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A New Model for a Non-Linear Electromagnetic Model with Self-Interacting Photons

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ABSTRACT

A new electromagnetism is still expected to be developed. Thus, under such a principle where nature works as a group, an electromagnetism beyond Maxwell is studied. It considers that light metric antecedes electric charge for founding the EM phenomena. Based on wholeness principle, Lorentz group and gauge invariance this electromagnetism sticked on light is proposed. Then, by electromagnetism it will be understood the physics derived from Lorentz group potential fields family (for simplicity other fields are not included). New electromagnetic fields, sectors, layers and regimes are developed. It yields a branch with transversal and longitudinal EM fields, granular and collective sectors, I -fields layers and four regimes (photonic, massive, neutral, charged) connected through a global photon. Their relationships are determined by a state equation identified as Global Maxwell equation. It contains new Gauss and Ampere laws, exclusive Faraday law for polarization and magnetization vectors, divergenceless magnetization vector having fields as sources, self-interacting photons mediated by a dimensionless coupling constant and other features. It is complemented by a Global Lorentz equation which besides the usual Lorentz force add forces depending on mass and on potential fields.

KEYWORDS

Light Invariance; Wholeness Principle; Global Maxwell; Global Lorentz; Photonic.

1 INTRODUCTION

Different principles have been building up physics. Inertia, minimal action, wave-particle, gauge symmetry, grandunification, supersymmetry, strings and others have set out different directions and comprehension. However, none of them is comparable with light invariance. It is out of sense: creates an absolute for nature and makes the speed of light transfinite.

Light invariance path is a challenge. In this work we shall explore a further aspect. We consider that its understanding is not complete and that there is still room for a new achievement on its track. After being developed through Maxwell equation [1], Relativity [2], Lorentz Group [3] we will investigate that there is a next step forward which will be called as Lorentz fields family. It identifies a kind of fields specie under light invariance. An interpretation where a given fields set under a common Lorentz group irreducible representation is considered as a primitive niche.

The objective here is to achieve such next comprehension coming from light invariance. Our assumption for that is that given a $(\frac{1}{2},\frac{1}{2})$ Lorentz group representation, one should associate to it a fields set $\{A_{\mu I}\}$ primordially than to just one field A_{μ}. Yang-Mills theory makes an initial example of a model where fields work as a group $\{A_{\mu}{}^{a}\}$. It considers vector fields working as a group in the adjoint representation by showing an a-fields association with a same coupling constant and with a diversity given through the group structure functions [4].

There is a fourth interpretation from light invariance relaying on a fields association physics. In order to support this thesis, our *ad hoc* is the wholeness principle [5]. Firstly, we would observe that it is in the nature mechanism. Our argument is that nature always moves in the wholeness direction. It is performed through conglomerates as particles, atoms, cells, bodies, ... galaxies. Then, by following this arrow of time the structure of this work will be to redefine the field theory understanding by mixing light invariance with wholeness.

Nevertheless, in order to propose this path one should first meditate more deeply on the presence of this wholeness principle. There are different reasons to support this principle in nature, as evolution theory, but we will choose to advocate first through quark confinement. Quark confinement brings an enigma for physics. Although various features were traced, such as color symmetry, jets, structure functions, flavors, asymptotic freedom, quarks were not observed [6]. Over the past 40 years various attempts have been made for proving confinement. Perhaps, one could summarize that there are no convincing results [7]. So we consider that similarly to light invariance, confinement is not something to be proved, but interpreted. It opens up a new cognitive logic. Our viewpoint is that confinement must be considered as a turning point of a reductionist sequence. It introduces the concept where there is a whole meaning for describing phenomena.

Our approach is that, instead of further looking for mathematical proof, the question may be raised how to interpret confinement. We consider it as a breakthrough in the reduction methodology that has been guiding physics during the last 2500 years. It states the reductionism fall. There is a new framework for physical interpretations. Rather than through the ultimate constituents of matter [8], perhaps physics should be calculated from another basic postulate: the notion of totality, or as we will call it, wholeness. Thus, a confinement interpretation is that the new abstraction is the whole. It says that being suppressed from the ultimate constituents physics should be understood under groupings. A view which move us from the ultimate constituents comprehension to a new origin based on the notion of fields grouping.

Complexity is another field that antagonizes the reductionism performance [9]. Complex system is considered as that one whose properties are not fully explained by an understanding of its parts. Its basic properties like correlation, cooperation, self-organization, order do not have a reductionist nature. So complexity joins confinement in order to say that physical laws should have antireductionist character. These two subjects are calling for a whole physics. The parts can not be seen isolately says confinement, the relationships between the parts are complex says complexity. Consequently, by stipulating the whole as the fundamental unity, instead of considering atomic Lagrangians being defined in terms of ultimate constituents of matter (as quarks and leptons), one should look for a type of Lagrangian which involves a diversity of fields organized through an integral model. We will identify them as 'whole Lagrangians', i.e. Lagrangians classified by the principle of wholeness.

Thus confinement and complexity lead us to identify the meaning of wholeness in physics. Another possible illustration of the idea of wholeness is through Quantum Mechanics [10]. The indeterminism and the probabilistic nature of quantum-mechanical measurements are implemented through the introduction of ensembles: a system is probed by means of an ensemble that reproduces a number of copies of the system itself. Results of measurements are statistical and observables are measured through expectation values, which means repetition of the measurement on a large number of copies of the system under observation. In short, Quantum Mechanics does not deal with an isolated system; it rather treats a collection of systems. The cosmology of universe expansion also works as another argument for the whole conception [11].

Nevertheless we have to take care of the meaning of grouping. Based on symmetry principle, physics can not treat every particle alone. By grouping may be interpreted: phenomenologically, as fields under common quantum numbers as Gell-Mann Eightfold Way [12] or as crystalline arrays in condensed matter [13]; theoretically, as sharing a multiplet [14] or a same Lorentz irreducible representation [15]. However, the question is how far these different arrangements consider that nature works as a group or not. Physically, the importance is: are they producing a whole or an isolated parts dynamics? The answer comes through two natures of symmetry: reductionist and antireductionist. For this, first observe that grandunification symmetry, it reductionistically associates quarks and leptons by incorporating them under a same SU(5) multiplet [16]-[17]; second, consider leptonic families where the lepton and its correspondent neutrino are associated as $\{L,\nu_L\}$, and given neutrino oscillations, it yields a complete interlaced leptonic family where every lepton become associated to each other, and so, the resulting grouping should be related through an antireductionist symmetry. Therefore, although multiplets and Lorentz group irreducible representations are both structures candidate to relate parts, at moment of doing physics, it will be important to define on which approach (reductionist/antireductionist) our model will be driven.

Our view is to assume that nature is a collective construction. We observe that there is an inevitable context introducing the meaning of wholeness in nature. Cosmology, biology, confinement, complexity, quantum mechanics, statistical mechanics, symmetry, unification are supporting that nature works as group. Under this concert of facts instead of searching only to the ultimate constituents physics one should also consider on the whole physics. An antireductionist physics is required. The challenge moves on how to interpret this antireductionist approach as beginning. Be able to define what is a fundamental grouping. Then, for supporting this line of thought, that nature acts as group, we have to do two enlargements. First, introduce a fourth light invariance interpretation saying that first than be considered through multiplets such fields primitive set should be encountered at each Lorentz group irreducible representation [18]. Second, modify the symmetry default between grouping and gauge theories which states that the number of potential fields must be equal to the number of group generators [19].

In this way, under this new cognitive logic, a next step shall be to introduce the 'whole question' through gauge symmetry. We should formulate on fields associations through gauge parameter. There is a whole physics to be tuned from light invariance and gauge symmetry stating that instead of ultimate constituents physics should be based on fields agglutinations. Searching for such a non-reductionist approach through a gauge model a possibility is to consider an initial set of fields transforming under a common gauge group as

$$A_{\mu I}(x) \rightarrow A'_{\mu I}(x) = U\,A_{\mu I}\,U^{-1} + \frac{i}{g}\partial_{\mu}U.U^{-1} \tag{1}$$

where $I = 1,...,N$ [20]. Eq. (1) is considered as a linear antireductionistic gauge symmetry. There are another antireductionist approaches based on polynomial and systemic gauge transformations [21]. These different possibilities are ways for associating different I -potential fields through a common gauge parameter.

The hypothesis here is that a whole primordial notion will be configured from Eq. (1). There is a new approach for physics phenomena to be understood. Something that introduces the possibility for enfolded fields and particles. However for developing this assumption a first necessary clause is to prove the existence of Eq. (1). Different origins based on Kaluza-Klein, supersymmetry, fibre bundle, σ -model formulations have already been studied [22]-[29].

The approach for extending gauge-field theories through the association of two or more gauge potentials transforming under a single Abelian or simple Lie group may be consistently justified in a Kaluza-Klein scenario. Starting off from a higher-dimensional gravity-matter theory and fine-tuning parameters so as to guarantee a spontaneous compactification, it can be shown [22], [23] that, once one performs a reduction on a coset space, G/H, the effective 4-dimensional theory that comes out exhibits 2 gauge fields that transform under the gauge group G. One can geometrically justify the independence of the vector fields by means of an existing torsion in the higher-dimensional original model. Also, based on ideas of supersymmetry, it has been shown [24], [25], [26], [27], that, upon the relaxation of the conventional constraint in the algebra of simple supersymmetry covariant derivatives, gauge supermultiplets appear that are independent and carry different gauge fields amongst their components. These gauge fields transform under the action of a common simple group.

In the framework of a fibre bundle formulation for gauge models, such an extension to gauge theories has already been discussed [28] and a precise geometrical meaning has been assigned to the various vector potentials that transform under the same group. One finds out that there is room for a single connection on the principal fibre bundle. The role of such a connection is played by the combination of vector potentials that transform inhomogeneously under the action of the gauge group. The other independent linear combinations of the potentials that transform homogeneously are interpreted as vectors of a vector bundle defined on the basis Minkowski space. Also, models formulated in terms of complex scalars coupled to gauge potentials that transform according to Eq. (1) have been discussed in [29], and it has been shown that the dynamics of these gauge vectors may, under certain circumstances, dynamically induce the appearance of CP^n -like nonlinear σ -models.

Our thesis is that nature contains a fundamental principle which is to be manifested through wholeness. It moves by taking the set as fundamental unity. Our line of thought is to study these fields associations through Eq. (1). There are different reasons for supporting Eq. (1) mathematically, and so, the next challenge becomes whether it really provides the whole features. Despite the possibility of taking different potential fields in a same group receive differently supports coming from geometry, superspace and σ -models, the main aspect is to understand how far it is able to accommodate the expected whole meaning. For this, it will be necessary to deduce integral mechanisms from Eq. (1). There is a systemic bridge between the part an the whole to be understood. Then, our research is that the principle of wholeness should be configured through the manifestation of properties as set, diversity, integration derived from a Lagrangian.

Considering this context, a fourth light invariance interpretation is taken and a new perspective emerges for analyzing the physical phenomena. Recapitulating, we have assumed here that there are confinement, complexity and others topics in opposition to the reductionism category, and so, instead of the reductionist sense where particles are the essential building blocks, now the whole becomes the fundamental unity. Maxwell will be the first choice of study. This because it already carries an association between $\vec{E}-\vec{B}$ fields, a fact suggesting that one should explore on this systemic behaviour first through the $(\frac{1}{2},\frac{1}{2})$ Lorentz family. The meaning of enfolded particles is expected [30]. Therefore, given Eq. (1), one expects to develop an abelian whole gauge model where the corresponding physical entities, laws and numbers will be expressing a non-reductionist behavior. Its challenge is to perform a relationship between the simple and complex. Generate the passage from reductionist view to an antireductionist view through a Global Maxwell equation.

The paper organizes such fourth light invariance interpretation as follows. In section 2, the wholeness principle is introduced through an abelian global gauge model. Following the development of Eq. (1), one notices the appearance of antireductionistic equations supported by global variables and global coefficients. These features allow the model be analyzed in terms of the non-reductionist approach. Then, at next section a set determinism is studied. It shows the presence of directive and circumstance symmetries controlling these $2N+7$ antireductionistic equations. Following that, the expected Global Maxwell equation is obtained and written in terms of new observables and introducing new EM regimes. It yields a state equation carrying whole properties like fields set, directive, circumstance, network. From it, a physical interpretation for potential fields in terms of lines of force emerges at section 5. At section 6, conservation laws are studied with the presence of different sources than electric charge. At section 7, a Global Lorentz force is derived showing potential fields and masses acting as forces. These sections introduce the idea that, this fourth interpretation leads to a global photon not necessarily depending on electric charge.

A Global Maxwell equation is configured based on the principle of wholeness. It introduces an EM beyond electric charge, potential fields as physical agents, global photon and set determinism. The corresponding $(\frac{1}{2},\frac{1}{2})$ EM features are analyzed at section 8, a new dispersion relation is obtained at section 9 and a macroelectromagnetism is analyzed at section 10. Then, at section 11, this antireductionist gauge symmetry is correlated to the Lorentz symmetry determining the photon singularity. Concluding, one figures out that this fourth light invariance interpretation leads to a singular-active-global photon producing a network EM physics with four interlaced regimes (photonic, massive, neutral, electric charged). Light turns candidate for a new energy source.

2 ABELIAN GLOBAL GAUGE MODEL

Eq. (1) allows for the existence of a gauge model involving different potential fields. Considering the abelian case and that these fields satisfy the Borscher's theorem [31], one can redefine them. To get a better transparency on symmetry, one should write the model, $A_{\mu I}{}' = A_{\mu I} + \partial_{\mu}\alpha$, in terms of the $\{D, X_i\}$ basis, where D_{μ} is defined as $D_{\mu} = \sum_I A_{\mu I}$, with $D_{\mu} \rightarrow D_{\mu}' = D_{\mu} + \partial_{\mu}\alpha$, and where $X_{\mu i}$ are potential fields: $X_{\mu 1} = A_{\mu 1} - A_{\mu 2}$, …, $X_{\mu(N-1)} = A_{\mu 1} - A_{\mu N}$, which are obviously gauge invariant, $i = 2,\ldots,N$. Geometrically, the potential fields $X_{\mu}^{\ i}$ arise from the torsion tensor of the higher-dimensional manifold that spontaneously compactify to $M^4 \times B^k$, where M^4 is the Minkowski space-time and B^k some k-dimensional internal space. Thus the origin of the potential fields can be traced back to the vielbein, spin-connection and Yang-Mills fields of higher-dimensional gravity-matter coupled theory spontaneously compactified on an internal space with torsion.

Thus, $\{D, X_i\}$ is called the constructor basis due to the fact that, under this field-referential, the gauge invariance origin for the Lagrangian terms becomes more immediate. Notice that D_{μ} is a genuine gauge field, while $X_{\mu i}$ correspond to a type of 'dark vector fields', in the sense that they exist (generate quanta) but are not detected by a gauge transformation. Working out the lagrangian in $\{D, X_i\}$ basis, we get

$$L = Z_{\mu\nu}Z^{\mu\nu} + \eta\tilde{Z}_{\mu\nu}Z^{\mu\nu} - m_{ij}^2 X_{\mu}^{\ i} X^{\mu j} + L_{g.f}. \qquad (2)$$

where this candidate for an abelian antireductionist Lagrangian contains contributions from three sectors which are the antisymmetric, symmetric and semi-topological sectors [32]. Stability restricts Eq. (2) to a square shape [33].

There is a global field strength $Z_{\mu\nu}$, which can be written as $Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)}$, with

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X_{\mu\nu}^{\ i} + \gamma_{[ij]} X_{\mu}^{\ i} X_{\nu}^{\ j}, \qquad (3)$$

$$Z_{(\mu\nu)} = \beta_i \Sigma_{\mu\nu}{}^i + \rho_i g_{\mu\nu} \Sigma_\alpha{}^{\alpha i} + \gamma_{(ij)} X_\mu{}^i X_\nu{}^j + \tau_{(ij)} g_{\mu\nu} X_\alpha{}^i X^{\alpha j} \tag{4}$$

where the basic field strengths are

$$D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu, \quad X_{\mu\nu}{}^i = \partial_\mu X_\nu{}^i - \partial_\nu X_\mu{}^i, \quad \Sigma_{\mu\nu}{}^i = \partial_\mu X_\nu{}^i + \partial_\nu X_\mu{}^i, \tag{5}$$

Notice that the coefficients d, α_i, $\gamma_{[ij]}$, β_i, ρ_i, $\gamma_{(ij)}$, $\tau_{(ij)}$, m_{ij} which are originated from gauge and Lorentz invariances. They are identified as the free coefficients of theory because they can take any value without the involved symmetries being broken. The total number of free coefficients carried by Eq. (2) is $\frac{1}{4}(3N^4 - 8N^3 + 13N^2 - 12N + 8)$ as Appendix A shows. Besides that, one needs the gauge fixing term considering that there is only one gauge parameter which means only one gauge fixing term expressed as $L_{g.f} = \frac{1}{\xi}\left[\partial_\mu\left(D^\mu + p_i X^{\mu i}\right)\right]^2$ [34].

A fundamental property on such fields set physics is that one can work under different fields referential systems [35]. Although Borscher's theorem guarantees that physics must be independent under fields re-parametrizations, the $\{D, X_i\}$ basis is not the physical basis. For this, one has to diagonalize the transversal sector. Then, one gets that physical fields are those which diagonalize the equations of motion (the physical masses are the poles of two-point Green functions). It is in terms of these fields that the corresponding measurable entities of the model must be defined, as the corresponding electric and magnetic fields.

Thus in order to diagonalize the transverse sector we have to introduce a matrix Ω. The physical basis $\{G_I\}$ is defined as

$$D_\mu = \Omega_{1I} G_\mu{}^I, \qquad X_{\mu i} = \Omega_{iI} G_\mu{}^I \tag{6}$$

where the Ω matrix is a function of these free coefficients [36].

Writing down the gauge transformation in terms of physical fields, one gets

$$G_{\mu I}(x) \to G'_{\mu I}(x) = G_{\mu I}(x) + \Omega^{-1}_{I1} \partial_\mu \alpha(x) \tag{7}$$

where every field transformation is specified by a weight Ω^{-1}_{I1} factor.

Eq. (6) yields the following transverse diagonalized gauge invariant Lagrangian

$$L(G) = Z_{[\mu\nu]} Z^{[\mu\nu]} + Z_{(\mu\nu)} Z^{(\mu\nu)} + \eta Z_{\mu\nu} \tilde{Z}^{\mu\nu} - m^2_{II} G_\mu{}^I G^{\mu I} + \xi_{IJ}\left(\partial_\mu G^{\mu I}\right)\left(\partial_\nu G^{\nu J}\right) \tag{8}$$

where the corresponding field strengths can be written in terms of more fundamental gauge invariant terms

$$Z_{[\mu\nu]} = b_I G_{\mu\nu}{}^I + z_{[\mu\nu]}, {}_{(\mu\nu)} = \beta_I S_{\mu\nu}{}^I + \rho_I g_{\mu\nu} S_\alpha{}^{\alpha I} + z_{(\mu\nu)} + g_{\mu\nu} w_{(\alpha}{}^{\alpha)} \tag{9}$$

with

$$G_{\mu\nu}{}^I = \partial_\mu G_\nu{}^I - \partial_\nu G_\mu{}^I \;, {}_{\mu\nu}{}^I = \partial_\mu G_\nu{}^I + \partial_\nu G_\mu{}^I \;,$$

$$z_{[\mu\nu]} = \gamma_{[IJ]} G_\mu{}^I G_\nu{}^J \;, \quad z_{(\mu\nu)} = \gamma_{(IJ)} G_\mu{}^I G_\nu{}^J \;, \quad w_{(\mu\nu)} = \tau_{(IJ)} G_\mu{}^I G_\nu{}^J \;. \tag{10}$$

L is parametrized by global free coefficients expressed as

$$b_I = d\Omega_{1I} + \alpha_i \Omega^i{}_I, \quad \gamma_{[IJ]} = \gamma_{[ij]} \Omega^i{}_I \Omega^j{}_J, \quad \beta_I = \beta_i \Omega^i{}_I,$$

$$\rho_I = \rho_i \Omega^i{}_I, \quad \gamma_{(IJ)} = \gamma_{(ij)} \Omega^i{}_I \Omega^j{}_J, \quad \tau_{(IJ)} = \tau_{(ij)} \Omega^i{}_I \Omega^j{}_J \;. \tag{11}$$

The relevance here is that field equations appear parametrized through global coefficients and with global variables as Eqs. (7) and (11) are showing. The terminology global assigned to the coefficients turns out to be appropriate because although being associated to a given flavor index I, their expression shows a global mix between the original free coefficients defined at Eq. (2). They mean a first indication of the existence of the totality conjecture. Notice that their expressions are depending on the number of involved fields. For the one-gauge fixing term, one gets $\xi_{IJ} = \frac{1}{\xi}\left(\Omega^1{}_I\Omega^1{}_J + p_i\Omega^1{}_I\Omega^i{}_J + p_i\Omega^i{}_I\Omega^1{}_J + p_i\, p_j\Omega^i{}_I\Omega^j{}_J\right)$, which yields a global gauge fixing expression. Fields normalization condition is $b_I b_J = b\delta_{IJ}$.

The corresponding N-equations of motion for $G_{\mu I}$ fields are

$$b_I\partial_\mu Z^{[\mu\nu]} + \frac{1}{2}m_I^2 G^\nu{}_I = J^\nu{}_I(G) \tag{12}$$

where

$$J^\nu{}_I(G) = \gamma_{[IJ]}G_\mu{}^J Z^{[\mu\nu]} + \gamma_{(IJ)}G_\mu{}^J Z^{(\mu\nu)} + \tau_{(IJ)}G_\nu{}^J Z_{(\mu}{}^{\mu)} + \eta\gamma_{[IJ]}E^{\nu\mu\rho\sigma}G_\mu{}^J Z_{[\rho\sigma]}$$
$$-\beta_I\partial_\mu Z^{(\mu\nu)} - \rho_I\partial_\nu Z_{(\mu}{}^{\mu)} + \eta b_I E^{\nu\mu\rho\sigma}\partial_\mu z_{[\rho\sigma]} \tag{13}$$

which yields a system with N-coupled equations. It contains a complex system. As a new fact, dynamically it also relates an individual-collective dynamics [37].

