta/\delta\nu\big|_{tot} = 8\pi h(\nu/c)^3$. Eventually the energy irradiated at any ν depends reasonably on the number n_ν of oscillators actually present in V at the given frequency. So the energy loss escaping from the space times has the form $8\pi h(\nu/c)^3 n_\nu$. Obviously $n_\nu \equiv n_{BF}$ is given in this case by the statistical distribution of bosons, already concerned in the previous section: so the result is nothing else but the well known Planck distribution function.

Is interesting the fact that the multiplicative factor of the Bose distribution in the Planck formula is the fingerprint of the definition of space time, eq (2,1).

14 NUMERICAL ESTIMATES

Some results hitherto exposed are well known and thus self-validated; it is useful therefore to examine the results that require numerical estimates to be fully assessed. In this respect, particular attention will be payed to the self-consistency of these outcomes. This section aims to link the features of the space time inferred as corollaries of the initial position (2,1) to that of our universe experimentally observed or at least estimated. The most intuitive approach to this purpose is to evaluate the formulas previously introduced with the current estimates representing our knowledge of the today universe and analyze the results. The key values resulting from the definitions of space time, eqs (2,1) and (12,9), are

$$V\nu = \frac{hG}{c^2} = 4.9\times10^{-61}\mathrm{m^3s^{-1}} \qquad \left(\frac{\zeta V\Lambda_t}{\xi}\right)^2 = hcG = 1.3\times10^{-35}\mathrm{m^6s^{-4}}. (14,1)$$

The key values that surrogate size, age, and visible matter contained in the region V of the space time are the estimates of radius Δr_u, age Δt_u and visible mass m_u available in the literature of the current cosmology:

$$\Delta r_u = 4.35\times10^{26}\mathrm{m} \quad \Delta t_u = 4.35\times10^{17}\mathrm{s} \quad m_u = 3\times10^{52}\mathrm{Kg}. (14,2)$$

The subscript u stands for "universe" and emphasizes just the cosmological meaning of these test values.

Replacing the variables present in the various equations with these data that describe the today universe, actually means regarding the space time as a "statistical mirror" of the universe. The hope of inferring physical information on the latter investigating the physical features of the former is justified by the fact of having already obtained a sensible background of known physical laws of the nature.

14.1 VOLUME AND MASS

The section 5 has shown tight correlation in the space time between volume increase and amount of matter: the formation of matter in a previously massless space time implies its volume increase. The problem arises now about how the volume progresses along with the amount of mass already formed. Examine the properties of the space time considering length units that do not implement conventional measure standards, but invariant physical lengths; this means considering that the lengths provide a natural way to describe the correlation between volume and mass. The model has introduced in the eqs (5,3) and (13,8) the Compton length and the black hole length, which have two important features: (i) they depend on the mass only and (ii) they coincide for $m=m_{Pl}$, being both equal to the Planck length l_{Pl}. For $m>m_{Pl}$ however λ_C is a decreasing function of m, i.e. it is suitable to describe a space time shrinking around the first seed of matter initially formed; λ_{bh} is instead an increasing function of m, i.e. it describes an expanding space time with increasing mass inside. Consider now the chance of expressing the space time volume via these lengths, e.g. $V=n_C^*\lambda_C^3$ or $V=n_{bh}^*\lambda_{bh}^3$ or even $V=n_C^*n_{bh}^*(\lambda_C\lambda_{bh})^{3/2}$; of course $n_C^*=n_C^*(\Delta t)$ and $n_{bh}^*=n_{bh}^*(\Delta t)$ and are arbitrary real numbers allowing in principle to describe any value of V as a function of Δt. The dependence of V on m results to be respectively $V=V(m^{-3})$ or $V=V(m^3)$, while being also possible V independent on m; e.g. $(\lambda_C\lambda_{bh})^{3/2}$ would describe evolution of space time independent of mass and thus in principle a possible universe without matter, filled by an appropriate radiation field only; the section 12 has in effect shown that is possible a space time characterized by virtual mass and charges only. Also, $V\propto\lambda_C^3$ would describe a space time with average density ρ increasing like $\rho\propto m^4$, whereas $V\propto\lambda_{bh}^3$ would imply $\rho\propto m^{-2}$. This is more than a formal approach. In a sense the Planck mass is the watershed between the elementary particles of the quantum physics and the large masses of the relativistic physics: it is sufficiently high to be regarded as the upper boundary of the former and sufficiently small to be the lower boundary of the latter. Consider indeed the eqs (5,3) and (13,8): as both equations depend on the mass only, one finds

$$\frac{\delta\lambda_C}{\delta\Delta t}=-\frac{h}{m^2c}\dot m\qquad\frac{\delta\lambda_{bh}}{\delta\Delta t}=\frac{2G}{c^2}\dot m\qquad\dot m=\frac{\delta m}{\delta\Delta t}.(14,3)$$

Moreover, the time dependence of V and ρ on m reads

$$\frac{\delta V}{\delta\Delta t}=\frac{\delta V}{\delta m}\dot m\propto -m^{-4}\dot m\qquad\frac{\delta\rho}{\delta\Delta t}=\frac{\delta\rho}{\delta m}\dot m\propto -m^3\dot m.(14,4)$$

$$\frac{\delta V}{\delta\Delta t}=\frac{\delta V}{\delta m}\dot m\propto m^2\dot m\qquad\frac{\delta\rho}{\delta\Delta t}=\frac{\delta\rho}{\delta m}\dot m\propto -m^{-2}\dot m(14,5)$$

For $m=m_{Pl}$ both λ_C and λ_{bh} coincide with the Planck length, crossing point in the fig 1, whereas at increasing length the mass has opposite behavior: i.e. it splits along curves with decreasing and increasing values. In other words λ_{bh} describes the ability of the space time to expand and form locally huge aggregates of several particles, λ_C describes the ability of the space time to shrink locally and form single particles. In fact one expects that the size of the quantum particles should reasonably of the order of magnitude of their Compton length; for example, the radius r_p of an isolated proton is estimated in the range 0.84-0.87 fm; the mass $1.672\times10^{-27}\,\mathrm{Kg}$ yields in effect the size 1.3 fm, i.e. about $2r_p$. Moreover Dirac has estimated the radius of an isolated electron of the order of its reduced Compton length. For this reason the region characterizing the quantum particles has been qualitatively sketched around the λ_C curve, whereas that leading to the large matter structures must be intended as an extrapolation of the λ_{bh} curve.

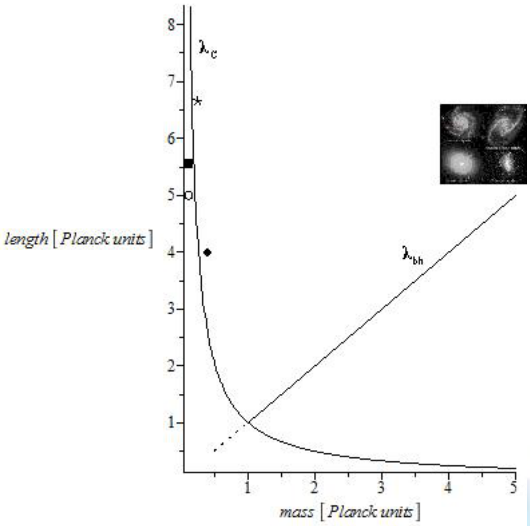

Fig 1: plot of λ_C and λ_{bh} as a function of m. The symbols along the respective curves represent the region within which quantum particles are formed at appropriate values of Planck length and Planck mass; the picture represents the formation of cosmic objects at large values of mass and length.

The common sense suggests that the tendency to form dense matter aggregates is a local feature of the space time; the large regions of vacuum between the matter islands suggest instead a black hole like behavior of the whole space time. Multiply both sides of the eq (9,2) by h/c and then by G/c^2; recalling the eqs (13,14), $\delta\varepsilon/\delta x$ turns into $\delta\varepsilon/\delta(n_C^* h/mc)$ and $\delta\varepsilon/\delta(n_{bh}^* mG/c^2)$. Regard the length coefficients n_C^* and n_{bh}^* as constants: this means that the lengths change as a function of m only. So one obtains two new Lagrange equations, one coming from λ_C and another from λ_{bh}, i.e. respectively

$$\frac{\delta}{\delta\Delta t}\frac{\delta\Delta\varepsilon}{\delta\dot{m}_r}=\frac{\delta\Delta\varepsilon}{\delta m_r}\qquad m_r=\frac{1}{m}\qquad \frac{\delta}{\delta\Delta t}\frac{\delta\Delta\varepsilon}{\delta\dot{m}}=\frac{\delta\Delta\varepsilon}{\delta m},(14,6)$$

which shows that m and its reciprocal m_r are readable as generalized coordinates likewise as the respective λ_{bh} and λ_C. Therefore not only these lengths but also the masses have physical meaning of generalized coordinates and thus should be expectedly suitable to describe the time evolution of the space time.

It is easy to show two examples of how can be implemented the eqs (14,6) to find important laws of the space time.

1) The ratios $\delta\varepsilon/\delta\dot{m}$ and $\delta\varepsilon/\delta m$ have physical dimensions l^2/t and v^2 respectively; put then $\delta\varepsilon/\delta\dot{m}=D$ and $\delta\varepsilon/\delta m=v_x^2$, so that the second Lagrange eq (14,6) reads $\delta D/\delta\Delta t=v_x^2$, being $\pm v_x$ the velocity component along an arbitrary x-axis. The double sign means that both components are expectedly allowed. As $v_x=\delta\Delta x/\delta\Delta t$, write thus $(\delta D/\delta\Delta t)\delta\Delta x^{-1}=v_x\delta\Delta t^{-1}$, having omitted the double sign by simplicity of notation. Multiply then both sides of this equation by the mass change δm; being $\delta D=D_2-D_1$ by definition, one obtains

$$D_2\frac{\delta m}{\delta\Delta x}=v_x\delta m+D_1\frac{\delta m}{\delta\Delta x}.$$

Multiply both sides by V_o^{-1}, being V_o a constant volume; as $v_x\delta m=\delta(mv_x)-m\delta v_x$, this result reads

$$D_2\frac{\delta C}{\delta\Delta x}=\delta(v_xC)+D_1\frac{\delta C}{\delta\Delta x}-C\delta v_x\qquad C=\frac{m}{V_o}.$$

Since all quantities just introduced are arbitrary, split this equation as follows:

$$D_2\frac{\delta C}{\delta\Delta x}=\delta J_x \quad D_1\frac{\delta C}{\delta\Delta x}-C\delta v_x=0 \quad J_x=v_xC.$$

Appears in this result the physical meaning of mass flux component J_x; also, with the minus sign of v_x the first result yields the Fick diffusion law, whereas the initial position $\delta\varepsilon/\delta\dot{m}$ is to be regarded as the definition of diffusion coefficient D. This result is important as it accounts for several physical gradient laws, e.g. the charge and heat transfer laws of Ohm and Fourier.

The second equation reads $\delta\log(C)/\delta\Delta x=\delta v_x/D_1$, i.e. with vector notation

$$\nabla\mu'=\frac{u}{\lambda}=\frac{1}{D_1}\delta v \quad \mu'=\log\left(\frac{C}{C_o}\right),$$

being λ an arbitrary length, C_o a constant density and u a unit vector arbitrary and dimensionless defined by δv. Multiply by k_BT both sides of the first equation: as the right hand side has the dimensions of a force, it is possible to write

$$\nabla\mu=-F \quad F=\frac{k_BT}{D_1}\delta v \quad \mu=-k_BT\log\left(\frac{C}{C_o}\right).$$

So one also finds the chemical potential μ of m, whereas the second equation is the famous Einstein equation linking mobility $\beta=velocity/force$ and D via k_BT. The second Fick law is a trivial consequence of the first one with the help of the continuity equation as a boundary condition.

The definition of diffusion coefficient $D=\delta\varepsilon/\delta\dot{m}$ does not exclude in principle even $D<0$; as it is known, $D>0$ describes the homogenization of a heterogeneous system (Fick laws), $D<0$ implies phase separation (e.g. spinodal decomposition).

