



# The efficiency of application of virtual cross-sections method and hypotheses MELS in problems of wave signal propagation in elastic waveguides with rough surfaces

A. S. Avetisyan, A. A. Hunanyan

Institute of Mechanics, National Academy of Sciences, Yerevan, Republic of Armenia

ara.serg.avetisyan@gmail.com

hunanyan.areg21@gmail.com

## ABSTRACT

The efficiency of virtual cross sections method and MELS (Magneto Elastic Layered Systems) hypotheses application is shown on model problem about distribution of wave field in thin surface layers of waveguide when plane wave signal is propagating in it. The impact of surface non-smoothness on characteristics of propagation of high-frequency horizontally polarized wave signal in isotropic elastic half-space is studied. It is shown that the non-smoothness leads to strong distortion of the wave signal over the waveguide thickness and along wave signal propagation direction as well.

Numerical comparative analysis of change in amplitude and phase characteristics of obtained wave fields against roughness of weakly inhomogeneous surface of homogeneous elastic half-space surface is done by classical method and by proposed approach for different kind of non-smoothness.

## Indexing terms/Keywords

Rough surface - Cross sections method - Surface geometric heterogeneities - Weakly inhomogeneous surface - Hypothesis-MELS - Modelling of boundary value problems

## Academic Discipline And Sub-Disciplines

Mechanics of Deformable Solids, Mathematics in Engineering

## SUBJECT CLASSIFICATION

Applied Mathematics and Physics

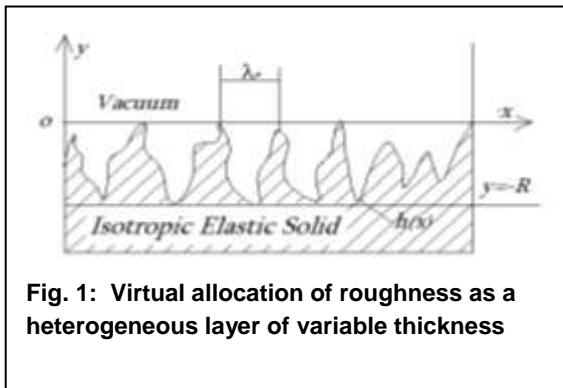
## TYPE (METHOD/APPROACH)

The use of combined method

## 1 INTRODUCTION

It is known that unlike Rayleigh wave it is impossible the localization of horizontally polarized (SH) waves near perfectly smooth mechanically free surface of isotropic elastic half-space [1,2]. From the other hand side, there exist localized high-frequency (short) waves near the contact line between elastic half-space and thin mechanically soft layer made from different material (see for example [2]). The localization of high frequency wave signals at smooth free from mechanical loads surface of waveguide also occurs in the presence of certain physical properties of material of the half-space (see for example [3,8, 9]). The case of smooth surfaces, unlike non-smooth ones, being idealized models can almost always be completely investigated. It is especially important for investigation of high-frequency wave signals (short waves) propagation in layered waveguides considering the actual heterogeneity (non-smoothness) of the surface layers, especially when the length of the wave signal is proportional to average surface roughness step. Accounting inhomogeneous surface layers of the layered waveguide significantly complicates the study of mathematical model of the problem, but makes it possible to reveal new near-surface wave effects and more accurately calculate the quantitative characteristics of the wave field. Most theoretical investigations of acoustic problems for not-smooth surfaces are done either by appropriate integral transformation [5,6] or by a perturbation method [7], deriving asymptotic estimates of wave characteristics. In [19,20] SH acoustic waves propagation in plate with rough surface is studied by method of integral transformations assuming that the average roughness height everywhere is a small part of the thickness of the waveguides (plates). In [8] the influence of dispersion due to roughness on surface acoustic waves and wave packets (in the frequency range of 30-200 MHz) for different degrees of nano-meter roughness on the surface of silicon (section-001) and (section- 111) is investigated. It is shown that the effect of the frequency dispersion induced by surface roughness will be significant for some relative characteristics of non-smooth surfaces. Previously unknown dispersion effects caused by crystalline anisotropy of surface layers is revealed. In [9] the problem of Love waves propagation in corrugated isotropic layer of homogeneous isotropic halfspace studied. The dispersion relation for the corrugated layer is derived as a function of wave amplitude, frequency and parameters of the corrugated section. In particular cases, the dispersion relation has been studied for corrugated layers bounded by periodic surfaces  $d_1 \cdot \cos(px)$  and  $d_2 \cdot \cos(px)$ . It should be noted also that in problems with complex statement about distribution of directed waves in waveguides, along with roughness of surfaces of the waveguide the reasons for arising new wave phenomena can be others (optical, electromagnetic, adhesive properties of bonded interfaces, etc.) [15, 16,21–24]. In some studies the boundary value problems are solved numerically often relying on algorithms involving finite element method on the basis of existing packages [25,26]. Such an approach of

course allows modeling of quite complicated boundary value problems, but it can suppress the general solution and conclusions.



**Fig. 1: Virtual allocation of roughness as a heterogeneous layer of variable thickness**

For identifying interaction effects of propagating wave signal with surface inhomogeneities it is natural to look for them in near-surface area of coupling to non-smooth surfaces. On this basis in [10,11] a model combination approach is proposed: by picking virtually out a homogeneous layer of variable thickness (or inhomogeneous thin layer with constant thickness) from coupling zone of piezoelectric half-spaces, and by introducing hypotheses about distribution of physical and mechanical fields and material characteristics over the layer thickness, the problem is reduced to investigation of wave phenomena occurring with more visual form of the dispersion equation and analytical solutions.

To identify wave effects (possible localization, distortion of the amplitude or phase function, possibility of internal resonance or interruption of frequency zones) emanating only from the heterogeneity of the surface of the waveguide, in this article is

being investigated the propagation of horizontally polarized (SH) shear wave signal in an isotropic elastic half-space, using the method of virtual sections and inputting hypothesis MELS [10,11].

## 2 THE PROBLEM STATEMENT

The propagation of high-frequency monochromatic horizontally polarized shear wave signal in isotropic elastic half-space occupying in Cartesian coordinate plane the domain  $\{|x| < \infty; -\infty < y \leq h(x); |z| < \infty\}$  with non-smooth, load free surface  $y = h(x)$  is investigated. The coordinate axis  $y = 0$  is directed along maximal tangential ledge of the non-smooth surface (Fig. 1). Another characteristic line of the surface heterogeneity (non-smooth) will be the tangent line of maximal trough  $y = -\max[h_{\max}(x) - h_{\min}(x)] = -R$ , of irregularities of the half-space surface.