The fundamental gauge invariance property for the whole abelian physical model is obtained from the Ω matrix invertibility condition

$$\Omega_{IK}\Omega^{-1}_{KJ} = \delta_{IJ}. \tag{14}$$

Notice that Eqs. (7) and (14) are enough for any proof on gauge invariance. They are able to show that field strengths $G^I_{\mu\nu}$, $\beta_I S^{\mu\nu I}$, $z_{[\mu\nu]}$, $z_{(\mu\nu)}$, $\omega_{(\mu\nu)}$ are gauge invariant and that the N-currents involved in Eq. (13) are neutral

A relevant result is on a gauge invariant mass term. Since 1938 with Stueckelberg [38], different efforts have been made in literature for introducing mass in physics. It is fundamental for London equation [39] and for the Landau-Ginzburg model of superconductivity [40]. Meanwhile, either through high derivatives [41]-[42]-[43], spontaneous symmetry breaking [44]-[45], technicolour [46]-[47], or dimensionalities different than four [48], they are imposing extra dependences, as Brout-Englert-Higgs fields [49]-[50]. The contribution here is that without imposing any ad hoc situations, Eqs. (7) and (14) derive a gauge invariant mass term $m^2_{II}G^I_\mu G^{\mu I}$ [51] and renormalizable [52], [53]. However it does not have on mind an explanation for the origin of the mass of particles [54]. It is just a mechanism for introducing a mass parameter in Lagrangian without violating gauge symmetry. Just for nomenclature, one identifies $m^2_{II} \equiv m^2_I$.

Another on-shell information can be derived from $\{D, X\}$-basis. Its corresponding equations can be related to the $\{G_I\}$-basis through the transformations

$$\Omega^{-1}_{I1}\left(\frac{\partial L}{\partial G_{\nu I}} - \partial_\mu\frac{\partial L}{\partial(\partial_\mu G_{\nu I})}\right) = 0, \qquad \Omega^{-1}_{Ii}\left(\frac{\partial L}{\partial G_{\nu I}} - \partial_\mu\frac{\partial L}{\partial(\partial_\mu G_{\nu I})}\right) = 0, \tag{15}$$

which produces the equations

$$\partial_\nu Z^{[\mu\nu]} + \eta\gamma_{[JK]}E^{\mu\nu\rho\sigma}\partial_\nu\left(G_\rho{}^J G_\sigma{}^K\right) = 0, \qquad \alpha_i\partial_\nu Z^{[\mu\nu]} + \frac{1}{2}m_i^2 G^\mu{}_i = J^\nu{}_i(G), \tag{16}$$

for $J^\nu{}_i(G) = \Omega^{-1}_{Ii}J^{\nu I}(G)$. A verification can be done by multiplying the first and second equation in (15) by d_I and Ω_{iI}, respectively. This says that Eqs. (12) and (16) are rotations of the same coupled equations derived from the Lagrangian (8).

The Poincaré lemma associated with the following covariant derivative $D_{\mu I} = b_I\partial_\mu + \gamma_{[IJ]}G_\mu{}^J$ gives

$$D_{\nu J}D_{\mu I}Z^{[\mu\nu]} = \left(b_J\gamma_{[IL]} - \gamma_{[JL]}b_I\right)\partial_\nu\left(G_\mu{}^L Z^{[\mu\nu]}\right) + \gamma_{[JL]}b_I\left(\partial_\nu G_\mu{}^L\right)Z^{[\mu\nu]}$$

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$$+\frac{1}{2}\left(\gamma_{[JK]}\gamma_{[IL]}-\gamma_{[JL]}\gamma_{[IK]}\right)G_{\nu}{}^{K}G_{\mu}{}^{L}Z^{[\mu\nu]} \tag{17}$$

where $\left[D_{\mu I}, D_{\nu I}\right]\neq 0$.

Field equations are complemented by identities. In principle, the presence of N-potential fields allows various combinations of identities as

$$[\nabla_{\mu}{}^{I},[\nabla_{\nu}{}^{J},\nabla_{\rho}{}^{K}]]+[\nabla_{\nu}{}^{J},[\nabla_{\rho}{}^{K},\nabla_{\mu}{}^{I}]]+[\nabla_{\rho}{}^{K},[\nabla_{\mu}{}^{I},\nabla_{\nu}{}^{J}]]=0 \tag{18}$$

where $\nabla_{\mu I}=\partial_{\mu}+g_{I}G_{\mu I}$. However, it yields that there are only N-independent gauge invariant expressions. They are the following N-Bianchi granular identities:

$$\partial_{\mu}G_{\nu\rho}{}^{I}+\partial_{\nu}G_{\rho\mu}{}^{I}+\partial_{\rho}G_{\mu\nu}{}^{I}=0. \tag{19}$$

There are also three more new collective identities in Bianchi style:

$$\partial_{\mu}z_{[\nu\rho]}+\partial_{\nu}z_{[\rho\mu]}+\partial_{\rho}z_{[\mu\nu]}=\gamma_{[IJ]}G_{\nu}{}^{I}G_{\mu\rho}{}^{J}+\gamma_{[IJ]}G_{\rho}{}^{I}G_{\nu\mu}{}^{J}+\gamma_{[IJ]}G_{\mu}{}^{I}G_{\rho\nu}{}^{J}, \tag{20}$$

$$\partial_{\mu}z_{(\nu\rho)}+\partial_{\nu}z_{(\rho\mu)}+\partial_{\rho}z_{(\mu\nu)}=\gamma_{(IJ)}G_{\mu}{}^{I}S_{\nu\rho}{}^{J}+\gamma_{(IJ)}G_{\nu}{}^{I}S_{\rho\mu}{}^{J}+\gamma_{(IJ)}G_{\rho}{}^{I}S_{\mu\nu}{}^{J}, \tag{21}$$

$$\partial_{\mu}w_{(\nu}{}^{\nu)}+2\partial_{\nu}w_{(\mu}{}^{\nu)}=\tau_{(IJ)}G_{\mu}{}^{I}S_{\nu}{}^{\nu J}+2\tau_{(IJ)}G_{\nu}{}^{I}S_{\mu}{}^{\nu J} \tag{22}$$

and a kinetic identity:

$$\partial_{\nu}G^{\nu\mu I}=\partial_{\nu}S^{\nu\mu I}-\eta^{\nu\mu}\partial_{\nu}S_{\alpha}{}^{\alpha I}. \tag{23}$$

The last identity is the local Noether theorem. It provides the following useful three relationships:

$$\partial_{\mu}J_{N}^{\mu}=0, \tag{24}$$

$$\Omega_{I1}\partial_{\nu}\frac{\partial L}{\partial(\partial_{\nu}G_{\mu I})}+J_{N}^{\mu}=0, \tag{25}$$

$$\Omega_{I1}\frac{\partial L}{\partial(\partial_{\mu}G_{\nu I})}\partial_{\mu}\partial_{\nu}\alpha(x)=0 \tag{26}$$

where J_{N}^{μ} for the model being studied is defined without any matter be included. Substituting (8) in (25), one gets $\partial_{\mu}Z^{[\mu\nu]}+\eta\gamma_{[IJ]}E^{\mu\nu\rho\sigma}\partial_{\mu}\left(G_{\rho}{}^{I}G_{\sigma}{}^{J}\right)=0$, which is the same expression as Eq. (16). It says that from the Noether information one detects that a U(1) group is also able to generate a current which carries its own gauge fields. Another Noether information is that Eq. (26) contains two solutions: $\Omega_{I1}\frac{\partial L}{\partial(\partial_{\mu}G_{\nu}{}^{I})}=0$, $\frac{\partial L}{\partial(\partial_{\mu}G_{\nu}{}^{I})}=G^{\mu\nu}{}_{I}$, which yields the Gauss law $\Omega_{I1}\partial_{\nu}G^{\mu\nu I}=-J_{N}^{\mu}$ [55].

A classical structure is defined. There is a total of $2N+7$ equations. They are the N-equations of motion associated to each field $G_{\mu I}$; N-Bianchi identities where each one is associated to a given field $G_{\mu I}$; three new Bianchi identities relating collective observables; one kinetic identity relating antisymmetric and symmetric sectors; and finally, there is the Noether theorem which adds three equations more. Thus one obtains a first clue that a non-reductionist dynamics is inserted in Eq. (1). There is an integral logic written through: coupled field equations with global parameters and variables defined in terms of a set; Bianchi identities involving collective fields as Eq. (20-22); Noether and gauge fixing expressions relating N-fields. Only through Eq. (19) is that the reductionist aspect is preserved. Consequently by putting together N -potential fields rotating under a same group a non-reductionistic dynamics is obtained. It yields a set determinism which will be studied at next section.

A next step is on analyzing the spectroscopy of the model. Based on arguments of gauge invariance, it seems

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more reasonable to start from a Lagrangian density built up in terms of antisymmetric, symmetric and semitopological sectors. It rewrites Eq. (8) in the following sectors:

$$\mathcal{L}(G) = \mathcal{L}_A + \mathcal{L}_S + \mathcal{L}_{st} \tag{27}$$

where

$$L_A = b_I b_J G_{\mu\nu}{}^I G^{\mu\nu J} + 2b_I \gamma_{[JK]} G_{\mu\nu}{}^I G^{\mu J} G^{\nu K} + \gamma_{[IJ]}\gamma_{[KL]} G_\mu{}^I G_\nu{}^J G^{\mu K} G^{\nu L}, \tag{28}$$

$$\begin{aligned} L_S &= (\beta_I \beta_J) S_{\mu\nu}{}^I S^{\mu\nu J} + (2\beta_I \rho_J + 4\rho_I \rho_J) S_\mu{}^{\mu I} S_\nu{}^{\nu J} + (2\beta_I \gamma_{(JK)}) S_{\mu\nu}{}^I G^{\mu J} G^{\nu K} \\ &+ (2\beta_I \tau_{(JK)} + 2\rho_I \gamma_{(JK)} + 8\rho_I \tau_{(JK)}) S_\mu{}^{\mu I} G_\nu{}^J G^{\nu K} \\ &+ (\gamma_{(IK)}\gamma_{(JL)} + 2\gamma_{(IJ)}\tau_{(KL)} + 4\tau_{(IJ)}\tau_{(KL)}) G_\mu{}^I G^{\mu J} G_\nu{}^K G^{\nu L} \end{aligned} \tag{29}$$

and

$$L_{st} = \eta E_{\mu\nu\rho\sigma}\left(2b_I \gamma_{[JK]} G^{\mu\nu I} G^{\rho J} G^{\sigma K} + \gamma_{[IJ]}\gamma_{[KL]} G^{\mu I} G^{\nu J} G^{\rho K} G^{\sigma L}\right) \tag{30}$$

which are showing a type of non-abelian-flavour model inside an abelian gauge model. However they mix spin-1 and spin-0. The former yield a $(1,0)\oplus(0,1)$ representation of the Lorentz group and carry pure spin-1; but the symmetric sector and the latter are mixing vectors and scalars.

Thus for reading off the involved particles and corresponding physics one should rewrite the Lagrangian in terms of the transverse and longitudinal projectors on the space of vector fields. For separating the spins initially organized by the symmetric and antisymmetric field strengths one should split in transverse and longitudinal pieces. For this, one rewrites the initial Eq. (8) as

$$L = L_K + L_{int} + \mathcal{L}_{td} \tag{31}$$

where

$$L_K = a_{(IJ)}(\partial_\mu G_\nu{}^I)(\partial^\mu G^{\nu J}) + b_{(IJ)}(\partial_\mu G_\nu{}^I)(\partial^\nu G^{\mu J}) + c_{(IJ)}(\partial_\mu G^{\mu I})(\partial_\nu G^{\nu J}) - m_I^2 G_{\mu I} G^{\mu I} \tag{32}$$

with

$$a_{(IJ)} = 2b_I b_J + 2\beta_I \beta_J, \quad b_{(IJ)} = -2b_I b_J + 2\beta_I \beta_J, \quad c_{(IJ)} = 4\beta_I \rho_J + 4\beta_J \rho_I + 16\rho_I \rho_J \tag{33}$$

and

$$L_{int} = L_{int}^{(3)} + L_{int}^{(4)} \tag{34}$$

where

$$L_{int}^{(3)} = a_{IJK}(\partial_\mu G_\nu{}^I) G^{\mu J} G^{\nu K} + b_{I(JK)}(\partial_\mu G^{\mu I}) G_\nu{}^J G^{\nu K}, \tag{35}$$

$$L_{int}^{(4)} = a_{(IJ)(KL)} G_\mu{}^I G^{\mu J} G_\nu{}^K G^{\nu L} + b_{(I_{(J}K)_{L)}} G_\mu{}^I G_\nu{}^J G^{\mu K} G^{\nu L} \tag{36}$$

with

$$\begin{aligned} &a_{IJK} = 4b_I \gamma_{[JK]} + 4\beta_I \gamma_{(JK)}, \quad b_{I(JK)} = 4\beta_I \tau_{(JK)} + 4\rho_I \gamma_{(JK)} + 16\rho_I \tau_{(JK)}, \\ &a_{(IJ)(KL)} = 2\gamma_{(IJ)}\tau_{(KL)} + 4\tau_{(IJ)}\tau_{(KL)}, \quad b_{(I_{(J}K)_{L)}} = \gamma_{[IJ]}\gamma_{[KL]} + \gamma_{(IJ)}\gamma_{(KL)}, \end{aligned} \tag{37}$$

and the total derivative term is

$$\mathcal{L}_{td} = \tilde{a}_{IJ}\,\varepsilon^{\mu\nu\rho\sigma}\partial_\mu (G_\nu{}^I \partial_\rho G_\sigma{}^J) \tag{38}$$

with

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$$\tilde{a}_{IJ} = 2b_I b_J \ . \tag{39}$$

Notice that, in this sort of extended model, it contains kinetic and interacting pieces separately gauge invariant. It also generates trilinear and quadrilinear vertices separately gauge invariants and with different coupling constants.

Nevertheless, for a physical interpretation we should open up a discussion whether we should find out the physical fields , $\{G_{\mu I}\}$, by diagonalizing the transverse sector or the piece of L defined by the anti-symmetric field strength. Since we understand that the piece of L built up in terms of the transverse sector feels contributions from both the symmetric and antisymmetric field strengths, it seems more sensible to define the Ω-matrix as the one that completely diagonalizes the piece of L with the transverse projector. In so doing, we are sure that all possible contributions to the spin-1 quanta have been accounted for. It yields

$$L = L_T + L_L + L_I \tag{40}$$

with

$$L_T = G_{\mu\nu I} G^{\mu\nu I} \tag{41}$$

$$L_L = \beta_I \beta_J S_\mu{}^{\mu I} S_\nu{}^{\nu J} + \beta_I S_{\mu\nu}{}^{I}\left(\rho_J g^{\mu\nu} S_\rho{}^{\rho J}\right) + \rho_I g_{\mu\nu} S_\rho{}^{\rho I}\left(\beta_J S^{\mu\nu J} + \rho_J g^{\mu\nu} S_\sigma{}^{\sigma J}\right) \tag{42}$$

and

$$L_I = L_I^A + L_I^S + L_I^{ST} \tag{43}$$

for

$$L_I^A = \left(b_I G_{\mu\nu}{}^{I} + Z_{[\mu\nu]}\right) z^{[\mu\nu]} \tag{44}$$

$$L_I^S = \left(\beta_I S_{\mu\nu}{}^{I} + \rho_I g_{\mu\nu} S_\rho{}^{\rho I} + Z_{(\mu\nu)}\right)\left(z^{(\mu\nu)} + g^{\mu\nu} w_{(\rho}{}^{\rho)}\right) \tag{45}$$

$$L_I^{ST} = \eta E_{\mu\nu\rho\sigma}\left(2b_I \gamma_{[JK]} G^{\mu\nu I} G^{\rho J} G^{\sigma K} + \gamma_{[IJ]}\gamma_{[KL]} G^{\mu I} G^{\nu J} G^{\rho K} G^{\sigma L}\right). \tag{46}$$

The explicitly transverse diagonalized associated system of equations of motion is

$$\partial_\nu\left(G^{\nu\mu}{}_I + b_I z^{[\nu\mu]}\right) + \frac{1}{2} m_I^2 G^\mu{}_I = J^\mu{}_I(G) \tag{47}$$

with

$$J^\mu{}_I(G) = J^{A\mu}{}_I(G) + J^{S\mu}{}_I(G) + J^{ST\mu}{}_I(G)$$

where

$$J^{A\mu}{}_I(G) = \gamma_{[IJ]} G_\nu{}^J Z^{[\nu\mu]},$$

$$J^{S\mu}{}_I(G) = \gamma_{(IJ)} G_\nu{}^J Z^{(\nu\mu)} + \tau_{(IJ)} G^{\mu J} Z_{(\nu}{}^{\nu)} - \beta_I \partial_\nu\left(z^{(\nu\mu)} + g^{\nu\mu} w_{(\rho}{}^{\rho)}\right)$$
$$- \rho_I \partial^\mu\left(z_{(\nu}{}^{\nu)} + g_\nu{}^\nu w_{(\rho}{}^{\rho)}\right) - \left(\beta_I \beta_J + \beta_I \rho_J + \beta_J \rho_I + 4\rho_I \rho_J + \frac{1}{4}\xi_{IJ}\right)\partial^\mu S_\nu{}^{\nu J},$$

$$J^{ST\mu}{}_I(G) = \eta\gamma_{[IJ]} E^{\mu\nu\rho\sigma} G_\nu{}^J Z_{[\rho\sigma]} + \eta b_I E^{\mu\nu\rho\sigma}\partial_\nu z_{[\rho\sigma]} \ . \tag{48}$$

Eq. (47) rewrites Eq. (12). A further simplification is by taking the kinetic identity (23), identity $a_{(IJ)} = \delta_{IJ}$ and by taking the Lorentz condition $(\beta_I + \rho_I)\partial_\nu G^{\nu I} = 0$. It yields

$$J^{S\mu}{}_I(G) = \gamma_{(IJ)} G_\nu{}^J Z^{(\nu\mu)} + \tau_{(IJ)} G^{\mu J} Z_{(\nu}{}^{\nu)} - \beta_I \partial_\nu \left(z^{(\nu\mu)} + g^{\nu\mu} w_{(\rho}{}^{\rho)}\right)$$

$$-\rho_I \partial^\mu \left(z_{(\nu}{}^{\nu)} + g_\nu{}^\nu w_{(\rho}{}^{\rho)}\right) - \left(3\rho_I \rho_J + \frac{1}{4}\xi_{IJ}\right)\partial^\mu S_\nu{}^{\nu J} \,. \qquad (49)$$

Eqs. (49) and (21-22) are relating the opportunity for the symmetric kinetic sector be suppressed in theory.

We should now discuss the corresponding physical masses. Rewriting Eq. (40), one gets

$$L(G) = L_{\text{free}} + L_I \,, \qquad (50)$$

then, using the matrix notation $G_\mu{}^t = (G_{\mu 1}, \ldots, G_{\mu N})$, one gets

$$L_{\text{free}} = \frac{1}{2} G_\mu{}^t (\mathsf{W} + m_I^2)\theta^{\mu\nu} G_\nu + \frac{1}{2} G_\mu{}^t (B\mathsf{W} + m_I^2)\omega^{\mu\nu} G_\nu \,. \qquad (51)$$

Although violates locality this decomposition is valid because it preserves Lorentz covariance, $G_\mu^T{}' = (\Lambda^{-1})^\rho{}_\mu G_\rho^T$ and $G_\mu^L{}' = (\Lambda^{-1})^\rho{}_\mu G_\rho^L$. Consequently it is valid to consider that the spin-1 quanta are accommodated in the first term and the spin-0 quanta at second term. The corresponding interacting term written at matrix form is

$$\mathsf{L}_I = z^t \partial^\mu G^\nu (G_\mu{}^t \lambda_T G_\nu + \varepsilon_{\mu\nu\rho\sigma} G^\rho \lambda_T G^\sigma) + t_T^t \partial^\mu G^\nu (G_\mu{}^t \Lambda_T G_\nu) + w_T^t \partial_\mu G^\mu (G_\nu{}^t \Lambda_T G^\nu)$$

$$+ v_T^t \partial_\mu G^\mu (G_\nu{}^t \Theta_T G^\nu) + (G^{\mu t}\Sigma_t G_\mu)^2 + (G^{\mu t}\Theta_T G_\mu)(G^{\nu t}\Sigma_t G_\mu)^2$$

$$+ (G^{\mu t}\Theta_T G_\mu)(G^{\nu t}\Gamma_T G_\nu) + \varepsilon^{\mu\nu\rho\sigma}(G_\mu{}^t \lambda_T G_\nu)(G_\rho{}^t \lambda_T G_\rho) \qquad (52)$$

where $z_T, \ldots, v_T$ are column matrices and $\lambda_T, \ldots, \Gamma_T$ are row matrices. Both are depending on the initial Lagrangian free coefficients.

The physical basis, $\{G_I\}$, stands for a field reference frame on the spin-1 quanta. Therefore, m_I^2 correspond to the physical masses of the transversal sector, $m_I^2 \equiv m_T^2$. Non-diagonalized scalar contributions are in the longitudinal sector. From Eqs. (7-8), one gets that there is necessarily one massless term and a relationship between the transverse (m_T^2) and longitudinal (m_L^2) masses. Analyzing the transverse sector, one reads off the diagonalized matrix m_I^2. It contains a zero and the other elements are depending on the free coefficients written in the initial Lagrangian. This means that tachyons can be avoided by controlling such coefficients. Analyzing the longitudinal sector, the mass spectroscopy is less immediate. The particles that it embodies display masses that are eigenvalues of the matrix $\left(B^{-1} m_I^2\right)$ where B is the longitudinal kinetic matrix defined from Eq. (51). However, sectors T and L are not completely independent. There is a relationship between the masses in both sectors. It is given by

$$m_1^2 \ldots m_N^2 = det(B) det\left(B^{-1} m_I^2\right) \qquad (53)$$

where Eq. (53) is showing that the presence of any null mass in sector-T will correspond to a massless quantum in sector-L [56]. A further analysis on the $\{G_{\mu I}\}$ spectroscopy is given in [57]. It studies on the fields set variety by showing that the $(\frac{1}{2}, \frac{1}{2})$ niche of potential fields contain quanta diversity (spin, mass, charges; C, P, T). A fact that turns realistic such fourth light invariance interpretation. Given a field set it is possible to discriminate a quanta diversity in every LG irreducible representation.