2) Consider now the first eq (14,6) and put

$$\Delta\varepsilon=\varepsilon_o+\frac{\delta a}{\delta\Delta t}\dot{m}_r+\frac{\delta^2 a}{\delta\Delta t^2}m_r$$

where ε_o is a constant and a is a function having physical dimensions of a square length that by definition depends neither on m_r nor on $\dot{m}_r$. This position is acceptable: replaced on the eq (14,6) one finds the identity $\delta^2a/\delta\Delta t^2\equiv\delta^2a/\delta\Delta t^2$. Moreover putting reasonably $\delta\Delta t=\delta\Delta x/v$, with v constant velocity, one finds

$$\frac{\delta^2 a}{\delta\Delta t^2}=v^2\frac{\delta^2 a}{\delta\Delta x^2}$$

i.e. again the Dâ€™Alembert wave equation (OXQ). Both results show that the proposed definition of $\Delta\varepsilon$ is sensible. Write now $\Delta\varepsilon$ explicitly as a function of m; being $\dot{m}_r=-\dot{m}/m^2$, one finds

$$\Delta\varepsilon'=-\frac{1}{m}\left(\frac{\delta a}{\delta\Delta t}\frac{\dot{m}}{m}-\frac{\delta^2 a}{\delta\Delta t^2}\right) \quad \Delta\varepsilon'=\Delta\varepsilon-\varepsilon_o$$

Note first that being $\Delta\varepsilon$ an uncertainty range, its size is not essential; so writing $\Delta\varepsilon'$ or $\Delta\varepsilon$ is physically irrelevant, as repeatedly shown. Is instead relevant the sign of the quantity in parenthesis, which necessarily has physical dimension of square momentum p. Since no hypothesis has been made on $\dot{m}$ and m, is particularly interesting the case where this sign is positive, in which case it is possible to write

$$\Delta\varepsilon'=-k^2\frac{p^2}{m} \quad p=\pm\frac{1}{k}\sqrt{\frac{\delta a}{\delta\Delta t}\frac{\dot{m}}{m}-\frac{\delta^2 a}{\delta\Delta t^2}} \quad \frac{\delta a}{\delta\Delta t}\frac{\dot{m}}{m}-\frac{\delta^2 a}{\delta\Delta t^2}>0$$

having introduced the proportionality factor k to define the corresponding momentum p. As $p=i\sqrt{m\Delta\varepsilon'}/k$ reads

$p = i\Delta\Pi/k$, because $\sqrt{m\Delta\varepsilon'}$ has physical dimensions of momentum range $\Delta\Pi = nh/\Delta x$, one finds

$$p = ih\frac{\Delta\psi}{\Delta x} \quad \Delta\psi = \frac{n}{k}.$$

The notation $\Delta\psi = \psi_2 - \psi_1$ emphasizes that, as repeatedly stated, n symbolizes a set of values and not a single specific value; so the arbitrary range $\Delta\psi$ of values of ψ is necessary to account for the arbitrary range of values of n corresponding to $\Delta\Pi$. The inequality simply remarks the condition consistent with ip . Also now, as done to infer the eq (13,2), the limit $\Delta \to 0$ yields the momentum operator of the wave mechanics. Appears clear once again why the range size $\Delta\varepsilon'$ is irrelevant by implementing the eqs (4,5).

14.2 MICRO- AND MACRO-SPACE TIME

Owing to the eqs (13,8) and (12,7), rewrite the eq (2,1) as

$$\frac{hG}{c^2} = V\nu = \frac{h}{mc}\frac{2mG}{c^2}\ell\nu \quad \ell\nu = \frac{c}{2};(14,7)$$

this means that the space time volume is also expressible via the fundamental lengths λ_C and λ_{bh} of the mass m , with the help of a further characteristic wavelength ℓ . Since there is no reason to reject physical meaning and implications of either length, according to the eqs (14,6) the conclusion is that both must be accepted: there is an expanding macro space time described by λ_{bh} and a shrinking micro space time described by λ_C . To this purpose, being ℓ arbitrary, it is possible at least in principle that, depending on m , is verified either condition:

$$\ell\frac{2mG}{c^2} = \left(\frac{h}{mc}\right)^2 \quad \ell = \frac{1}{2}\frac{h^2}{m^3G} \quad \nu = \frac{m^3Gc}{h^2},(14,8)$$

in which case V is governed by the eq (5,3) only, or

$$\ell\frac{h}{mc} = \left(\frac{2mG}{c^2}\right)^2 \quad \ell = \frac{4m^3G^2}{hc^3} \quad \nu = \frac{hc^4}{8m^3G^2},(14,9)$$

in which case V is governed by the eq (13,8) only.

Nothing excludes therefore that in fact the space time as a whole grows as supermassive black hole. This feature, in principle not required, worths attention: it promotes the growth of the space time as it implies that neither mass nor energy escape outside V .

On the one hand, invoking the effective occurrence of black hole behavior of V means acknowledging the best growth condition of the space time.

On the other hand, this boundary condition requires internal mass continuously created through the energy existing in V as long as its size is allowed to grow.

Eventually it is also reasonable to suppose that the successful growth of the space time volume till to the internal formation of huge and complex matter structures is subordinate to the arising of an energy trigger sufficient to generate a first nucleus of gravitational mass in an initial massless space time.

Having shown that the eq (2,1) entails by itself the existence of energy density η inherent the definition (1,1) of space time, suppose that an energy quantum fluctuation is allowed to occur at an arbitrary time t_0 in V where $\eta = \eta_0$: let the time profile of η ramp up till to the time $t_{\max}$, after which it ramps down till to vanish at the time t_{end} . It is necessary that during the time range $t_{end} - t_0$, the amount of energy created by the fluctuation enables the conversion $\eta_{t_0} + \delta\eta \to \rho c^2/V$ of energy density into mass density sufficient to fulfill the black hole condition: whatever the specific mechanism of mass creation might be, e.g. the gamma-gamma process [25], this is indeed the most advantageous prerequisite for the further growth and evolution of V .

With the initial black hole condition the boundary of the early universe would intuitively be a sharp interface between an empty region external to V and an internal region V related to η_{t_0} and filled with matter, radiation and virtual particles of quantum vacuum created and soon annihilating; the attractive action of the self-gravity makes this boundary well defined and unsurmountable.

Without the initial black hole condition, the sharp boundary would be instead an extended diffusion region through which virtual particles, radiation and matter escape and diffuse outside V; hold in this respect the considerations of the section 13.7 and 14.1. In the presence of losses, the residual fluctuation energy could be inadequate to prevent valuable mass/energy escaping outside V, which therefore could even stop expanding.

An early energy sufficient to trigger the black hole condition, therefore, is required for the space time to start its growth process even after the successive ramp down of the fluctuation energy: without this condition, the growth evolution would abort as a mere perturbation transient with a sterile time profile.

All this does not require hypotheses additional to the eq (2,1): the eqs (5,2) and (5,1) show that the definition of space time contains all basic ingredients useful to support this point of view, simply implementing the quantum concept of fluctuation energy. Indeed ν implies the possible existence of higher harmonics $n\nu_n$ defined by an integer number n of shorter wavelengths still contained in the volume V.

The longest steady wavelength allowed in V is $\lambda_{\max} = 2\Delta x$; as stated in the section 13.2, it is made of two half wavelengths with nodes at the boundaries of Δx. Moreover shorter steady wavelengths $\Delta x/n$ are also possible, because with n integer even these latter have nodes at the boundaries of Δx. So, whatever the propagation rate v of the wave in Δx might be, the possible frequencies are $v/\lambda_{\max} = v/2\Delta x = \nu/2$ and $n\nu$ with $\nu = v/\Delta x$. Hence the energy expectable in Δx^3 is $(n+1/2)h\nu$, different from zero even for $n=0$. In fact this result is expectable according to the eqs (8,3), (13,9) and (13,10). Clearly all $n>0$ correspond to the allowed frequencies describing the quantum fluctuation, whereas the zero point energy and the higher harmonic energies are intrinsic features of the space time and its uncertainty corollary (4,5) described by the frequency ν early introduced in the eq (2,1).

To find how V grows contextually with the increasing of n, are necessary two equations linking ν and V. The first equation links directly ν to V via the eqs (5,1) written as a function of V; i.e. $h\nu = h^2G/c^2V$ yields

$$\eta = \left(n+\frac{1}{2}\right)\left(\frac{h}{cV}\right)^2 G \quad \varepsilon = \eta V \quad n = n(\Delta t).(14,10)$$

The second equation links this result with the eqs (10,4) and (5,1); i.e.

$$\frac{\alpha^2}{9G}\left(\frac{c}{\Delta t}\right)^2 = \left(n+\frac{1}{2}\right)\frac{(c\nu)^2}{G}$$

reads

$$\frac{\alpha}{3}\frac{c}{\Delta t} = \pm c\nu\sqrt{n+\frac{1}{2}}.(14,11)$$

This equation is significant as $\pm\nu$ shows equal chances of ν or $\bar{\nu}$ i.e., according to the eq (6,1), of forming matter and antimatter during the evolution of the space time; in lack of further information, one must conclude that matter and antimatter form with the same probability. Hold all considerations carried out in the section 6, in particular the fact that the negative frequency implies negative Δt according to the CPT theorem.

The eq (10,4) describes the progressive decrease of energy density as a function of time and calculates the temperature at various times. Indeed

$$T = \sqrt[4]{\frac{\alpha^2}{9G}\left(\frac{c}{\Delta t}\right)^2\frac{1}{\theta}} \quad \theta = \frac{8\pi^5 k_B^4}{15(hc)^3} = 7.6\times10^{-16}\,\frac{\mathrm{J}}{\mathrm{m^3K^4}},(14,12)$$

where θ is the black body constant. This is particularly interesting, not only to verify the validity of the eq (10,4) but also because the temperature is the fingerprint of the processes occurring in the universe at various stages of its life. An example is the formation of matter structures: since the chemical binding energy is of the order of some eV, it is clear that neither chemical compound nor solid matter are stable at $T > 10^5\mathrm{K}$. An analogous reasoning holds for nuclear constituents like neutrons and protons; the T profile elucidates the steps through which the universe evolved till today.

The eq (14,12) can be immediately verified even without need of further data. Consider Δt at the present time, i.e. Δt_u seconds after the "big bang". Introducing $\Delta t = \Delta t_u$ of the eqs (14,2), one finds

$$\eta_{cmbr} = 4.2\times10^{-14}\frac{\mathrm{J}}{\mathrm{m}^3} \quad T_{cmbr} = \sqrt[4]{\frac{\alpha^2}{9\theta G}\left(\frac{c}{\Delta t_u}\right)^2} = 2.73\mathrm{K};(14,13)$$

the notation stresses that this energy density does not refer to any mass, as it in effect appears in the eq (10,4), but reasonably to the radiation field. This is just the well known temperature corresponding to the measured CMBR.

At the beginning, say at the Planck time, let be $n = 0$: the space time is in its own ground state of zero point energy ε_{t_0} of density $\eta_{t_0} = (c\nu)^2/2G$. Next, the energy fluctuation rises n to values different from zero. Since according to the eq (5,4) $mc^2 > h\nu$, the creation of mass requires at least $mc^2 = h\nu + h\nu/2$ taking into account just the zero point energy; so the eq (14,10) requires $n \geq 1$. Hence the matter era was allowed to occur after the radiation era, when the quantum fluctuation had already provided trigger energy additional to the zero point energy of the early radiation field. Thereafter just $n\nu$ provides the conditions for the space time mass and volume growths.

The next subsections highlight these introductory remarks; the formulas describing the evolution of the space time will be calculated with parameters characterizing our universe at various times to which correspond pertinent values of n. The numerical outcomes of the next sections aim to assess the results obtainable from the propositions just introduced.

14.3 THE PLANCK TIME $n = 0$

The early size of the space time at the beginning of the Planck era should expectedly be of the order of the Planck length. Is interesting the fact that in effect the frequency defined by $V\nu$ of the eq (2,1) corresponds to a steady wavelength of the same order of magnitude of the size $\Delta x = l_{Pl}$ defining the initial V_{Pl}: replacing $V_{Pl} = (hG/c^3)^{3/2}$ in the eq (2,1) yields $\nu_{Pl} = (hG/c^2)/(hG/c^3)^{3/2} = t_{Pl}^{-1}$. At this time the eq (5,1) calculates

$$\eta_{n=0} = \frac{(c/t_{Pl})^2}{2G} = \frac{1}{2}\frac{\varepsilon_{Pl}}{l_{Pl}^3} = 4.0\times10^{112}\frac{\mathrm{J}}{\mathrm{m}^3} \quad \varepsilon_{n=0} = \eta_{n=0}l_{Pl}^3 = 2.7\times10^9\,\mathrm{J}.$$

The temperature and pressure are of the order of

$$T_{n=0} = \sqrt[4]{\left(\frac{c}{t_{Pl}}\right)^2\frac{1}{2\theta}} = 8.3\times10^{31}\mathrm{K} \quad P_{n=0} = 1.3\times10^{112}\mathrm{Pa}.$$

At this high radiation density it is reasonable to expect the refraction index $\mathrm{n} > 1$; this suggests that the related radiation wavelength consistent with $\nu_{Pl} = t_{Pl}^{-1}$ and V_{Pl} inferred from the eq (2,1) at $n = 0$, reads actually $\lambda_v = v/\nu_{Pl} = vt_{Pl}$.

The equivalent mass of the radiation energy field compatible with $n = 0$ results to be $m_{Pl}/2$. It is worth noticing that calculating the eq (13,8) with this mass one finds $2(m_{Pl}/2)G/c^2 = l_{bh}$, i.e. l_{bh} is just the size of the Planck length l_{Pl}. The fact that the black hole condition is fulfilled is not surprising, considering that the radiation field just introduced is entirely confined within the available V_{Pl}; indeed, even admitting $v \to c$, the result would be that λ_v tends to the upper limit of the Planck length ct_{Pl} defining the initial volume V_{Pl} of the space time. Thus, whatever $v < c$ might be, this result is compatible with unsteady early wavelengths λ_v, which however run anyway within V_{Pl}: this confirms that no wave escapes outside V_{Pl}. Replacing $\nu = v/\lambda$ in the eq (14,11) and taking for brevity one sign only, one finds

$$\frac{\sqrt{2}}{3}\frac{\alpha}{t_{Pl}} = \frac{v}{\lambda};(14,14)$$

considering in particular a steady zero point wave consistent with v, i.e. a half wave with nodes at the boundaries of V_{Pl}, this expression reads

$$\lambda_v = 2l_{Pl} \qquad \frac{v}{c} = \alpha\sqrt{\frac{8}{9}}.(14,15)$$

Combine the eqs (13,8) and (12,12) replacing m of the former with m_0 of the latter; one obtains $\lambda_{bh} = 2V\zeta\Lambda_t/c^2$. Moreover, owing to the eq (12,4), this result reads $\lambda_{bh} = 2\zeta\rho_\Lambda VG/c^2$; so $\lambda_{bh} = 2m_\Lambda G\zeta/c^2$, being $m_\Lambda = \rho_\Lambda V$. Recalling the eqs (6,3), put now

$$\zeta = \alpha \qquad \alpha = \frac{e^2}{hc} = -\frac{e\bar{e}}{hc};(14,16)$$

noting that by definition $G/c^3 = l_{Pl}^2/h$, one finds thus $\lambda_{bh} = 2m_\Lambda e^2 l_{Pl}^2/h^2$. Hence

$$\frac{e^2}{\lambda_{bh}} = \frac{1}{2}\frac{h^2}{m_\Lambda l_{Pl}^2};$$

also, considering the Compton length $\lambda_C = h/m_\Lambda c = l_{Pl}$ of the virtual particle of mass m_Λ, this result reads

$$-\frac{e\bar{e}}{\lambda_{bh}} = \frac{m_\Lambda c^2}{2} = \frac{m_{Pl}c^2}{2}.$$

At the left hand side, appears the Coulomb interaction of one couple of virtual charges; here this energy also emphasizes that it is effectively related to one half equivalent Planck mass previously introduced. This result supports the position (14,16), which must be regarded as due to $-e\bar{e}$ in a Planck volume electrically neutral. In effect after having found that the size of V at $n=0$ is compatible with the Planck length, it is natural to expect λ_C related to l_{Pl}.