The problem for horizontally polarized displacement in the half-space is separated from that for plain deformation, and the displacement  $w(x, y, t)$  of shear wave (SH) satisfies to the equation

$$\frac{\partial^2 w(x, y, t)}{\partial x^2} + \frac{\partial^2 w(x, y, t)}{\partial y^2} = c_t^{-2} \frac{\partial^2 w(x, y, t)}{\partial t^2}, \quad (1)$$

in which  $c_t^2 = G/\rho$  is the speed of shear bulk wave in the medium.

The normal to the geometrically inhomogeneous surface  $y = h(x)$  which is free from mechanical loads reads as follows

$$\vec{n}(h(x)) = \left\{ h'(x) / \sqrt{1 + [h'(x)]^2}; 1 / \sqrt{1 + [h'(x)]^2}; 0 \right\}, \quad (2)$$

the boundary condition  $\sigma_{zx}(x, y, t) \cdot n_x(h(x)) + \sigma_{xy}(x, y, t) \cdot n_y(h(x)) = 0$  takes the form

$$h'(x) \cdot \left. \frac{\partial w(x, y, t)}{\partial x} \right|_{y=h(x)} + \left. \frac{\partial w(x, y, t)}{\partial y} \right|_{y=h(x)} = 0. \quad (3)$$

Naturally, it is merely a model problem about propagation of high-frequency horizontally polarized monochromatic shear wave signal in isotropic elastic layer-waveguide  $\{|x| < \infty; y \leq |h(x)|; |z| < \infty\}$  the surfaces  $y = \pm h(x)$  of which is non-smooth and free from mechanical loads, considered in short-wave approximation  $\lambda_0 \ll H_0$ , where  $\lambda_0$  is the wave length, and  $H_0$  is the basic depth of the elastic waveguide.

In the case of short-wave approximation along with condition (3) we have to indicate also a condition on the wave field at infinity, i.e. when  $y \rightarrow -\infty$ .

Moreover, if the attenuation condition is satisfied at infinity  $y \rightarrow -\infty$



$$w(x, y, t) \Big|_{y \rightarrow -\infty} \rightarrow 0, \quad (4)$$

then localized wave exists on the surface  $y = h(x)$  of the half-space due to presence of surface geometric heterogeneities. If the wave field is not attenuated at infinity, then presence of surface geometric heterogeneity does not lead to localization of wave signal.

Equation (1) with boundary condition (3) and the condition of attenuation at infinity (4) form a mathematical boundary value problem.

### 3 SOLUTION OF THE PROBLEM

To reveal the effects of heterogeneity of the half-space surface on the amplitude-frequency characteristic of the propagating plane wave signal  $w(x, y, t) = W_0 A(y) \times \exp\{i(k_0 x - \omega_0 t)\}$ , and then assess the accuracy of the method of sections and the virtual hypotheses MELS [10,11], the solution of the formulated boundary value problem will be implemented in two ways. The first way is the direct solution.

#### 3.1 Representation Of the Solution As Complex Waves

Physical heterogeneity of the environment or geometrical heterogeneity of the waveguide surface naturally lead to interaction of amplitude-phase of the plane monochromatic wave signal  $w(x, y, t) = W_0 A(y) \times \exp\{i(kx - \omega t)\}$ . Due to geometric heterogeneity of the waveguide surface the propagating wave is presented as follows

$$w(x, y, t) = W_0 \exp U(x, y) \times \exp\{i[\theta(x, y) - \omega t]\}, \quad (5)$$

where  $U(x, y) = \ln[A_w(x, y)]$  is the logarithm of the amplitude function,  $A_w(x, y)$  is the amplitude function, and  $\{\theta(x, y) - \omega t\}$  is the phase function of the propagating wave. Then from the linear boundary value problem (1), (3) and (4) we come to the nonlinear interaction of the logarithm of the amplitude function  $U(x, y)$  and phase function  $\{\theta(x, y) - \omega t\}$  [13,14].

Since the presence of the half-space non-smoothness (see (3)) leads to amplitude-phase interaction, the solutions of obtained boundary value problem are presented with complex characteristics, namely wave attenuation coefficient  $\alpha \approx \sqrt{1 - \eta^2/k^2} = \alpha_1 + i\alpha_2$  and wave number  $k \approx k_1 + ik_2$ . Here  $\eta \approx \omega / ((k_1 + ik_2)c_t) = \eta_1 + i\eta_2$  is the complex relative phase velocity. Satisfying to the complex representation of the wave

$$w(x, y, t) = W_0 \exp(\alpha_1 k_1 - \alpha_2 k_2) y \times \exp(-k_2 x) \times \exp\{i[k_1 x + (\alpha_1 k_2 + \alpha_2 k_1) y - \omega t]\}, \quad (6)$$

from conditions (3) and (4) we obtain complex dispersion equation  $ik_1 h'(x) + i(\alpha_1 k_2 + \alpha_2 k_1) + (\alpha_1 k_1 - \alpha_2 k_2) - k_2 h'(x) = 0$ , (7)

from which we obtain conditions for existence of non-trivial solution

$$\alpha_1 \equiv 0; \alpha_2 = -h'(x); \Rightarrow \alpha = \alpha_1 + i\alpha_2 = 0 - ih'(x);$$

$$k_2 \equiv 0; \Rightarrow \eta_2 \equiv 0; \Rightarrow k_1 = k(x) = (\omega_0/c_t) \left\{1 + [h'(x)]^2\right\}^{-1/2} \quad (8)$$

Thus, for propagation of plane shear wave signal  $w_0(x, y, t) = W_0 \times \exp\{i(k_0 x - \omega_0 t)\}$  in elastic half-space with non-smooth surface  $y = h(x)$ , from the propagating plane body waves are being formed not attenuated wave field as along the depth of the elastic half-space, as along the direction of the propagation

$$w(x, y, t) = w_+(x, y, t) + w_-(x, y, t), \quad (9)$$

$$w_{\pm}(x, y, t) = W_0 \times \exp\left\{\mp ik_0 h'(x) \left[1 + [h'(x)]^2\right]^{-1/2} \cdot y\right\} \times \exp\left\{i \left[\pm k_0 \left[1 + [h'(x)]^2\right]^{-1/2} \cdot x - \omega_0 t\right]\right\}, \quad (10)$$

or in expanded form



Unlike half-space with smooth boundary, the propagation of plane shear wave signal in half-space with non-smooth surface brings to distortion of permanent amplitude on plane fronts of propagating and reflecting waves, i.e.  $k_0 x \pm \omega_0 t = 0$ . Thus,

$$w_{\pm}(x, y, t) = W_0 \times \exp \left\{ ik_0 [\pm x - h'(x) \cdot y] \cdot \left[ 1 + [h'(x)]^2 \right]^{-1/2} \right\} \times \exp \{-i\omega_0 t\}.$$