Concluding, the objective of this section was to investigate on the wholeness principle. In 1962, Cabbibo and Ferrari also built up a model with more than one potential fields [58]. Their proposal was to work through U(1) $\otimes$ U(1). Differently, the thesis here is that by associating fields under a common gauge parameter one produces an antireductionistic behaviour. For this, instead of a multiplicity of abelian gauge symmetries like U$(1)^n$, it introduces through Eq. (7) the fields network meaning. Its entities are not more longer isolated, instead, they feed up a global system with the following global properties: physical fields $G_{\mu I}$ are defined globally as variables which are linear combinations of the initial $A_{\mu I}$, coefficients as $\gamma_{[IJ]}$ depending on initial free coefficients, coupled equations, Noether laws relating a N-set and also Green functions whose poles expression will be depending on parameters associated to the entire Lagrangian. Therefore, Eq. (8) is more than a mechanism with different massive potential fields without requiring the Higgs mechanism or a QED plus $(N-1)$ Proca equations. It contains an integral symmetry. Its proposal is not anymore for being restricted for generating interactions, but also, to build up a global interrelated system. It conducts an antireductionist abelian whole unity through gauge symmetry that works differently from Cabbibo, Ferrari and Higgs performances. The corresponding $(2N+7)$ classical equations are establishing an integral system where these equations are interconnected as a whole and expressing that physics is more than a problem of fragmentation. However, there is something more than just being a fields complex dynamics. There is a network meaning with a set determinism to be understood.

3 SET DETERMINISM

Given such whole structure one has to analyze its corresponding dynamics. For this, one has to examine the involved symmetries. These $2N+7$ integral equations develops two kinds of symmetry associated to gauge invariance. They are the directive symmetry and the circumstantial symmetry. Their qualitative difference is that while the director appears as a natural instruction from the gauge parameter, the circumstantial will depend on relationships between the so-called global parameters.

Whole gauge theories introduce the symmetry management. They will develop a new feature in gauge theory which is the organization context. Provide two controls on the symmetry action which are the directive and the circumstantial behaviours. We will study on it through five cases: genuine gauge field, current conservation, global symmetry, $SO(N)$, Ward-Takahashi. Firstly, Eq. (1) management produces a field set determinism where only one genuine gauge field coordinates $2N+7$ classical equations. This genuine gauge field shows up at $\{D, X_i\}$ basis where D_μ appears as the only one field depending on the gauge parameter. This means that the directive symmetry implies that Eq. (7) contains just one massless potential field while for the other fields the corresponding masses will be depending on the symmetry circumstances stipulated by the free parameters.

As the current conservation is consequence of a given symmetry implementation, our second case is to investigate on these symmetries by analyzing the corresponding conserved currents. The presence of a common gauge parameter results in the Ward and Noether informations which are respectively associated to action and Lagrangian invariances. They represent what we call as an instruction from the director symmetry. They yield just one conserved current with N-global contributions. From action invariance

$$\int d^4x \delta G_{\mu I} \frac{\delta S}{\delta G_{\mu I}} = 0 \tag{54}$$

one gets through Eq. (31)

$$\begin{aligned}
\partial_\mu J_W^\mu = \Omega_{I1} \frac{\delta S}{\delta G_{\mu I}} = \Omega^I{}_1 \Big(\big(2a_{(IJ)} + 2b_{(IJ)} + 2c_{(IJ)}\big) \partial_\mu \partial^\mu \partial_\nu G^{\nu J} + 2m_I^2 \partial_\mu G^\mu{}_I \\
+ b_{I(JK)} \partial_\mu \partial^\mu \big(G_\nu{}^J G^{\nu K}\big) + a_{IKJ} \partial_\mu \partial_\nu \big(G^{\mu J} G^{\nu K}\big) - 2b_{K(IJ)} \partial_\mu \big(G^{\mu J} \partial_\nu G^{\nu K}\big) \\
- a_{KIJ} \partial_\mu \big(G_\nu{}^J \partial^\mu G^{\nu K}\big) - a_{KJI} \partial_\mu \big(G_\nu{}^J \partial^\nu G^{\mu K}\big) \\
- \Big(2a_{(IJ)(KL)} + 2a_{(KL)(IJ)} + 2b_{(I_{(K^J)_L})} + 2b_{(K_{(I^L)_J})}\Big) \partial_\mu \big(G^{\mu J} G_\nu{}^K G^{\nu L}\big) \Big) = 0
\end{aligned} \tag{55}$$

which says that the directive instruction is a conserved current involving mass terms, longitudinal kinetic terms and trilinear and quadrilinear vertices. Considering Lagrangian invariance Eqs. (24-26) introduce the Noether current. Notice that they do not necessarily coincide as in usual QED (Noether is an on-shell information, while Ward is off-shell). The relevance here is that their instructions are derived directly from the gauge parameter.

Now let us analyze on conserved currents through the presence of the circumstantial symmetry. We should try to impose N-conservation possibilities by rewriting Eq. (47) as

$$\partial_\mu J^\mu{}_I(G) = \frac{1}{2} m_I^2 \partial_\mu G^\mu{}_I \tag{56}$$

with

$$J^{\mu}{}_{I}(G) = n_{IJ}^{(1)}\partial^{\mu}\partial_{\nu}G^{\nu J} + n_{IJK}^{(1)}G^{\mu J}\partial_{\nu}G^{\nu K} + n_{IJK}^{(2)}G_{\nu}{}^{J}\partial^{\mu}G^{\nu K} + n_{IJK}^{(3)}G_{\nu}{}^{J}\partial^{\nu}G^{\mu K} +$$
$$+ n_{IJK}^{(4)}\eta E^{\mu\nu\rho\sigma}G_{\nu}{}^{J}\partial_{\rho}G_{\sigma}{}^{K} + n_{IJKL}^{(1)}G^{\mu J}G_{\nu}{}^{K}G^{\nu L} + n_{IJKL}^{(2)}\eta E^{\mu\nu\rho\sigma}G_{\nu}{}^{J}G_{\rho}{}^{K}G_{\sigma}{}^{L}$$

where

$$n_{IJ}^{(1)} = -\frac{1}{2}\xi_{IJ} - 2\beta_I\beta_J - 2\beta_I\rho_J - 2\beta_J\rho_I - 8\rho_I\rho_J$$
$$n_{IJK}^{(1)} = -\beta_I\gamma_{(JK)} + 2\beta_K\tau_{(IJ)} + 2\rho_K\gamma_{(IJ)} + 8\rho_K\tau_{(IJ)}$$
$$n_{IJK}^{(2)} = b_K\gamma_{[IJ]} - 2\beta_I\tau_{(JK)} + \beta_K\gamma_{(IJ)} - 2\rho_I\gamma_{(JK)} - 8\rho_I\tau_{(JK)}$$
$$n_{IJK}^{(3)} = -b_K\gamma_{[IJ]} - \beta_I\gamma_{(JK)} + \beta_K\gamma_{(IJ)}$$
$$n_{IJK}^{(4)} = -2b_I\gamma_{[JK]} + 2b_K\gamma_{[IJ]}$$
$$n_{IJKL}^{(1)} = \gamma_{[IK]}\gamma_{[JL]} + \gamma_{(IJ)}\tau_{(KL)} + \gamma_{(IK)}\gamma_{(JL)} + \gamma_{(KL)}\tau_{(IJ)} + 4\tau_{(IJ)}\tau_{(KL)}$$
$$n_{IJKL}^{(2)} = \gamma_{[IJ]}\gamma_{[KL]} \quad . \qquad (57)$$

Taking the partial derivative, yields

$$\partial_{\mu}J^{\mu}{}_{I}(G) = n_{IJ}^{(1)}\partial_{\mu}\partial^{\mu}\partial_{\nu}G^{\nu J} + n_{IJK}^{(1)}\partial_{\mu}\left(G^{\mu J}\partial_{\nu}G^{\nu K}\right) + n_{IJK}^{(2)}\partial_{\mu}\left(G_{\nu}{}^{J}\partial^{\mu}G^{\nu K}\right) +$$
$$+ n_{IJK}^{(3)}\partial_{\mu}\left(G_{\nu}{}^{J}\partial_{\nu}G^{\mu K}\right) + n_{IJK}^{(4)}\eta E^{\mu\nu\rho\sigma}\partial_{\mu}\left(G_{\nu}{}^{J}\partial_{\rho}G_{\sigma}{}^{K}\right) +$$
$$+ n_{IJKL}^{(1)}\partial_{\mu}\left(G^{\mu J}G_{\nu}{}^{K}G^{\nu L}\right) + n_{IJKL}^{(2)}\eta E_{\mu\nu\rho\sigma}\partial_{\mu}\left(G_{\nu}{}^{J}G_{\rho}{}^{K}G_{\sigma}{}^{L}\right) \quad . \qquad (58)$$

Consequently, the classical decoupling of the longitudinal sector $\partial^{\mu}G_{\mu I} = 0$ will depend on the circumstantial adjusting of the values of the global free parameters. For proving this, one has to study explicitly every case involving a given number of potential fields. In Appendix B the cases $N = 2,3$ are performed. In principle, the model contains opportunities for N-conserved currents.

Conserved currents are showing that, differently from usual gauge theories, Eq. (1) introduces the meaning of symmetry management. It says that for systematizing a coherent whole one has to read off its directive and circumstantial instructions. The evolution equations will depend on both aspects where while the former means a guideline of an overall plan originated from the gauge parameter the latter establishes gauge strategies derived from the free coefficients. They are respectively a natural consequence of sharing a common gauge parameter (as Ward and Noether information) and stem from possibilities given by global free parameters relationships (a gauge strategy) based on a so-called volume of circumstances as Appendix A calculates.

A third case to study is on global symmetry. Carrying on the investigation over these whole symmetries, let us consider the most general global symmetry

$$\delta G_{\mu I} = A_{IJ}G_{\mu}{}^{J} \qquad (59)$$

with the corresponding Ward identity

$$\int d^4x A_{IJ}G_{\mu}{}^{J}\frac{\delta S}{\delta G_{\mu I}} = 0 = \int d^4x\partial_{\mu}J^{\mu} \,. \qquad (60)$$

Calculating it explicitly, one gets

$$\int d^4x = \partial_{\mu}\left(w_{IJ}^{(1)}G^{\mu I}\partial_{\nu}G^{\nu J} + w_{IJ}^{(2)}G_{\nu}{}^{I}\partial^{\mu}G^{\nu J} + w_{IJK}^{(1)}G^{\mu I}G_{\nu}{}^{J}G^{\nu K}\right) +$$
$$+ w_{IJ}^{(3)}\left(\partial_{\mu}G^{\mu I}\right)\left(\partial_{\nu}G^{\nu J}\right) + w_{IJ}^{(4)}\left(\partial_{\mu}G_{\nu}{}^{I}\right)\left(\partial^{\mu}G^{\nu J}\right) + w_{IJ}^{(5)}G_{\mu}{}^{I}G^{\mu J} +$$

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$$+w^{(2)}_{IJK}G_\mu{}^I G^{\mu J}\partial_\nu G^{\nu K}+w^{(3)}_{IJK}G_\mu{}^I G_\nu{}^J\partial^\mu G^{\nu K}+w_{IJKL}G_\mu{}^I G^{\mu J}G_\nu{}^K G^{\nu L}=0$$

where

$$w^{(1)}_{IJ}=2\,A^K{}_I\left(b_{(JK)}+c_{(JK)}\right),\qquad w^{(2)}_{IJ}=2\,A^K{}_I\,a_{(JK)},$$

$$w^{(3)}_{IJ}=-2A^K{}_J\left(b_{(IK)}+c_{(IK)}\right),\qquad w^{(4)}_{IJ}=-2A^K{}_J\,a_{(IK)},$$

$$w^{(5)}_{IJ}=-2A_{JI}\,m_J^2,\qquad w^{(1)}_{IJK}=A^L{}_I\,b_{L(JK)}+A^L{}_J\,a_{LIK},$$

$$w^{(2)}_{IJK}=-2A^L{}_I\,b_{K(JL)}-A^L{}_K\,b_{L(IJ)},\qquad w^{(3)}_{IJK}=-A^L{}_I\,a_{KLJ}-A^L{}_J\,a_{KIL}-A^L{}_K\,a_{LJI},$$

$$w_{IJKL}=-2A^M{}_I\left(a_{(JM)(KL)}+a_{(KL)(JM)}+b_{(J_{(K}{}^{M)}{}_{L)}}+b_{(K_{(J}{}^{L)}{}_{M)}}\right). \tag{61}$$

Therefore, for this Eq. (59) symmetry to be implemented, the last six parameters not connected with a total derivative must vanish. A necessary condition is $\det A=0$. However the implementation condition will depend on circumstances between the global parameters. In Appendix D the case $N=2$ is studied.

Another case is to consider a set of symmetries

$$\int d^4x A_{IJK}G_\mu{}^J\frac{\delta S}{\delta G_{\mu K}}=0=\int d^4x\partial_\mu J^\mu{}_I\,. \tag{62}$$

It yields,

$$\int d^4x=\partial_\mu\left(r^{(1)}_{IJK}G^{\mu J}\partial_\nu G^{\nu K}+r^{(2)}_{IJK}G_\nu{}^J\partial^\mu G^{\nu K}+r^{(1)}_{IJKL}G^{\mu J}G_\nu{}^K G^{\nu L}\right)+$$

$$+r^{(3)}_{IJK}\left(\partial_\mu G^{\mu J}\right)\left(\partial_\nu G^{\nu K}\right)+r^{(4)}_{IJK}\left(\partial_\mu G_\nu{}^J\right)\left(\partial^\mu G^{\nu K}\right)+r^{(5)}_{IJK}G_\mu{}^J G^{\mu K}+$$

$$+r^{(2)}_{IJKL}G_\mu{}^J G^{\mu K}\partial_\nu G^{\nu L}+r^{(3)}_{IJKL}G_\mu{}^J G_\nu{}^K\partial^\mu G^{\nu L}+r_{IJKLM}G_\mu{}^J G^{\mu K}G_\nu{}^L G^{\nu M}=0$$

where

$$r^{(1)}_{IJK}=2A_I{}^L{}_J\left(b_{(KL)}+c_{(KL)}\right),\qquad r^{(2)}_{IJK}=2A_I{}^L{}_J\,a_{(KL)},$$

$$r^{(3)}_{IJK}=-2A_I{}^L{}_K\left(b_{(JL)}+c_{(JL)}\right),\qquad r^{(4)}_{IJK}=-2A_I{}^L{}_K\,a_{(JL)},$$

$$r^{(5)}_{IJK}=-2A_I{}^L{}_J\,d_{KL},\qquad r^{(1)}_{IJKL}=A_I{}^M{}_J\,b_{M(KL)}+A_I{}^M{}_K\,a_{MJL},$$

$$r^{(2)}_{IJKL}=-2A_I{}^M{}_J\,b_{L(KM)}-A_I{}^M{}_L\,b_{M(JK)},$$

$$r^{(3)}_{IJKL}=-A_I{}^M{}_J\left(a_{LMK}+a_{LKM}\right)-A_I{}^M{}_L\,a_{MKJ},$$

$$r_{IJKLM}=-2A_I{}^N{}_J\left(a_{(KN)(LM)}+a_{(LM)(KN)}+b_{(K_{(L}{}^{N)}{}_{M)}}+b_{(L_{(K}{}^{M)}{}_{N)}}\right). \tag{63}$$

As a fourth case, let us consider the SO(N) case $G_{\mu I}=R_I{}^P G_{\mu P}$ with $R=e^{iw_a t_a}$. From Eq. (59), a relevant application for this circumstantial symmetry is by imposing a certain field symmetry on the Lagrangian. Rewriting Eq. (13) as

$$L_K=\left(\partial_\mu G_\nu\right)^t A\left(\partial^\mu G^\nu\right)+\left(\partial_\mu G_\nu\right)^t B\left(\partial^\nu G^\mu\right)+\left(\partial_\mu G^\mu\right)^t C\left(\partial_\nu G^\nu\right)+G_\mu^t M G^\mu \tag{64}$$

one has the conditions

$$R^t AR = A, \qquad R^t BR = B, \qquad R^t CR = C, \qquad R^t M R = M. \tag{65}$$

Considering an infinitesimal rotation, we obtain for the interacting terms

$$(t_A)_{pi} a_{iqr} + (t_A)_{qi} a_{pir} + (t_A)_{ri} a_{pqi} = 0,$$

$$(t_A)_{ip} a_{iqrs} + (t_A)_{iq} a_{pirs} + (t_A)_{ir} a_{pqis} + (t_A)_{is} a_{pqri} = 0. \tag{66}$$

The corresponding directive and circumstantial symmetries are implemented and enlarged vectorial fields are introduced with charged vectorial fields [59].

A fifth case is that one where the Ward-Takahashi identity freezes the longitudinal part from one of the N-involved fields. Considering the director symmetry: $\delta G_{\mu I} = \Omega_{I1}^{-1}\partial_\mu \alpha$, $\delta\psi = i\alpha\psi$, $\delta\overline{\psi} = -i\alpha\overline{\psi}$ (fermions included), and the gauge fixing term $L[G_I] = \sigma_I \partial_\mu G^{\mu I}$ one derives the expression [52]

$$[\frac{i}{\alpha}\sigma_I \sigma_J \Omega_{I1} W \partial_\mu G_I^\mu - i\Omega_{I1}\partial_\mu \frac{\delta\Gamma}{\delta G_{\mu I}} + g(\overline{\psi}\frac{\delta\Gamma}{\delta\overline{\psi}} - \psi\frac{\delta\Gamma}{\delta\psi})] = 0. \tag{67}$$

which yields the following system of equations

$$\Omega_{I1}\partial^\mu \Gamma_{\mu\nu,IJ} = \Delta_{\nu J}, \tag{68}$$

where

$$\Gamma_{\mu\nu,IJ} \equiv \frac{\delta^2\Gamma}{\delta G_{\mu I}\,\delta G_{\nu J}} \quad \text{and} \quad \Delta_{\nu I} = \frac{\sigma_I}{\alpha} W \partial_\nu \delta(x-y). \tag{69}$$

Then, working out the above expressions one gets as directive, that even for such abelian generalization a massless photon is preserved when it interacts with electron and positron and also with another vector fields and that the propagator longitudinal part corresponding to a generic field $G_{\mu I}$ do not suffer radiation correction. As circumstance, that some infinities can be cancelled by adding graphs.

The five cases studied are showing the notion of symmetry management. As a novelty, there appears the meaning of circumstance, which introduces the chances of interfering on symmetry. Circumstances means the space to do physical modelling without violating gauge symmetry. For instance, the kinetic L-sector be isolated. It is possible to isolate symmetric kinetic sector either by taking $\partial_\mu G_I^\mu = 0$ or by substituting Eqs. (20-23) in Eq. (12). It says that by adjusting parameters the symmetric kinetic sector can be decoupled as Appendix C shows or transformed into the antisymmetric kinetic sector and interactions terms.

Thus, the gauge parameter is transformed from compensating fields and from taking degree of freedom into an instrument for set orientation and for creating chance. The fact to be observed here is that from this set determinism one derives two types of symmetries. Eq. (7) provides instructions to Eq. (8), such as the compulsory existence of one massless field together with circumstantial possibilities for physics be done. It yields a whole physics playing with symmetry possibilities being defined with the notions of directive through gauge parameter and circumstance through global free coefficients. Consequently, the $2(N+7)$ equations derived at last section should be read off as a conglomerated of fields under a set determinism where the directive laws are related directly to the gauge parameter (Bianchi identities, Noether current, Ward identities), while the circumstantial laws bring the meaning of chance (choice on degrees of freedom, conserved currents, etc).

A gauge organizing principle appears. An association fields, saying that any whole gauge model contains directive and opportunities, where an organization for establishing the fields set $\{G_{\mu I}\}$ behaviour can be derived without gauge invariance being violated. There is a management control through the directive symmetry and expectatives through the circumstantial symmetry. The realization of a whole physical system will be depending not only on compensating interactions but also through organization. It means the capacity of guiding and interfering in the whole system. Given the initial $4N$-potential fields, one gets M-measurable entities and S-sources with an organization capability being managed by gauge symmetry.

Thus, these antireductionist equations are not only relating interdependent fields, but providing another concept of determinism. There is a set dynamics embedded in a symmetry management. It introduces another concept of determinism where one can not predict the future state precisely, and like quantum mechanics and chaos, whole gauge models work with an uncertain causality. A dynamics where instead of probability introduces the concept of chance.

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However, differently it shows a determinism based on organization. As a new concept, it introduces a causality coordinated by the notions of directive and circumstance. As consequence, the whole approach generates a state equation with a gauge organizing principle based on directive and chance, which must be our next fact to be explored. We have to write it in terms of physical variables.

4 GLOBAL MAXWELL EQUATION

The systemic principle is under consideration. It says that nature general tendency is to build up wholeness. So, prior to entropy, nature moves in direction of possible whole structures as atoms, molecules, bodies, ..., planetary systems, galaxies. Therefore, in terms of field theory, our compromise will be with a set of fields $\{G_{\mu I}\}$, as Eq. (8) states. Given the $(\frac{1}{2},\frac{1}{2})$ niche, there are coupled equations with a global flow carrying granular and collective variables and global conservation laws to be studied. We should now rewrite section 2 in terms of correspondent observables.

We have now to look for the state equation corresponding to the $(\frac{1}{2},\frac{1}{2})$ niche. As clue, it will correspond to an equation involving particles with spin 1 and 0; as result, a fundament from this model is that given fields with a LG nature, it is possible to differentiate them dynamically and through discrete symmetries saying that the $(\frac{1}{2},\frac{1}{2})$ niche contains particles with different quantum numbers [57]. But now, we wish to explore this diversity through EM fields.

Being an experimental science, the first physics's challenge for a given model is to promote new entities to be measured. In fact, generate physical entities is a first symmetry consequence, even before interactions. The possibility of including different potential fields in the same gauge group introduces other types of electric and magnetic fields. Eqs. (9) and (10) develop new fundamental entities for defining the measuring process. Dimensional analysis and gauge invariance select N-pairs of generic electric and magnetic fields with the same Maxwell structure

$$\vec{E}_I = G_{0iI}, \quad \vec{B}_I = \frac{1}{2}E_{ijk}G_{jiI} \tag{70}$$

and also, composite electric and magnetic fields

$$\vec{e} = \gamma_{[IJ]}\phi^I G_i{}^J, \quad \vec{b} = \frac{1}{2}\gamma_{[IJ]}E_{ijk}G_j{}^I G_i{}^J \tag{71}$$

defined from potential fields

$$G^{\mu I} \equiv \left(\phi^I, \vec{G}^I\right). \tag{72}$$

Being composite fields, as Eq. (71) shows, $\vec{e}$ and $\vec{b}$ are a new type of measurable fields. They will have macroscopic consequences, although defined as collective microscopic fields. These collective fields are a first manifestation of the wholeness principle at field theory.