These results can be obtained directly from the eq (14,9), as ℓ yields

$$v = \frac{c}{2\ell} = \frac{hc^4}{8m^3G^2}.$$

Expressing without loss of generality m as a function of the Planck mass as $m = qm_{Pl}$ via an arbitrary dimensionless coefficient q, the result is $v = (8q^3 t_{Pl})^{-1}$. If in particular $m = m_{Pl}/2$ i.e. $q = 1/2$, one finds again $v_{n=0} = t_{Pl}^{-1}$ and thus the other results as well; this confirms the first position of the eq (14,15) and the eq (14,16).

14.4 $n=1$

The eq (14,10) yields

$$\Delta\varepsilon = \varepsilon - \varepsilon_{Pl} = \left(n+\frac{1}{2}\right)\left(\frac{h}{c}\right)^2\frac{G}{V} - \left(\frac{c}{t_{Pl}}\right)^2\frac{V_{Pl}}{2G}.$$

With the position $c^2V/Gh = \tau = v^{-1}$ one infers

$$\Delta\varepsilon = \frac{h}{2}\left(\frac{2n+1}{\tau} - \frac{\tau}{t_{Pl}^2}\frac{V_{Pl}}{V}\right) \qquad \Delta t = \frac{h}{\Delta\varepsilon} = \frac{2\tau t_{Pl}^2}{(2n+1)t_{Pl}^2 - \tau^2(V_{Pl}/V)},$$

so that, expressing τ as a function of t_{Pl} via a proportionality factor β, one finds thanks to the eq (13,8)

$$\tau = \beta t_{Pl} \qquad V = \frac{Gh\beta t_{Pl}}{c^2} = \beta l_{Pl}^3 \qquad m = \frac{V^{1/3}c^2}{2G} = \frac{\beta^{1/3}l_{Pl}c^2}{2G} \qquad 1 < \beta < 2n+1.(14,17)$$

For $n=1$, expectedly soon after the Planck time, $\tau < 3t_{Pl}$ means that $\tau >: t_{Pl}$, so that by consequence $V >: V_{Pl}$ and $m >: m_{Pl}/2$. The space time at $\Delta t >: t_{Pl}$ is basically similar to that at $n=0$. Analogous considerations hold for $n=2$ and $n=3$.

14.5 $n>3$

Let us calculate n necessary to create one full Planck mass. As $l_{Pl}c^2/2G = m_{Pl}$ by definition, the third eq (14,17) reads $2m_{Pl}G/c^2 = \beta^{1/3}l_{Pl}$ and yields $\beta^{1/3}=2$; in turn $\beta=8$ requires $n=4$ and $\tau=8t_{Pl}$. Also, with $V=8l_{Pl}^3$ and $\Delta x = 2l_{Pl}$, the eq (14,10) yields $\eta = 9c^7/(128G^2h) = 5.2\times10^{111}\mathrm{J/m^3}$, one order of magnitude smaller than that at $n=0$. All this happens at the time given by the eq (10,4), i.e. $\tau_m \approx 8\times10^{-37}\mathrm{s}$; the subscript stands for "matter".

The beginning of the matter era is assumed in the present model as the time where one Planck mass was allowed to form. The reason is that the Planck units define the fundamental quantities that constitute the physical laws regardless of conventional measure standards, which are actually formal agreements between humans only; the concept of Planck mass is on the contrary inherent itself the eq (2,1), being by definition

$$\frac{hG}{c^2} \equiv \frac{(m_{Pl}G)^2}{c^3} \qquad m \equiv m_{Pl} \qquad m \equiv \overline{m}_{Pl}.(14,18)$$

The fact that the Planck mass appears here as that of matter and antimatter, in agreement with ν and $\overline{\nu}$ of the eq (14,11), deserves attention. The symbol m has been formally introduced in the early eqs (4,1) that specify the physical dimensions of momentum and energy; moreover m, latently hidden in h and G, has been extracted in the section 3 first and in the sections 5 and 12 next from the Compton length, without hypotheses "ad hoc" but also without explicit reference to the real world around us. Also, the mass could be even defined as $m = \lim_{v\to0} p/v$ via the first eq (5,4). Actually, however, the concept of mass becomes explicitly inherent the definition of space time (1,1) via the Planck mass only; this justifies the idea that the first occurring of matter in the space time having physical significance coincides with the presence of one full Planck mass, defined uniquely by the constants present in the position (1,1) coherently with both expected chances m_{Pl} and $\overline{m}_{Pl}$.

On the one hand, this idea clarifies the physical meaning of the result $m_0/2$ previously found: one half Planck mass, and its increasing value subsequently allowed until τ_m, can be related to nothing else but that of virtual particles.

On the other hand, being the Planck mass very large with respect to that of the elementary particles, one m_{Pl} implies actually the formation of several real quantum particles concurrently formed at the threshold time τ_m and disseminated throughout the space time volume, according to the quantum curve λ_C of the figure 1.

It is not surprising therefore that, according to the eq (5,2), the presence of one Planck mass has an important consequence: the sudden volume expansion simply because of the presence of mass originated from the radiation field, already emphasized as $V \to V_0$ in that equation. This is not exactly a superluminal expansion: rather with the presence of mass the space time changes its properties and turns into a new space time with different size and geometry, as it is further shown in the next subsection. This is not even a "change of state", unless one intends with this ambiguous terminology the formation of a "dual phase" system, real plus virtual particles, replacing a "single-phase" system of early virtual particles only.

Anyway it is possible to calculate the entity of this change knowing n.

Reasoning as before, put again $n=4$ and $\nu = v/\lambda$ with $\lambda = 2\beta^{1/3}l_{Pl} = 4l_{Pl}$ in analogy with the eq (14,15); the eqs (14,11) and (5,2) yield

$$\frac{\alpha}{3} = \sqrt{\frac{9}{2}}\frac{v}{c} \qquad \frac{V_0}{V} = \frac{\varepsilon_{Pl}}{h\nu} = 12\sqrt{\frac{9}{2}}\frac{l_{Pl}\varepsilon_{Pl}}{hc\alpha} = 4\times10^4 \qquad \frac{v}{c} = \frac{\sqrt{2}}{9}\alpha = 10^{-3};(14,19)$$

so $\mathrm{n} = c/v$ implies a very high volume increase V_0/V of the order of 10^4 according to the eq (5,4). With the formation of one Planck mass at the time $\tau = 8t_{Pl}$ starts the inflationary era, at the temperature given by the eq (14,12):

$T = 2\times10^{27}\mathrm{K}$. The inflationary epoch is reported in the literature at estimated time $10^{-36}\mathrm{s}$ after the big bang. At this time the universe was surely opaque because of the high refraction index due to Planck mass equivalent particles of matter crowding the small space time volume, of the order of $8l_{Pl}^3$. At increasing times, the energy density decreased correspondingly to the increase of V.

14.6 THE TIME LINE OF THE SPACE TIME

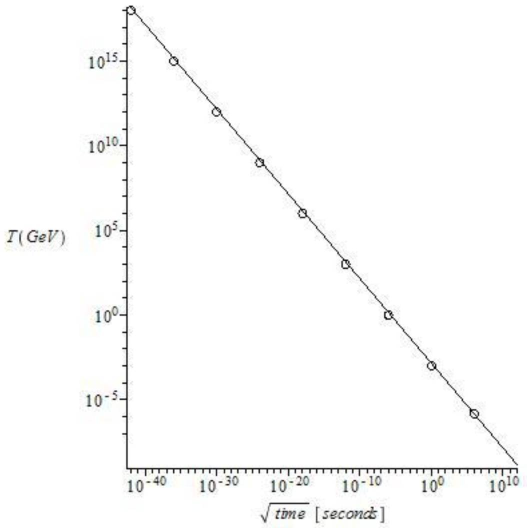

Fig 2: log log plot of the eq (14,12): $T = \kappa/\sqrt{\Delta t}$ is calculated as a function of Δt with $\kappa = (\alpha^2 c^2/9G\theta)^{1/4}$. The time Δt is expressed in seconds. The circles represent the literature estimates reported in [26].

The eq (10,4) preliminarily tested in the eq (14,13) is now more systematically assessed at some particular time values significant for their implications. First at the Planck era: the value obtained at $n = 0$ is well acknowledged. Next at the time of formation of matter, estimated in the literature in the range $10^{27} \leftrightarrow 10^{28}\mathrm{K}$; also this range of temperatures agrees with the value just calculated.

Consider then the grand unification time, estimated in the interval $10^{-43} \to 10^{-36}\mathrm{s}$; the equation calculates $\Delta t = 10^{-38}\mathrm{s}$ at the typical temperature of $10^{15}\mathrm{GeV}$; also, the electroweak symmetry breaking and the quark epoch are estimated in the time interval $10^{-12} \to 10^{-6}\mathrm{s}$; the model calculates $4\times10^{-8}\mathrm{s}$ at the typical temperature of $1\mathrm{GeV}$.

These sensible values are better assessed through a global standpoint thanks to the data summarized in the Fermilab Photograph 85-138CN [26]; these data concern the whole universe time line rather than estimated time intervals characterizing single events. The result is the log log plot of the fig 2, showing that effectively the eq (14,12) represents well the temperature evolution and thus the energy density itself as a function of time. It is crucial at this point to explain why the quantum fluctuation was in fact able to trigger the subsequent self-sustained evolution of the universe.

14.7 THE MATTER ERA

To follow the evolution of the space time, the previous considerations suggest a sustainable hint: the fact that the primordial zero point radiation energy at $n = 0$ was confined in the Planck space time region, is compatible with energy field and mass still confined inside V at any subsequent time. This allows calculating V as a function of time via the eq (13,8) only, which introduces the total amount of mass M_u compatible with the radius Δr_u fulfilling the black hole requisite. In this way the whole space time is in fact a super-massive black hole and thus an isolated system, whose internal pressure is consistent with the condition that neither matter nor electromagnetic energy can escape outside V: the boundary of the space time behaves in fact as a self gravity driven barrier for any internal particle. As done to infer the eq (14,15), replace once more $\lambda_{bh} = 2\Delta r_u$ in the eq (13,8) calculated with the values of the eqs (14,2); so $\Delta r = mG/c^2$ yields

$$M_u = \frac{c^2\Delta r_u}{G} = 5.9\times10^{53}\mathrm{Kg} \quad M_u c^2 = \frac{c^4\Delta r_u}{G} = 5.3\times10^{70}\mathrm{J}.(14,20)$$

This value of M_u provides a first way to estimate the volume V_u at the present time Δt_u via the eq (5,1): putting $\nu = \nu_{bh} = c/\lambda_{bh}$ in this equation, one finds $\eta_u = M_u c^2/V_u = (c\nu_{bh})^2/G$. Trivial manipulations yield

$$V_u = M_u G\left(\frac{2\Delta r_u}{c}\right)^2 = 3.3\times10^{80}\mathrm{m}^3(14,21)$$

and thus, thanks to the eq (5,1),

$$\rho_u = \left(\frac{c}{2\Delta r_u}\right)^2 G^{-1} = 1.8\times10^{-27}\frac{\mathrm{Kg}}{\mathrm{m}^3} \quad \eta_u = \left(\frac{c^2}{2\Delta r_u}\right)^2 G^{-1} = 1.6\times10^{-10}\frac{\mathrm{J}}{\mathrm{m}^3}.$$

Compare these values with that calculated implementing directly Δr_u

$$V_u = \frac{4\pi}{3}\Delta r_u^3 = \frac{4\pi}{3}\left(\frac{M_u G}{c^2}\right)^3 = 3.7\times10^{80}\mathrm{m}^3,(14,22)$$

and

$$\langle\rho_u\rangle = \frac{M_u}{V_u} = 1.6\times10^{-27}\frac{\mathrm{Kg}}{\mathrm{m}^3} \quad \langle\eta_u\rangle = \langle\rho_u\rangle c^2 = 1.5\times10^{-10}\frac{\mathrm{J}}{\mathrm{m}^3}(14,23)$$

Note two implicit and independent assumptions made to infer these results: V_u has been calculated in the eq (14,21) putting $\nu_{bh} = c/2\Delta r_u$, i.e. a refraction index $\mathrm{n} = 1$, whereas it has been calculated in the eq (14,23) via a hyper spherical geometry of the space time. The agreement shows that these assumptions, clearly independent each other, are both fulfilled. In particular:

-The matter and radiation energy density of the space time at the time Δt_u justifies the propagation rate of light equal to c.