The distorted wave field in the half-space  $\{|x| < \infty; -\infty < y \leq h(x); |z| < \infty\}$  will take the following form

$$w(x, y, t) = W_0 \times \left\{ \begin{array}{l} \exp \left\{ ik_0 [x - h'(x) \cdot y] \cdot \left[ 1 + [h'(x)]^2 \right]^{-1/2} \right\} + \\ + \exp \left\{ -ik_0 [x + h'(x) \cdot y] \cdot \left[ 1 + [h'(x)]^2 \right]^{-1/2} \right\} \end{array} \right\} \times \exp \{-i\omega_0 t\}. \quad (11)$$

From relations (9)-(11) it becomes obvious that the distortion of propagating and reflecting waves occurs along the unit tangent vector of the non-smooth surface:  $\vec{\tau}(h(x)) = \left\{ 1/\sqrt{1+[h'(x)]^2}; \mp h'(x)/\sqrt{1+[h'(x)]^2}; 0 \right\}$ . (12)

Moreover, the presence of non-smoothness  $h'(x) \neq 0$  on the half-space surface generates vibrations in the direction of “-y” of the form

$$\cos \left[ k_0 h'(x) \left[ 1 + [h'(x)]^2 \right]^{-1/2} y \right] - i \cdot \sin \left[ k_0 h'(x) \left[ 1 + [h'(x)]^2 \right]^{-1/2} y \right], \quad (13)$$

and the presence of non-smooth surface  $h'(x) \neq 0$  leads to the distortion of the flat front of propagation in the direction “±x”, i.e.  $k_0 x \mp \omega_0 t = 0$  of the form

$$\cos \left[ k_0 x \cdot \left[ 1 + [h'(x)]^2 \right]^{-1/2} \right]. \quad (14)$$

Let  $\omega_0 = const$  is the eigen-frequency of the wave signal source for the established dynamics, and the speed of bulk shear wave  $c_t = \sqrt{G_0/\rho_0} = const$  in the half-space material. Then, dispersion will occur either in wavelength or wave number

$$\lambda(x) = \lambda_0 \cdot \sqrt{1+[h'(x)]^2}, \quad k(x) = k_0 \left[ 1 + [h'(x)]^2 \right]^{-1/2}. \quad (15)$$

It follows from (15) that  $\lambda(x) \geq \lambda_0$ , i.e. the surface heterogeneity increases the wavelength with respect to the length of the wave signal. Then the corresponding frequency of oscillation will be  $\omega(x) = \omega_0 \left[ 1 + [h'(x)]^2 \right]^{-1/2} \leq \omega_0$ .

The nature of the change in the wavelength, or self-induced oscillation frequency (dispersion) for the propagation of the wave signal is definitely determined by the nature of the surface heterogeneity and depends on the relative linear characteristics of non smoothness:  $\lambda_*$  is the average pitch and  $R$  is the maximal height of the ledge of surface roughness.

The analytical solution and discussion of the results were convincingly possible due to maximal prostate of the selected model boundary value problem. In the case of complicated mathematical boundary value problem corresponding to geometric heterogeneity (docking of two rough surfaces with or without adhesive) or when it is necessary to take into account arising physical inhomogeneity of the material of the waveguide, the method of direct solutions will not always be of help.

### 3.2 Solution By the Method Of Vertical Sections And Applying Hypotheses MELS

For a more complete detection of surface wave phenomena near the non-smooth surface of the half-space let us resolve the problem by the method of virtual sections and introducing the hypotheses MELS [10,11]. Since surface nonsmoothness as a geometric heterogeneity is localized in the near-surface area of the half-space, along the line  $y = -R$ , where  $-R$  is the maximal valley of non smoothness, let us virtually cut the homogeneous near-surface layer of





variable thickness  $\{|x| < \infty; -R \leq y \leq h(x); |z| < \infty\}$ . Then we also will have the homogeneous half-space  $\{|x| < \infty; -\infty < y \leq -R; |z| < \infty\}$ , which is in full contact with the virtually cut layer (Fig. 1).

It is obvious that in the resulting layered waveguide along with equation (1) in two separated areas, boundary condition (3) on the mechanically free, nonsmooth surface  $y = h(x)$  and conditions of the wave amplitude attenuation at infinity, on the plane of the virtual section  $y = -R$ , the conditions of conjugation of the elastic field

$$w_1(x, y, t)|_{y=-R} = w_2(x, y, t)|_{y=-R}; \quad \frac{\partial w_1(x, y, t)}{\partial y} \Big|_{y=-R} = \frac{\partial w_2(x, y, t)}{\partial y} \Big|_{y=-R}, \quad (16)$$

must be satisfied.

The solution in half-space attenuating at infinity has the known form  $w_1(x, y, t) = W_{01} \exp(\alpha ky) \cdot e^{i(kx - \omega t)}$ . Taking into account the thinness of introduced layer of variable thickness, let us represent the solution in it by means of shear displacements on the surfaces of introduced layer:

$$w_2(x, y, t) - w_1(x, -R, t) = g_w(x, y) \cdot [w_2(x, h(x), t) - w_1(x, -R, t)]. \quad (17)$$

Here, the shear wave field distribution function along the variable thickness of the layer is taken to be

$$g_w(x, y) \square \frac{sh[\alpha k(y + R)]}{sh[\alpha k(h(x) + R)]}, \quad (18)$$

in order to satisfy conditions of conjugation (16) on the plane of virtual sections. For this it is necessary that on the surface of the layer  $g_w(x, h(x)) = 1$  and  $g_w(x, -R) = 0$ . Moreover, taking into account that in homogeneous halfspace have the known form  $w_1(x, y, t) = W_{01} \exp(\alpha ky) \cdot e^{i(kx - \omega t)}$ , in homogeneous layer  $\{|x| < \infty; -R \leq y \leq h(x); |z| < \infty\}$  it must be expressed by wave harmonics  $\exp(\pm \alpha ky)$ .

Then, the elastic shear in the layer will take the form  $w_2(x, y, t) = W_{01} \{1 + sh[\alpha k(y + R)]\} \cdot \exp(-\alpha kR) \cdot \exp\{i(kx - \omega t)\}$ , when  $y \in [-R, h(x)]$  (19)

For investigation of existence of nontrivial solutions, from the boundary conditions (3) we obtain the dispersion equation

$$ikh'(x) \{1 + sh[\alpha k(h(x) + R)]\} + \alpha k \cdot ch[\alpha k(h(x) + R)] = 0. \quad (20)$$

Obviously, in the case of waveguide smooth surface, i.e. when  $h'(x) \equiv 0$ , equations (20) and (7) has only the trivial solution  $\alpha k = 0$  corresponding to non dispersion and non attenuation along the depth of the shear waves.

