Modeling of Temperature Measurement Using Newton's Law of Cooling

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Abstract

The room temperature of Physics Department University of Port Harcourt at 7am was taken and the body temperature of the researcher was also taken at that time. Temperature of the body was taken at regular intervals and compared with that obtained using Newton's law of cooling with given initial conditions. Comparison showed that it fits well with minimal approximation.

Keywords

Newton's Law of Cooling, Modeling, Measurement, Environment

Introduction

Modeling generally is the representation or abstraction of an actual object or situation. It shows the relationship and the interrelationship of action and reaction in terms of cause and effect. Mathematical or symbolic modeling is therefore, representing a reality into a set of axioms that best interprets such a reality (Buck and Buck 1976). The passage or steps before arriving at a suitable mathematical model is not too easy and in many cases represent major scientific advances. Temperature measurement and its interpretation is extensively used by scientist to determine; if a patient is in stable condition or not, temperature range for certain experiments to be carried out, effective practice of agriculture and many more. The use of temperature measurement for scientific investigation cannot be over emphasized. Thermometers measure the temperature of a system by making use of the change in some physical properties with temperature. Some of these physical properties are ; the change in volume of a liquid, the change in length of a solid, the change in pressure of a gas at constant volume and the change in volume of a gas at constant pressure. Others are the change in electric resistance of a conductor and the change in colour of a very hot body (Serway 1986). A temperature scale can be established for a given substance using one of these physical quantities. The most popular of all the thermometers is the one that consist of a glass bulb connected to a glass capillary tube. The glass tube is filled with a volume of mercury that expands into the capillary tube when heated. A mercury thermometer cannot be used below the freezing point of mercury. A universal thermometer whose readings are independent of the substance used is needed, although the gas thermometer meets this requirement, its control for validating the measurement is of utmost necessity because wrong use or measurement of temperature and its interpretation can lead to colossal damage or destruction. The need to validate or control the measurement of temperature stem from the fact that very many thermometers constructed or invented has various degrees of limitations. The limitations are primarily the range of temperatures it can measure and its sensitivity. The inability of the existing thermometers to measure higher temperatures necessitated Isaac Newton to perform an experiment in which he established a practical scale that can measure temperature of bodies with higher melting point which was reported by Ruffner (1963). In the work of Bohren (1991), Newton states that the excess of degree of heat were in geometric progression when the time are in an arithmetical progression. The principle is now known as Newton's law of cooling. This law of cooling leads to classical equation of exponential decline over time, which can be applied to many phenomena in science and engineering, such as determining the time it takes a capacitor to discharge and in radioactive decay. Hurley (1974), Boyce and Diprima (1992), Jaffe (1989) and Smith (1978) used the Newton's law of cooling to determine the time of death in the investigation of a homicide or accidental death. The aim of this work is to use Newton's law of cooling to verify temperature of a human body (palm) measured in the laboratory with mercury thermometer. This in our view will guide us in minimizing errors arising from the use of temperature measuring instruments.

Materials and Methods

Materials

Mercury thermometer

Stop watch

Human body (palm)

Methods

The mercury thermometer was used to measure the temperature of the environment inside the electronics laboratory of University of Port Harcourt, Choba, Rivers State and showed to be $30.95^0 C$ at about 7:00am, that of the palm was also recorded as $35.4^0 C$ at the same time. The procedure is repeated five times at an interval of two hours and the measurements recorded.

The relations

$$T_k = (T + 273.15)K \quad (1)$$

$$T_F = (1.8T + 32)^0 F \quad (2)$$

and

$$T_R = (1.8T_k)^0 R \quad (3)$$

respectively, are the Kelvin (K), degree Fahrenheit ($^0 F$) and degree Rankine ($^0 R$) equivalents.