-The Planck volume, before the inflationary era, was calculated as l_{Pl}^3; the change of geometry inherent a space time containing mass with respect to that containing radiation only, already remarked when introducing the eq (5,2), is confirmed here.

-The distinctive factor $4\pi/3$, in principle not required and necessarily introduced here to fit the two results, has a simple explanation: the matter curves the space time.

-The curvature is such to justify the Euclidean value of π here implemented.

-The total energy due to the presence of the CMBR field calculated with the eq (14,13) is $\eta_{cmbr}V_u = 1.6\times10^{67}\mathrm{J}$.

Is interesting in this respect a third independent way to calculate again V_u.

Implement the eqs (12,12) and (12,13) replacing $m_0 = M_u$; the former equation yields

$$\xi_0 = \frac{M_u}{m_{Pl}} = 1.1\times10^{61},(14,24)$$

whereas the latter with the positions $V = V_u$ and $\Delta t_\Lambda = \Delta t_u$ yields $m_0\Delta t_\Lambda^2 G/V_u = 0.02$ and implies $\zeta < 0.02$. In effect the eq (14,16) has already shown that actually ζ coincides with the fine structure constant α at the Planck time; being ζ by definition a constant, its initial value still holds also at the present time. So, owing to the eq (14,16), the second eq (12,12) yields the values

$$\Lambda_t = 1.5\times10^{-35}\mathrm{s}^{-2}(14,25)$$

still compatible with the respective uncertainty inequalities. This result is significant, as it implies $\sqrt{\Lambda_t} > \Delta t_\Lambda^{-1}$: in effect

$\Delta t_u\sqrt{\Lambda_t}=1.7.$ It is significant the chance of verifying once more the eq (12,15); replacing again Δt_Λ and Δr_Λ with Δt_u and Δr_u, one finds via Λ_t

$$\Delta t_u=\left(\frac{\Delta r_u}{c\Lambda_t}\right)^{1/3}=4.6\times10^{17}\mathrm{s},(14,26)$$

which agrees well with the estimated value (14,2) and thus confirms the aforesaid equation. These results legitimate the strategy of calculating the previous equations with the estimates (14,2). Is remarkable the fact that replacing the value of Λ_t and $\zeta=\alpha$ in the eq (12,12), the value of m_0 put equal to M_u of the eq (14,20) yields

$$V_u=\frac{M_uG}{\alpha\Lambda_t}=3.67\times10^{80}\mathrm{m}^3.(14,27)$$

Also this value, calculated regardless of Δr_u, agrees with that calculated directly via the eq (14,20), without requiring any assumption on the geometry of the space time. Clearly here the values of Δt_u and V_u have been inferred via the formulas of the quantum vacuum.

The consistency of the eq (14,21), (14,22) and (14,27), in principle not required, is encouraging: the black hole condition (13,8) inherent the eq (14,20) is a mere linear relationship between mass and length, so it has no direct link with the assumption of spherical space time independently asserted in the eqs (14,22) and (14,27). Moreover this value of V fulfills also the condition (12,13), since $\alpha<:M_u\Delta t_u^2G/V_u=0.02$; this supports the volume geometry implemented in the eq (13,12), i.e. the physical meaning of ξ_0, and links the next results with the eqs (13,20).

The value of $\langle\rho_u\rangle$ fits that of one mass unit per unit volume, which includes of course the visible mass, i.e. the stars, plus other possible forms of ordinary matter, e.g. dust or black holes; all masses concur to attain the density value consistent with the black hole boundary condition, crucial for a lossless growth of the space time.

Note that the value of M_u is about twenty times that estimated via m_u accounting for the visible mass only, which is in fact about 5 % of the total M_u only.

All this reveals however that even considering $\eta_{cmbr}+m_uc^2/V=7.3\times10^{-12}\mathrm{J/m}^3$, the energy density hitherto introduced is still much smaller than $\langle\rho_u\rangle$. This evidence and the fact that $M_u>>m_u$, deserve a careful explanation.

First of all, the chance of writing identically $M_u\equiv2m_u+(M_u-2m_u)$ suggests splitting the density equation (14,21) into the sum of two terms $\langle\rho_u\rangle=\rho'_u+\rho''_u$; splitting accordingly $(c/\Delta r_u)^2$ as well, one finds

$$\frac{8\pi G\rho''_u}{3}+\frac{4\pi G\rho'}{3}=(1-q)\left(\frac{c}{\Delta r_u}\right)^2+q\left(\frac{c}{\Delta r_u}\right)^2\qquad\rho''_u=\frac{m_u}{V_u}\qquad\rho'_u=\frac{(M_u-2m_u)}{V_u}.(14,28)$$

The second equation shows that ρ''_u refers to the visible gravitational mass only. So, with the further position

$$4\pi G\rho'=\Lambda_E+q'\left(\frac{c}{\Delta r_u}\right)^2,$$

where Λ_E has the physical dimensions of Λ_t, the result is

$$\frac{8\pi G\rho''_u}{3}+\frac{\Lambda_E}{3}=H^2+k\left(\frac{c}{\Delta r_u}\right)^2\qquad k=1-q-q'\qquad q=\chi^2\qquad H^2=q\left(\frac{c}{\Delta r_u}\right)^2.$$

In this way the first equation agrees with the eq (13,20) putting $q=\chi^2$ and coincides with the first Friedman equation including the Einstein cosmological constant Λ_E. Moreover, even without hypotheses on q and q', in principle $1-q-q'$ can be expectedly positive or negative; so it is easy to recognize this factor as the relativistic coefficient k of $k(c/a)^2$, reported in the literature with $k=0,\pm 1$. Eventually the physical meaning of $c/\Delta r_u$ is proportional via q to the Hubble function $H=\dot{a}/a$, which shows that the uncertainty range Δr_u plays the role of the scale length a reported in the literature. So, replacing again Δr_Λ with Δr_u in the first eq (12,16) as done previously, one finds $H > \approx c/\Delta r_u$ and thus

$$H < \approx \frac{1}{\Delta t_u} \qquad \Delta t_u < \approx H^{-1};$$

the time inequality $\Delta t_u < \approx 1/H$ of the Hubble constant with respect to the age of the universe, also well known, appears in fact to be nothing else but the fingerprint of the uncertainty. The first eq (10,7) previously found is instead the second Friedman equation, where however the pressure appears with the $\pm$ sign. It appears therefore that both Friedman equations account only partially for the the picture here inferred.

Despite hold in principle all considerations about the chance of an expanding or shrinking space time depending on the values of the mass m_u and H, a simple reasoning shows however that in fact is correct the former chance. According to the eq (14,21), $\delta\eta_u/\delta V_u$ yields $\delta\eta_u=(\delta M_u/M_u-\delta V_u/V_u)\eta_u$; as shown in the section 10, the corresponding pressure change δP_u results respectively positive or negative like the sign of $\delta\eta_u$, i.e. depending on whether $\delta M_u > \rho_u\delta V_u$ or $\delta M_u < \rho_u\delta V_u$. As

$$\frac{\delta M_u}{M_u}=\frac{\delta\Delta r_u}{\Delta r_u}=\frac{1}{3}\frac{\delta V_u}{V_u},(14,29)$$

then $\delta\eta_u=-2\eta_u\delta V_u/3V_u$ shows that a negative pressure inside the space time tends to push outwards the boundary and to increase V_u. Despite these results justify the way of splitting M_u as in the eq (14,28), the implications of this position will appear more clearly in the next subsection 14.8. Yet, let us remark:

(i) The inequality (12,13) legitimates the eq (14,16) that identifies $\zeta\equiv\alpha$.

(ii) The eq (12,14) yields

$$\eta_H=2\frac{\Lambda_t c^2}{G}=4.0\times 10^{-8}\,\frac{\mathrm{J}}{\mathrm{m}^3}:(14,30)$$

owing to the considerations about the eq (12,14), this means that inside any unit volume of the space time contributing to V_u there is the energy field

$$\varepsilon_H=4\times 10^{-8}\,\mathrm{J}=252\,\mathrm{GeV}(14,31)$$

pervading uniformly all space time volume; indeed no space coordinate appears in these equations, whose physical meaning will be highlighted below.

14.8 TIME AVERAGES AND GROWTH RATE

The first part of this section concerns the growth rate of the mass allowed to form in the space time; then the volume, density and energy density of the space time are also described.

1) The most intuitive definition of average rate $\langle\dot{M}\rangle$ with which new mass progressively forms in the space time, is

$$\langle\dot{M}\rangle=\frac{M_u}{\Delta t_u}=1.4\times 10^{36}\,\frac{\mathrm{Kg}}{\mathrm{s}}.(14,32)$$

This value is proportional to $\Delta r_u/\Delta t_u$ according to the eq (14,20) and agrees with that provided by the eq (14,29), which reads $\rho_u \delta V_u = 3\delta M_u$; multiplying both sides by $c/\delta\Delta r_u$ and implementing the eq (14,20), $c\rho_u \delta V_u/\delta\Delta r_u = 3cM_u/\Delta r_u$ yields

$$3\frac{cM_u}{\Delta r_u} = 3\frac{c^3}{G} = 3\frac{m_{Pl}}{t_{Pl}} = 1.2\times10^{36}\frac{\text{Kg}}{\text{s}}.(14,33)$$

The agreement is not accidental: it shows that effectively M_u and Δr_u increase as a function of Δt in order to fulfill the eq (13,8); in principle $\langle \dot{M} \rangle$ does not reveal explicitly this requirement, as instead the eq (14,33) does. Is not surprising the fact that the increment $\rho_u \delta V_u$ of mass pertinent to the increase δV_u of space time volume is proportional, but not equal, to δM_u: the eq (13,8) relates indeed δM_u to $\delta\Delta r_u$, whereas the mass production is a volume process that scales with Δr_u^3 and not linearly with Δr_u. It is evident that the factor 3 agrees with the Euler homogeneous function theorem. The calculation carried out with m_u, i.e. considering the visible mass only, would have given both average results an order of magnitude lower than that inferred via the fundamental constants.

A linear process with respect to M_u is instead the average production rate of total energy $M_u c^2$, as indeed the first eq (14,20) yields $\Delta r_u/c = M_u G/c^3$; so the equation obtained multiplying both sides by the Planck power c^5/G yields $(c^5/G)(\Delta r_u/c) = M_u c^2$, which is nothing else but the eq (14,20) itself. Hence the average energy growth rate is

$$\frac{c^4}{G}\left\langle\frac{\Delta r_u}{\Delta t_u}\right\rangle = \frac{M_u}{\Delta t_u}c^2 = \langle \dot{M} \rangle c^2.(14,34)$$

In the case where V of the eq (2,1) describes the size scale of the whole space time, the pertinent mass and time imply expectedly large values of $\langle \dot{M} \rangle$ to allow M_u after the radiation era; in fact $M_u >> m_u$ shows that the space time must create ordinary matter m_u plus some additional equivalent mass, much more relevant than m_u itself, that will be concerned in the next section.

Consider now the mass growth scale of a single particle moving throughout the quantum vacuum. If the mass creation rate concerns one particle only, multiplying both sides of the eq (2,1) by $m_\gamma \nu$ one finds

$$\gamma = \frac{m_\gamma{}^2 G}{V\nu} = \frac{(m_\gamma c)^2}{h} = \frac{h}{\lambda_\gamma^2} \quad \gamma = \text{n}m_\gamma\nu \quad \lambda_\gamma = \frac{h}{m_\gamma c};(14,35)$$

the physical dimensions of γ are $mass/time$. As expected, the space scale of the local mass production process is controlled by the Compton length of m_γ. Multiply both sides of the eq (14,35) by Δx^2; since $\nu\Delta x$ yields a velocity v_x one finds $m_\gamma v_x \Delta x = \gamma\Delta x^2$, whence $\gamma\Delta x^2 >: h$ because $(m_\gamma v_x)\Delta x >: h$. For instance the order of magnitude $\gamma \approx 1\text{Kg/s}$ yields $\Delta x \approx 10^{-17}\text{m}$, to which correspond $\nu_\gamma \approx 10^{25}\text{s}^{-1}$ and $m_\gamma \approx 10^{-25}\text{Kg}$, i.e. $m_\gamma c^2 \approx 50\text{GeV}$.

These results highlight the orders of magnitude of ε_γ and time range $\Delta t_\gamma \approx h/\varepsilon_\gamma$ expectable when the mass scale is that of an elementary particle; yet a more careful analysis is necessary to find the correct value of the microscopic mass production rate γ. To this purpose are useful the eqs (5,1) and (5,5) that yield $\varepsilon_\gamma = m_\gamma c^2 = \text{n}h\nu$ with $\text{n} > 1$; then

$$\varepsilon_\gamma^2 = hc^2\gamma \quad \Delta t_\gamma \approx \frac{1}{c}\sqrt{\frac{h}{\gamma}},(14,36)$$

The local value of mass production rate in the space time depends therefore on ε_γ, or analogously on the time length Δt_γ

during which the mass m_γ is allowed to form from the quantum vacuum; also,

$$\varepsilon_\gamma = 48\sqrt{\mathrm{n}\, m_\gamma \nu}\,\mathrm{GeV}.(14{,}37)$$

To find the scale factor between the eqs (14,32) and the eq (14,35), consider that in the eq (14,35) the sought mass production rate at left hand side is controlled by the characteristic length λ_γ^2 only; this suggests that just this square length is the sought scale factor. To correlate the eqs (14,32) and (14,35), write then

$$\frac{\lambda_\gamma^2}{l_{ref}^2}\frac{M_u}{\Delta t_u} = \frac{h}{\lambda_\gamma^2}.$$

The right hand side is the concerned $\mathrm{n}m_\gamma\nu$; at the left hand side appears the ratio $(\lambda_\gamma/l_{ref})^2$ times $\langle\dot{M}\rangle$, being l_{ref} a reference length. The scale law is therefore such that for $\lambda_\gamma = l_{ref}$ the left hand side yields again the eq (14,32) and the right hand side the corresponding λ_γ; so, it is useful to define $l_{ref} = 1\,m$ in order that for $\lambda_\gamma \neq l_{ref}$ the left hand side takes the meaning of the eq (14,35). Hence, one finds the reasonable values

$$\lambda_\gamma = 4.7\times10^{-18}\mathrm{m} \quad \mathrm{n}_\gamma m_\gamma \nu_\gamma = 30\mathrm{Kg/s} \quad \varepsilon_\gamma = 4.2\times10^{-8}\mathrm{J} = 264\mathrm{GeV} \quad \Delta t_\gamma = 1.5\times10^{-26}\mathrm{s}.$$

The last equation yields the interaction time range; the value of ε_γ is reasonably close to that of the quantum vacuum energy field of the eq (14,31).