Simplifying (20) we will arrive at

$$\alpha k = -ikh'(x) \frac{1 + sh[\alpha k(h(x) + R)]}{ch[\alpha k(h(x) + R)]}. \quad (21)$$

For ultrasonic wavelengths, i.e. when  $\lambda_0 \square 2\pi R_{\max} \alpha (1 + h(x)/R_{\max})$ , from (21) it follows that the wave interaction argument  $\alpha k(h(x) + R) \rightarrow 0$ . For wave signals "longer" than the height of maximal ledge  $R_{\max}$ , i.e. when  $\lambda \square 2\pi R \alpha (1 + h(x)/R)$ , from (21) it follows that the wave interaction argument  $\alpha k(h(x) + R) \square 1$ . It is easy to see that in both cases the factor on the right hand side of (21) tends to unity without changing the sign and the equation itself acquires the form (7), naturally with bounded solutions (8):

$$\alpha = \alpha_1 + i\alpha_2 = 0 - ih'(x); \quad k_0(x) = k_0 \{1 + [h'(x)]^2\}^{-1/2};$$

$$\omega_0(x) = \omega_0 [1 + [h'(x)]^2]^{-1/2} \leq \omega_0; \quad \lambda_0(x) = \lambda_0 \cdot \sqrt{1 + [h'(x)]^2}. \quad (22)$$

Therefore, introducing unknown function  $\alpha = -i\gamma(x)$  the solution of (21) will take the form

$$\alpha = \mp ih'(x); \quad k_{n^*}(x) = \pm n\pi \left[ (h(x) + R) \right]^{-1} \cdot [h'(x)]^{-2}; \quad n \in \mathbb{Z} \quad (23)$$

In fact, the heterogeneity of the half-space surface when signal (19) with parameters (22) is propagating generates similarity oscillations due to the ratio of the wave signal length and variable thickness of the virtually selected layer

$$w_{n^*}(x, y, t) = W_{0n} \left\{ 1 - sh \left[ i \frac{n\pi(1 + y/R)}{[1 + h(x)/R] \cdot h'(x)} \right] \right\} \cdot \exp \left( i \frac{n\pi}{[1 + h(x)/R] \cdot h'(x)} \right) \times \exp \left\{ \pm i \frac{n(\lambda_0/R)}{2[1 + h(x)/R] \cdot [h'(x)]^2} (k_0 x - \omega_0 t) \right\} \quad (24)$$

The length and frequency of the generated oscillations are represent by relations

$$\lambda_{n^*}(x) = \left[ 2(h(x) + R)/n \right] \cdot [h'(x)]^2; \quad \omega_{n^*}(x) = \pm n\pi c_t \cdot \left[ (h(x) + R) \right]^{-1} \cdot [h'(x)]^{-2}, \quad (25)$$

respectively. It becomes obvious from (25) that these in-layered oscillations do not exist if  $h'(x) \equiv 0$ , or simply vanish when  $h(x) \rightarrow -R$  or in the case of large numbers of forms  $n \gg 1$ .

Thus, considering the characteristics of the dispersions (23) in the nearsurface selected layer of thickness  $y \in [-R; h(x)]$ , for elastic shear we obtain

$$w_*(x, y, t) = \sum_{n=1}^{\infty} \left[ w_{n^*}(x, y, t) + \bar{w}_{n^*}(x, y, t) \right]. \quad (26)$$

Thus, using the method of virtual sections and introducing hypothesis MELS, from (17) and (18) new, commensurable with linear dimensions of the nonsmoothness oscillations of length  $\lambda \approx |\alpha k (h(x) + R)|$  are revealed. At that naturally all possible modes of length  $\lambda_{n^*}(x)$  from (24) occur. Note that in the weakly inhomogeneous surface of the half-space when the variable thickness of near-surface layer is much smaller than the wavelength of the signal, i.e.  $(h(x) + R) \ll \lambda_0$ , and in the case of strictly inhomogeneous surface of the half-space when the variable thickness of near-surface layer is much bigger than the wavelength of the signal, i.e.  $(h(x) + R) \gg \lambda_0$ , we get the same wave fields with characteristics (22).

#### 4 Equivalent Dynamic Bearing Load On Surface

The presence of inhomogeneous surface leads to interaction of plain wave with non-smoothness. Introducing the equation of motion (1) in the virtually cut layer  $-R \leq y \leq h(x)$  in the form

$$G \frac{\partial^2 w}{\partial x^2} + \frac{\partial \sigma_{yz}}{\partial y} = \rho \frac{\partial^2 w}{\partial t^2}, \quad (27)$$

and integrating it over the variable thickness of layer we derive the difference between mechanical tension on the virtual surface  $y = -R$  which is equivalent to presence of the cut layer of variable thickness in the near-surface layer

$$\sigma_{yz} \Big|_{-R}^{h(x)} = \int_{-R}^{h(x)} \left\{ -G \frac{\partial^2 w_*}{\partial x^2} + \rho \frac{\partial^2 w_*}{\partial t^2} \right\} \cdot dy = G \int_{-R}^{h(x)} \frac{\partial^2 w_*}{\partial y^2} \cdot dy. \quad (28)$$

Taking into account surface conditions (3) and (16) and expression (26), (24) for shear in the cut layer  $-R \leq y \leq h(x)$  and (9), as well as (11) for shear in the homogeneous half-space  $-\infty < y \leq -R$ , for equivalent load we will obtain

$$\Delta\sigma = \sigma_{yz} \Big|_{-R}^{h(x)} = -G \left\{ h'(x) \cdot \frac{\partial w_*}{\partial x} \Big|_{y=h(x)} + \frac{\partial w}{\partial y} \Big|_{y=-R} \right\}. \quad (29)$$

Consequently, the problem of propagation of shear wave signal in the halfspace with non-smooth surface is equivalent to the problem of propagation of monochromatic shear signal in the half-space loaded with dynamic force  $|\Delta\sigma| = \sigma_{yz} \Big|_{-R}^{h(x)} + \bar{\sigma}_{yz} \Big|_{-R}^{h(x)}$  and smooth surface  $y = -R$ .

## 5 Internal Resonance

It was shown above that when shear wave signal is propagating in waveguide with non-smooth surface, established oscillations with characteristics (8) and (15) occur. On the other hand, due to the presence of non-smoothness commensurable with wavelength of propagating signal, oscillations in the near-surface area  $-R \leq y \leq h(x)$  are generated characterized by relations (23) and (25). Since in the first version of solution (11) the induced frequency has the form

$$\omega(x) = \omega_0 \left[ 1 + [h'(x)]^2 \right]^{-1/2} \leq \omega_0, \text{ then it will be resonant at}$$

$$\omega_r(x) = \omega_0 \left[ 1 + [h'(x)]^2 \right]^{-1/2} \approx \omega_0 \quad (30)$$

or at the critical points, where  $h'(x) \approx 0$ .