Table 1: Readings of Temperature of human palm using Thermometer

Average room temperature $[30.95^0 C, 304.1K, 87.71^0 F, 547.38^0 R]$					
S/N	Time(hrs)	Measurement using mercury thermometer			
		$^0 C$	K	$^0 F$	$^0 R$
1	0:00	33.40	306.55	92.12	551.79
2	2:00	34.70	307.85	94.46	554.13
3	4:00	35.00	308.15	95.00	554.67
4	6:00	35.40	308.55	95.72	555.39
5	8:00	35.50	308.65	95.90	555.57
6	10:00	35.80	308.95	96.44	556.11

Validation of the measurement using Newton's law of cooling

Newton's law of cooling states that the surface temperature of an object changes at a rate proportional to the difference between the temperature of the object and that of the surrounding environment (Math24.net). It's translation into mathematical form is

$$\frac{dT}{dt} = -k(T - T_e) \quad (4)$$

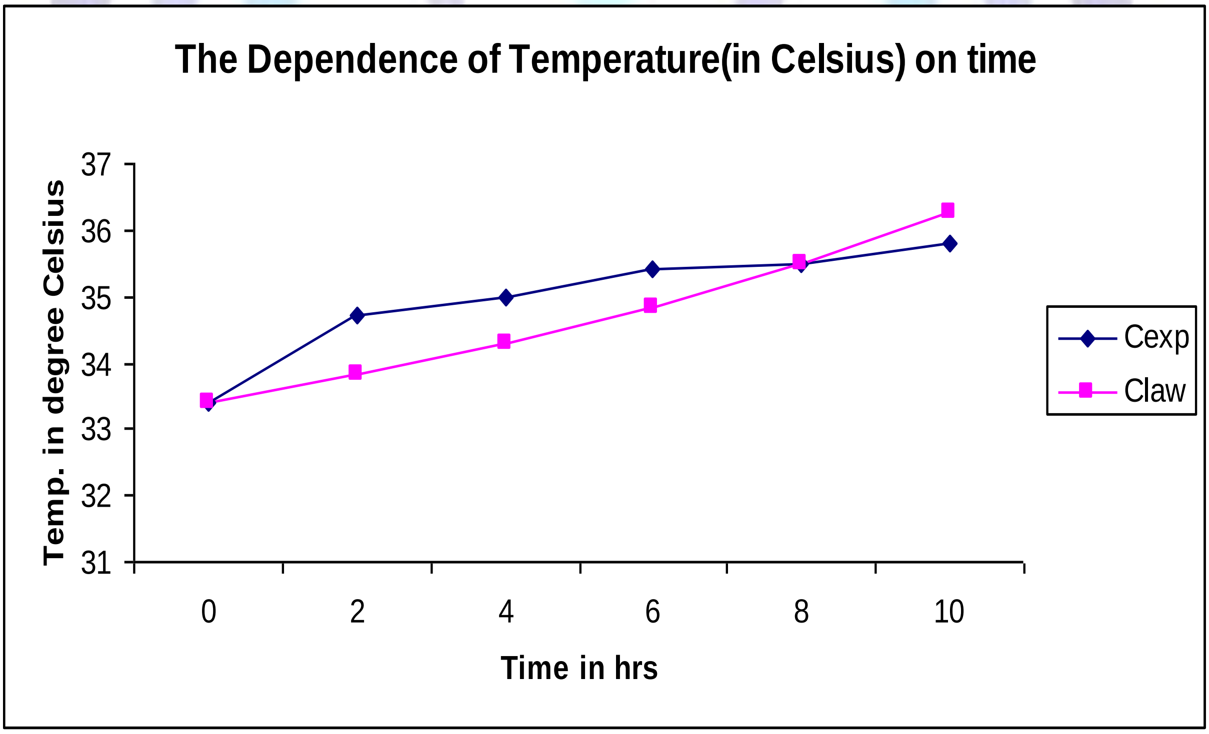
where T is temperature of the object at time t, T_e is temperature of the environment and k is constant.