In the eqs (5,5) and following equations, n played the role of refraction index whose effect was to slow down the propagation rate of the electromagnetic waves in a medium with respect to c. Here its physical meaning is conceptually similar: as increasing n means decreasing ν and increasing ε_γ, it follows that n controls the energy field associated to the mass creation rate. In particular, appears once again the necessity of $\mathrm{n} > 1$ to allow the formation of mass in agreement with the eqs (5,2), (5,1) and (5,4). Anyway, whenever $m\nu \neq 0$ implies the production of mass, there is a non-null product $\mathrm{n}m$ times the rate ν associated to the energy ε_γ and to the velocity of the particles involved by the mass creation process itself. This is explained admitting the interaction of the particle with the energy field ε_γ via n.

2) The simplest and most intuitive definition of volume growth rate is the average value of $V_u/\Delta t_u$, calculated at the time Δt_u neglecting the initial Planck tiny volume. With the help of the eq (14,27) one finds

$$\langle\dot{V}\rangle = \frac{V_u}{\Delta t_u} = \frac{M_u G}{\alpha\Lambda_t \Delta t_u}.(14{,}38)$$

Differentiating the eqs (14,20), one finds with the help of the eq (14,29)

$$\dot{\rho}_u = \frac{\dot{M}_u}{V_u} - \frac{M_u}{V_u^2}\dot{V}_u = -2\frac{c^2}{GV_u}\Delta\dot{r}_u \quad \dot{M}_u = \frac{\delta M_u}{\delta\Delta t} \quad \Delta\dot{r}_u = \frac{\delta\Delta r_u}{\delta\Delta t} \quad \dot{V}_u = \frac{\delta V_u}{\delta\Delta t}.(14{,}39)$$

The first result is found more shortly differentiating the eq (14,21): this yields $\dot{\rho}_u = -(6c^2/4\pi G)\Delta\dot{r}_u/\Delta r_u^3 = -2(c^2/GV_u)\Delta\dot{r}_u$, so that $\dot{\rho}_u c^2 = -2\rho_u c^2\Delta\dot{r}_u/\Delta r_u$ according to the eq (14,20). However this last way of inferring $\dot{\rho}_u$ does not emphasize the two contributions concurring to $\dot{\rho}_u$: one positive due to the increase of M_u and one negative due to the increase of V_u. It appears that, owing to the sign of $\dot{\rho}_u$, the latter contribution overcomes the former. An interesting result is found calculating $\langle\dot{M}\rangle$ directly via the eq (13,8); with $\dot{M}_u = (c^2/G)\Delta\dot{r}_u$, the first equation yields

$$\Delta\dot{r}_u = 3\frac{G}{c^2}\frac{m_{Pl}}{t_{Pl}} = 3c \qquad \frac{\Delta r_u}{\Delta t_u} \approx 3.3c.$$

In effect the calculated value of $\Delta\dot{r}_u$ agrees reasonably with the estimate inferred from the eqs (14,2). Other average quantities are

$$\dot{V}_u = \frac{m_{Pl}}{t_{Pl}}\frac{9}{\rho_u} \quad \dot{\rho}_u = -\frac{6}{V_u}\frac{m_{Pl}}{t_{Pl}} \quad \frac{m_{Pl}}{t_{Pl}} = \frac{c^3}{G},(14,40)$$

whose values calculated with the help of the estimates (14,2) are

$$\dot{V}_u = 2.3\times10^{63}\,\frac{\mathrm{m}^3}{\mathrm{s}} \quad \dot{\rho}_u = -6.6\times10^{-45}\,\frac{\mathrm{Kg}}{\mathrm{m}^3\mathrm{s}}.$$

Note that the second eq (14,40) calculates an energy density loss

$$\dot{\rho}_u c^2 = -5.9\times10^{-28}\,\frac{\mathrm{J}}{\mathrm{m}^3\mathrm{s}},$$

formally due to the fact that the increase of V_u as a function of time overcomes the energy increase in V_u, so that the net result is a time decrease of energy density.

The crucial point is now to understand what this loss actually means: on the one hand the black hole feature of the space time excludes any chance of mass and energy escaping outside V, on the other hand just the expansion of the space time promoted by this advantageous feature causes the rarefaction of energy/matter contained in each elementary unit volume forming the whole V_u. In other words, the problem is to explain why the formation of new mass sufficient to ensure the eq (13,8) at any time, implies however the dilution effect described by the eq (13,8) itself. The next two sections will show how to explain this apparent oxymoron, while also explaining what actually "loss" does mean.

14.9 THE GRAVITATIONAL BINDING ENERGY

The position of the eq (14,20) is now implemented noting that

$$M_u c^2 = \frac{c^4\Delta r_u}{G} = \frac{M_u^2 G}{\Delta r_u},(14,41)$$

which implies $M_u c^2 - 0.6M_u^2 G/\Delta r_u > 0$. So the classical binding energy ε_{be}^{cl} of the eq (13,18) calculated with $M = M_u$ and $\Delta r = \Delta r_u$ is insufficient to balance $M_u c^2$ and thus to justify the mass M_u itself. Size and mass of the space time need an additional amount of energy to fulfill the eq (13,8), which could be for example that coming from the initial quantum energy fluctuation previously concerned. This is hypothetically possible, once being sure that no energy anyhow created within V is dissipated outwards. However the early peak power should be exceedingly high: during the very short time transient compatible with the quantum uncertainty driven violation of the energy conservation, should be generated at least the missing amount $0.4M_u c^2$ of energy at the time Δt_u. But, being $0.4M_u c^2 \approx 2\times10^{70}\,\mathrm{J}$, an uncertainty time lapse of the order of $h/0.4M_u c^2$, much shorter than the Planck time, is unreasonable. More sensible appears nevertheless the chance prospected by the relativistic binding energy equation $-\varepsilon_{be}^{rel}$ of the eq (13,19), which modifies favorably the classical conclusion. The condition

$$M_u c^2 + \varepsilon_{be}^{rel} = 0(14,42)$$

prospects the chance of gravitational binding energy compatible with $M_u > m_u$ required by the eq (14,20) to allow the black hole feature of the space time: otherwise stated, the quantum fluctuation is required to provide not the total energy of the eq (14,42) but an initial seed of mass only, after which the gravitational binding energy starts acting. In this respect, the eqs (13,18) yields

$$M_u = \frac{2}{3}\frac{m^2 G}{c^2\Delta r_u}\frac{1}{\sqrt{1-2mG/c^2\Delta r_u}} \quad M_u = m + ...;(14,43)$$

i.e. an appropriate value of mass m in the whole space time volume described by Δr_u makes this equation compatible with M_u, which of course includes m itself. Whatever the dots stand for, the equivalent mass exceeding m is due to the binding gravitational effect defined by the solution of this equation. Calling m_{dm} this solution, the result is

$$m_{dm} = 2.8\times10^{53}\text{Kg}.(14,44)$$

However: if m_{dm} accounts for the arising of total gravitational binding energy $-M_u c^2$ necessary to balance that of the eq (14,20), what else justifies m_{dm} itself ? Let us suppose now that m_{dm} consists of the masses of particles and antiparticles formed with equal proportions in the space time; so assume $m_{dm} = m'_{dm} + \overline{m}'_{dm}$, being obviously $m'_{dm} = \overline{m}'_{dm} = m_{dm}/2$. The physical meaning of m_{dm} is highlighted implementing again the eq (13,18) to calculate first which mass m'' is related to m'_{dm}. The same equation, aimed now simply to evaluate the expression

$$m'' = \frac{2}{3}\frac{(m'_{dm})^2 G}{c^2 \Delta r_u}\frac{1}{\sqrt{1-2m'_{dm}G/c^2\Delta r_u}} \qquad m'_{dm} = \overline{m}'_{dm} = \frac{m_{dm}}{2},(14,45)$$

yields an interesting result: $m'' = 3.2\times10^{52}\text{Kg} = m_u$. So the mass m'_{dm} is consistent with a numerical value of m'' that reasonably represents the visible mass m_u of the eqs (14,2). This also suggests that $\overline{m}'_{dm}$ would have given the antimatter partner $\overline{m}_u$ of the visible mass m_u contextually formed according to the eqs (14,11) and (14,18). The eq (14,45) written more expressively as

$$m_u = \frac{2}{3}\frac{m'^2_{dm}G}{c^2\Delta r_u}\frac{1}{\sqrt{1-2m'_{dm}G/c^2\Delta r_u}} \qquad \overline{m}_u = \frac{2}{3}\frac{\overline{m}'^2_{dm}G}{c^2\Delta r_u}\frac{1}{\sqrt{1-2\overline{m}'_{dm}G/c^2\Delta r_u}}(14,46)$$

clarifies the link between m_{dm} and M_u via m_u:

the total m_{dm} accounts for $M_u > m_{dm}$, i.e. for the full energy $M_u c^2$ required by the eq (13,8) once implementing the mere literature estimate of Δr_u; the smaller m'_{dm} and $\overline{m}'_{dm}$ separately account for the formation of the estimated visible mass $m_u < m'_{dm}$ and its corresponding $\overline{m}_u < \overline{m}'_{dm}$. On the one hand this result is due to the form of the eq (13,18), which allows increasing values of M as long as $y \rightarrow 1/2$; on the other hand this result confirms that m_u, which we are made of, consists actually of ordinary matter only.

Even so, however, the total mass balance in the space time is still incomplete because

$$m_u + |\overline{m}_u| + m'_{dm} + |\overline{m}'_{dm}| + m_{bal} = M_u \qquad m_{bal} = 2.5\times10^{53}\text{Kg}:(14,47)$$

i.e. a further ancillary mass m_{bal} is still necessary to get M_u in addition to m_u and m'_{dm} plus their respective antiparticles just introduced. Here the mass balance is expressed with the notation of the third eq (6,2); of course m_{bal} is uniquely defined even writing the last equation as

$$(m_u + \overline{m}_u) + (m'_{dm} + \overline{m}'_{dm}) + m_{bal} = M_u,$$

which emphasizes the possible interaction and annihilation of the concerned matter and antimatter with release of the corresponding energy gap. In effect, the problem is just this: no stable matter structures are possible if matter and antimatter are allowed to interact. A possible way to account for the presence of stable structures of either kind is to separate them to prevent their interaction. In effect the mechanism through which matter and antimatter disconnect each other has been already described in a previous paper [27], although not intentionally aimed to the present purposes. It is worth summarizing this mechanism here not only for completeness, but mostly to show that even its explanation is still included in the frame of the eq (2,1); moreover this point answers the priority questions about where are $\overline{m}_u$ and $\overline{m}'_{dm}$ and why the bulk of the space time appears consisting of matter only.

14.10 MATTER AND ANTIMATTER

The starting point is the value $\eta_u = 1.5\times10^{-10}\text{J/m}^3$ of energy density calculated in the eq (14,23), showing that each unit volume element V_0 of space time, say $V_0 = 1\text{m}^3$, contains one mass unit $m_p = 1.6\times10^{-27}\text{Kg}$. As any V_0

is a statistical micro-scale mirror of V_u, by definition η_u of the eq (14,23) includes even the possible interactions between the various mass elements concurring to m_p in their respective V_0, e.g. their Coulomb interaction. Despite the global charge in any local region of V_u is statistically null, in general each V_0 could contain charges of both signs plus particles and respective antiparticles mutually interacting: in short, everything contributes to η_u. Consider thus one V_0 among the many $n_{tot} = V_u/V_0$ in any core point of the space time: all actual particles concurring to m_p are statistically regarded as a unique composite body of matter. Specific details about the actual particle/antiparticle content of V_0 are not influential for the present reasoning; is crucial instead the concept of quantum delocalization, according which m_p, whatever it might be made of, could actually be in any place of V_u. In particular, it could be even gravitationally stuck just at the external boundary of the space time region consistently with the black hole behavior of V_u. Then the Newton shell theorem shows that all mass M_u acts on such external m_p as if it would be concentrated at the geometrical center of the hyperspherical V_u, i.e. at distance Δr_u from the boundary. This appears in fact also here, i.e. the result

$$G\frac{M_u m_p}{\Delta r_u} = 1.5\times 10^{-10}\,\mathrm{J} \quad (14,48)$$

could have been expected: the left hand side is by definition nothing else but $m_p c^2$, which multiplied by n_{tot} yields just $M_u c^2$, whereas the right hand side times n_{tot} is of course the numerical value of the eq (14,20). Note that it is not necessary to think m_p physically moving throughout the space time volume; according to the quantum character of the present model, any V_0 and its energy content is actually delocalized everywhere in V_u, thus even at its outermost boundary from which however it cannot escape. So the eq (14,48) shows that m_p could be displaced from any bulk state V_{0b} to any surface state V_{0s} of the space time without gravitational energy change with respect to the result (14,23).