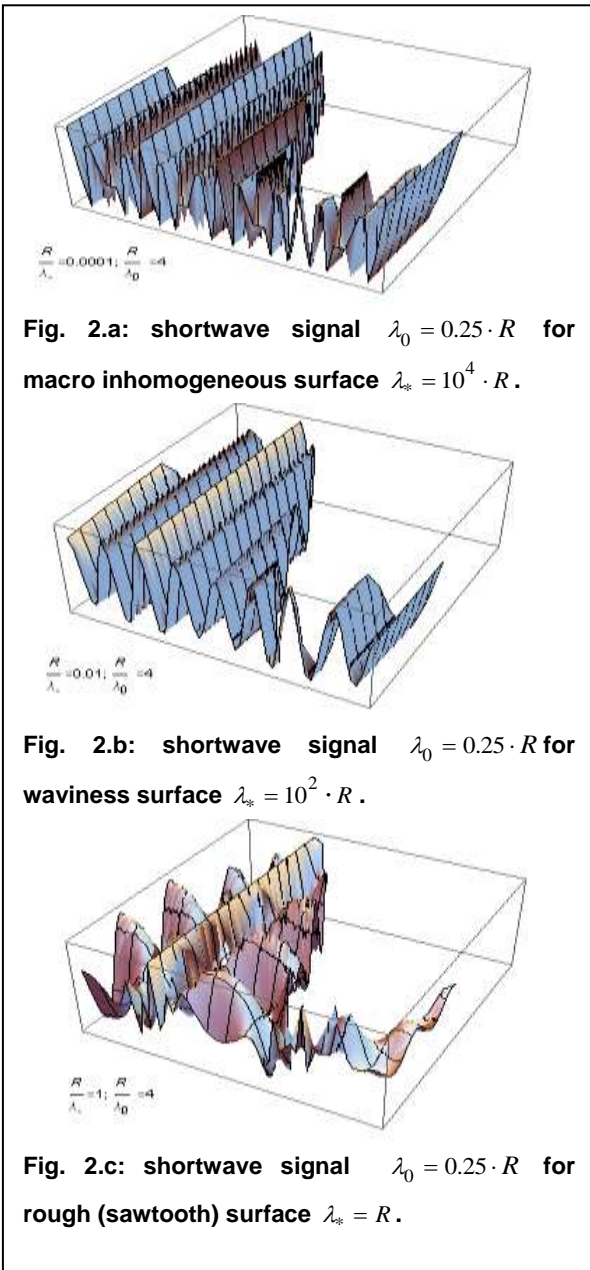
In the case of corrugated (sinusoidal) surface of the half-space when  $h(x) = (R/2) \cdot [\sin(2\pi x/\lambda_*) - 1]$  will have  $h'(x) = (\pi R/\lambda_*) \cdot \cos(2\pi x/\lambda_*)$ , consequently induced frequency will be resonant when  $h'(x) = (\pi R/\lambda_*) \cdot \cos(2\pi x/\lambda_*) \approx 0$ . Therefore in the case of macro non-smoothness of the half-space surface when the average pitch of the irregularities is not bigger than the maximum height of the ledges, i.e.  $\lambda_*/R_{\max} \geq 10^4$ , we have

$$\max [h'(x)]^2 \ll 10^{-7} \text{ for all wavelengths of the signal.}$$

Thus, in this case the surface of the half space behaves as an ideally smooth, and internal resonance occurs in the excitations of eigen-frequencies  $\omega_r(x) \approx \omega_0$ .

From the other hand side, for this heterogeneity and in cases when the linear parameters of corrugated (sinusoidal) surface correspond to other classifications of non-smooth surface, for instance, surface waviness  $\lambda_*/R_{\max} \ll 50 \div 100$  or surface roughness  $\lambda_*/R_{\max} \leq 10$ , then the internal resonance can occur in separate segments of the configurations of the surface.

Local internal resonances may occur in surrounding areas of the critical points of the surface roughness, near the tops of the ledges or near pits troughs where the derivative of then non-smoothness tends to zero:  $|h'(x)| \rightarrow 0$ . In the case of corrugated surface of half-space these zones  $\Delta x_m \approx (1+m) \cdot \lambda_*/4 \pm \varepsilon(o)$ ,  $m \in \mathbb{Z}$ .



For virtually cut near-surface layer  $-R \leq y \leq h(x)$  in the case when the non-smoothness is commensurable with the wavelength of the propagating signal, oscillations generated in the near-surface zone characterized by multiple



frequencies  $\omega_{n^*}(x) = \pm n\pi c_t \cdot [(h(x) + R)]^{-1} \cdot [h'(x)]^{-2}$  are revealed. Equating value  $\omega_{n^*}(x)$  to the value of frequency of established waves  $\omega(x) = \omega_0 [1 + [h'(x)]^2]^{-1/2} \leq \omega_0$ , we will obtain a condition of internal resonance with respect to the ratio of the wavelength and geometric characteristics of non-smoothness:

$$\lambda_0 \approx \frac{2}{n} \cdot \frac{(h(x) + R) \cdot [h'(x)]^2}{\sqrt{1 + [h'(x)]^2}} \quad (31)$$

In the case of corrugated (sinusoidal) surface when  $h(x) = (R/2) \cdot [\sin(2\pi x/\lambda_*) - 1]$  the internal resonance will occur at

$$n \approx \frac{R}{\lambda_0} \frac{[\sin(\xi(x)) + 1] \cdot [(\pi R/\lambda_*) \cdot \cos(\xi(x))]^2}{\sqrt{1 + [(\pi R/\lambda_*) \cdot \cos(\xi(x))]^2}} \square F[(R/\lambda_*); (R/\lambda_0); (\xi(x))], \quad (32)$$

where  $\xi(x) \square 2\pi x/\lambda_* \in [0; 2\pi]$  is the scaling argument along the surface.

## 6 NUMERICAL COMPARATIVE ANALYSIS

From solution of (11) it follows that in the near-surface layer  $-R \leq y \leq h(x)$ , as well as throughout the depth of the half-space a single dispersive, not attenuated form of oscillations is propagating. If the presence of surface heterogeneity generates oscillations along the variable thickness of the layer of the form