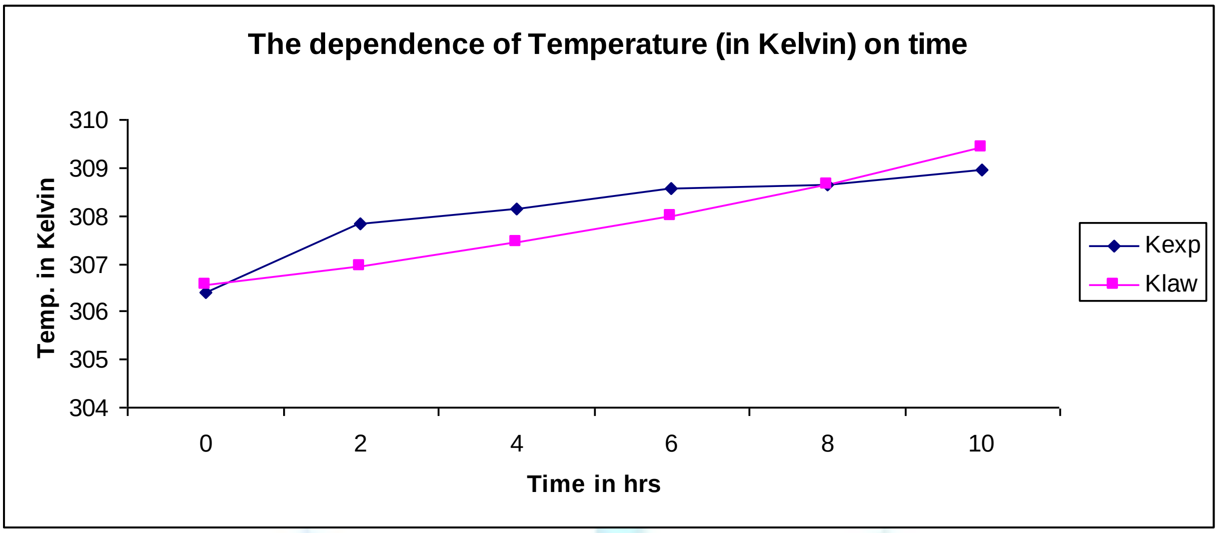
Solving equation (4) and imposing the boundary conditions $T(0) = 33.4$ and $T(8) = 35.5$, we get

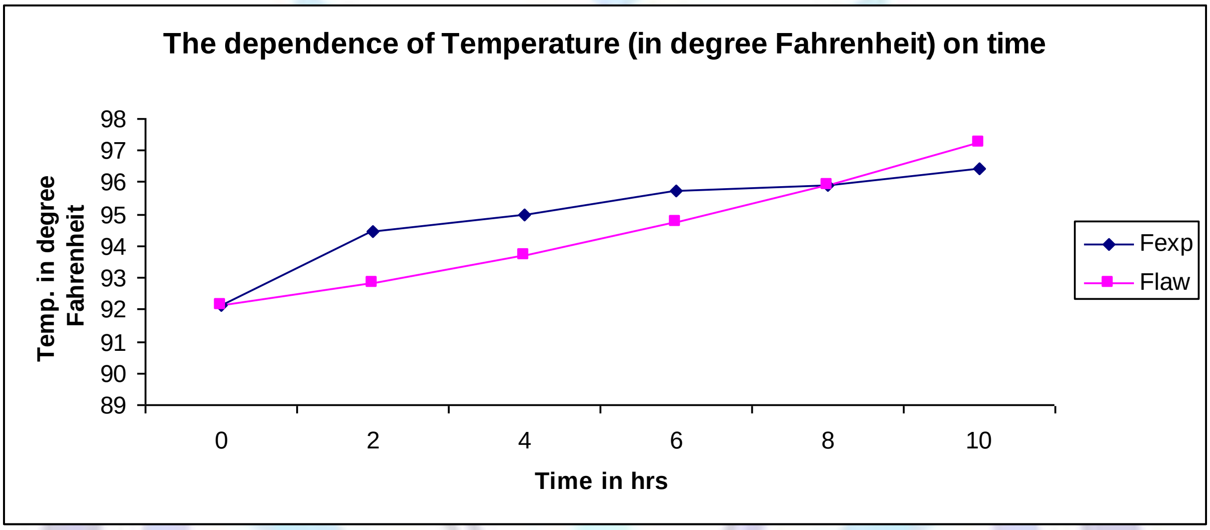
$$T(t) = 30.95 + 2.45e^{0.0773799t} \qquad (5)$$