The existence of surface states V_{0s} occupied indifferently by m_p or $\overline{m}_p$ according to the eq (14,48) has several implications, e.g. this conclusion is related to the growth of V_u: indeed m_p and $\overline{m}_p$ in these surface states replicate the bulk structure of the space time at its boundaries. Hence the increase of V_u does not mean simply swelling; rather the delocalization affects the growth process via the progressive formation of new external layers replicating the internal structure of V outside its boundary [27].

In other words, the space time clones itself at the boundary without cost of gravitational energy thanks to the bulk $\leftrightarrow$boundary quantum delocalization, while new core mass structures are also continuously created at the expense of the gravitational binding energy already existing. The initial trigger of this continuously renewing creative process was the early quantum fluctuation energy in the space time at the Planck era. These considerations suggest that V_0 is not a mere statistical parameter, rather it has a relevant physical meaning.

The plain energy balance of masses in V, eq (14,47), waives the fact that m_u/n_{tot} and $\overline{m}_u/n_{tot}$ reasonably contributing to m_p and $\overline{m}_p$ and coexisting in the same V_0 cannot account for the formation of stable matter structures. Is however evident the link between m_p and $\overline{m}_p$ with the respective addends of the eq (14,47), noting that $n_{tot}m_p/2$ and $n_{tot}\overline{m}_p/2$ are separated just by the energy gap $2(n_{tot}m_p/2) = M_u$; nothing thus excludes that the elementary volumes V_0 consist actually of V_0 and $\overline{V}_0$ having physical meaning of allowed states for particles in positive or negative energy levels, whereas $M_u c^2$ corresponds to the energy gap between matter and antimatter content in the V_0 and $\overline{V}_0$ states. Moreover if V_0 and $\overline{V}_0$ are randomly distributed throughout V_u, it remains still true that the space time is statistically homogeneous and isotropic on large scale, despite the different ways of energy level occupation in the respective volume elements. To highlight the formation of matter structures of either kind, rewrite therefore the eq (14,47) identically, but in a different form and with a different physical meaning, as

$$(m_u + m'_{dm} + m_{de}) + \left(|\overline{m}_u| + |\overline{m}'_{dm}| + m_{bal} - m_{de}\right) = M_u = n_{tot}m_p/2 - n_{tot}\overline{m}_p/2 :(14{,}49)$$

the two addends at left hand side of the eq (14,49) define now the energy gap at the right hand side. This form emphasizes the idea that each volume element of the space time contains either one m_p or one $\overline{m}_p$, regarding the two addends at the left hand side as referred to particles and antiparticles occupying their respective V_0 or $\overline{V}_0$. To this purpose m_{bal} has been split into quantities, m_{de} and $m_{bal} - m_{de}$, involving separately matter and antimatter via the new equivalent mass m_{de}, thus presumably with $m_{bal} < m_{de}$ in order to have in the second addend negative energy states only. Clearly all bulk states of the space time are by definition occupied, since they result just averaging the total energy M_uc^2 in the whole V_u. Eventually, regard the eq (14,49) as follows

$$m_u + m'_{dm} + m_{de} = M_u - \mu \quad \mu = |\overline{m}_u| + |\overline{m}'_{dm}| + m_{bal} - m_{de} \quad 0 < \mu < M_u .(14{,}50)$$

Here the equivalent mass μ, whatever its numerical value might be, takes the physical meaning of an energy subtracted to the total energy M_uc^2 at right hand side, whereas at the left hand side appear positive energy states only; the third position is self evident because the residual energy cannot be greater than the total available energy M_u.

This reasoning suggests an interesting interpretation of the eq (14,50), i.e. the chance that part of the total energy M_uc^2 is utilized to excite $\overline{m}_uc^2$ or $\overline{m}'_{dm}c^2$ from negative energy states to ordinary matter state. The result

$$\frac{m_u + m'_{dm} + m_{de}}{M_u - \mu} = 1$$

calculates the relative proportions of the three terms contributing to the residual energy $M_u - \mu$ once knowing m_{de}. To this purpose it is necessary to describe what happens when part of the total energy excites the antimatter from its negative energy state to the positive energy state of ordinary matter. Three remarks are useful in this respect.

(i) The reasoning underlying the Dirac sea prospects the chance that $\overline{m}_u$ and $\overline{m}'_{dm}$, once excited, leave behind the respective holes of antimatter. On the one hand, is missing in the present model the weird requirement of an infinite number of negative states of the Dirac sea; the freshly formed m_u and m'_{dm} occupy $\overline{V}_0$ previously containing the respective antiparticles. On the other hand, however, just for this reason the holes contextually formed have no empty core cells to be occupied; so the only chance for these antimatter holes is to occupy surface negative energy states at the outer boundary Δr_u of the space time; this transfer, occurring at zero cost of gravitational energy as previously seen, is in fact nothing else but the mere quantum delocalization of particles and antiparticles looking for empty states to be occupied. In conclusion: with this mechanism of relocation and reorganization implied by the chance of segregation of holes at the surface of V_u, the antimatter is progressively detached from the matter and expelled outwards at the boundaries of the space time. Hence neither $\overline{m}_u$ nor $\overline{m}'_{dm}$ appear longer at the left hand side of the eq (14,50), they simply concur to determine the value of the excitation term μ.

(ii) According to the eqs (14,46), if $\overline{m}_u = 0$, then $\overline{m}'_{dm} = 0$ too, because their gravitational effects are mutually linked. Of course this does not mean that the antiparticles disappear from the space time, in fact they are simply displaced, but that if is missing the gravitational binding energy of the former because of the shell theorem, then is also missing the analogous effect of the latter: the Newton theorem predicts that an external shell of matter does not affect the gravitational behavior of the internal shells of mass where is located our measurement point. As the antimatter at the surface states does not concur to the gravitational binding energy, it comprehensibly does not appear in the eqs (14,45) and (14,43); so it is enough to assume that $\mu = 2m_u$ only.

It is worth noticing that actually this is not the only chance possible, i.e. in principle this statement could be reverted asserting that $\overline{m}_u = 0$ as a consequence of having excited $\overline{m}'_{dm}$ to the state of ordinary dark matter; yet exciting $\overline{m}_u$ to the state of ordinary matter requires less energy, $2m_uc^2$ instead of $2m'_{dm}c^2$, so the chance $M_u - 2m_u$ seems preferable to $M_u - 2m'_{dm}$. In effect the assumption of minimum excitation energy of antimatter states is easily confirmed recalling the eqs (14,39).

(iii) It is clear now the physical meaning of density loss concerned in the eq (14,39): as expected there is no mass loss outside V_u, but simply equivalent mass spent to trigger the mechanism that excites the antimatter to the state of ordinary matter. Otherwise stated, the negative value of $\dot{\rho}_u$ due to the space time expansion rate corresponds to the loss of equivalent mass $2\overline{m}_u$ necessary to decouple the antimatter from the matter. Implement thus the eqs (14,39) and (14,50) to demonstrate that really the energy balance implied by the excitation mechanism is synchronized with and allowed by the expansion rate; it is thus necessary to show that $\mu = 2\overline{m}_u$. The first eq (14,39) reads $\dot{\rho}_u = -2\rho_u \Delta\dot{r}_u/\Delta r_u$. The factor $2\rho_u$ suggests that the density change $\delta\rho_u \approx \dot{\rho}_u \Delta t_u$ at the time Δt_u corresponds to the dilution of the total mass present in V_u, which implies an analogous effect for that in both V_0 and $\overline{V}_0$ cells occupied by matter in positive and negative energy states. If however the lost density of the eq (14,50) corresponds to the antiparticles promoted to the positive energy states only, then the amount of equivalent mass transferred reads $-\rho_u \Delta\dot{r}_u \Delta t_u/\Delta r_u$, i.e. $\delta\rho_u/2$ only. So

$$\frac{M_u - \mu}{V_u} = -\frac{\dot{\rho}_u \Delta t_u}{2}$$

yields $\mu = (\rho_u + \dot{\rho}_u \Delta t_u/2)V_u$. In effect the eqs (14,23), (14,22) and (14,40) yield $\mu = 6.1\times10^{52}\,\mathrm{Kg}$, i.e. just $2m_u$.

In conclusion: this mechanism of hole segregation at the surface of V_u splits progressively the antimatter to the boundaries of the space time; it has gravitational effect null on the core space time, whereas its probability of destructive interaction with ordinary matter is also averted. Therefore the values of Δr_u and M_u as a function of Δt_u fit the idea of a well controlled evolution of the space time, where the energy balance proceeds according to the best growth condition and in order to ensure a growth rate allowing the contextual separation of matter and antimatter necessary for the formation of stable aggregates of matter.

But there is more. It is enough to recall the first eq (14,39), that reads $\dot{\rho}_u c^2 = -2c^2 \rho_u \Delta\dot{r}_u/\Delta r_u$ because $c^2/G = M_u/\Delta r_u$ according to the eq (14,20). Differentiate with respect to time the eq (14,20); being $\Delta\dot{r}_u = \dot{M}_u G/c^2$, one finds $\dot{\rho}_u c^2 = -2\rho_u M_u G/(\Delta r_u \Delta t_u)$ according to the eq (14,32). Multiply now both sides of this result by V_u and write the energy loss at left hand side as $\dot{\rho}_u c^2 V_u = -\Delta\varepsilon/\Delta t$ by dimensional reasons; one finds thus

$$\frac{\Delta\varepsilon}{\Delta t} = \dot{\rho}_u c^2 V_u = -2G\rho_u \frac{M_u V_u}{\Delta t_u \Delta r_u}$$

Note that the eq (14,20) allows expressing V_u both as $4\pi\Delta r_u^3/3$ and identically as $4\pi(GM_u/c^2)^3/3$ via the total mass. Rewrite therefore the last equation replacing $\rho_u = M_u/(4\pi\Delta r_u^3/3)$ and $V_u = 4\pi(GM_u/c^2)^3/3$. One finds

$$\frac{\Delta\varepsilon}{\Delta t} = -2G\frac{M_u^2(M_u G/c^2)^3}{\Delta t_u \Delta r_u^4}$$

Apparently all this seems a trivial way to rewrite the first eq (14,39) implementing the eq (14,20). It is not so. Write in general $MG = \Delta r^3\omega^2$ being ω an arbitrary frequency, as it is evident by mere dimensional reasons; in particular, it is known that the arbitrary parameters Δr and ω determine the length a frequency scale of an orbiting system, in which case M is clearly the reduced mass of a gravitational system with the center of mass at rest, as it could be rigorously demonstrated via the Lagrange equations (9,3). Rewrite thus the last equation via this result also in the case of the space time expansion; replacing $\Delta r = \Delta r_u$, the eqs (14,40) yield

$$\frac{\Delta\varepsilon}{\Delta t} = -\left(\frac{2\Delta r_u}{c\Delta t_u}\right)G\frac{M_u^2\Delta r_u^4\omega^6}{c^5} \qquad \Delta r_u = 3.3c\Delta t_u.$$

Note that according to the estimates (14,2) the coefficient 6.6 resulting in parenthesis of the formula is very close to literature value 32/5 reported in the classical relativity for the energy loss due to the generation of gravitational waves. This result has been obtained reasoning on macroscopic quantities like ρ_u and V_u and respective time changes. A more accurate analysis on quantum basis shows however that the energy lost from gravitational systems via gravitational waves is quantized [19]. Yet this result shows that the gravitational waves are generated not only by bound orbiting systems, but also by the expansion itself of the space time; here ω is not related to the orbital period, it is the frequency related to the mere energy density loss during the expansion of the space time. Comparing these results one finds $\omega = c/\Delta r_u$, i.e. ω is about one third the Hubble parameter.