The symmetric sector also develops new gauge invariant terms. It yields the following scalars, vectors and tensors

$$\sigma = \beta_I \sigma^I, \quad \vec{\sigma} = \beta_I \vec{\sigma}_i{}^I, \quad \sigma_{ij} = \beta_I \sigma_{ij}{}^I, \quad \sigma_i{}^i = \beta_I \sigma_i{}^{iI};$$

$$\text{with} \quad \sigma^I = S_{00}{}^I, \quad \vec{\sigma}_i{}^I = S_{0i}{}^I, \quad \sigma_{ij}{}^I = S_{ij}{}^I, \quad \sigma_i{}^{iI} = S_i{}^{iI},$$

and

$$s = \gamma_{(IJ)}\phi^I\phi^J, \quad \vec{s}_i = \gamma_{(IJ)}\phi^I G_i{}^J, \quad s_{ij} = \gamma_{(IJ)}G_i{}^I G_j{}^J, \quad s_i{}^i = \gamma_{(IJ)}G_i{}^I G^{iJ},$$

and

$$\theta = g_{00}\rho_I S_\alpha{}^{\alpha I}, \quad \theta_{ij} = g_{ij}\rho_I S_\alpha{}^{\alpha I}, = g_{00}\tau_{(IJ)}G_\alpha{}^I G^{\alpha J}, \quad r_{ij} = g_{ij}\tau_{(IJ)}G_\alpha{}^I G^{\alpha J}, \tag{73}$$

where $g_{00} = 1$, $g_{ij} = -\delta_{ij}$.

The antisymmetric field tensor is read off as

$$Z_{[\mu\nu]} = b_I \begin{bmatrix} 0 & E_x{}^I & E_y{}^I & E_z{}^I \\ -E_x{}^I & 0 & -B_z{}^I & B_y{}^I \\ -E_y{}^I & B_z{}^I & 0 & -B_x{}^I \\ -E_z{}^I & -B_y{}^I & B_x{}^I & 0 \end{bmatrix} + \begin{bmatrix} 0 & e_x & e_y & e_z \\ -e_x & 0 & -b_z & b_y \\ -e_y & b_z & 0 & -b_x \\ -e_z & -b_y & b_x & 0 \end{bmatrix} \quad (74)$$

where Eq. (74) introduces elements for the space-time scenario with granular and collective domains. Its basic entities are generic Faraday variables $\vec{E}_I$ and $\vec{B}_I$, which have the usual Maxwell character, and $\vec{e}$, $\vec{b}$, which have a cooperative character.

Similarly one gets the symmetric tensor

$$Z_{(\mu\nu)} = \begin{bmatrix} \sigma & \sigma_x & \sigma_y & \sigma_z \\ \sigma_x & & & \\ \sigma_y & & [\sigma_{ij}] & \\ \sigma_z & & & \end{bmatrix} + \begin{bmatrix} s & s_x & s_y & s_z \\ s_x & & & \\ s_y & & [s_{ij}] & \\ s_z & & & \end{bmatrix} +$$

$$+ \begin{bmatrix} \theta + r & 0 & 0 & 0 \\ 0 & -\theta_{11} - r_{11} & 0 & 0 \\ 0 & 0 & -\theta_{22} - r_{22} & 0 \\ 0 & 0 & 0 & -\theta_{33} - r_{33} \end{bmatrix}. \quad (75)$$

where Eq. (75) introduces elements for a symmetric EM sector. Given that any symmetric tensor can be expressed as $S_{\mu\nu} = \bar{S}_{\mu\nu} + \frac{1}{4}(\mathrm{tr}S)\eta_{\mu\nu}$ where $\bar{S}_{\mu\nu}$ is a traceless matrix, one gets for Eq. (75) the Lorentz decomposition (1,1)+(0,0), saying that its components contains the following spin content $\underline{0} \oplus \underline{1} \oplus \underline{2} \oplus \underline{0}$. Consequently, while Maxwell is only associated to spin 1, Eq. (75) provides a symmetric electromagnetism where differently from gravity, one derives a spin-2 behaviour (and others) without associating a rank-2 field $h_{\mu\nu}$. It shows that from potentials field $G_{\mu I}$ $(\underline{0}, \underline{1})$, one obtains EM fields containing spin 0, 1 and 2, as for instance, σ, $\vec{\sigma}$, σ_{ij}.

We are looking for an evolution on our comprehension of the EM phenomena. EM started with electric charge, EM fields, Maxwell equations, non-minimal EM interactions with magnetic moments, QED photon field, and now, one reverses this path through a fourth interpretation to light invariance principle. As expected, there are more EM fields than usual Maxwell. Following the lemma that one should consider a symmetry at its most, Eqs. (74)-(75) are showing that before implementing interactions the first symmetry proposal is to find out the maximum number of physical objects. Spinors works as this example, where studying all possible Lorentz group irreducible representations, one finds out an entity which does not live on space-time. In our case, from an antireductionist gauge symmetry, one is able to build up physical entities beyond Maxwell, which kinematics and dynamics are challenging us.

Kinematically, the above EM fields Lorentz transformations introduces new invariants as $Z_{[\mu\nu]}Z^{[\mu\nu]} = 2[(\vec{B}+\vec{b})^2 - (\vec{E}+\vec{e})^2]$, $Z_{(\mu\nu)}Z^{(\mu\nu)} = 2(\vec{s}+\vec{\sigma})^2 + (\sigma_{ii} + s_{ii} + \theta_{ii} + r_{ii})^2$, $Z_{[\mu\nu]}\tilde{Z}^{[\mu\nu]} = 8(\vec{E}\cdot\vec{B} + \vec{E}\cdot\vec{b} + \vec{B}\cdot\vec{e})$, $Z_{(\mu\nu)}\tilde{Z}^{(\mu\nu)} = 8(\vec{\sigma}+\vec{s})^2$ where $\vec{E} = a_I\vec{E}^I$, $\vec{B} = b_I\vec{B}^I$. The trace is also invariant $Z^{(\alpha}{}_{\alpha)}{}' = Z^{\alpha}{}_{\alpha}$. Also $G_{\mu\nu I}G^{\mu\nu I} = \vec{E}_I{}^2 - \vec{B}_I{}^2$, $z_{[\mu\nu]}z^{[\mu\nu]} = -2(\vec{e}^{\,2} - \vec{b}^{\,2})$, $z_{[\mu\nu]}\tilde{z}^{[\mu\nu]} = 4\vec{e}\cdot\vec{b}$, and so on. As a new result, it shows that the granular and collective sectors are connected through relativity, as relationships $G_{\mu\nu I}z^{[\mu\nu]} = -2(\vec{E}_I\cdot\vec{e} - \vec{B}_I\cdot\vec{b})$, $G_{\mu\nu I}\tilde{z}^{[\mu\nu]} = -2(\vec{E}_I\cdot\vec{b} - \vec{B}_I\cdot\vec{e})$ and the corresponding symmetric relationships are showing [60].

Dynamically, we are pursuing for an equation representing the antireductionist performance. Maxwell fields do not occupy a special room anymore, other variables candidates to a dynamics become possible. We have to consider how

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these variables defined at Eqs. (74)-(75) are described. There is a new electromagnetic scenario for being understood. An electromagnetic extension shall be obtained by substituting Eqs. (74), (75) in (47). A state equation to be expressed from variational principle having the group of fields $\{Z_{[\mu\nu]}, Z_{(\mu\nu)}\}$ as block variables. It yields the so-called Global Maxwell equation which can be separated in two sectors: **Granular sector:**

$$\vec{\nabla}\cdot\left(\vec{E}_I + b_I\vec{e}\right) + \frac{1}{2}m_I^2\phi_I = \rho_I(G), \tag{76}$$

$$\vec{\nabla}\times\left(\vec{B}_I + b_I\vec{b}\right) + \frac{1}{2}m_I^2\vec{G}_I = \frac{\partial}{\partial t}\left(\vec{E}_I + b_I\vec{e}\right) + \vec{J}_I(G), \tag{77}$$

$$\vec{\nabla}\times\vec{E}_I + \frac{\partial}{\partial t}\vec{B}_I = 0, \tag{78}$$

$$\vec{\nabla}\cdot\vec{B}_I = 0, \tag{79}$$

where the internal sources $\rho_I(G)$ and $\vec{J}_I(G)$ are defined at Eq. (48). Later on, they will be rewritten in terms of $\vec{E}_I$ and $\vec{e}$. As Faraday, one gets EM fields depending on potential fields pairs $\{\Phi_I, \vec{G}_I\}$. Eqs. (78-79) provide the solutions $\Phi_I' = \Phi_I + \frac{\partial}{\partial t}f_I$ and $\vec{G}_I' = \vec{G}_I + \vec{\nabla}f_I$ which are consistent with the gauge transformation proposed by Eq. (7).

Collective sector:

$$\vec{\nabla}\times\vec{e} + \frac{\partial}{\partial t}\vec{b} = \gamma_{[IJ]}\left(\vec{G}^I\times\vec{E}^J - \phi^I\vec{B}^J\right), \tag{80}$$

$$\vec{\nabla}\cdot\vec{b} = \gamma_{[IJ]}\vec{G}^I\cdot\vec{B}^J, \tag{81}$$

where a collectivism based on induction laws is derived from Eq. (20).

We have to understand on Eq. (4). These equations should not be taken as a surprise. First, two space vectors can always be thought as components of a skew-symmetric rank-2 tensor $A^{\mu\nu}$, which writes through the field equation $\partial_\mu A^{\mu\nu} = \sigma^\nu$, where $A^{\mu\nu} = (\vec{e}, \vec{b})$ complements Eqs. (76)-(77). Similarly for $(\vec{b} - \vec{e})$, one gets 80)-(81). So, these apparently new equations are just following that all natural phenomena shall be described by equations which possess the Lorentz group as their symmetry group. Second, there is a systemic logic to be understood. We have to analyze how this determinism proceed. Observe on its capacity to manage with the antireductionist symmetry. So, it derives a $(\frac{1}{2}, \frac{1}{2})$ physics ruled by the notions of set,organization, directive, circumstance, network. Where every field from Eqs. (5)-(6) is now embedded in a whole. Also there is a dynamics relating the individual and the collective fields. Third, on that fields $\vec{E}_I$, $\vec{B}_I$, $\vec{e}$, $\vec{b}$ and others are originated from internal charges ρ_I and $\vec{J}_I(G)$ and not from electric charge.

As a fourth aspect, there are two interpretations for these equations. A first one, separate in two sectors as above. A second one, based on vectorial analysis where the Helmoltz prescription says that a given vector is defined through the divergent and rotational operators; then, by adding both sectors one gets four equations which well-defined variables are the vectors $\vec{E}_I + b_I\vec{e}$ and $\vec{B}_I + b_I\vec{b}$. Later on, at chapter 10, this second interpretation will be associated to macroscopic fields. Finally, notice that every term in above Global Maxwell equation is gauge invariant, for instance, $\gamma_{(IJ)}\phi_I' = \gamma_{(IJ)}\phi_I + \gamma_{(IJ)}\Omega_{I1}^{-1}\frac{\partial}{\partial t}\alpha$ is invariant due to (14).

A next step is to analyze on these equations. We are not more playing with a pair $\vec{E} - \vec{B}$ coupled with electric charge, but there is a state equation with two sectors, I -EM layers, four EM-regimes, explicit potential fields and a global photon to be understood. The first systematization is that Eq. (4) provides two classes of equations. The first ones are (76)-(79) with a granular nature and having the Maxwell's equations as particular case. The second class of equations exhibits a collective nature written in terms of $\vec{e}$, $\vec{b}$ variables, with charge and current defined by point-like fields being summed over. It is interesting to compare them with Born-Infeld where a qualitative difference is that while eqs (80)-(81) develops a dynamical interpretation for the collective vector fields the Born-Infeld contributions are just algebraic,

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$H_i = \frac{\partial L_{BI}}{\partial B_i}$ and $D_i = \frac{\partial L_{BI}}{\partial E_i}$ [64].

Then, from this antireductionist abelian model, one obtains a Global Maxwell state equation. It introduces new terms on Gauss [61], Ampère [62], Faraday's laws [63] and a new photon interpretation. These equations are introducing an EM fields dynamics without electric charge dependence. They provide an I-layers dynamics where each one corresponds to every $G_{\mu I}$ field from the original $(\frac{1}{2},\frac{1}{2})$ niche. Each layer contains its own Gauss and Ampère laws relating granular and collective fields with non-linear sources. Induction laws are also extended for new EM fields. A new photon emerges, taking the layer $I=1$, Maxwell is reobtained and $G_{\mu 1}$ corresponds to a massless photon field which is not more passive but with a non-linear source $\rho_1(G)$. It appears a global photon which function is to manage this I-layers physics by the directive symmetry.

We should now observe some features on dynamics, photon behaviour, phenomenology. A first one is on dynamics, instead of the time evolution of a given field, it contains the dynamics of the whole system. It develops an individual-collective dynamics relating the granular ($\vec{E}_I - \vec{B}_I$) and the collective ($\vec{e} - \vec{b}$) fields. At following sections we will study the corresponding conservation laws and forces. A second aspect is on the photon, it is associated to the directive symmetry while other fields behave under the circumstantial symmetry. There is a new photon behaviour to be considered. The third one is on possible phenomenologies, Eq. (4) will sustain three planes of phenomenology. The subtlest layer depends just on gauge invariant conglomerates of potential fields, as $\gamma_{[IJ]}\vec{G}^J$. These clusters lie down in the region of Bohm-Aharanov effect and quantum mechanics wave function. The second layer means the usual Maxwell sector with new couplings for ($\vec{E}_1 - \vec{B}_1$) plus an enlargement with other flavour fields ($\vec{E}_I - \vec{B}_I$). The third one corresponds to a tissue made of collective fields, and the fourth with longitudinal fields with different spins. The last layer is associated to macroscopic EM fields studied at section 10.

There is still a second collective sector to be considered, the longitudinal identities at Eqs. (21)-(22). However their relationships are just algebraic. The relevance of these Bianchi-longitudinal-identities is that they connect the symmetric kinetic and interacting sectors. Depending on the *ansatz* (at this classical level it is not necessary to study the *ansatz* stability) there are different possibilities to relate them. The first one is $\gamma_{(IJ)} = \beta_I \beta_J$, a circumstance which makes the identity (21) give the following relationships:

$$\frac{\partial}{\partial t}s = \beta_I \phi^I \sigma, \qquad 2\frac{\partial}{\partial t}\vec{s} + \vec{\nabla}s = 2\beta_I \phi^I \vec{\sigma} - \beta_I \vec{G}^I \sigma,$$

$$\frac{\partial}{\partial t}s_{ij} + \nabla_i s_j + \nabla_j s_i = \beta_I \phi^I \sigma_{ij} + \beta_I G_i^{\ I} \sigma_j + \beta_I G_j^{\ I} \sigma_i,$$

$$\nabla_i s_{jk} + \nabla_j s_{ki} + \nabla_k s_{ij} = \beta_I G_i^{\ I} \sigma_{jk} + \beta_I G_j^{\ I} \sigma_{ki} + \beta_I G_k^{\ I} \sigma_{ij} \,. \tag{82}$$

Adjusting the relationships $\tau_{(IJ)} = \rho_I \rho_J$, Eq. (22) yields:

$$\frac{\partial}{\partial t}r + 2\frac{\partial}{\partial t}w - 2\vec{\nabla}\cdot\vec{w} = \rho_I \phi^I \theta + 2\rho_I \phi^I \tilde{\sigma} + 2\rho_I \vec{G}^I \cdot \vec{\tilde{\sigma}}\,, \tag{83}$$

$$\vec{\nabla}r + 2\frac{\partial}{\partial t}\vec{w} + 2\nabla_k w_i^{\ k} = -\rho_I \vec{G}^I \theta + 2\rho_I \phi^I \vec{\tilde{\sigma}} + 2\rho_I G_k^{\ I} \tilde{\sigma}_i^{\ k}\,, \tag{84}$$

$$\frac{\partial}{\partial t}r_{ij} - 2\delta_{ij}\frac{\partial}{\partial t}w + 2\delta_{ij}\vec{\nabla}\cdot\vec{w} = \rho_I \phi^I \theta_{ij} - 2\delta_{ij}\rho_I \phi^I \tilde{\sigma} - 2\delta_{ij}\rho_I \vec{G}^I \cdot \vec{\tilde{\sigma}}\,, \tag{85}$$

$$\nabla_k r_{ij} - 2\delta_{ij}\frac{\partial}{\partial t}\vec{w} - 2\delta_{ij}\nabla_l w_k^{\ l} = \rho_I G_k^{\ I}\theta_{ij} - 2\delta_{ij}\rho_I \phi^I \vec{\tilde{\sigma}} - 2\delta_{ij}\rho_I G_l^{\ I}\tilde{\sigma}_k^{\ l}\,, \tag{86}$$

where $\tilde{\sigma} = \rho_I S_{00}^{\ \ I}$, $\vec{\tilde{\sigma}} = \rho_I S_{0i}^{\ \ I}$, $\tilde{\sigma}_{ij} = \rho_I S_{ij}^{\ \ I}$.

For $\rho_I = \beta_I$ circumstantial relationships, the identity (22) transform into:

$$\frac{\partial}{\partial t}r+2\frac{\partial}{\partial t}s-2\vec{\nabla}\cdot\vec{s}=\rho_I\phi^I\theta+2\rho_I\phi^I\sigma+2\rho_I\vec{G}^I\cdot\vec{\sigma}, \tag{87}$$

$$\vec{\nabla}r+2\frac{\partial}{\partial t}\vec{s}+2\nabla^k s_{ik}=-\rho_I\vec{G}^I\theta+2\rho_I\phi^I\vec{\sigma}+2\rho_I G^{kI}\sigma_{ik}, \tag{88}$$

$$\frac{\partial}{\partial t}r_{ij}-2\delta_{ij}\frac{\partial}{\partial t}s-2\delta_{ij}\vec{\nabla}\cdot\vec{s}=\rho_I\phi^I\theta_{ij}-2\delta_{ij}\rho_I\phi^I\sigma-2\delta_{ij}\rho_I\vec{G}^I\cdot\vec{\sigma}, \tag{89}$$

$$\nabla_k r_{ij}-2\delta_{ij}\frac{\partial}{\partial t}s_k-2\delta_{ij}\nabla^l s_{kl}=\rho_I G_k{}^I\theta_{ij}-2\delta_{ij}\rho_I\phi^I\sigma_k-2\delta_{ij}\rho_I G^{lI}\sigma_{kl}\,. \tag{90}$$

For completeness of this section the kinetic identity (23) will be rewritten in two ways, i.e. either as

$$\beta_I\vec{\nabla}\cdot\vec{E}^I=-\vec{\nabla}\cdot\vec{\sigma}-\frac{\partial}{\partial t}\sigma_k{}^k, \tag{91}$$

$$\beta_I\vec{\nabla}\times\vec{B}^I=\beta_I\frac{\partial}{\partial t}\vec{E}^I-\frac{\partial}{\partial t}\vec{\sigma}+\vec{\nabla}\sigma+\vec{\nabla}\sigma_k{}^k+\nabla_k\sigma^{ik}, \tag{92}$$

or as

$$\rho_I\vec{\nabla}\cdot\vec{E}^I=\frac{\partial}{\partial t}\tilde{\sigma}-\frac{\partial}{\partial t}\theta-\vec{\nabla}\cdot\tilde{\vec{\sigma}}, \tag{93}$$

$$\rho_I\vec{\nabla}\times\vec{B}^I=\rho_I\frac{\partial}{\partial t}\vec{E}^I-\frac{\partial}{\partial t}\tilde{\vec{\sigma}}+\vec{\nabla}\theta+\nabla_k\tilde{\sigma}^{ik}. \tag{94}$$

Eqs. (82)-(94) provide useful kinematic relationships for decoupling the symmetric kinetic sector.

A Global Maxwell equation is introduced. Eq. (4) proposes a step forward in the EM comprehension. From usual Maxwell one notices that the electromagnetic fields behave as a set $\{\vec{E},\vec{B}\}$, and so, a step forward should be to consider a set of potential fields $\{G_{\mu I}\}$. It yields that, besides the historical Maxwell sector there are other EM aspects and laws. However, we should be cautious for considering that Eq. (4) means an extension to Maxwell equations. We should just highlight that up to now we have expressed a mathematical expression derived from wholeness principle with gauge invariance. For joining to the observational Maxwell circle, at least three next steps are required. First, understand on its contributions to the Maxwell limitations; observe on the model consistency (positive hamiltonian, renormalizability, unitarity); for finally, construct its phenomenological implications as the four EM regimes that it proposes. With this plan of work, a next step will be to discuss on Eq. (4) contribution to the potential fields physicality.

5 POTENTIAL FIELDS AS LINES OF FORCE

Maxwell theory was a great triumph. However, although Maxwell equations have been considered as the theory for electric charges [65], their main message was not about properties of the electric charge, but about the existence of a strange constraint in nature. From Franklin (1750) [66] to Hertz (1886), electromagnetism had been a subject developed between the electric charge and the electromagnetic waves observations. But, when the Michelson-Morley experiment (1887) [67] made a rupture, there was something more than the incorporation of optics into the theory of electromagnetism. There was a hidden contract in nature. Implicitly, Maxwell theory had grasped an explicit nature determination: light invariance.

Certainly light invariance is a strange dogma. However, due to the fact that it was discovered experimentally, there is no other choice than to follow it. Theoretical abstractions were derived, and so, astonishingly, they provide consistent predictions. Maxwell equations, Relativity and Lorentz group show that, light invariance is not just a principle supported by an experimental background, but also with theoretical-experimental consequences as electromagnetic waves, matter-energy convertibility, spin etc. Through these results physics become more confident on following such principle, although its nature still remains interrogative. We could say that light invariance is not more a postulate but a determination to do physics. It contains the physics DNA.

Therefore, given light invariance as the basis for exploring the physics phenomena, we have to take into account not only Maxwell and Relativity, but also the Lorentz group (LG). It makes a bridge between space-time and internal symmetries. The Lorentz group global symmetry, $\Lambda^{\mu}{}_{\nu} = \left(e^{\frac{1}{2}\omega^{\alpha\beta}\sum_{\alpha\beta}} \right)^{\mu}{}_{\nu}$, goes further than Lorentz transformations and defines a Lie Algebra. Physical properties obtained from this algebra are showing that there are more consequences coming from group algebra than from Lorentz transformations. By associating covariant fields transforming through LG we are giving rules for systematizing physics. Its global symmetry commands not only the space-time transformations but also develops four features for the physical processes. They are the fields spin representations (irreducible group representations), relativistic equations (Klein-Gordon, Maxwell, Proca, Weyl, Dirac, Rarita-Schwinger, Pauli-Fierz), gauge symmetry (cancelation of spurious spins from LG) and CPT theorem (LG and spin statistics).

The effort of this work has been to discover a path through light invariance to go beyond Maxwell. Observe that, first Relativity produces the equation $E = mc^2$ showing differently from Larmor equation that radiation can come from matter and not only from electric charge, and second, that LG generates Lagrangians that are beyond of Maxwell domain. However, both are not able to add any new term to Maxwell equation. Besides all these new channels for describing physical processes that they have been discovering, Relativity and LG are not able to go further than Maxwell equation. The basic question “electric charge or light - which comes first?” remains with electric charge as primordial, now supported by Dirac equation.