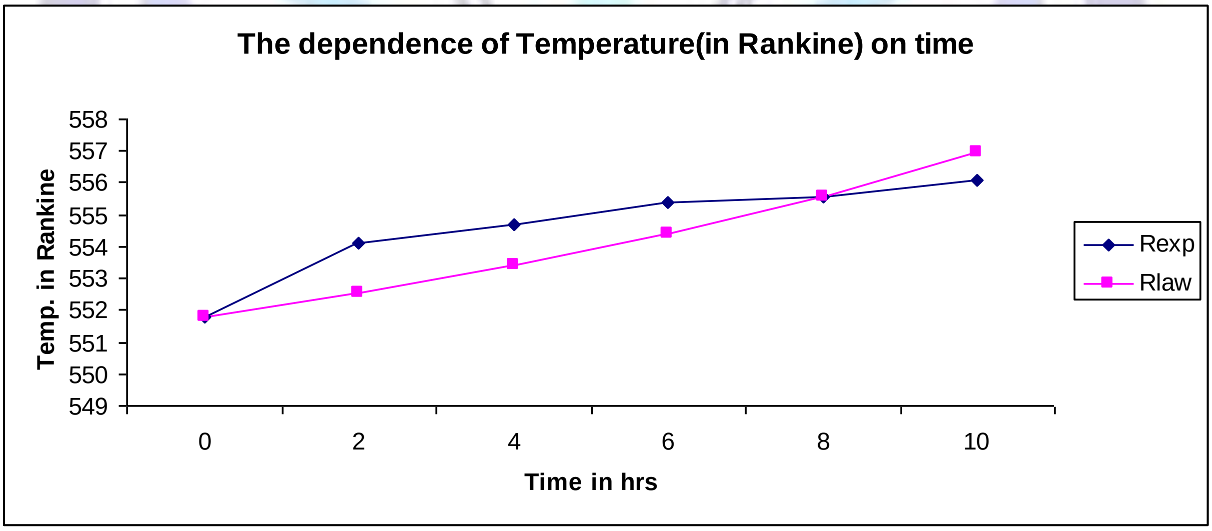
Table 2: Readings of Temperature of human palm using Newton's law of cooling

Average room temperature $[30.95^0 C, 304.1K, 87.71^0 F, 547.38^0 R]$					
S/N	Time(hrs)	Modeling using Newton's law of cooling			
		^0C	K	^0F	^0R
1	0:00	33.40	306.55	92.12	551.79
2	2:00	33.81	306.96	92.86	552.53
3	4:00	34.29	307.44	93.72	553.39
4	6:00	34.85	308.00	94.73	554.40
5	8:00	35.50	308.65	95.90	555.57
6	10:00	36.26	309.41	97.27	556.94









Discussion

From the readings of the thermometer and that gotten from the Newton's law of cooling, it is clear that provided the temperature of the environment is known and that of the object to be measured is known at a given time, the temperature of the body and future temperatures can be determined using the model. As shown in the graphs and tables, the Newton's law of cooling to a minimal approximation fits well in the determination of temperature of a body provided the temperature is not fluctuating within a given period.

Conclusion

A universal temperature measuring thermometer whose readings are independent of the substance used is available but its control for validating the measurement necessitated the study however, the model can only tackle increasing or decreasing temperature over time but may be difficult to accommodate fluctuating temperature within a given time.

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