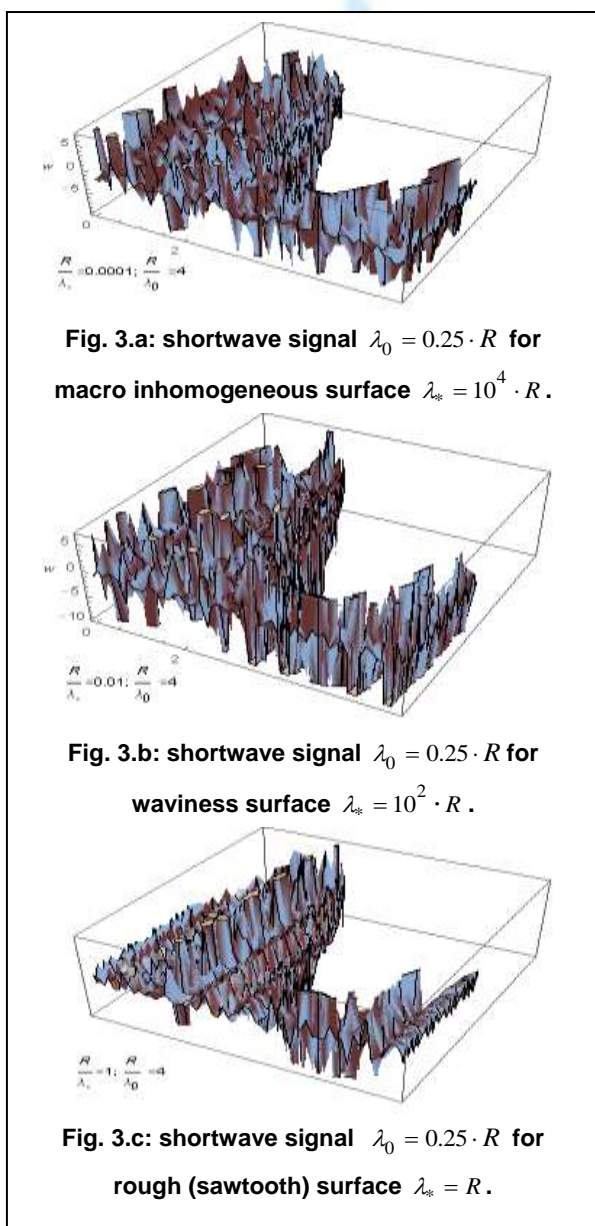
$$\cos \left[ k_0 h'(x) \left[ 1 + [h'(x)]^2 \right]^{-1/2} y \right] - i \cdot \sin \left[ k_0 h'(x) \left[ 1 + [h'(x)]^2 \right]^{-1/2} y \right]$$

then these heterogeneities distort the plane fronts of wave signal  $\pm ik_0 x \rightarrow \pm ik_0 \left[ 1 - \sqrt{1 + [h'(x)]^2} \right] \cdot x$  along directions of propagation and reflection.

For different characteristic inhomogeneities of the surface of the half-space; macro inhomogeneity  $\lambda_*/R_{\max} \square 10^4$ , waviness  $\lambda_*/R_{\max} \square 10^2$ , roughness  $\lambda_*/R_{\max} \square 10$ , or sawtoothness  $\lambda_*/R_{\max} \square 1.0$ , forms of the wave surface corresponding to solution of (11) are shown in Fig. 2.a; b; c. For similar parameters, forms of the wave surfaces corresponding to solution of (26) taking into account (24) are shown in Fig. 3.a; b; c.

From Fig. 2 and Fig. 3 it is visible that in the case of the surface sawtooth inhomogeneity, the wave surface is distorted more quickly and dramatically compared with weakly inhomogeneous surface.

From solution (26) with (24) it is obvious that the new approach reveals the effect of surface inhomogeneities analytically, and the wave field is represented as a package of waveforms. It can be seen as well that in the case of weakly inhomogeneous surfaces when the wavelength of the signal is much bigger than the height of the surface roughness



**Fig. 3.a:** shortwave signal  $\lambda_0 = 0.25 \cdot R$  for macro inhomogeneous surface  $\lambda_* = 10^4 \cdot R$ .

**Fig. 3.b:** shortwave signal  $\lambda_0 = 0.25 \cdot R$  for waviness surface  $\lambda_* = 10^2 \cdot R$ .

**Fig. 3.c:** shortwave signal  $\lambda_0 = 0.25 \cdot R$  for rough (sawtooth) surface  $\lambda_* = R$ .



$\lambda \square 2\pi R\alpha(1+h(x)/R)$ , solutions of the distortion of phase function in directions of propagation and reflection of

wave are the same  $\pm \pm ik_1 x \rightarrow \pm ik_1 \left[ 1 - \sqrt{1 + [h'(x)]^2} \right] \cdot x$

in both approaches, while the distortions of the amplitude along the thickness of the virtually cut thin layer in the near-surface area are different:

$$\Delta W = \left\{ \cos \left[ k_1 h'(x) (y + R) \right] - 1 \right\} \times \exp \left\{ ik_1 h'(x) R \right\}.$$

In the particular case of corrugated (sinusoidal) surface of the half-space  $h(x) = (R/2) \cdot \left[ \sin(2\pi x/\lambda_0) - 1 \right]$  where  $\lambda_0$  is the average pitch, and  $R$  is the maximum height of the ledge (or the maximum depth) of the non-smooth surface, distortions of the amplitude functions from non-smooth surface along the depth of the half-space for different types of weakly inhomogeneous surfaces  $\lambda \square \max \{ \lambda_0; R \}$  are plotted in Fig. 2.

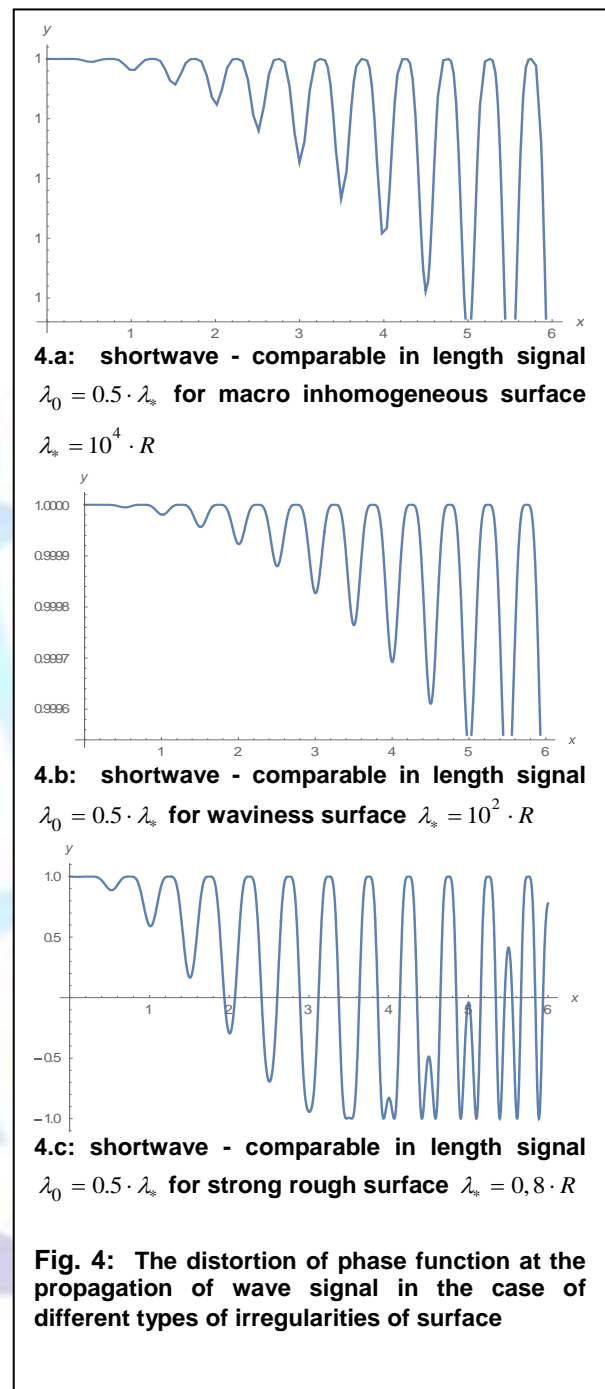
Note, that for macro roughness  $\lambda_0/R \geq 1000$  or surface waviness  $\lambda_0/R \square 50 \div 1000$ , the weak inhomogeneity of the surface is governed by the ratio of the wavelength of the signal and the average pitch of the irregularities is  $p \square \lambda_0/\lambda_*$ , while in cases of roughness  $\lambda_0/R \leq 50$  or even greater and sawtoothness  $\lambda_0/R \leq 1$ , the weak heterogeneity is governed by the ratio of the signal wavelength and the maximal height of non-smoothness is  $q \square \lambda_0/R$ . It turns out that in cases of weakly inhomogeneous macro roughness or waviness of the surface (thick waveguide) in the near-surface zone the elastic shear does not differ qualitatively, and the quantitative relative difference is of order  $10^{-12} \div 10^{-8}$  (Fig. 2a).