Consider then the residual amount of energy $M_u c^2 - 2m_u c^2$ still available after having excited the content of the antimatter cells $\overline{V}_0$, whatever they might be made of, from the negative energy state to the ordinary state. This term is that introduced to make the eq (14,28) compliant with the eq (13,20); now are clearer the considerations of the section 14.4. Rewrite the eq (M03) including however the masses gravitationally active only, i.e. the masses at the left hand side; one finds thus

$$\frac{m_u + m'_{dm} + m_{de}}{M_u - 2m_u} = 1 \qquad 2m_u = |\overline{m}_u| + |\overline{m}'_{dm}| + m_{bal} - m_{de}$$

The second equation allows calculating m_{de} with the help of the eqs (14,47), (14,45), (14,44) and (14,2):

$$m_{de} = m_{bal} + m'_{dm} - m_u = 3.6\times10^{53}\mathrm{Kg},$$

whereas the first equation yields the relative abundance ratios

$$\frac{m_u}{M_u - 2m_u} = 0.05 \qquad \frac{m'_{dm}}{M_u - 2m_u} = 0.26 \qquad \frac{m_{de}}{M_u - 2m_u} = 0.69;(14{,}51)$$

These results agree with the well known abundance ratios of the so called dark matter and dark energy with respect to the visible mass in the universe. The eqs (14,51) yield also

$$\frac{m_u}{V_u} = 8.1\times10^{-29}\frac{\mathrm{Kg}}{\mathrm{m}^3} \qquad \frac{m'_{dm}}{V_u} = 3.8\times10^{-28}\frac{\mathrm{Kg}}{\mathrm{m}^3} \qquad \frac{m_{de}}{V_u} = 9.7\times10^{-28}\frac{\mathrm{Kg}}{\mathrm{m}^3}$$

and

$$m_u c^2 = 2.7\times10^{69}\mathrm{J} \qquad m'_{dm}c^2 = 1.3\times10^{70}\mathrm{J} \qquad m_{de}c^2 = 3.2\times10^{70}\mathrm{J}.(14{,}52)$$

Refer these energies to the common volume V_u containing them; one finds

$$\frac{m_u c^2}{V_u} = 7.3\times10^{-12}\frac{\mathrm{J}}{\mathrm{m}^3} \qquad \frac{m'_{dm}c^2}{V_u} = 3.5\times10^{-11}\frac{\mathrm{J}}{\mathrm{m}^3} \qquad \frac{m_{de}c^2}{V_u} = 8.7\times10^{-11}\frac{\mathrm{J}}{\mathrm{m}^3}.(14{,}53)$$

15 DARK MATTER AND DARK ENERGY

The eqs (14,53) suggest the chance of correlating the dark quantities and the pressures introduced in the eqs (13,20) and (13,17). Multiply tentatively the two energy densities of the ordinary and dark matter by 2/3, and the dark energy density by 1/3; so

$$P_m = 4.9\times10^{-12}\mathrm{Pa} \qquad P_{dm} = 2.3\times10^{-11}\mathrm{Pa} \qquad P_{de} = 2.9\times10^{-11}\mathrm{Pa}.$$

Note that

$$P_m + P_{dm} = 2.9\times10^{-11}\mathrm{Pa},$$

i.e. with the multiplicative coefficients proposed here the sum of matter pressures is very close to the dark energy pressure P_{de}. The results of the section 10 show how to regard these pressures, i.e. according to the eqs (13,17) and (13,18) leading to the reasonable eq (13,20): on the one hand the dark energy determines the negative pressure of light acting inside the space time volume against its boundary, which therefore tends to swell; on the other hand the matter, visible and dark, determines the gravitational pressure tending to contract the size of V. In the former case the active source justifying P_{de} is the photon energy density $\eta_{de} = U_{de}/V_u$. Nevertheless the results of the section 14.4 show that actually the former slightly overcomes the latter. Thus one could even suppose that the exceeding swelling effect could be due to the CMBR: the eq (14,13) has calculated $\eta_{cmbr} = 4.2\times10^{-14}\mathrm{J/m}^3$ and thus $\eta_{cmbr}/3 = 1.4\times10^{-14}\mathrm{Pa}$, which is anyway radiation pressure and thus a swelling pressure. More realistically, the fact that the absolute values of the pressures P_{de} and

$P_m + P_{dm}$ result almost equal suggests three remarks:

(i) the dark and ordinary quantities are near the thermodynamic equilibrium, so it is difficult to affirm that $P_{cmbr} << P_{de}$ is surely the decisive unbalancing effect responsible of the expansion;

(ii) the dark quantities satisfy the same pressure/energy density relationships as the ordinary matter and electromagnetic radiation;

(iii) despite this similarity with the ordinary radiation and matter, peculiar properties characterize the dark matter and energy and make the latter physically different from the former.

Trusting that the agreement between the values of pressure is not accidental and collecting this preliminary information, examine more closely the tentative link in fact introduced by assigning the coefficients 1/3 and 2/3 to the energy densities of the eqs (14,53). As concerns the radiation existing in the space time, regard the dark energy basically as an electromagnetic radiation field in a black body cavity of volume V_u; in fact the energy density of the microwave background field and its temperature have been calculated correctly just implementing the hypothesis of black body radiation field. This attempt is sensible thanks to the results of the section 13.7 that introduce in the frame of the eq (2,1) the physics of a photon gas in a cavity: the CMBR concerns the residual fossil field originated in the early stages of the space time life, whereas the data of the eqs (14,53) concern the today dark radiation field in the space time. Let therefore U_{de} be the total average energy of the photon gas in a cavity of volume V_u: thus $\eta_u = U_{de}/V_u$ and $P_{de} = U_{de}/3V_u$. The problem is how to implement these data considering the well known thermodynamics of the photon gas in equilibrium with the walls of a cavity.

Implementing the results of the section 13.7, one finds that the photon density due to the number N_{de} of photons in V_u and the related internal energy density due to U_{de} are given by the well known formulas

$$\frac{N_{de}}{V_u} = 16\pi\zeta(2,2)\left(\frac{k_B T_{de}}{hc}\right)^3 \qquad \frac{U_{de}}{V_u} = \frac{8\pi^5 hc}{15}\left(\frac{k_B T_{de}}{hc}\right)^4,$$

being $\zeta(2,2) = 1.202$ the Riemann function and T_{de} the average temperature of the photon gas. Equating U_{de}/V_u to η_{de} according to the present assumption, one finds

$$\frac{U_{de}}{V} = \frac{m_{de}c^2}{V_u} = 3.24\times10^{-23}\left(\frac{k_B T_{de}}{hc}\right)^4 = 8.7\times10^{-11}\,\frac{\mathrm{J}}{\mathrm{m}^3} \qquad k_B T_{de} = 2.6\times10^{-22}\,\mathrm{J}.$$

The value of $k_B T_{de}$ allows calculating the number density N_{ph} and total number $N_{ph}^{tot} = N_{ph}V_u$ of photons with the help of the eq (14,21)

$$N_{ph} = \frac{N_{de}}{V_u} = 1.3\times10^{11}\,\mathrm{m}^{-3} \qquad N_{ph}^{tot} = 4.8\times10^{91};(15,1)$$

the average energy per photon is

$$E_{ph} = \frac{U_{de}}{N_{de}} = \frac{\pi^4}{30\zeta(2,2)}k_B T_{de} = 8.2\times10^{-22}\,\frac{\mathrm{J}}{\mathrm{photon}}.(15,2)$$

Hence $E_{ph} \approx 3k_B T_{de}$, as if each photon would be a classical oscillator with kinetic and potential energy in a crystal lattice instead of an ideal gas of free photons.

On the one hand is recognizable a physical interpretation underlying these results, on the other hand just this peculiar conclusion could be the key to understand the physical difference between ordinary and dark quantities.

Try to investigate the idea of a structure formed by photons trapped by dark matter particles to explain why the dark energy behaves like photons without being however visible, whereas the dark matter becomes invisible itself despite its basic similarly to the ordinary matter. Otherwise stated, the peculiarity of the dark quantities seems not due to the exotic nature of unknown particles constituting them, but to the peculiar arrangement of known particles.

This suggests the formation of gravitational mircro/nano systems disseminated throughout the space time volume and formed by a core of dark matter particle surrounded by a cloud of photons; the chance of binding a photon cloud is provided by the mass of the core particle, regarded like a nucleation site on which cluster the photons via gravitational effect.

In effect an orbiting system formed by photons curling around a matter core is possible [17]; this result has been inferred as a limit case of the light beam bending as a consequence of the quantum uncertainty. This point is so crucial, that it is considered here implementing again the eqs (9,4) and (9,5).

Let m'^{o}_{dm} be the mass of each dark particle concurring to the total mass m'_{dm} calculated in the eqs (14,45) and (14,44): now $m_g \equiv m'^{o}_{dm}$ is the source of gravitational field, whereas the electron orbiting mass m is replaced by the equivalent mass E/c^2 of the photons trapped by m'^{o}_{dm}. Owing to the eq (13,8), it is easy to show that the same equation (9,3) leading to the Bohr atom and to the light beam bending is also compatible with the quantum gravitational levels of the system dark particle/photons; the Bohr radius takes the physical meaning of orbiting distance Δr such that $\Delta r < 2m'^{o}_{dm}G/c^2$. If this condition is fulfilled, then the photons around m'^{o}_{dm} do no longer behave as a free gas, they form a bound system having potential and kinetic energies governed by the gravitational field of m'^{o}_{dm}. Write thus the black hole condition as

$$\frac{(nh)^2}{m\vartheta} < \frac{2m'^{o}_{dm}G}{c^2} \quad \text{i.e.} \quad \frac{(nh)^2}{Gm^2 m'^{o}_{dm}} = \frac{2q^2 m'^{o}_{dm}G}{c^2} \quad q^2 < 1,$$

where $q^2 < 1$ is the parameter ensuring the initial inequality necessary to describe the trapping of photons in the gravitational field of m'^{o}_{dm}; expectedly $n = 1$ represents the most favorable choice to fulfill the inequality, yet for completeness keep explicitly n. The second equation yields

$$nh = \pm\frac{\sqrt{2}qGm'^{o}_{dm}m}{c}.(15,3)$$

Now replace m with the energy $|E|/c^2$ of the photons in this last result and in the energy equation (9,4). The former substitution yields

$$|E| = \pm\frac{nhc^3}{\sqrt{2}qGm'^{o}_{dm}};(15,4)$$

the latter substitution yields

$$E = -\frac{G^2m^3(m'^{o}_{dm})^2}{2(nh)^2} = -\frac{G^2|E^3|(m'^{o}_{dm})^2}{2c^6(nh)^2},(15,5)$$

whose solution is

$$E_{tr} = -\sqrt{2}\frac{nhc^3}{m'^{o}_{dm}G} \quad \Delta r = \frac{1}{2}\frac{m'^{o}_{dm}G}{c^2} = \frac{(nhc^2)^2}{m'^{o}_{dm}GE^2_{tr}} \quad q = -\frac{1}{2}.$$

The subscript tr stands for trapped, with reference to the structure of the "trapped photon/dark matter" system; the given value of q fulfills the initial inequality required for a photon bound state and determines uniquely E of the eqs (15,4) and (15,5). The equalities of Δr express the orbital photon distance from m'^{o}_{dm} via both eqs (9,4) and (13,8), which of course coincide: indeed replacing $m'^{o}_{dm}G$ with Ze^2/m according to the eq (9,5) and E^2_{tr} with $(mc^2)^2$, obviously Δr yields the Bohr radius. This check confirms that Δr is the distance between m'^{o}_{dm} and the photons. As every particle of dark mass m'^{o}_{dm} traps one or more non-interacting photons, E_{tr} is the energy of all N_{tr} photons forming the cloud around m'^{o}_{dm}; it is simply $E_{tr} = N_{tr}E_{ph}$.

It is possible to summarize these results in two equations for E_{tr} and Δr as follows

$$m'^{o}_{dm}c^2 = \frac{\Delta r}{\sqrt{2}}\frac{c^4}{G} \quad E_{tr} = -\frac{hc}{\sqrt{2}\Delta r} \quad n = 1.$$

Having two equations for three unknowns, a guess is necessary:

if $\Delta r = l_{Pl}/\sqrt{2}$, then one finds $m'^{o}_{dm}c^2 = \varepsilon_{Pl}/2$, whereas $E_{tr} = -hc/l_{Pl} = -\varepsilon_{Pl}$. The number $N_{dm'}$ of dark particles determining the value of m'_{dm}, eq (14,45), is thus according to the eqs (14,52)

$$N_{dm} = \frac{2m'_{dm}c^2}{\varepsilon_{Pl}} = 5.1\times10^{60}.$$

The factor 2 indicates that this number includes both m'_{dm} and $\overline{m}'_{dm}$ according to the eqs (14,45) and (14,46). One half of N_{dm} is however effective as concerns the gravitational binding energy; so it is easy to link this result to the previous equations. In particular:

(i) the fact that $\Delta r < l_{Pl}$ means that even for $n = 0$ there is within the Planck volume l^3_{Pl} a trapped structure; in effect, the energy balance $\varepsilon_{Pl}/2 + m'^{o}_{dm}c^2 + E_{tr} = 0$. In lack of an appropriate trigger, thanks to the assumed configuration of dark matter and photon, the state of Planck space time is that of global energy equal to zero, like that of the eq (14,42); the quantum fluctuation perturbs this situation, as previously explained.

(ii) according to the eq (15,2) the number of photons trapped in each cloud is $E_{tr}/E_{ph} = 6\times10^{30}$, which is reasonably consistent with the number of m'^{o}_{dm} particles and with the eq (15,1): indeed $(6\times10^{30})\times(5.1\times10^{60}) = 3.1\times10^{91}$ fits well the total number of photons N^{tot}_{ph} constituting the dark trapped energy.

Owing to the fact that $m'^{o}_{dm}c^2 = \varepsilon_{Pl}/2$ whereas $E = -\varepsilon_{Pl}$, it was found in the section 14.4 that the side of V was $\beta^{1/3}l_{Pl}$, eq (14,17); thinking $\beta^{1/3} = \sqrt{2}$, which fulfills the inequality $2^{3/2} < 3$, one infers that the size of the photon/dark mass structure is also compatible with the volume existing at $n = 1$.