Nevertheless there is a conflict to be considered. First, the Lorentz group clarify the A_μ presence through the $(\frac{1}{2},\frac{1}{2})$ representation. It indicates that, a comprehension beyond Maxwell should be to incorporate the photon field as a physical agent, as LG does with spinors. Second, just considering a massless particle with spin-1, one heuristically associates the expression $\mathrm{W} A^{\mathrm{T}}_{\mu} = 0$, from which, one derives the Maxwell equations $\partial_\mu F^{\mu\nu} = 0$ and $\partial_\mu \tilde{F}^{\mu\nu} = 0$, showing how Maxwell can come out from light first than electric charge. Both work as clauses, determining that potential fields should be more than subsidiary fields derived from Bianchi identities. In practice, as time goes by the potential fields presence became more and more relevant for physics be done. They show how to express the theory degrees of freedom, reveal symmetries as gauge invariance, derive from $L = \frac{1}{2}mv^2 - e\overleftarrow{v}\cdot\overleftarrow{A} + e\phi$ the force basic expression $m\ddot{x} = e\overleftarrow{E} + e\overleftarrow{v}\times\overleftarrow{B}$, implement Maxwell in Quantum Mechanics by redefining the canonical momentum $\vec{p} = m\vec{v} + q\vec{A},$ develop spin presence as a reaction to LG rotations. QED also is based on A_μ field [68]. They are indicating that the electron sees the photon field. Scalar electrodynamics also contains a conserved current with a explicit dependence on potential fields. Phenomenologically, it produce the Bohm-Aharanov [69] and Aharanov-Casher effects [70]. There are also different electro and magneto-optics effects as Faraday, Kerr, Pockels, Cotton-Mouton which have been studied for more than a century showing the interaction of light and matter [71].

Thus, a basic challenge for a model beyond Maxwell is to include the potential fields as physical agents. Something is missing. A new interpretation becomes necessary. There is something more to be done than including monopoles, non-linearity and so on. It is necessary a deeper comprehension on light invariance symmetry. Our view is that there is still a fourth interpretation to be given to light invariance where after Maxwell equation, Relativity and Lorentz group one should consider fields under the wholeness principle. Interpret that the interdependence between $\vec{E} - \vec{B}$ fields indicates something beyond than introducing displacement current and electric charge. Consider that, more than relativize physical entities, light invariance primordially promotes physical associations as $\{\vec{E},\vec{B}\}$, $\{t,\vec{x}\}$, $\{E,\vec{p}\}$, $\{\omega,\vec{k}\}$, $\{\phi,\vec{A}\}$. A fact saying that, for light invariance signature the meaning of association is more fundamental. Based on that, we should go further on the meaning of associations. Interpret that, instead of light invariance be initially correlated to the photon field, it should be to a niche of potential fields.

Considering as fundamental grouping a set of fields $\{A_{\mu I}\}$ at $(\frac{1}{2},\frac{1}{2})$ representation, one supposes that before Maxwell equation, there is a state equation corresponding to such irreducible representation family, which function is to network fields under a same Lorentz nature. As result, for $I = 1$ Maxwell is reobtained, and as new result the photon field is introduced as an explicit variable. Something appears. There are news EM agents. Eqs. (76)-(77), (80)-(81) are showing potential fields as physical variables. These equations explicitly show the presence of $G_{\mu I}$ fields not more as subsidiary fields, but carrying their own self-determination. The *zeitgeist* based on external sources is changed, now one moves to an interpretation where potential fields are their own sources. It develops an $(\frac{1}{2},\frac{1}{2})$ electromagnetism which replaces the

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electric charge by an EM based on light metric and with selfinteracting photons not depending on electric charge [72].

The meaning of potential fields as lines of force appears. Historically, a qualitative change happened when Faraday introduced the concept of lines of force, up to then, physics was catalogued at Newtonian category where the phenomena was only related to matter presence. Faraday was revolutionary, it opened physical laws to a subtle behaviour, and magically, his induction law brought civilization to the industrial age just by correlating fields. However, although the $\vec{E}-\vec{B}$ fields become realistic, the $(\phi-\vec{A})$ fields remained without physicality. Classically there are three sets of equations from where the potential fields presence can be understood as lines of force. They are the equations of motion, conservation laws and forces. Eq. (8) provides three arguments to the potential fields meaning. The first one is that nonlinear equations as $\mathrm{W}G^{I}_{\mu}=J^{I}_{\mu}(G)$ are developing fields as own sources; second, comes out through the conservation laws, where section 6 will identify internal charges $\{J^{N}_{\mu}, J^{I}_{\mu}, J^{c}_{\mu}, J^{B}_{\mu}, J^{J}_{\mu}, \Theta_{\mu\nu}\}$ as fields sources; and by third, the force expressions at Eqs. (160-161) are showing $\rho_I(G)$ and $\vec{J}_I(G)$ as acting elements.

Complementing there are also five phenomenological possibilities for verifying the presence of potential fields lines of force. First, is on the presence of field-balls as $\gamma_{[IJ]}\vec{G}^{J}$. Second, by defining $G_{\mu I}=\partial_{\mu}\varphi_{I}$ one obtains a dynamics just depending on potential fields. Third, through the explicit interaction between EM fields and potential fields as $\vec{E}_I\Phi_I$, $\vec{E}_I\times\vec{G}_I$, $\vec{B}_I\times\vec{G}_I$ and so on as section 8 writes. Fourth, is their presence on the Global Lorentz force as section 7 studies. And by fifth, is that the four EM regimes (electric, photonic, massive, neutral) will be directly depending on potential fields. Their physical existence will be studied at sections 6-8.

A new physicality appears. There is a physics beyond Maxwell to be understood which opens a new passage for exploring nature behaviour through potential fields. It is supported by charges and currents explicitly depending on the potential fields and their self-sources. There is an antireductionist approach, where any field $G_{\mu I}$ interacts with itself and with other fields, and also, with spins, masses and charges derived from the field set $\{G_{\mu I}\}$. As result, similarly to Faraday lines, from Eq. (4) one is able to generate potential fields lines of force. At next section, we will start by exploring more on the energy flux through potential fields lines of force.

6 CONSERVATION LAWS

A physics that precedes electric charge is introduced by a $(\frac{1}{2},\frac{1}{2})$ whole electromagnetism. Different sources from electric charge are creating an EM with potential fields as line of force. It indicates that, although in the historical process, charge came first than light, now this process can be inverted. Although the classical definition of electromagnetism considers the action of electric and magnetic fields over particles with a charge and/or magnetic moment [73], a $(\frac{1}{2},\frac{1}{2})$ whole physics antecedes. there is a systemic flux to be investigated.

In order to go further than electric charge it is necessary to systematize on new sources. They will be showing a fields system working as a group derived from just one gauge parameter. There is a migration carrying various fields and sources where through an whole abelian gauge model new Faraday lines of force get through space in terms of currents flow and continuity equations. Then, we have to explore that right-hand terms developed in previous sections.

The first N-conserved charges are given by

$$\frac{\partial}{\partial t}\rho_I(G)+\vec{\nabla}\cdot\vec{J}_I(G)=\frac{1}{2}m_I^2\left(\frac{\partial}{\partial t}\phi_I+\vec{\nabla}\cdot\vec{G}_I\right) \tag{95}$$

which define a bunch of internal charges and currents expressed by Eq. (47). Notice that the Maxwell sector, $I=1$ and $m_I^2=0$, obeys a first conservation law between nonlinear terms.

Eqs. (80-81) introduce another conservation. It is identified as a collective source, because they are the inhomogeneous terms for collective fields,

$$\frac{\partial}{\partial t}\rho_c+\vec{\nabla}\cdot\vec{J}_c=0$$

with

$$\rho_c=\gamma_{[IJ]}\vec{G}^{I}\cdot\vec{B}^{J}, \quad \vec{J}_c=-\gamma_{[IJ]}\left(\vec{G}^{I}\times\vec{E}^{J}-\phi^{I}\vec{B}^{J}\right). \tag{96}$$

Eqs. (95) and (96) define the internal charges in Eqs. (4) right hand side. These equations are showing that there is the possibility of expressing charges in terms of fields. They support how fields modify and sustain their own existence. In Eq. (95) they promote a nonlinear behaviour, while in Eq. (96) they give rise to fields with a granular nature working as source for collective fields.

A third conservation law comes from collective Bianchi identities. From Eq. (20), one gets

$$\partial_\mu J^\mu_B = 0, \quad \text{where the source is} \quad J^B_\mu = \varepsilon_{\mu\nu\rho\sigma}\gamma_{[IJ]}G^\nu_I G^{\rho\sigma}_J \,. \tag{97}$$

Notice that the symmetric Bianchi identities are algebraic, they do not correspond to any charge.

Noether current at Eq. (24) expresses a fourth continuity equation [74]. It provides a charge conservation associated to the group of fields. It is a step forward to Maxwell displacement current and corresponding electric charge conservation by including an internal charge conservation.

A relevant fact is that these four equations are not expressing the electric charge, but charges and three-current densities which are depending on fields. These continuity equations are supporting on the existence of abelian fields depending on internal sources. Observe that, at usual Maxwell theory only entities as the Poynting vector are expressed through fields, whereas here charges and currents are also defined. The relevant aspect is that these self-internal charges act as origin for EM fields: $\rho_I(G)$ acts as source for granular fields and $\rho_c(G)$ acts as source for collective fields. Consequently, given a photonic charge $\rho_1(G)$ one obtains that photonic EM fields $\vec{E}_1 - \vec{B}_1$ are generated. It yields an electromagnetism where light is not more passive, but is producing its own fields.

The fifth conservation law is given by the second type of global conservation associated to the improved energy momentum tensor

$$\partial_\mu \Theta^{\mu\nu} = 0\,. \tag{98}$$

Although Eq. (98) is guaranteed by translation invariance, one should calculate it in order to verify the consistency of the equations being introduced. Deriving the energy momentum tensor $T_{\mu\nu}$ of the global electromagnetic field, we get

$$T_{\mu\nu} = T^A_{\mu\nu} + T^S_{\mu\nu} + T^{ST}_{\mu\nu} + T^M_{\mu\nu} \tag{99}$$

with

$$T^A_{\mu\nu} = 4b_I Z_{[\mu\rho]}\partial_\nu G^{\rho I} - \eta_{\mu\nu}L^A\,, \tag{100}$$

$$T^S_{\mu\nu} = 4\beta_I Z_{(\mu\rho)}\partial_\nu G^{\rho I} + 4\rho_I Z^{(\rho}{}_{\rho)}\partial_\nu G^I_\mu - \eta_{\mu\nu}L^S\,, \tag{101}$$

$$T^{ST}_{\mu\nu} = 2b_K E_{\mu\alpha\beta\rho} z^{[\alpha\beta]}\partial_\nu G^{\rho K} + 2b_K E_{\mu\alpha\beta\rho} G^{\alpha K}\partial_\nu z^{[\rho\beta]} - \eta_{\mu\nu}L^{ST} \tag{102}$$

$$T^M_{\mu\nu} = \eta_{\mu\nu} m_I^2 G_{\rho I} G^{\rho I} \tag{103}$$

and

$$\partial^\mu\left(T^A_{\mu\nu} + T^S_{\mu\nu} + T^{ST}_{\mu\nu} + T^M_{\mu\nu}\right) = 0\,. \tag{104}$$

Working out the improved tensor we obtain

$$\Theta_{\mu\nu} = \Theta^A_{\mu\nu} + \Theta^S_{\mu\nu} + \Theta^{ST}_{\mu\nu} + \Theta^M_{\mu\nu} \tag{105}$$

with

$$\Theta^A_{\mu\nu} = 4Z_{[\mu\rho]}Z_{[\nu}{}^{\rho]} - \eta_{\mu\nu}L^A \tag{106}$$

$$\Theta^S_{\mu\nu} = 4Z^{(\rho}{}_{\rho)} w_{(\mu\nu)} + 4Z_{(\mu\rho)}\left(\beta_I S_\nu{}^{\rho I} + z_{(\nu}{}^{\rho)}\right) - 4\rho_I\left(\left(\partial_\mu Z^{(\rho}{}_{\rho)}\right)G^I_\nu + \left(\partial_\nu Z^{(\rho}{}_{\rho)}\right)G^I_\mu\right)$$
$$-4\beta_I\left(\partial^\rho\left(Z_{(\mu\rho)}G^I_\nu\right) + \partial^\rho\left(Z_{(\nu\rho)}G^I_\mu\right)\right) + 4\beta_I\partial_\rho\left(Z_{(\mu\nu)}G^{\rho I}\right) + 4\rho_I\eta_{\mu\nu}\partial_\rho\left(Z^{(\sigma}{}_{\sigma)}G^{\rho I}\right)$$
$$-\eta_{\mu\nu}L^S \tag{107}$$

$$\Theta^{ST}_{\mu\nu} = 4\eta E_{\mu\alpha\rho\sigma} b_I G_\nu{}^{\alpha I} z^{[\rho\sigma]} + 4\eta E_{\mu\alpha\rho\sigma} z_{[\nu}{}^{\alpha]} Z^{[\rho\sigma]} - \eta_{\mu\nu}L^{ST} \tag{108}$$

$$\Theta^M_{\mu\nu} = -2m_I^2 G_{\mu I} G_\nu{}^I + \eta_{\mu\nu} m_I^2 G_{\rho I} G^{\rho I} \ . \tag{109}$$

Given that gauge invariance is not broken by the introduction of a gauge fixing term we will include it in Eq. (105),

$$\Theta^{gf}_{\mu\nu} = \eta_{\mu\nu}\xi_{IJ}\left(\partial_\rho G^{\rho I}\right)\left(\partial_\sigma G^{\sigma J}\right) + 2\eta_{\mu\nu}\xi_{IJ} G_\rho{}^I \partial^\rho\partial_\sigma G^{\sigma J} - 2\xi_{IJ} G_\mu{}^I \partial_\nu\partial_\rho G^{\rho J} +$$
$$-2\xi_{IJ} G_\nu{}^I \partial_\mu\partial_\rho G^{\rho J} \ . \tag{110}$$

Verifying the property of symmetry, one checks that [73]

$$\Theta_{\mu\nu} = \Theta_{\nu\mu} \ , \tag{111}$$

as obviously expected. Taking the derivative of Eq. (105), the conservation law Eq. (98) is explicitly shown. The dilatation current $\partial_\mu D^\mu = \Theta_\mu{}^\mu$ can be worked out into

$$\Theta_\mu{}^\mu = 2m_I^2 G_{\mu I} G^{\mu I} + \partial_\mu\left(4\beta_I Z_{(\rho}{}^{\rho)} G^{\mu I} - 8\beta_I Z^{(\mu}{}_{\rho)} G^{\rho I} + \right.$$
$$\left. + 8\rho_I Z_{(\rho}{}^{\rho)} G^{\mu I} + 4\xi_{IJ} G^{\mu I}\partial_\rho G^{\rho J}\right). \tag{112}$$

Considering that the total derivative can be reabsorbed, one gets that only the mass term breaks the dilatation invariance, although the Poincaré invariance is preserved.

For physical reasons the energy momentum tensor should be rewritten in terms of transverse and longitudinal pieces. It reads

$$\theta_{\mu\nu} = \theta^T_{\mu\nu} + \theta^L_{\mu\nu} + \theta^{I-A}_{\mu\nu} + \theta^{I-S}_{\mu\nu} + \theta^{I-ST}_{\mu\nu} \tag{113}$$

where

$$\theta^T_{\mu\nu} = 4G_{\mu\rho I} G_\nu{}^{\rho I} - \eta_{\mu\nu}L^T \ , \tag{114}$$

$$\theta^L_{\mu\nu} = \left(4\beta_I\beta_J + 4\beta_I\rho_J + 4\beta_J\rho_I + 16\rho_I\rho_J\right) S_{\mu\nu}{}^I S_\sigma{}^{\sigma J}$$
$$-\left(4\beta_I\beta_J + 4\beta_I\rho_J + 4\beta_J\rho_I + 16\rho_I\rho_J\right)\partial_\mu\left(S_\sigma{}^{\sigma J} G_\nu{}^I\right)$$
$$-\left(4\beta_I\beta_J + 4\beta_I\rho_J + 4\beta_J\rho_I + 16\rho_I\rho_J\right)\partial_\nu\left(S_\sigma{}^{\sigma J} G_\mu{}^I\right)$$
$$+\left(4\beta_I\beta_J + 4\beta_I\rho_J + 4\beta_J\rho_I + 16\rho_I\rho_J\right) g_{\mu\nu}\partial^\rho\left(S_\sigma{}^{\sigma J} G_\rho{}^I\right) - \eta_{\mu\nu}L^L \ , \tag{115}$$

$$\theta^{I-A}_{\mu\nu} = 4b_I G_{\mu\rho}{}^I z_{[\nu}{}^{\rho]} + 4b_I G_{\nu\rho}{}^I z_{[\mu}{}^{\rho]}] + 4z_{[\mu\rho]} z_{[\nu}{}^{\rho]} - \eta_{\mu\nu}L^{I-A} \ , \tag{116}$$

$$\theta_{\mu\nu}^{I-S} = 4\beta_I \left(z_{(\mu\rho)} + g_{\mu\rho} w_{(\sigma}{}^{\sigma)}\right) S_\nu{}^{\rho I} + 4Z_{(\mu\rho)} z_{(\nu}{}^{\rho)}$$
$$+4\rho_I \left(z_{(\rho}{}^{\rho)} + 4w_{(\sigma}{}^{\sigma)}\right) S_{\mu\nu}{}^I + 4Z_{(\rho}{}^{\rho)} w_{(\mu\nu)}$$
$$-4\beta_I \partial^\rho \left(\left(z_{(\mu\rho)} + g_{\mu\rho} w_{(\sigma}{}^{\sigma)}\right) G_\nu{}^I\right) - 4\beta_I \partial^\rho \left(\left(z_{(\nu\rho)} + g_{\nu\rho} w_{(\sigma}{}^{\sigma)}\right) G_\mu{}^I\right)$$
$$+4\beta_I \partial^\rho \left(\left(z_{(\mu\nu)} + g_{\mu\nu} w_{(\sigma}{}^{\sigma)}\right) G_\rho{}^I\right) - 4\rho_I \partial_\mu \left(\left(z_{(\rho}{}^{\rho)} + 4w_{(\sigma}{}^{\sigma)}\right) G_\nu{}^I\right)$$
$$-4\rho_I \partial_\nu \left(\left(z_{(\rho}{}^{\rho)} + 4w_{(\sigma}{}^{\sigma)}\right) G_\mu{}^I\right) + 4\rho_I \eta_{\mu\nu} \partial_\rho \left(\left(z_{(\rho}{}^{\rho)} + 4w_{(\sigma}{}^{\sigma)}\right) G^{\rho I}\right) - \eta_{\mu\nu} L^{I-S} ,$$

and

$$\theta_{\mu\nu}^{I-ST} = \theta_{\mu\nu}^{ST} . \tag{117}$$

There is a flow described through energy density, Poynting vector and stress tensor. The differential laws are

$$\frac{\partial}{\partial t} U + \vec{\nabla} \cdot \vec{S} = 0, \qquad \frac{\partial}{\partial t} S_i - \partial_j T_{ij} = 0 . \tag{118}$$

The energy density of the system is

$$U = U_T + U_L + U_M + U_I + U_{gf} \tag{119}$$

where

$$U_T = 2\vec{E}_I \cdot \vec{E}^I + 2\vec{B}_I \cdot \vec{B}^I ,$$

$$U_L = \sigma_k{}^k \sigma_l{}^l + \theta_{kl}\theta^{kl} + \theta^2 - 3\sigma^2 - 2\sigma\sigma_k{}^k - 4\sigma_k{}^k \tilde{\sigma} - 4\sigma\tilde{\sigma} + 2\sigma_{kl}\theta^{kl} - 2\sigma\theta - 4\tilde{\sigma}\theta_k{}^k$$
$$-4\tilde{\sigma}\theta + 4\frac{\partial}{\partial t}\left(\left(\beta_I + \rho_I\right)\left(\sigma_k{}^k + \sigma + \theta\right) + \rho_I \theta_k{}^k\right)\phi^I$$
$$-4\vec{\nabla} \cdot \left(\left(\beta_I + \rho_I\right)\left(\sigma_k{}^k + \sigma + \theta\right) + \rho_I \theta_k{}^k\right)\vec{G}^I , \tag{120}$$

$$U_M = m_I^2 \left(\phi_I \phi^I + \vec{G}_I \cdot \vec{G}^I\right), \tag{121}$$

and,

$$U_I = U_{I-A} + U_{I-S} + U_{ST} , \tag{122}$$

for

$$U_{I-A} = 4b_I \vec{E}^I \cdot \vec{e} + 4b_I \vec{B}^I \cdot \vec{b} + 2e \cdot e + 2b \cdot b , \tag{123}$$

$$U_{I-S} = 2\sigma_{kl} s^{kl} + 4\vec{\sigma} \cdot \vec{s} - 6\sigma s + 2\sigma_{kl} r^{kl} - 2\sigma r - 4\sigma_k{}^k w - 4\sigma w - 4\tilde{\sigma} s_k{}^k - 4\tilde{\sigma} s$$
$$-4\tilde{\sigma} r_k{}^k - 4\tilde{\sigma} r + 2\theta_{kl} s^{kl} - 2\theta s + 2\theta_{kl} r^{kl} + 2\theta r - 4\theta_k{}^k w - 4\theta w + s_{kl} s^{kl}$$
$$+2\vec{s} \cdot \vec{s} - 3s^2 + 2s_{kl} r^{kl} - 2sr - 4s_k{}^k w - 4sw + r_{kl} r^{kl} + r^2 - 4r_k{}^k w - 4rw$$

$$+4\frac{\partial}{\partial t}\left(\rho_I s_k{}^k \phi^I + (\beta_I+\rho_I)s\phi^I + \rho_I r_k{}^k \phi^I + (\beta_I+\rho_I)r\phi^I\right)$$

$$-4\vec{\nabla}\cdot\left(\rho_I s_k{}^k \vec{G}^I + 4\beta_I \vec{s}\phi^I + (\beta_I+\rho_I)s\vec{G}^I + \rho_I r_k{}^k \vec{G}^I + (\beta_I+\rho_I)r\vec{G}^I\right), \tag{124}$$

$$U_{ST} = 0\,, \tag{125}$$

and,

$$U_{gf} = \xi_{IJ}\left(\vec{\nabla}\cdot\vec{G}^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right) - \xi_{IJ}\left(\frac{\partial}{\partial t}\phi^I\right)\left(\frac{\partial}{\partial t}\phi^J\right) + 2\xi_{IJ}\phi^I\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\phi^J + \vec{\nabla}\cdot\vec{G}^J\right)$$

$$-2\xi_{IJ}\vec{\nabla}\cdot\left(\vec{G}^I\left(\frac{\partial}{\partial t}\phi^J\right) + \vec{G}^I\left(\vec{\nabla}\cdot\vec{G}^J\right)\right). \tag{126}$$