In the case of rough irregularity of the surface (Fig. 2b), the elastic shear is strongly distorted along the depth of the half-space when  $\lambda_0/R \square 0,5 \div 2$ , and especially in the near-surface area  $-R \leq y \leq h(x)$  significant quantitative differences between solutions (11) and (26) arise.

Distortions of the phase functions from non-smooth surface along of the signal wavelength  $\lambda_0 = 0,5 \cdot \lambda_*$  is plotted in Fig. 4 for different types of surface inhomogeneities.

In the case of macro inhomogeneous surface, the phase distortion can be neglected and is established along the propagation of the wave signal quite slowly (Fig. 4.a). From comparison of the distortion graphs (Fig. 4a, 4b, 4c) it is seen that the waviness and the roughness are more sensitive for high frequency wave signals. It is also seen that in all cases the distortion of the phase functions starts immediately after the signal passage of the first ledge of the non-smooth surface.

The analysis of possible internal resonance is also carried out numerically in particular case of corrugated (sinusoidal) surface of the half-space  $h(x) = (R/2) \cdot \left[ \sin(2\pi x/\lambda_0) - 1 \right]$ . From condition of coincidence of minimal frequency of multiple induced harmonics  $\omega_{n*}(x) = \pm n\pi c_t \cdot \left[ (h(x) + R) \right]^{-1} \cdot [h'(x)]^{-2}$  with the main frequency distortion of the amplitude  $\omega(x) = \omega_0 \left[ 1 + [h'(x)]^2 \right]^{-1/2} \leq \omega_0$ , we get the resonant wavelength of the signal.



For macro inhomogeneity when  $\lambda_*/R_{\max} \ll 10^4$  and for waviness when  $\lambda_*/R_{\max} \ll 10^2$ , in half-space internal resonance can arise when ultrashort wave signals propagate correspondingly for  $\lambda_0 \ll 10^{-11} \lambda_*$  and  $\lambda_0 \ll 10^{-5} \lambda_*$ . If the surface is rough  $\lambda_*/R_{\max} \ll 10$  or sawtooth  $\lambda_*/R_{\max} \ll 1.0$ , then the internal resonance occurs for wavelengths  $\lambda_0 \ll 10^{-2} \lambda_*$  and  $\lambda_0 \ll 3,6 \cdot \lambda_*$  (Fig. 5.a–5.d), correspondingly, commensurable with the sizes of the non-smoothness.

## 7 CONCLUSION

The use of virtual sections and the introduction of hypothesis MELS unlike direct solution allows to identify near-surface wave effects of surface inhomogeneities and obtain analytical solution in the near-surface areas. Surface heterogeneities of homogeneous elastic waveguide convert flat wave signal to a non-attenuated oscillatory motion into the waveguide and distort the flat front of the signal. The localization of horizontally polarized shear (SH) shortwave (high frequency) signals occurs in the near-surface areas of variable thickness virtually cut layer.

Analytical and numerical comparative analysis show that independent from heterogeneity types in the case of weakly inhomogeneity of the waveguide surface, solutions fully coincide. At comparable lengths of the wave signal with the characteristic dimensions of the non-smooth surface of the waveguide due to the presence of surface heterogeneities in the near-surface layer occur oscillations which in cases of certain wave signal lengths can cause internal resonance.