16 DISCUSSION

The invariance of c is one among the pillars of the relativity. Pillar of the present model is instead the invariance of a group of constants merged in the position (1,1). The model, which has "ab initio" character because it starts from this unique position, has shown that the Planck units are not mere numerical inputs alternative to the conventional measure standards, useful to carry out calculations only; appropriate combinations of these units account for known physical laws and introduce new ideas. The space time is more than a fundamental concept of the modern physics; it is a real entity expressible through a formula: the particular combination (1,1) of units exemplifies a possible way to reveal its own physical properties. All previous considerations have been implemented via elementary algebraic steps to provide information on the features of the universe. The strategy to this purpose was in principle simple: to extract as much information as possible form the eq (2,1). The final goal was to understand as a consequence of this unique initial intuition how the space time evolves to form energy and matter aggregates starting from an initial energy field. Most important is the chance of having inferred from the definition of space time two straightforward corollaries, the Lorentz invariance and the statistical formulation of space time uncertainty, on which are rooted the quantum and relativistic theories, as previous papers have shown [15]. Common root means that in fact quantum mechanics and relativity simply diversify their formalism implementing a unique idea; so their diversity is apparent only, being mostly a methodological issue rather than a conceptual conflict. The eq (4,4) highlights that the formulation of physical problems is possible without concerning specific reference systems, so the uncertainty in its most agnostic proposition is the quantum equivalent of the concept of covariancy. Moreover there is no necessity of tensor calculus, because the local coordinates are conceptually disregarded in the present theoretical frame; but just for this reason all reference systems are indistinguishable and thus equivalent in any physical problem.

The fact that several results here exposed were already found implementing the eqs (4,5) only [15,28] is not surprising, as the latter are the most straightforward consequence of the eq (2,1). Hence the model proposed in this paper represents a step even more fundamental than the quantum uncertainty itself: the eqs (4,5), formerly postulated as a basic principle of the nature, actually appear to be a corollary of an even more general concept, the physical definition of space time. From the space time standpoint the quantum uncertainty is a necessity, not a successful postulate. It is interesting that the evolution of the universe governed by the uncertainty shows actually an inherent synchronism between mass formation, size growth and time running evidenced in the sections 14.7 and 14.8.

No hypothesis "ad hoc" has been made in the model; everything was based on the position (1,1) and its dimensional root only: inferring the Lagrange equations, the concept of action and that of entropy together with the laws of thermodynamics means having reached the foundations of the modern physics, i.e. the conceptual frame from which everything follows.

Is attracting the idea of a unique conceptual root that underlies both quantum physics and relativity, yet this idea requires modifying the concept of interval: it must be compliant with the Heisenberg principle, which in turn makes the relativity based on such intervals compliant the quantum physics in a natural way.

In effect the uncertainty ranges provide typical outcomes not only of quantum physics, like the dual behavior of matter and the De Broglie momentum, but also of general relativity; e.g. the gravitational waves [19], the light beam bending, the red shift and the perihelion precession in the same conceptual frame based on the eqs (9,4) and (9,5) apparently pertinent to the Bohr atom only.

The idea of searching appropriate combinations of Planck constants is fruitful; further information on the features of the space time are obtained involving even α, which introduces the electric charge into the physical arena: in combination with the (1,1) it introduces the electromagnetism and thus the weak interactions and eventually the strong interactions as well [28]. A few remarks clarify this point.

Regard $e^2/hc=\alpha$ likewise as $hG/c^2=V\nu$, despite the physical definition at the left hand side corresponds now to one numerical constant at the right hand side only; the reasoning is in principle identical to that leading to the eq (5,1) and (6,3), which is now further commented.

Start with the identity $e^2/(hc\Delta r)=\alpha/\Delta r$, so that $e^2/\Delta r=hc/\Delta r'$ with $\Delta r'=\Delta r/\alpha$ whatever the reference system defining Δr might be; plugging the numerical value of α into the new length $\Delta r'$, results defined the energy $hc/\Delta r'$ whose physical meaning is however nothing else but the Coulomb interaction. The eqs (6,3) generalize this result: the repulsive energy $e^2/\Delta r$ between similar charges implies also the attractive energy $-e\bar{e}/\Delta r$ between opposite charges, which leads to the CPT theorem.

The section 9.2 has clearly shown that the range sizes are inessential as concerns quantum eigenvalues and typical relativistic consequences like the light beam bending: so is inessential the fact of having defined the interaction between charges Δr apart via the the concept of energy implementing another range $\Delta r'$; what is crucial is the analytical form resulting at the left hand side for the given interaction and its physical meaning of energy at the right hand side. Otherwise stated, in any problem involving interacting charges is relevant the Δr^{-1} energy dependence, not the size of Δr itself or that of $\Delta r'$ merely defining this energy. This is also evident, for example, putting $\Delta r'$ and $\Delta p'_r$ in the eq (9,3): the result would be identical because anyway $\Delta r'\Delta p'_r$ would be replaced by $n'h$, however with $n'\equiv n$ for the reasons explained in the section 4.

To highlight further this concept, write according to the eqs (4,5) $e^2/\Delta r=c\Delta p'_r/n$ as $ne/(\Delta r\Delta t)=cF'/e$, having put $F'_r=\Delta p'_r/\Delta t$. It is immediate to recognize that this result reads in general $|B|\propto|i|/\Delta r$, where $|i|=ne/\Delta t$ is the current due to the flow of ne electric charges and $|B|$ the modulus of a new field defined by $|B|=F'_r/e$. This is nothing else but the Biot-Savart law.

Combine the eqs (5,1) and (6,3); it is possible to write $e^2/\lambda=(e\lambda)^2/\lambda^3=(c\nu)^2V/G$, because in principle λ^3 identifies a volume. The fact that both V and λ are arbitrary, allows writing $(e\lambda)^2=(cV\nu)^2/G$; so with the help of the eq (2,1) one finds

$$e\lambda=\pm\frac{h}{c}\sqrt{G},$$

where the double sign shows that the relationship holds for charges of both signs. Consider now the particular case where λ is the Compton length introduced in the eq (5,3); then this equation reads $e/m=\pm\sqrt{G}$. Hence $e_1e_2=\pm m_1m_2G$, being e_1 and e_2 two different amounts of charges to which correspond two different masses m_1 and m_2 [28]; the double sign depends on that of the charges. Now it is possible to divide both sizes of this last equation by the arbitrary length $\Delta x'$ defined in the eqs (5,10) and (5,11) to infer

$$\frac{e_1e_2}{\Delta x'}=\pm U=\pm G\frac{m_1m_2}{\Delta x'}.$$

On the one hand this result remarks once more the analogy between the expressions of the Coulomb and Newton expressions; on the other hand, recalling the considerations of the sections 5 and 9.4, it also takes into account the relativistic corrections necessary for the plain classical expressions of Newton and Coulomb. Eventually it also confirms the possibility of the anti-gravity correspondingly to both signs well known for the Coulomb law only. This suggests that $e\lambda$ must have a particular importance just for $\lambda=\lambda_C$. Consider thus $e\lambda_+$ and $e\lambda_-$ with λ_C corresponding to the respective signs; since $e\lambda_+ - e\lambda_- = 2h\sqrt{G}/c$, it is possible to define $e\lambda/2 = (e\lambda_+ - e\lambda_-)/2 = h\sqrt{G}/c$. In effect

$$\mu_B = \frac{eh}{2mc}$$

is the well known Bohr magneton, whose magnetic dipole character appears clearly in this derivation.

Another example is carried out multiplying the energy density η of the eq (5,1) by α; one finds with the help of the eq (2,1) $\eta\alpha = (c\nu e)^2/(hcG) = (c\nu e)^2/(c^3 V\nu)$. Also, multiplying both sides by 2/3 and recalling the results of the sections 10 and 5, $2\eta\alpha/3 = P\alpha = 2(c\nu e)^2/(3c^3\nu V)$ reads $P_\alpha V\nu = \varepsilon_\alpha \nu = 2e^2\dot{v}^2/3c^3$; this is because $c\nu$ has the physical dimensions of an acceleration here implemented for brevity, i.e. it can be written as $\dot{v}$ of the particle carrying the charge e. The numerical value of α has been merged into that of P_α and ε_α, likewise as in the previous example it was merged in Δr; also now, since both P and ε are arbitrary, the new quantities including α are still arbitrary values of pressure and energy without loss of generality. So, being the subscripts mere inessential notation, the result is the well known Larmor equation describing the energy loss or gain rate $|W|$ by an accelerating charge

$$|W| = \frac{2}{3}\frac{e^2\dot{v}^2}{c^3}.$$

The result describes both energy gain of a charge accelerated by an external field, in which case W has positive sign, or energy loss by irradiation, in which case W has negative sign. Of course this is a non-relativistic result because of the simplified way of defining the acceleration; however the result previously exposed to express explicitly Δx as shown for instance in the eqs (7,4) and (5,11), indicates the way to generalize easily the calculation of $\Delta\ddot{x}$ to correct appropriately and generalize the Larmor equation. Clearly this is not the main point to be concerned here; rather is crucial to emphasize the simplicity of the steps necessary to get this result.

Shortness and straightforwardness are not "per se" requirements of a physical model; however, a direct pathway to reach the result of interest certainly indicates that physical intuition and mathematical approach are adequate to fit the real essence of the problem. This point of view, already exemplified in the section to infer the black body formula, has been in fact followed throughout the development of the present model.

The paper could be stopped at the end of the section 13: to show the validity of the eq (2,1) would have been enough the corollary of wave/corpuscle behavior of the matter. The dual nature of the particles is so weird that obtaining it as a corollary of the apparently vague and naive position (1,1) is a crucial test to validate the basic motivation of the paper. Particular attention has been however payed also to the cosmological implications of the model: trusting on the chance of regarding the space time as "statistical mirror" of the universe is a further crucial check of the present model, in the frame of which also relativistic results have been inferred without additional hypotheses, see for instance the eqs (5,1) and (13,8) and (9,6) among the others.

But the model provides more than the simple comparison with basic concepts well acknowledged. Consider the two slit diffraction experiment, where the electron seems to pass simultaneously through both of them; the explanation, incompatible with the mere corpuscular nature of the electron, forced to postulate the wave nature of the electron as well, consistently with the wavelike diffraction pattern. It is interesting to see what the present way of reasoning contributes to this important result.

Once having acknowledged that the quantum uncertainty excludes local coordinates, look at the eqs (4,4) and consider what the concurrent lack of a specific reference system means for this experiment where the electron moves through slits at rest. The experiment must clearly hold in any reference system. The position of the electron depends on the reference system, e.g. the electron could be on the origin of one of them, R, but in general anywhere in another of them, R'. Yet R and R' must be not only equivalent, but also physically indistinguishable: likewise as it is impossible to think electron 1 and electron 2 in a many electron system, it is also impossible to distinguish the reference systems R and R' with an analogous quantum motivation. This statement is more compelling than their simple equivalence, which excludes one privileged reference system while acknowledging however that R and R' are different. The quantum uncertainty, instead, rules out even the chance of regarding the electron at the origin of R and elsewhere in R' just because the concept of distance is excluded once having disregarded the local coordinates.

The only chance is that the electron must be everywhere with respect to the former, and thus identically everywhere with respect to the latter as well.

This is the most agnostic physical meaning of indistinguishable reference systems: from the fact that the concept of delocalization is synonym of "everywhere" in the allowed range, follows by necessity also the concept of "wavelike corpuscle" evidenced via simple algebraic manipulations as a corollary of the definition of space time. It also follows that the so called EPR paradox is actually due to an unphysical statement: without the concept of distance is meaningless to distinguish two particles at "superluminal" or "relativistic" distances; an analogous explanation holds for the Aharonov-Bohm effect, whose "here and/or there" must be replaced by "everywhere".

An interesting remark concerns the transformation of $\Delta x \Delta p_x$ in R into $\Delta x' \Delta p'_x$ in R'. Suppose that Δx is introduced in R as length $\Delta l = \sqrt{(c\Delta t)^2 - \Delta x^2}$ wherever it appears in the formulation of any physical problem of special relativity. So in R' holds $\Delta l' \equiv \Delta l$ and thus $\Delta p_l \equiv \Delta p'_l$. It means that the momenta allowed by Δp_l in R remain unchanged in R', whereas the respective numbers of allowed states are unchanged as well. This statement is stronger than the mere fact that in general $\Delta x \Delta p_x$ and $\Delta x' \Delta p'_x$ are indistinguishable whatever their specific numbers of states might be; otherwise stated $\sqrt{(c\Delta t)^2 - \Delta x^2} = \sqrt{(c\Delta t')^2 - \Delta x'^2}$ implies that even though Δx changes for any reason, the momenta of the system of particles remain actually unchanged in R and R'. This is clearly a momentum symmetry of the system with respect to the coordinates; the changes of these latter, whatever they might be, leave unaffected the conjugate momenta. The same reasoning and conclusion hold also for Δt and $\Delta \varepsilon$. These considerations fulfill the Noether theorem: in the present model condition necessary but not sufficient for its validity is that the numbers of states characterizing the eigenvalues of a system remain unchanged.

One could inquire at this point about the numerical values of the fundamental constants, e.g. to hypothesize how new exotic universes with different values of these constants could be made. The exercise of simulating the properties of these universes is in principle simple: it is enough to carry out the same calculations via new values of the light speed or gravity constant, while replacing the Planck constant in the eq (4,2) as well. This would allow following the behavior and the evolution of a universe grown on a different kind of space time. But this attempt would actually be an ineffective curiosity not experimentally verifiable, thus a useless and hopeless effort. What is however crucial is that another universe could in principle exist, as nothing compels that the respective values of fundamental constants are necessarily the ones we know: all concepts hitherto exposed, e.g. the black hole length or the space time interaction with matter or the dual behavior of matter, identically hold regardless of the specific numerical values of these constants.

17 CONCLUSION

The paper has described the physical laws that govern the space time starting uniquely from a combination of fundamental constants describing space and time coordinates. The theoretical model has been developed mostly through a deductive analytical approach. The part of mere calculation has been limited to the minimum necessary to compare basic ideas inferred "ab initio" and cosmological properties. Most of the results hold in principle regardless of the specific numerical values of the fundamental constants, primarily important the uncertainty and the quantization. The universe we know appears to be simply the one, among those possible, characterized just by the given values of these constants.

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