The total flux density is

$$\vec{S} = \vec{S}_T + \vec{S}_L + \vec{S}_M + \vec{S}_I + \vec{S}_{gf}\,, \tag{127}$$

where

$$\vec{S}_T = 4\vec{E}_I \times \vec{B}^I\,, \tag{128}$$

$$\vec{S}_L = 4\vec{\sigma}\sigma_k{}^k + 4\sigma\vec{\sigma} + 4\sigma_k{}^k \vec{\vec{\sigma}} + 4\sigma\vec{\vec{\sigma}} + 4\sigma_k\theta_i{}^k + 4\vec{\vec{\sigma}}\theta_k{}^k + 4\vec{\vec{\sigma}}\theta$$

$$+4\frac{\partial}{\partial t}\left((\beta_I+\rho_I)\sigma_k{}^k\vec{G}^I + (\beta_I+\rho_I)\sigma\vec{G}^I + \beta_I\theta_k{}^k\vec{G}^I + (\beta_I+\rho_I)\theta\vec{G}^I\right)$$

$$-4\vec{\nabla}\left((\beta_I+\rho_I)\sigma_k{}^k\phi^I + (\beta_I+\rho_I)\sigma\phi^I + \rho_I\theta_k{}^k\phi^I + \rho_I\theta\phi^I\right) - 4\nabla_k\left(\beta_I\theta_i{}^k\phi^I\right), \tag{129}$$

$$\vec{S}_M = 2m_I^2\phi_I\vec{G}^I\,, \tag{130}$$

and,

$$\vec{S}_I = \vec{S}_{I-A} + \vec{S}_{I-S} + \vec{S}_{ST}\,, \tag{131}$$

for

$$\vec{S}_{I-A} = 4b_I\vec{E}^I \times \vec{b} + 4b_I\vec{B}^I \times \vec{e} + 4e \times b\,, \tag{132}$$

$$\vec{S}_{I-S} = 4\sigma_{ik}s^k + 4\vec{\sigma}s + 4\sigma_k s_i{}^k + 4\sigma\vec{s} + 4\sigma_k r_i{}^k + 4\sigma_k{}^k\vec{w} + 4\sigma\vec{w} + 4\vec{\vec{\sigma}}s_k{}^k + 4\vec{\vec{\sigma}}s + 4\vec{\vec{\sigma}}r$$

$$+4\vec{\vec{\sigma}}r_k{}^k + 4\theta_{ik}s^k + 4\theta_k{}^k\vec{w} + 4\theta\vec{w} + 4s_{ik}s^k + 4s\vec{s} + 4s_k r_i{}^k + 4s_k{}^k\vec{w} + 4s\vec{w}$$

$$+4r_k{}^k\vec{w} + 4r\vec{w} + 4\frac{\partial}{\partial t}\left(\rho_I s_k{}^k\vec{G}^I + (\beta_I+\rho_I)s\vec{G}^I + \rho_I r_k{}^k\vec{G}^I + (\beta_I+\rho_I)r\vec{G}^I\right)$$

$$-4\rho_I\vec{\nabla}\left(r_k{}^k\phi^I + r\phi^I + s_k{}^k\phi^I + s\phi^I\right) + 4\beta_I\nabla_k\left(\vec{s}G^{kI} + s^k\vec{G}^I - r_i{}^k\phi^I - s_i{}^k\phi^I\right), \tag{133}$$

$$\vec{S}_{ST}=0\,, \tag{134}$$

and,

$$\vec{S}_{gf}=2\xi_{IJ}\vec{G}^{I}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\phi^{J}+\vec{\nabla}\cdot\vec{G}^{J}\right)-2\xi_{IJ}\phi^{I}\vec{\nabla}\left(\frac{\partial}{\partial t}\phi^{J}+\vec{\nabla}\cdot\vec{G}^{J}\right). \tag{135}$$

The global Maxwell's stress tensor is

$$T^{ij}=T_{T}^{ij}+T_{L}^{ij}+T_{M}^{ij}+T_{I}^{ij}+T_{gf}^{ij}\,, \tag{136}$$

with

$$T_{T}^{ij}=4E^{i}{}_{I}E^{jI}+4B^{i}{}_{I}B^{jI}-\delta^{ij}\left(2\vec{E}_{I}\cdot\vec{E}^{I}+2\vec{B}_{I}\cdot\vec{B}^{I}\right), \tag{137}$$

$$\begin{aligned}
T_{L}^{ij}&=4\sigma^{ij}\sigma_{k}{}^{k}+4\sigma\sigma^{ij}+4\sigma_{k}{}^{k}\tilde{\sigma}^{ij}+4\sigma\tilde{\sigma}^{ij}+4\sigma^{j}{}_{k}\theta^{ik}+4\tilde{\sigma}^{ij}\theta_{k}{}^{k}+4\tilde{\sigma}^{ij}\theta\\
&-4\nabla^{i}\left((\beta_{I}+\rho_{I})\sigma_{k}{}^{k}G^{jI}+(\beta_{I}+\rho_{I})\sigma G^{jI}+\rho_{I}\theta_{k}{}^{k}G^{jI}+\rho_{I}\theta G^{jI}\right)\\
&-4\nabla^{j}\left((\beta_{I}+\rho_{I})\sigma_{k}{}^{k}G^{iI}+(\beta_{I}+\rho_{I})\sigma G^{iI}+\rho_{I}\theta_{k}{}^{k}G^{iI}+\rho_{I}\theta G^{iI}\right)\\
&+4\beta_{I}\nabla_{k}\left(\theta^{ij}G^{kI}-\theta^{ik}G^{jI}-\theta^{jk}G^{iI}\right)+4\beta_{I}\frac{\partial}{\partial t}\left(\theta^{ij}\phi^{I}\right)\\
&+\eta^{ij}\Big(-\sigma_{k}{}^{k}\sigma_{l}{}^{l}-2\sigma\sigma_{k}{}^{k}-\sigma^{2}-2\sigma_{kl}\theta^{kl}-2\sigma\theta-\theta_{kl}\theta^{kl}-\theta^{2}\\
&+4\frac{\partial}{\partial t}\left((\beta_{I}+\rho_{I})\sigma_{k}{}^{k}\phi^{I}+(\beta_{I}+\rho_{I})\sigma\phi^{I}+\rho_{I}\theta_{k}{}^{k}\phi^{I}+\rho_{I}\theta\phi^{I}\right)\\
&+4\vec{\nabla}\cdot\left((\beta_{I}+\rho_{I})\sigma_{k}{}^{k}\vec{G}^{I}+(\beta_{I}+\rho_{I})\sigma\vec{G}^{I}+\rho_{I}\theta_{k}{}^{k}\vec{G}^{I}+\rho_{I}\theta\vec{G}^{I}\right)\Big),
\end{aligned} \tag{138}$$

$$T_{M}^{ij}=-2m_{I}^{2}G^{i}{}_{I}G^{jI}+\eta^{ij}\left(m_{I}^{2}\phi_{I}\phi^{I}-m_{I}^{2}\vec{G}_{I}\cdot\vec{G}^{I}\right), \tag{139}$$

and,

$$T_{I}^{ij}=T_{I-A}^{ij}+T_{I-S}^{ij}+T_{ST}^{ij}\,, \tag{140}$$

for

$$\begin{aligned}
T_{I-A}^{ij}&=4b_{I}\left(E^{iI}e^{j}+E^{jI}e^{i}\right)+4b_{I}\left(B^{iI}\vec{b}^{j}+B^{jI}\vec{b}^{i}\right)+4e^{i}e^{j}+4\vec{b}^{i}\vec{b}^{j}\\
&-\delta^{ij}\left(4b_{I}\vec{E}^{I}\cdot\vec{e}+4b_{I}\vec{B}^{I}\cdot\vec{b}+2\vec{e}\cdot\vec{e}+2\vec{b}\cdot\vec{b}\right),
\end{aligned} \tag{141}$$

$$\begin{aligned}
T_{I-S}^{ij}&=4\sigma^{i}{}_{k}s^{jk}+4\sigma^{j}{}_{k}s^{ik}+4\sigma^{i}s^{j}+4\sigma^{j}s^{i}+4\sigma^{j}{}_{k}r^{ik}+4\sigma_{k}{}^{k}w^{ij}+4\sigma w^{ij}+4\tilde{\sigma}^{ij}s_{k}{}^{k}\\
&+4\tilde{\sigma}^{ij}s+4\tilde{\sigma}^{ij}r_{k}{}^{k}+4\tilde{\sigma}^{ij}r+4\theta^{i}{}_{k}s^{jk}+4\theta_{k}{}^{k}w^{ij}+4\theta w^{ij}+4s^{i}{}_{k}s^{jk}+4s^{i}s^{j}\\
&+4s^{j}{}_{k}r^{ik}+4s_{k}{}^{k}w^{ij}+4sw^{ij}+4r_{k}{}^{k}w^{ij}+4rw^{ij}\\
&+4\beta_{I}\frac{\partial}{\partial t}\left(s^{ij}\phi^{I}-s^{i}G^{jI}-s^{j}G^{iI}+r^{ij}\phi^{I}\right)
\end{aligned}$$

$$
\begin{aligned}
&-4\rho_I \nabla^i \left(s_k{}^k G^{jI} + sG^{jI} + r_k{}^k G^{jI} + rG^{jI}\right)\\
&-4\rho_I \nabla^j \left(s_k{}^k G^{iI} + sG^{iI} + r_k{}^k G^{iI} + rG^{iI}\right)\\
&+4\beta_I \nabla_k \left(s^{ij} G^{kI} - s^{ik} G^{jI} - s^{jk} G^{iI} + r^{ij} G^{kI} - r^{ik} G^{jI} - r^{jk} G^{iI}\right)\\
&+\eta^{ij}\left(-2\sigma_{kl} s^{kl} + 4\vec{\sigma}\cdot\vec{s} - 2\sigma s - 2\sigma_{kl} r^{kl} - 2\sigma r - 2\theta_{kl} s^{kl} - 2\theta s - 2\theta_{kl} r^{kl}\right.\\
&-2\theta r - s_{kl} s^{kl} + 2\vec{s}\cdot\vec{s} - s^2 - 2s_{kl} r^{kl} - 2sr - r_{kl} r^{kl} - r^2\\
&\left.+4\rho_I \frac{\partial}{\partial t}\left(s_k{}^k \phi^I + s\phi^I + r_k{}^k \phi^I + r\phi^I\right) + 4\rho_I \vec{\nabla}\cdot\left(s_k{}^k \vec{G}^I + s\vec{G}^I + r_k{}^k \vec{G}^I + r\vec{G}^I\right)\right),
\end{aligned}
\tag{142}
$$

$$
T_{ST}^{ij} = 0\,, \tag{143}
$$

and,

$$
\begin{aligned}
T_{gf}^{ij} &= 2\xi_{IJ}\left(G^{iI}\nabla^j + G^{jI}\nabla^i\right)\left(\frac{\partial}{\partial t}\phi^J + \vec{\nabla}\cdot\vec{G}^J\right) + \eta^{ij}\left(-\xi_{IJ}\left(\frac{\partial}{\partial t}\phi^I\right)\left(\frac{\partial}{\partial t}\phi^J\right)\right.\\
&-2\xi_{IJ}\left(\frac{\partial}{\partial t}\phi^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right) - \xi_{IJ}\left(\vec{\nabla}\cdot\vec{G}^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right)\\
&\left.-2\xi_{IJ}\left(\phi^I\frac{\partial}{\partial t} + \vec{G}^I\cdot\vec{\nabla}\right)\left(\frac{\partial}{\partial t}\phi^J + \vec{\nabla}\cdot\vec{G}^J\right)\right).
\end{aligned}
\tag{144}
$$

These conservation laws are manifesting the meaning of the whole. New features are introduced through an antireductionistic physics through a global system organized by the abelian gauge parameter. First, Eqs. (95)-(144) are physics laws for a joined system with a global physics compensation between the set of observables; second, that these internal charges are expressing on the existence of 'fields stones' as J_μ^I, and later, Eq. (161) show how these fields stones act as forces (remember that the passage from Newton laws to Maxwell equations brought another nature behaviour through fields, and so, as a next contribution to this fields environment, the above equations introduce fields as source); third, they are explicitly showing that gauge symmetry goes beyond to the usual lemma where the number conservation laws must be equal to the number of generators [75]; fourth, they are flowing a systemic behaviour consistent with the fact that internal charges and currents expressions emerge earlier than the introduction of the electric charge; and finally, new aspects that extend Maxwell theory come into play which will be analyzed below.

We are now going to consider three cases. A first concrete consequence is to compare the polarization and magnetization vectors coming from this fourth interpretation to light invariance and the usual Maxwell constitutive approach. Comparing literature the $\vec{e}$ - $\vec{b}$ contributions in Eq. (4) differ from the standard case for $\vec{P}$ - $\vec{M}$ [76]. Although both pairs presence coincide at left hand side of Global Maxwell, discrepancies appear for the corresponding expressions of energy, energy-flow and stress tensor. We have denoted in italic these new contributions. They are showing that by describing polarization and magnetization vectors from first principles in the original Lagrangian makes a difference with respect to introducing them by hand in the equations of motion. As an example of a new physical contribution, Eq. (132) shows a term $\vec{e}\times\vec{b}$ for changing momentum.

Another consideration is on the positivity of the hamiltonian. The existence of a stable ground state is required for perturbative excitations to be well defined. An unbounded-from-below hamiltonian would mean the existence of ghost-tachyons and a divergent partition function. For analysing on these aspects, it is necessary to consider just the kinetic terms. From Eq. (119), one reads that the transverse and massive sectors are positively defined. However the longitudinal sector has mixed contributions. An analysis is required. For this, one describes U_L in function of potential fields. When you discard in U_L the positive terms and the divergence term, which can be integrated over, you get

$$U_L \sim 8\beta_I\rho_J\left(\vec{\nabla}\cdot\vec{G}^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right)+\left(8\beta_I\beta_J-8\beta_I\rho_J+8\beta_J\rho_I+16\rho_I\rho_J\right)\frac{\partial}{\partial t}\left(\phi^I\vec{\nabla}\cdot\vec{G}^J\right)$$

$$+\left(8\beta_I\beta_J+8\beta_I\rho_J+8\beta_J\rho_I-16\rho_I\rho_J\right)\frac{\partial}{\partial t}\left(\phi^I\frac{\partial}{\partial t}\phi^J\right)$$

$$+\left(-8\beta_I\beta_J+8\beta_I\rho_J-24\beta_J\rho_I-32\rho_I\rho_J\right)\left(\frac{\partial}{\partial t}\phi^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right)$$

$$+\left(-12\beta_I\beta_J-24\beta_J\rho_I+32\rho_I\rho_J\right)\left(\frac{\partial}{\partial t}\phi^I\right)\left(\frac{\partial}{\partial t}\phi^J\right). \quad (145)$$

Considering the relationship $\beta_I = 2\rho_I$

$$U_L \sim 6\rho_I\rho_J\left(\vec{\nabla}\cdot\vec{G}^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right)+48\rho_I\rho_J\frac{\partial}{\partial t}\left(\phi^I\vec{\nabla}\cdot\vec{G}^J\right)+48\rho_I\rho_J\frac{\partial}{\partial t}\left(\phi^I\frac{\partial}{\partial t}\phi^J\right)$$

$$-96\rho_I\rho_J\left(\frac{\partial}{\partial t}\phi^I\right)\left(\vec{\nabla}\cdot\vec{G}^J\right)-64\rho_I\rho_J\left(\frac{\partial}{\partial t}\phi^I\right)\left(\frac{\partial}{\partial t}\phi^J\right) \quad (146)$$

and the Lorentz condition, $\rho_I\vec{\nabla}\cdot\vec{G}^I=-\rho_I\frac{\partial}{\partial t}\phi^I$, one gets

$$U_L \sim 48\left(\frac{\partial}{\partial t}\Phi\right)^2, \quad \Phi\equiv\rho_I\phi^I\ . \quad (147)$$

Another proof come outs from the following vectorial identities:

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\vec{E}_I{}^2+\vec{B}_I{}^2\right)+\vec{\nabla}\cdot\left(\vec{E}_I\times\vec{B}^I\right)=\vec{E}^I\cdot\left(b_I\vec{\nabla}\times\vec{b}-b_I\frac{\partial}{\partial t}\vec{e}-J_I\right), \quad (148)$$

$$\frac{1}{2}b_I\frac{\partial}{\partial t}\left(\vec{e}^{\,2}+\vec{b}^{\,2}\right)+b_I\vec{\nabla}\cdot\left(\vec{e}\times\vec{b}\right)=\vec{e}\cdot\left(\vec{\nabla}\times\vec{B}_I-\frac{\partial}{\partial t}\vec{E}_I\right)-b_I\vec{b}\cdot\vec{J}_c, \quad (149)$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left(\left(\vec{E}_I+b_I\vec{e}\right)^2+\left(\vec{B}_I+b_I\vec{b}\right)^2\right)+\vec{\nabla}\cdot\left(\left(\vec{E}^I+b^I\vec{e}\right)\times\left(\vec{B}_I+b_I\vec{b}\right)\right)=$$

$$=-b_I\left(\vec{B}^I+b^I\vec{b}\right)\cdot\vec{J}_c-\left(\vec{E}^I+b^I\vec{e}\right)\cdot J_I \quad (150)$$

where $J_I=\vec{J}_I(G)+\vec{j}_I-\frac{1}{2}m_I^2\vec{G}_I$. Substituting Eqs. (6) in Eq. (118), one gets

$$\frac{\partial}{\partial t}U'+\vec{\nabla}\cdot\vec{S}'=\rho_{source}$$

with

$$U'=U_L+U_M+U_{I-S}+U_{gf}\ , \qquad \vec{S}'=\vec{S}_L+\vec{S}_M+\vec{S}_{I-S}+\vec{S}_{gf}\ ,$$

$$\rho_{source}=-4b_I\left(\vec{B}^I+b^I\vec{b}\right)\cdot\vec{J}_c+4\left(\vec{E}^I+b^I\vec{e}\right)\cdot J_I\ , \quad (151)$$

with the normalized condition $b_Ib^I=1$. Eq. (151) provides a more general proof for hamiltonian positivity by isolating the transverse and antisymmetric sectors. Integrating over time one gets an expression for U_L which does not depend on kinetic terms.

The third result is on the consequences from the longitudinal decoupling on the components of the energy-momentum tensor. Based on the conserved currents derived circumstantially at section 3, implying $\partial_\mu G_I^\mu = 0$, a consistent result on freezing the kinetic longitudinal term is obtained. Appendix C shows that longitudinal physical entities as charges and currents, energy, Poynting vector, stress tensor are cancelled. As consequence, the unitarity for this abelian whole model is considered. After Eqs. (145)-(151) be showing on a positive Hamiltonian, renormalizability be verified through power counting and Ward identity [52], [53], this longitudinal decoupling works as a first argument for unitarity (a further perturbative study is under development).

The sixth conservation law concerns to the total angular momentum $J_{k\lambda}^\mu = L_{k\lambda}^\mu + S_{k\lambda}^\mu$. It contains the orbital angular momentum plus the spin contribution. It gives $\partial_\mu J_{k\lambda}^\mu = 0$ where $L_{k\lambda}^\mu = x_k \theta_\lambda^\mu - x_\lambda \theta_k^\mu$, $S_{k\lambda}^\mu = \frac{\partial L}{\partial \partial_\mu G_{\rho I}} (\Sigma_{k\lambda})_\rho^\sigma G_{\rho I}$ which will be worked out in a next work where we are going to study on a neutral electromagnetism under spin influence.

Summarizing, different charges are derived from this antireductionist abelian gauge symmetry. They are saying that there is something more than electric charge. Usually there are two types of charges basing gauge theories which are the Noether and topological charges [77]. The new fact here is that there is a branch of charges. They are revealing how from the symmetry management one derives directive charges as $\{J_\mu^N, J_\mu^C, J_\mu^B, J_\mu^J, \Theta_{\mu\nu}\}$ and circumstantial charges $\{J_\mu^I\}$. As consequence, there is something more than electric charge for the EM phenomena be described. While displacement current allowed electric charge conservation, the introduction of the fields set propitiates opportunities for various interactions $J_\mu^A G_{I,A}^\mu$, where A means an index expressing these various conserved charges possibilities. As result, they are characterizing a behaviour where individual fields are associated to systemic charges.

7 GLOBAL LORENTZ FORCE

This section will deal with the meaning of force. Special relativity defines force through the equation $\partial_\mu \Theta^{\mu\nu} = f^\nu$ which means an exchange of energy and momentum between two systems made by fields and external sources. Following this methodology we have to investigate on Eq. (8) consequences. For coupling the interaction of fields with external sources one has to discuss the gauge invariance alternatives. The simplest scalar interaction term involving N-external currents $j_{\mu I}$ is of the form $G^{\mu I} j_{\mu I}$, where the $j_{\mu I}$ source does not necessarily correspond to the electric charge. This leads us to the following Lagrangian density

$$L = L_0 + G^{\mu I} j_{\mu I} \tag{152}$$

Considering the basis $\{D, X_i\}$, the sources that produce the fields are given by $j_{\mu I} = \Omega_{1I} j_\mu + \Omega_{iI} j_\mu{}^i$ where the external current j_μ is coupled to the D_μ genuine gauge field, while $j_\mu{}^i$ to $X_\mu{}^i$. Then, the Noether theorem provides a continuity equation $\partial_\mu j^\mu = 0$ which is enough for the complete Lagrangian to be gauge invariant; therefore the $j_\mu{}^i$ currents are not required to be conserved. The corresponding field equations become

$$\partial_\nu \left(G^{\nu\mu}{}_I + b_I z^{[\nu\mu]} \right) + \frac{1}{2} m_I^2 G^\mu{}_I = J^\mu{}_I(G) + j^\mu{}_I \,, \tag{153}$$

and as result, the force equation is derived from Eq. (152). From Appendix D, one gets

$$f^\nu = f_L^\nu + f_M^\nu + f_E^\nu \tag{154}$$

with

$$f_L^\nu = 4G^{\nu\mu I} j_{\mu I}, \qquad f_M^\nu = -2m_I^2 G^\nu{}_I \left(\partial^\mu G_\mu{}^I\right), \qquad f_E^\nu = 4G^{\nu I}\left(\partial^\mu J_{\mu I}(G)\right) \tag{155}$$

Eq. (154) is the fundamental equation. Besides the usual Lorentz type, it gives rise to a massive and environmental nature. Notice that Eqs. (155) are introducing global force with variables as external sources, potential fields, masses, densities $\rho_I(G)$, $\vec{J}_I(G)$, continuity equations.