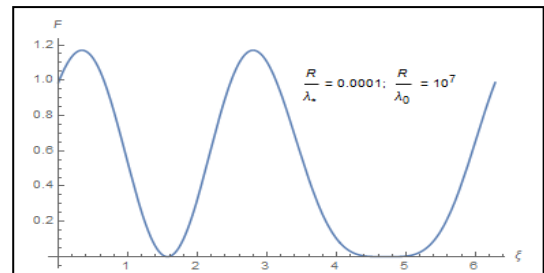


Fig. 5.1: resonant length  $\lambda_0 \ll 10^{-11} \lambda_*$

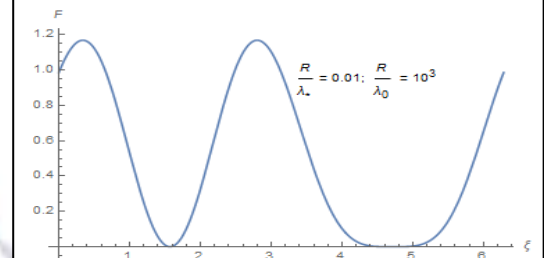


Fig. 5.2: resonant length  $\lambda_0 \ll 10^{-5} \lambda_*$

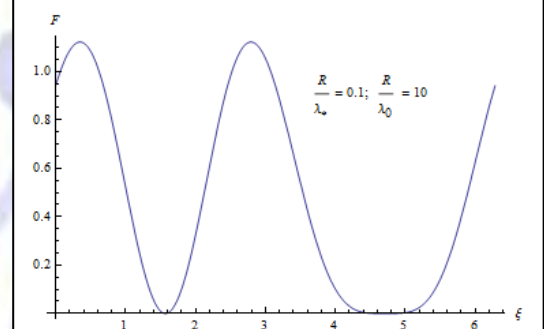


Fig. 5.3: resonant length  $\lambda_0 \ll 10^{-2} \lambda_*$

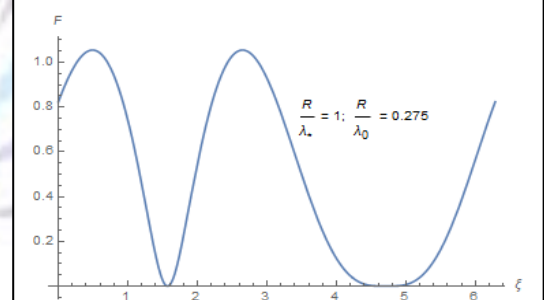


Fig. 5.4: resonant length  $\lambda_0 \ll 3,6 \cdot \lambda_*$

## REFERENCES

- [1] Biryukov, S.V., Gulyaev, Y.V., Krylov, V., Plessky, V., Surface acoustic waves in inhomogeneous media, Springer Series on Wave Phenomena, Vol. 20, 1995, 388
- [2] L Brekhovskikh, Waves in Layered Media 2e, Applied mathematics and mechanics, Vol. 16, Elsevier Science, 2012, 520
- [3] D. Royer, E. Dieulesaint, Elastic Waves in Solids I: Free and Guided Propagation, Springer Science & Business Media, 2000, 374



- [4] Flannery CM, von Kiedrowski H., Effects of surface roughness on surface acoustic wave propagation in semiconductor materials, *Ultrasonics*. 2002 Vol. 40 (Issues 1-8): 83-87.
- [5] D. H. Berman, J. Acoust., Computations of rough surface reflection coefficients in a waveguide *Soc. Am.* Vol. 96, 2834-2843 (1996).
- [6] Carbone G, Scaraggi M, Tartaglino U., Adhesive contact of rough surfaces: comparison between numerical calculations and analytical theories., *Eur Phys J E Soft Matter.* 2009, Vol.30(Issues 1), 65-74
- [7] Svetovoy V.B., Palasantzas G, Influence of surface roughness on dispersion forces, *Adv Colloid Interface Sci.*, 2015; 216 C:1-19.,
- [8] Flannery CM, von Kiedrowski H., Effects of surface roughness on surface acoustic wave propagation in semiconductor materials, *Ultrasonics*. 2002 Vol. 40 (Issues 1-8): 83-87.
- [9] S. S. Singh, Love Wave at a Layer Medium Bounded by Irregular Boundary Surfaces, *Journal of Vibration and Control*, 2010; Vol.16., 1177
- [10] Avetisyan A.S., The boundary problem modelling of rough surfaces continuous media with coupled physico mechanical fields, *Reports of NAS of Armenia*, (2015), vol. 115, №2, pp. 119-131.
- [11] Avetisyan A.S., On the formulation of the electro-elasticity theory boundary value problems for electro-magneto-elastic composites with interface roughness, *Proc. of NAS of Armenia, Mechanics*, (2015), vol. 68, №2, pp. 29-42.
- [12] Catherine Potel, Michel Bruneau, Ludovic C. Foze N'Djomo, Damien Leduc, Mounsi Echcherif Elkettani, Jean-Louis Izbicki, Shear horizontal acoustic waves propagating along two isotropic solid plates bonded with a non-dissipative adhesive layer: Effects of the rough interfaces, *J. App. Phys.*, vol 118, (2015), pp. 118-134
- [13] Bakirtas I. et Maugin G. A., Ondes de surface SH pures en elasticite inhomogene, *Journal de Mechanique Theoretique et Appliquee*, v. 1, № 6, 1982, p. 995–1013.
- [14] Avetisyan A.S., On propagation of electroelastic monochromatic wave in inhomogeneous piezoelectric layer, *Proc. of NAS of Armenia, Mechanics*, vol. 41, №5, 1988, pp. 34-40. (In russian)
- [15] Banerjee S. and Kundu T., "Elastic wave propagation in sinusoidally corrugated waveguides", *J. Acoust. Soc. Am.* 119(4), 2006–2017 (2006).
- [16] D. Qing-tian and Y. Zhi-chun, "Propagation of guided waves in bonded composite structures with tapered adhesive layer," *Appl. Math. Modell.* 35, 5369–5381 (2011).
- [17] T. Krasnova, P.-A. Jansson, and A. Boström, "Ultrasonic wave propagation in an anisotropic cladding with a wavy interface," *Wave Motion*, vol. 41, 163–177 (2005).
- [18] Catherine Potel, Michel Bruneau, Ludovic C. Foze N'Djomo, Damien Leduc, Mounsi Echcherif Elkettani, and Jean-Louis Izbicki, Shear horizontal acoustic waves propagating along two isotropic solid plates bonded with a non-dissipative adhesive layer: Effects of the rough interfaces, *Journal of Applied Physics*, vol. 118, 224904 (2015)
- [19] T. Valier-Brasier, C. Potel, and M. Bruneau, "Modes coupling of shear acoustic waves polarized along a one-dimensional corrugation on the surfaces of an isotropic solid plate," *Appl. Phys. Lett.* 93, 164101 (2008).
- [20] T. Valier-Brasier, C. Potel, and M. Bruneau, "Shear acoustic waves polarized along the ridged surface of an isotropic solid plate: Mode coupling effects due to the shape profile," *J. Appl. Phys.* 108(7), 074910 (2010).
- [21] M. Castaings, "SH ultrasonic guided waves for the evaluation of interfacial adhesion," *Ultrasonics* 54, 1760–1775 (2014)
- [22] A. R. McGurn and A. A. Maradudin, "Speckle correlations in the light scattered by a dielectric film with a rough surface: Guided wave effects," *Phys. Rev.* B58(8), 5022–5031 (1998).
- [23] W. A. Kuperman and H. Schmidt, "Rough surface elastic wave scattering in a horizontally stratified ocean," *J. Acoust. Soc. Am.* 79(6), 1767–1777 (1986).
- [24] V. E. Ostashev, D. K. Wilson, and S. N. Vecherin, "Waveguide sound propagation in a turbulent atmosphere above randomly rough ground," *Proc. Meet. Acoust.* 14, 045004 (2012).
- [25] V. Vlasie, S. de Barros, M. Rousseau, L. Champaney, H. Duflo, and B. Morvan, Mechanical and acoustical study of a structural bond: Comparison theory/numerical simulations/experiment," *Eur. J. Mech. A/ Solids* 25, 464–482 (2006).
- [26] J. Koreck, C. Valle, J. Qu, and L. J. Jacob, "Computational characterization of adhesive layer properties using guided waves in bonded plates," *J. Nondestruct. Eval.* 26, 97–105 (2007).





### Author' biography with Photo



Ara Sergey Avetisyan

Professor of Mathematics and Physics – from 2010,

Corresponding Member of National Academy of Sciences of Armenia – from 2006,

Chief Editor of Proceedings of NAS of Armenia, ser. Mechanics – from 2005,

Doctor of Mathematical and Physical Sciences – from 1998.