Three kinds of forces emerge. Each term in Eq. (155) corresponds respectively to a granular, massive and environmental force. We should first prove that each of them is gauge invariant. For the first and second terms the proof is immediate. Rewriting Eq. (54) as $J^{A}_{\mu I}(G) = \Omega_{Ii} J^{A^i}_{\mu}(G)$, $J^{S}_{\mu I}(G) = \Omega_{Ii} J^{S^i}_{\mu}(G)$, $J^{ST}_{\mu I}(G) = \Omega_{I1} J^{ST^1}_{\mu}(G) + \Omega_{Ii} J^{ST^i}_{\mu}(G)$, we obtain $\partial^\mu J_{\mu I}(G) = \Omega_{Ii}\partial^\mu J_{\mu}{}^{i}(G)$ which shows that the third term is also invariant. Consequently, every contribution can be measured separately. A general relationship is $f^{\nu}_{M} + f^{\nu}_{E} = 4G^{\nu I}\partial^\mu j_{\mu I}$. The usual Lorentz force will be reobtained under the condition $\partial_\mu j^{\mu I} = 0$.

Another indication about a whole physics is obtained. It is the physical existence of an environment just depending on potential fields. From Eq. (5), there are four contributions for environmental forces:

$$\vec{f}^{A}_{E} + \vec{f}^{S-I}_{E} = 4\vec{G}^{I}\Big\{n^{(1)}_{IJK}\partial_\mu\left(G^{\mu J}\partial_\nu G^{\nu K}\right) + n^{(2)}_{IJK}\partial_\mu\left(G_\nu{}^{J}\partial^\mu G^{\nu K}\right)$$
$$+ n^{(3)}_{IJK}\partial_\mu\left(G_\nu{}^{J}\partial_\nu G^{\mu K}\right) + n^{(1)}_{IJKL}\partial_\mu\left(G^{\mu J}G_\nu{}^{K}G^{\nu L}\right)\Big\},$$
$$\vec{f}^{S-K}_{E} = 4n^{(1)}_{IJ}\vec{G}^{I}\partial_\mu\partial^\mu\partial_\nu G^{\nu J},$$
$$\vec{f}^{ST}_{E} = 4\vec{G}^{I}\Big\{n^{(4)}_{IJK}\eta E^{\mu\nu\rho\sigma}\partial_\mu\left(G_\nu{}^{J}\partial_\rho G_\sigma{}^{K}\right) + n^{(2)}_{IJKL}\eta E_{\mu\nu\rho\sigma}\partial_\mu\left(G_\nu{}^{J}G_\rho{}^{K}G_\sigma{}^{L}\right)\Big\}. \tag{156}$$

Notice that above equations do not depend on any external parameter as charge or mass. Eq. (4) implies that $\vec{f}^{A}_{E}$ corresponds to a sector depending only on $\gamma_{[IJ]}$ coefficients.

Eq. (154) can also be written through a flavour series $f^{\nu} = \sum_{I=1}^{N} f^{\nu}{}_{I}$, which shows that for $I = 1$ the usual Lorentz force is reobtained. It is because the last two terms are canceled by a massless genuine gauge field and Noether theorem. However for $I \neq 1$ there are different regions with forces beyond Lorentz. There is a possibility for the photon to connect other aspects of matter not depending on electric charge, as W's weak force. Eq. (154) shows that instead of four interactions, one can decompose the number of interactions depending on the number of flavours. Notice a photon field presence in all these sectors.

For a better understanding, we should describe these new forces in terms of components. This gives

$$f^{0} = 4\vec{j}_I\cdot\vec{E}^{I} + 4\phi_I\left(\frac{\partial}{\partial t}\rho^{I}(G) + \vec{\nabla}\cdot\vec{J}^{I}(G)\right) - 2m_I^2\phi_I\left(\frac{\partial}{\partial t}\phi^{I} + \vec{\nabla}\cdot\vec{G}^{I}\right) \tag{157}$$

and

$$\vec{f} = \vec{f}_L + \vec{f}_M + \vec{f}_E \tag{158}$$

with

$$\vec{f}_L = 4\rho_I\vec{E}^{I} + 4\vec{j}_I\times\vec{B}^{I}, \tag{159}$$

$$\vec{f}_M = -2m_I^2\vec{G}_I\left(\frac{\partial}{\partial t}\phi^{I} + \vec{\nabla}\cdot\vec{G}^{I}\right), \tag{160}$$

$$\vec{f}_E = 4\vec{G}_I\left(\frac{\partial}{\partial t}\rho^{I}(G) + \vec{\nabla}\cdot\vec{J}^{I}(G)\right). \tag{161}$$

Notice that strengths are proportional to charges, masses and environmental couplings like $\gamma_{(IJ)}$. Besides a granular sector associated to test charges there is a network dynamics. Eqs. (7) comes out from the interdependence relationships produced by a whole model. Eq. (158) shows that different lines of force emerge which are not anymore from charge to charge. They are presenting how fields act on fields and on mass without any external source.

A new significance for force appears. Eq. (158) contains different sources for transmitting momentum. Particles can be deflected not only from test charges as in Eq. (159). A force can also be originated from a given field $\vec{G}_I$ interacting with its own masses as Eq. (160) shows. Eq. (161) means an environmental force where its variables $\rho_I(G)$ and $\vec{J}_I(G)$ do not act over individual objects, but over gauge invariant conglomerates of potential fields it is a force derived from the fields cooperation aspect.

Eq. (160) defines a classical massive interaction. It offers a microscopic reason for the relationship between mass, force and fields. For instance, consider a inverse square law depending on a potential fields solution $\vec{G}_I\left(\frac{\partial}{\partial t}\phi^I + \vec{\nabla}\cdot\vec{G}^I\right) = -\frac{a}{r^{2n}}$. Considering the static limit, one gets the solution $\frac{G_I^2}{2} = a\frac{r^{-2n+1}}{2n-1}$.

For a better understanding of Eq. (161), one should be divided in three parts: $\vec{f}_E = \vec{f}_E^A + \vec{f}_E^S + \vec{f}_E^{ST}$, with $\vec{f}_E^S = \vec{f}_E^{S-K} + \vec{f}_E^{S-I}$. It gives,

$$\vec{f}_E^A = 4\gamma_{[IJ]}\vec{G}^I\left(\frac{\partial}{\partial t}\left(\vec{G}^J\cdot\vec{E}_T\right) + \vec{\nabla}\cdot\left(\phi^J\vec{E}_T + \vec{G}^J\times\vec{B}_T\right)\right), \tag{162}$$

$$\vec{f}_E^{S-K} = 4\vec{G}^I\left(\vec{\nabla}^2 - \frac{\partial^2}{\partial t^2}\right)\left\{(\beta_I+\rho_I)\left(\sigma_j{}^j + \sigma\right) + (\beta_I + 4\rho_I)\theta + \frac{1}{2}\xi_{IJ}\left(\frac{\partial}{\partial t}\phi^J + \vec{\nabla}\cdot\vec{G}^J\right)\right\}, \tag{163}$$

$$\begin{aligned}\vec{f}_E^{S-I} = 4\vec{G}^I\Bigg(&\frac{\partial}{\partial t}\left(\gamma_{(IJ)}\vec{G}^J\cdot(\vec{s}+\vec{\sigma}) + \tau_{(IJ)}\phi^J\left(\sigma_j{}^j + \sigma + s_j{}^j + s + 4\theta + 4r\right)\right.\\ &\left.+\gamma_{(IJ)}\phi^J(s+\sigma+\theta+r) + \beta_I\vec{\nabla}\cdot\vec{s} - \rho_I\frac{\partial}{\partial t}s_j{}^j - (\beta_I+\rho_I)\frac{\partial}{\partial t}s - (\beta_I+4\rho_I)\frac{\partial}{\partial t}r\right)\\ &+\vec{\nabla}\cdot\Big(-\gamma_{(IJ)}\phi^J(\vec{s}+\vec{\sigma}) + \tau_{(IJ)}\vec{G}^J\left(\sigma_j{}^j + \sigma + s_j{}^j + s + 4\theta + 4r\right) + \gamma_{(IJ)}\vec{G}^J(\theta + r)\\ &+\gamma_{(IJ)}G_j{}^J\left(s^{ij}+\sigma^{ij}\right) + \beta_I\frac{\partial}{\partial t}\vec{s} - \beta_I\nabla_j s^{ij} + \rho_I\vec{\nabla}s_j{}^j + \rho_I\vec{\nabla}s + (\beta_I + 4\rho_I)\vec{\nabla}r\Big)\Bigg),\end{aligned} \tag{164}$$

$$\vec{f}_E^{ST} = 8\eta\gamma_{[IJ]}\vec{G}^I\left(-\frac{\partial}{\partial t}\left(\vec{G}^J\cdot\vec{B}_T\right) + \vec{\nabla}\cdot\left(\vec{G}^J\times\vec{E}_T - \phi^J\vec{B}_T\right)\right). \tag{165}$$

Environmental forces say that matter transformations depend exclusively on fields. Eqs. (7) are expressing mathematically the grouping meaning of fields. Notice that every term can separatedly be measured.

A consequence from this network of fields is on the presence of induced forces. Eqs. (4) and (158) have interconnected relationships which produce induced forces. For instance, let us consider from Eq. (160) the expressions for the set involving just two fields $\vec{f}_1 = \vec{0}$, $\vec{f}_2 = -2m_2^2\vec{G}_2(\frac{\partial\phi_2}{\partial t} + \vec{\nabla}\cdot\vec{G}_2)$. Then, one gets for $\vec{f}_2$ an implicit dependence on $(\phi_1, \vec{G}_1)$ through Eq. (4). Induced forces here means an interconnected causality.

8 $(\frac{1}{2},\frac{1}{2})$ EM PHYSICALITY

After classical equations, conservation laws and forces being studied a next aspect is to analyze on the corresponding $(\frac{1}{2},\frac{1}{2})$ EM physicality. Historically, different trials have happened in order to improve Maxwell equations. Today there are nearly 31 models beyond Maxwell [79]. Each of them has developed special features. Between various efforts we would emphasize de Broglie-Proca massive photon [80], [81], Bohr-Infield non-linearity [64], Euler-Heisenberg vacuum polarization [82], Dirac monopoles [83], Podolsky high derivatives [41], Chern-Simmons with Levi-Civita [48], Kalb-Rammond scalar photon-dark energy [84], [85], Wilczek axionic [86], Jackiw Lorentz symmetry violation [87], Gambini-Pullin loop quantum gravity [88], Meyers-Pospelov strings [89]. However, they are not being able to replace QED, which have been remained as the standard theory for electromagnetism. And so, the basic discussion whether light or electric charge comes first? - conserves the old answer where QED introduces the photon field coupled to the electric charge.

There is a concern on the foundations of electromagnetism. Under this challenge we would like to understand what are Eq. (1) real contributions to EM phenomena. Given such principle where nature works as a group this work has proposed to cross Maxwell frontier through a fourth interpretation to light invariance. Eq. (4) was entitled as a Global Maxwell equation, but its crucial test is about the possibilities to go beyond Maxwell. It contains a mathematical support, proposes a whole electromagnetism, introduces potential fields physically, new conserved charges, but up to now, it is just a conjecture. We still need to analyse how far it is able to surpass Maxwell limitations. Then, after sections 1-7 be developed, the next effort on this text shall be to understand whether it provokes just a mathematical formulation or it is really able to go beyond Maxwell limitations.

A new EM scenario appears. Based on a fields set $\{G_{\mu I}\}$ a branch of electromagnetic fields $\{\gamma_{[IJ]}\phi^J, \gamma_{[IJ]}\vec{G}^J; \overleftarrow{E}_I - \overleftarrow{B}_I, \overleftarrow{e} - \overleftarrow{b}; \sigma_I, \overleftarrow{\sigma}_I, \ \sigma_I^{ij}, s, \overleftarrow{s}, s^{ij}, \theta, \theta_i^i, r, r_i^{\ i}\}$ is introduced containing four types of observables: conglomerates of potential fields as $\gamma_{[IJ]}\phi^J$, granular $\vec{E}_I$ and $\vec{B}_I$, $\vec{e}$ and $\vec{b}$ collective fields, and longitudinal observables also with collective and granular aspects. The existence of these observables is spread through I -fields layers, two sectors (granular and collective) and four physical regions (photonic, massive, neutral, charged). They build up the Global Maxwell equation containing ordinary Maxwell and new terms. Their equations modify the usual Coulomb, Ampère, Faraday, Lorentz laws and introduce new Bianchi identities. And we have to ask: Maxwell frontier is being crossed?

Theoretically, Maxwell equations are changed and we should analyse the possibilities for these new laws. New charges and currents appear to be measured with an explicit dependance on potential fields. Experimentally, a main difficulty for any post Maxwell model is on the precisions tests of QED being related by Kinoshita table [90]. The difference here is that Lorentz and antireductionist gauge invariances introduce new couplings constants, as $gG_{\mu 1}$, where g is not necessarily related to the fine structure constant. New parameters can be introduced as Eq. (4) shows. This crucial fact allows to investigate on the existence of new EM laws without being limited by QED high precisions. Physically, one derives an EM which contains the displacement current, light invariance, but surpass the electric charge requirement.

We should now consider on the existence of new laws. Two facts are protecting their proposals. First, being able to offer an argument to answer the Kinoshita restrictions. Second, we get space for proposing new laws by noticing that the dates of these 19th century publications are in striking contrast with those related to the sophisticated, multi-parametrized and cosmological detections of our current days. Initially, there is a new Coulomb law in Eq. (76) where the Maxwell electric field works as its own source. It introduces a non-linear Gauss law, where a first physical consequence is on the existence of fields not necessarily depending on electric charge and not necessarily depending on the square inverse law. The expression of these these granular electric fields will be depending on the number of involved fields in the initial set [91]. Follows that, a new Ampère law is obtained adding another displacement current depending on $\vec{e}$ and on potential fields $\vec{J}_I(G)$. This result brings an experimental challenge for Ampère law be tested through cluster data. Faraday's law is also generalized for induction laws depending on collective and potential fields and not depending on electric charge and for collective fields. There is a new EM to be understood. They leave room for an EM not more charge-based and not restricted to EM fields.

A first good signal on the chances of these new equations is from the comparison with the Maxwell constitutive equations. Experiments tell that there are constitutive relations to be derived from an initial Lagrangian. They show that many materials respond to an electromagnetic field by setting up globally neutral charge and current distributions. Then, for characterizing the charge and current distributions of material response, one has to introduce by hand the polarization and magnetization vectors, which are macroscopic variables. Therefore, Maxwell equations are defined through four vectors $\vec{E}$, $\vec{D}$, $\vec{B}$ and $\vec{H}$, where the displacement field $\vec{D}$ and magnetic field strength $\vec{H}$ are heuristically introduced. So, a space is left for arguing about an origin for $\vec{D}$ and $\vec{H}$ from first principles. What is the physical significance of the displacement $\vec{D}$? A step forward given by Eq. 4 is that such antireductionistic gauge invariance can define not only Maxwell fields, but also collective fields, and so, an analogy between the polarization and magnetization vectors ($\vec{P}$ and $\vec{M}$) and the self-cooperations ($\vec{e}$ and $\vec{b}$) is performed. Eqs. (76)-(77) are characterizing a potential field origin for $\vec{P}$ and $\vec{M}$ vectors and a nonlinear behavior. These equations offer an *ab initio* reason given by $\vec{D}_I \equiv \vec{E}_I + b_I \vec{e}$, $\vec{H}_I \equiv \vec{B}_I + b_I \vec{b}$. The antireductionist symmetry says that, instead of this lacking variable being considered just as a phenomenological response function, the polarization and magnetization vectors can be defined from initial potential fields. As in the Born-Infeld case, it produces an expression also valid in vacuum.

The relevant result is that through the whole gauge principle there is an origin and a contribution at heuristic

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constitutive phenomenology installed at Maxwell equation. However, it brings coincidences and differences. Although $(\vec{P}-\vec{M})$ and $(\vec{e}-\vec{b})$ pairs coincide in some terms at Eq. (4), they differ when the theory is computed further as section 6 and 7 show. Which pair fits better with the first principles of field theory and light invariance? Answer appears through the respective Lagrangian, conservation laws and forces. For this, we should compare each pair contribution on $U, \vec{S}, T_{ij}, \vec{f}$ expressions, and so, from previous sections one notices that $(\vec{e}-\vec{b})$ pair splits more terms than $(\vec{P}-\vec{M})$. The collective fields derived from primordial potential fields introduce new energy and momentum terms, and, a global force where $\vec{e}$ and $\vec{b}$ are variables for changing momentum differently from the usual product of the dipole momentum with the first derivative of the field. They also introduce not by hand a nonlinear behavior at the Global Maxwell equation. As conclusion, from first principles, Eq. (4) rewrites usual Maxwell constitutive laws and adds new terms.

A second contribution is on the so-called massive photon. Since de Broglie (1922) and Proca (1936), a list of the extensions of Maxwell's EM with mass have been studied. The discussion of extended EM models with the appearance of massive photons and photinos [93], [94] become a relevant matter. In 1941 Schrödinger emphasized the link between photon mass and a finite range of static forces [95]. Meanwhile, there is a continuous concern on the upper limit of photon mass [96]. The difference here is that Eqs. (76), (77) are introducing an unification between massless photon and massive potential fields. It yields a model that although does not reply London's equations it contains implications on Meissner effect with contributions on powers of the magnetic fields, $\vec{\nabla}^2\vec{B}_I+\alpha_{IJ}\vec{B}^J+\beta_{IJ}W\vec{B}^J+\alpha_{IJK}\vec{\nabla}(\vec{B}^J\cdot\vec{B}^K)+\alpha_{IJKL}\vec{\nabla}(\vec{B}^J\cdot\vec{B}^K\wedge\vec{B}^L)+g(\vec{G},\vec{B},\vec{e},\vec{b})=0$ where $\alpha_{IJ,...}$ are depending on the free coefficients and masses [97].

A third contribution is on non-linearity. 1912 was done the first attempt to make Maxwell's equations non-linear through Mie's theory [98]. Non-linear theories constitute a branch of classical and quantum field theories relevant in a variety of theoretical and observational, see for instance [100] and references therein. As nonlinear terms, we will identify those proportional to fields which have a space-time evolution. At this scenario, Eq. (4) provides self-sources, but for electric and magnetic fields. There are three independent contributions, where each one means a gauge invariant block in Eq. (4). They are

$$\rho_I(G)=\rho_I^A(G)+\rho_I^S(G)+\rho_I^{ST}(G) \qquad (166)$$

and

$$\vec{J}_I(G)=\vec{J}_I^A(G)+\vec{J}_I^S(G)+\vec{J}_I^{ST}(G). \qquad (167)$$

Analyzing the antisymmetric sector, one observes two independent charges

$$\rho_I^A(G)=\gamma_{[IJ]}\vec{G}^J\cdot\left(b_K\vec{E}^K+\vec{e}\right) \qquad (168)$$

which are showing electric and composite electric fields acting as their own sources. This means fields with feedback. Four terms contribute to the currents

$$\vec{J}_I^A(G)=\gamma_{[IJ]}\phi^J\left(b_K\vec{E}^K+\vec{e}\right)+\gamma_{[IJ]}\vec{G}^J\times\left(b_K\vec{B}^K+\vec{b}\right) \qquad (169)$$

which are explicitly showing a vectorial causality where the currents are proportional to any type of electric field and perpendicular to any type of magnetic field. Explicitly, Eqs. (168-169) are showing the presence of potential fields.

The symmetric sources depend on kinetic and interacting terms $\rho_I^S(G)=\rho_I^{S-K}(G)+\rho_I^{S-I}(G)$ where

$$\rho_I^{S-K}(G)=-(\beta_I+\rho_I)\frac{\partial}{\partial t}\sigma_j{}^j-(\beta_I+\rho_I)\frac{\partial}{\partial t}\sigma-(\beta_I+4\rho_I)\frac{\partial}{\partial t}\theta$$
$$-\frac{1}{2}\xi_{IJ}\left(\frac{\partial^2}{\partial t^2}\phi^J+\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{G}^J\right),$$

$$\rho_I^{S-I}(G)=\gamma_{(IJ)}\phi^J(s+\sigma+\theta+r)+\tau_{(IJ)}\phi^J\left(\sigma_j{}^j+\sigma+s_j{}^j+s+4\theta+4r\right)$$
$$+\gamma_{(IJ)}\vec{G}^J\cdot(\vec{s}+\vec{\sigma})+\beta_I\vec{\nabla}\cdot\vec{s}-\rho_I\frac{\partial}{\partial t}s_j{}^j-(\beta_I+\rho_I)\frac{\partial}{\partial t}s-(\beta_I+4\rho_I)\frac{\partial}{\partial t}r. \qquad (170)$$

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For currents, one derives $\vec{J}_I^S(G) = \vec{J}_I^{S-K}(G) + \vec{J}_I^{S-I}(G)$ where

$$\vec{J}_I^{S-K}(G) = (\beta_I + \rho_I)\vec{\nabla}\sigma_j{}^j + (\beta_I + \rho_I)\vec{\nabla}\sigma + (\beta_I + 4\rho_I)\vec{\nabla}\theta + \frac{1}{2}\xi_{IJ}\left(\vec{\nabla}\frac{\partial}{\partial t}\phi^J + \vec{\nabla}\vec{\nabla}\cdot\vec{G}^J\right),$$

$$\vec{J}_I^{S-I}(G) = -\gamma_{(IJ)}\phi^J(\vec{s} + \vec{\sigma}) + \tau_{(IJ)}\vec{G}^J\left(\sigma_j{}^j + \sigma + s_j{}^j + s + 4\theta + 4r\right) + \gamma_{(IJ)}\vec{G}^J(\theta + r)$$

$$+\gamma_{(IJ)}G_j{}^J\left(s^{ij} + \sigma^{ij}\right) + \beta_I\frac{\partial}{\partial t}\vec{s} - \beta_I\nabla_j s^{ij} + \rho_I\vec{\nabla}s_j{}^j + \rho_I\vec{\nabla}s + (\beta_I + 4\rho_I)\vec{\nabla}r \qquad (171)$$

with photons self-interacting currents. Observe that terms with second derivative at ρ_I^{S-K} and $\vec{J}_I^{S-K}$ are converted to interactive terms through the kinetic identity, Eq. (23) and the symmetric Bianchi identities.

The semi-topological contribution carries energy through the following charges and currents

$$\rho_I^{ST}(G) = -2\eta\gamma_{[IJ]}\vec{G}^J\cdot\left(b_K\vec{B}^K + \vec{b}\right) + 2\eta b_I\vec{\nabla}\cdot\vec{b} \qquad (172)$$

$$\vec{J}_I^{ST}(G) = 2\eta\gamma_{[IJ]}\vec{G}^J\times\left(b_K\vec{E}^K + \vec{e}\right) - 2\eta\gamma_{[IJ]}\phi^J\left(b_K\vec{B}^K + \vec{b}\right) - 2\eta b_I\frac{\partial}{\partial t}\vec{b} - 2\eta b_I\vec{\nabla}\times\vec{e} \qquad (173)$$

where one notices that, inversely, Eq. (173) shows currents proportional to magnetic fields and perpendicular to electric fields. There is also information in Eq. (172) on the existence of a self-magnetic field divergence term as a source.

Non-linear laws are derived from this systemic signature. Plasma [100], condensed matter, astrophysics, are requiring a non-linear electrodynamics based on first principles. Originally Born-Infeld have been the heart of this description. Eqs. (166)-(167) are introducing that an essential product from Eq. (4) is that its observables get mathematical and physical existence without external sources. For better comprehension, we should take as comparison with kinks, domain walls, solitons, monopoles, instantons and mesons, where they can be taken as the best well-known structures to illustrate configurations of fields that are no longer excited by perturbing the system with external sources [101]. The difference here is that under this global view no topological property is necessary to support non-perturbative configurations in the absence of external sources.

Another evident but necessary argument on the consistency of Eq. (4) is to analyze if source equations automatically will be fulfilled for all $t > 0$ due to the homogeneous equations, once they are satisfied by the initial data at $t = 0$. To check this, we differentiate (78) and use (79). It follows indeed that $\vec{\nabla}\cdot\vec{B}_I(t,\vec{x}) = 0$ once if it is true at $t = 0$. Differentiating (76) and using (77) we get $\vec{\nabla}\cdot\frac{\partial}{\partial t}\vec{E}_I - \frac{\partial}{\partial t}\rho_I(G) = 0$.

A fourth result is a second type of Faraday law about collective laws. Eqs. (80)-(81) develops an exclusive Faraday law between collective fields. We should make a comparison with the usual non-abelian case, with the weak interaction case. Considering that the Z^0 and $W^{\pm}$-particles associated to a Yang-Mills field–strength, one gets the following relationship $\vec{\nabla}\times\vec{E}_a + \frac{\partial}{\partial t}\vec{B}_a = -g\,f_{abc}\left(\vec{A}^b\times\vec{E}^c - \phi^b\vec{B}^c\right)$ where $\vec{E}_a$ and $\vec{B}_a$ are fields associated to a weak charge. However, probably the Yang-Mills law is not so much discussed because of weak interactions being related to microscopic effects. Nevertheless it should generate an isospin current. Then, we would say that the existence of an exclusive Faraday law with sources is not a missing view but depends more on interpretation than on experimental evidence. A fact were Eq. (80) can be supported by the YM case.

Eq. (81) introduces monopoles not as particles but as configurations. This means that instead of searching to Dirac monopoles one should observe on fields set. This equation is experimentally possible. Materials with spontaneous polarization (piezoelectricity) or spontaneous magnetization are candidates. There is a literature supporting the detection of a divergenceless magnetization, $\vec{\nabla}\cdot\vec{M} = g$, as domain walls in ferromagnetism and of a exclusive Faraday law as in magnetic nuclear resonance [102]. Another situation is with spin ice [103]. Complementing, we should observe that, the Gauss law for the non-Abelian magnetic field might, at first glance, also suggests that magnetic monopoles come out due to the non linear structure. It gives $\vec{\nabla}\cdot\vec{B}_a = -g\,f_{abc}\vec{A}^b\cdot\vec{B}^c$ where $\vec{B}_a$ denotes a non-abelian magnetic field. Then a similarity between Eq. (81) and non-abelian case appears. However, this result is not true in general. Indeed 't Hooft and Polyakov showed that scalars must be present and it is their coupling to the Y-M potentials that induce, for a class of local symmetry groups, soliton-like solutions that carry non-trivial magnetic charge [104], [105] and, still more important, only in the phase of spontaneously broken symmetry.

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In a special case, Eqs. (4) can be written concisely as just two equations. By introduction of a complex vector field $\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right)$ in 3+1 dimensions, one confirms the similarity between the granular/collective electric and magnetic fields [106]:

$$\vec{\nabla}\cdot\left(\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right)\right) = 0\,,$$

$$\vec{\nabla}\times\left(\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right)\right) = i\frac{\partial}{\partial t}\left(\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right)\right). \quad (174)$$

Eqs. (174) are equations that display Poincaré symmetry, conformal symmetry (with respect to space-time transformations preserving angles and not just lengths) and the duality rotation symmetry $\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right) \to e^{i\theta}\left(\left(\vec{E}_I + b_I \vec{e}\right) + i\left(\vec{B}_I + b_I \vec{b}\right)\right)$.

Thus, after considering new Coulomb, Ampère, Faraday, collective, duality laws, from the above results it is possible to evince the Maxwell modified laws. They are showing features where a new EM phenomena should be considered through the Lorentz symmetry and the antireductionist gauge symmetry. Both be responsible to cross Maxwell frontier and introduce Eq. (4) not more as a mathematical expression. It becomes candidate to be explored experimentally.

9 DISPERSION RELATIONS

A further topic to study is the wave-like behavior of Eq. (4). Dispersion relations can be tested through the behavior of light in materials [107]. We should first explore the model in terms of its most fundamental variables. The homogeneous equations provide the usual solutions

$$\vec{E}_I = -\vec{\nabla}\phi_I - \frac{\partial}{\partial t}\vec{G}_I\,, \qquad \vec{B}_I = \vec{\nabla}\times\vec{G}_I\,. \quad (175)$$

Substituting (175) in (4), it yields

$$-\vec{\nabla}\cdot\left(\vec{\nabla}\phi_I + \frac{\partial}{\partial t}\vec{G}_I\right) - b_I\gamma_{[JK]}\vec{\nabla}\cdot\left(\phi^J\vec{G}^K\right) + \frac{1}{2}m_I^2\phi_I = \rho_I(G),$$

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\vec{G}_I + \vec{\nabla}\left(\frac{\partial}{\partial t}\phi_I + \vec{\nabla}\cdot\vec{G}_I\right) - b_I\gamma_{[JK]}\frac{\partial}{\partial t}\left(\phi^J\vec{G}^K\right) +$$

$$+ b_I\gamma_{[JK]}\left(\vec{\nabla}\cdot\vec{G}^J + \vec{G}^J\cdot\vec{\nabla}\right)\vec{G}^K + \frac{1}{2}m_I^2\vec{G}_I = \vec{J}_I(G), \quad (176)$$

with

$$\rho_I(G) = n_{IJ}^{(1)}\frac{\partial^2}{\partial t^2}\phi^J + n_{IJ}^{(1)}\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{G}^J + n_{IJK}^{(1)}\phi^J\frac{\partial}{\partial t}\phi^K + n_{IJK}^{(2)}\phi^J\frac{\partial}{\partial t}\phi^K + n_{IJK}^{(3)}\phi^J\frac{\partial}{\partial t}\phi^K$$

$$+ n_{IJK}^{(1)}\phi^J\vec{\nabla}\cdot\vec{G}^K + n_{IJKL}^{(1)}\phi^J\phi^K\phi^L - n_{IJKL}^{(1)}\phi^J\vec{G}^K\cdot\vec{G}^L - n_{IJK}^{(2)}\vec{G}^J\cdot\frac{\partial}{\partial t}\vec{G}^K$$

$$+ n_{IJK}^{(3)}\left(\vec{G}^J\cdot\vec{\nabla}\right)\phi^K - \eta n_{IJK}^{(4)}\vec{G}^J\cdot\left(\vec{\nabla}\times\vec{G}^K\right) + \eta n_{IJKL}^{(2)}\vec{G}^J\cdot\left(\vec{G}^K\times\vec{G}^L\right),$$

$$\vec{J}_I(G) = -n_{IJ}^{(1)}\vec{\nabla}\frac{\partial}{\partial t}\phi^J - n_{IJ}^{(1)}\vec{\nabla}\vec{\nabla}\cdot\vec{G}^J + n_{IJK}^{(3)}\phi^J\frac{\partial}{\partial t}\vec{G}^K - n_{IJK}^{(2)}\phi^J\vec{\nabla}\phi^K +$$

$$-\eta n_{IJK}^{(4)}\phi^J\vec{\nabla}\times\vec{G}^K + \eta n_{IJKL}^{(2)}\phi^J\vec{G}^K\times\vec{G}^L + n_{IJK}^{(1)}\vec{G}^J\frac{\partial}{\partial t}\phi^K + n_{IJK}^{(1)}\vec{G}^J\vec{\nabla}\cdot\vec{G}^K +$$

$$+ n_{IJKL}^{(1)}\vec{G}^J\phi^K\phi^L - n_{IJKL}^{(1)}\vec{G}^J\vec{G}^K\cdot\vec{G}^L - \eta n_{IJK}^{(4)}\vec{G}^J\times\frac{\partial}{\partial t}\vec{G}^K + n_{IJK}^{(2)}\vec{G}_j{}^J\vec{\nabla}\vec{G}^{jK} +$$

$$+ n_{IJK}^{(3)}\left(\vec{G}^J\cdot\vec{\nabla}\right)\vec{G}^K - \eta n_{IJK}^{(4)}\vec{G}^J\times\vec{\nabla}\phi^K + 2\eta n_{IJKL}^{(2)}\vec{G}^J\times\vec{G}^K\phi^L\,. \quad (177)$$

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Notice that the coefficients are defined in Eq. (57) and Lorentz condition was not considered.

Describing the wave equations in terms of measurable fields, one gets

$$\left(\frac{\partial^2}{\partial t^2}-\vec{\nabla}^2\right)\left(\vec{E}_I+b_I\vec{e}\right)=b_I\vec{\nabla}\times\left(\vec{\nabla}\times\vec{e}+\frac{\partial}{\partial t}\vec{b}\right)-\vec{\nabla}\rho_I(G)-\frac{\partial}{\partial t}\vec{J}_I(G)-\frac{1}{2}m_I^2\vec{E}_I,$$

$$\left(\frac{\partial^2}{\partial t^2}-\vec{\nabla}^2\right)\left(\vec{B}_I+b_I\vec{b}\right)=b_I\gamma_{[JK]}\frac{\partial}{\partial t}\left(\vec{G}^J\times\vec{E}^K-\phi^J\vec{B}^K\right)-b_I\vec{\nabla}\vec{\nabla}\cdot\vec{b}$$

$$+\vec{\nabla}\times\vec{J}_I(G)-\frac{1}{2}m_I^2\vec{B}_I \qquad (178)$$

where the right hand side contains interactive terms. Observe that, differently from the usual Maxwell case, Eqs. (176) and (177) do not coincide. Given that the variables $(\phi_I,\vec{G}_I)$ and $(\vec{E}_I,\vec{B}_I,\vec{e},\vec{b})$ do not travel under the same equation, we get different causalities. This means that the different types of forces in Eq. (178) will not propagate with the same velocity.

A next step is to study these wave equations by taking the plane wave solutions

$$\phi^I=\phi_0^I e^{i\left(\vec{k}\cdot\vec{x}-wt\right)},\quad \vec{G}^I=\vec{G}_0^I e^{i\left(\vec{k}\cdot\vec{x}-wt\right)} \qquad (179)$$

which yields

$$\vec{E}^I=\vec{E}_0^I e^{i\left(\vec{k}\cdot\vec{x}-wt\right)},\quad \vec{B}^I=\vec{B}_0^I e^{i\left(\vec{k}\cdot\vec{x}-wt\right)},\quad \vec{e}=\vec{e}_0 e^{2i\left(\vec{k}\cdot\vec{x}-wt\right)},\quad \vec{b}=\vec{b}_0 e^{2i\left(\vec{k}\cdot\vec{x}-wt\right)}$$

with

$$\vec{E}_0^I=i\left(-\vec{k}\phi_0^I+w\vec{G}_0^I\right),\quad \vec{B}_0^I=i\vec{k}\times\vec{G}_0^I,$$

$$\vec{e}_0=-\gamma_{[IJ]}\phi_0^I\vec{G}_0^J,\quad \vec{b}_0=-\frac{1}{2}\gamma_{[IJ]}\vec{G}_0^I\times\vec{G}_0^J\,. \qquad (180)$$

Thus, the ensuing equation connecting w and k is obtained from the following $4N$ equations:

$$\vec{k}\cdot\left(\vec{k}\phi_I-w\vec{G}_I-2ib_I\gamma_{[JK]}\phi^J\vec{G}^K\right)=\rho_I(G)-\frac{1}{2}m_I^2\phi_I\,,$$

$$-\vec{k}\times\left(\vec{k}\times\vec{G}_I+ib_I\gamma_{[JK]}\vec{G}^J\times\vec{G}^K\right)=w\left(-\vec{k}\phi_I+w\vec{G}_I+2ib_I\gamma_{[JK]}\phi^J\vec{G}^K\right)+\vec{J}_I(G)-\frac{1}{2}m_I^2\vec{G}_I$$

with

$$\rho_I(G)=-n_{IJ}^{(1)}w^2\phi^J+n_{IJ}^{(1)}w\vec{k}\cdot\vec{G}^J-i\left(n_{IJK}^{(1)}+n_{IJK}^{(2)}+n_{IJK}^{(3)}\right)w\phi^J\phi^K+$$

$$+i\left(n_{IJK}^{(1)}+n_{IKJ}^{(3)}\right)\phi^J\vec{k}\cdot\vec{G}^K+in_{IJK}^{(2)}w\vec{G}^J\cdot\vec{G}^K+in_{IJK}^{(4)}\eta\vec{k}\cdot\left(\vec{G}^J\times\vec{G}^K\right)$$

$$+n_{IJKL}^{(1)}\phi^J\phi^K\phi^L-n_{IJKL}^{(1)}\phi^J\vec{G}^K\cdot\vec{G}^L+n_{IJKL}^{(2)}\eta\vec{G}^J\cdot\left(\vec{G}^K\times\vec{G}^L\right)$$

$$\vec{J}_I(G)=-n_{IJ}^{(1)}w\vec{k}\phi^J+n_{IJ}^{(1)}\vec{k}\left(\vec{k}\cdot\vec{G}^J\right)-i\left(n_{IKJ}^{(1)}+n_{IJK}^{(3)}\right)w\phi^J\vec{G}^K+$$

$$+i\left(n_{IJK}^{(1)}+n_{IKJ}^{(3)}\right)\vec{G}^J\left(\vec{k}\cdot\vec{G}^K\right)-in_{IJK}^{(2)}\vec{k}\phi^J\phi^K+in_{IJK}^{(2)}\vec{k}\left(\vec{G}^J\cdot\vec{G}^K\right)+$$

$$+i\left(n^{(4)}_{IKJ}-n^{(4)}_{IJK}\right)\eta\phi^J\vec{k}\times\vec{G}^K+in^{(4)}_{IJK}\eta w\vec{G}^J\times\vec{G}^K+n^{(1)}_{IJKL}\vec{G}^J\phi^K\phi^L+$$

$$-n^{(1)}_{IJKL}\vec{G}^J\left(\vec{G}^K\cdot\vec{G}^L\right)+\left(n^{(2)}_{IJKL}+n^{(2)}_{IKLJ}+n^{(2)}_{ILJK}\right)\eta\phi^J\vec{G}^K\times\vec{G}^L \tag{181}$$

where the coefficients such as $n^{(1)}_{IJ}$ are written in Eq. (4).

A next study is to discuss the corresponding dispersion relations. Taking the particular case involving just linear potentials, one gets the following system

$$M_{IJ}\begin{pmatrix}\phi_J\\ \vec{G}_J\end{pmatrix}=\begin{pmatrix}R_I\\ \vec{T}_I\end{pmatrix}$$

where the dispersion relation matrix M_{IJ} is

$$M_{IJ}(\omega;\overleftarrow{k})=\begin{pmatrix}\left(\vec{k}^2+\frac{1}{2}m_J^2\right)\delta_{IJ}+w^2n^{(1)}_{IJ} & -w\vec{k}_b\left(\delta_{IJ}+n^{(1)}_{IJ}\right)\\ -w\vec{k}_a\left(\delta_{IJ}+n^{(1)}_{IJ}\right) & -\left(\vec{k}^2-w^2+\frac{1}{2}m_J^2\right)\delta_{ab}\delta_{IJ}+\vec{k}_a\vec{k}_b\left(\delta_{IJ}+n^{(1)}_{IJ}\right)\end{pmatrix}$$

(182)

Eq. (182) means the linear Global Maxwell equation in the momentum space and R_I and $\vec{T}_I$ are sources associated to potential fields respectively. Notice that a means a space index ($a=1,2,3$).

Different scenarios as dispersion from plasma [108], quantum gravity [89], spontaneous symmetry breaking of Lorentz invariance [109], loop quantum gravity [110] have been shown on dispersion relations beyond Maxwell. The wholeness principle introduces another case. The corresponding poles equation is

$$A_{4N}\left(w^2\right)^{4N}+A_{4N-1}\left(w^2\right)^{4N-1}+\ldots+A_0=0 \tag{183}$$

where A_M denotes coefficients depending on (k^2, m_I^2, global coefficients). It yields, $w^2=f(k)$, with the corresponding group velocity $v_g=\dfrac{f'(k)}{2\sqrt{f(k)}}$. Eq. (183) is showing on the possibilities for new dispersion relations. The nature of the waves associated to these poles is given through a linear combination of potential fields. A relationship obtained from the eigenvectors in the diagonalization of Eq. (182).

Thus it is fundamental to explore the possibilities for traveling faster than the speed of light. These will depend on M_{IJ} nature. For the diagonal M_{IJ} matrix case, under the condition

$$\left(\delta_{IJ}+n^{(1)}_{IJ}\right)\partial_\mu G^{\mu I}=0, \tag{184}$$

there is one-to-one correspondence between poles and potential fields

$$\phi_I=\frac{-R_I}{w^2-k^2-\frac{1}{2}m_I^2}\quad,\quad \vec{G}_I=\frac{\vec{T}_I}{w^2-k^2-\frac{1}{2}m_I^2} \tag{185}$$

which implies a dispersion relation bounded by c. The corresponding residues are consistent because the R_I sign is irrelevant. Thus physically it becomes important to explore the non-diagonal case. Eq. (184) becomes a frontier. It says that the whole dynamics introduced by Eq. (1) coincides with the usual Maxwell light for the coefficients conditions

$$\xi_{IJ}=-4\beta_I\beta_J-4\beta_I\rho_J-4\beta_J\rho_I-16\rho_I\rho_J+2\delta_{IJ}\quad, \tag{186}$$

or when the Eq. (59) conserved internal current is considered. On other hand, without violating any basic principle as Lorentz transformations or hamiltonian positivity, one gets dispersion relations greater than c. In Appendix E we explore such circumstances for the case $I=1,2$.

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The Michelson-Morley invariant result is preserved and expanded. Eq. (E.2) shows the relationship $k^2 = w^2$ as a directive and provides circumstances for waves traveling faster than c . Notice that these velocities are corresponding to potential fields waves and not to observable waves.

Returning to Eq. (43) and taking the particular plane wave and linear potential solutions, one gets

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + \frac{1}{2}m_I^2\right)\vec{E}_I = 0, \qquad \left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + \frac{1}{2}m_I^2\right)\vec{B}_I = 0. \tag{187}$$

Concluding, for the particular linear solution considered here, one gets that while $\vec{E}_I$ and $\vec{B}_I$ are bounded by c, combinations of ϕ_I and $\vec{G}_I$ can move faster. Therefore, taking the linear approximation Eq. (182), one derives that while $\vec{f}_L$ travels bounded by c (Eq. (187)), $\vec{f}_M$ can take a greater speed (Appendix E) and $\vec{f}_E$ with mixed effects.

Longitudinal waves can also be derived from the kinetic identity

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\sigma_k{}^k = \vec{\nabla}^2\sigma - 2\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{\sigma} + \nabla_i\nabla_k\sigma^{ik}$$

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\theta = \frac{\partial^2}{\partial t^2}\tilde{\sigma} - 2\frac{\partial}{\partial t}\vec{\nabla}\cdot\tilde{\vec{\sigma}} + \nabla_i\nabla_k\tilde{\sigma}^{ik}$$

For transversality conditions, one gets the k-Global Maxwell equations

$$\begin{aligned}
&i\vec{k}\cdot\left(\vec{E}_I + 2b_I\vec{e}\right) = \rho_I(G) - \frac{1}{2}m_I^2\phi_I \\
&i\vec{k}\times\left(\vec{B}_I + 2b_I\vec{b}\right) = -iw\left(\vec{E}_I + 2b_I\vec{e}\right) + \vec{J}_I(G) - \frac{1}{2}m_I^2\vec{G}_I \\
&i\vec{k}\times\vec{E}_I - iw\vec{B}_I = 0 \\
&i\vec{k}\cdot\vec{B}_I = 0 \\
&2i\vec{k}\times\vec{e} - 2iw\vec{b} = \gamma_{[IJ]}\left(\vec{G}^I\times\vec{E}^J - \phi^I\vec{B}^J\right) \\
&2i\vec{k}\cdot\vec{b} = \gamma_{[IJ]}\vec{G}^I\cdot\vec{B}^J
\end{aligned} \tag{188}$$

where

$$\rho_I^A(G) = \gamma_{[IJ]}b_K\vec{G}^J\cdot\vec{E}^K + \gamma_{[IJ]}\vec{G}^J\cdot\vec{e}$$

$$\vec{J}_I^A(G) = \gamma_{[IJ]}b_K\phi^J\vec{E}^K + \gamma_{[IJ]}\phi^J\vec{e} + \gamma_{[IJ]}b_K\vec{G}^J\times\vec{B}^K + \gamma_{[IJ]}\vec{G}^J\times\vec{b}$$

$$\rho_I^S(G) = \rho_I^{S-K}(G) + \rho_I^{S-I}(G)$$

$$\rho_I^{S-K}(G) = \left(2(\beta_I + \rho_I)\beta_J + 2(\beta_I + 4\rho_I)\rho_J + \frac{1}{2}\xi_{IJ}\right)\left(w^2\phi^J - w\vec{k}\cdot\vec{G}^J\right)$$

$$\rho_I^{S-I}(G) = i\left(2(\beta_I+\rho_I)\gamma_{(JK)} - 2\left(\gamma_{(IJ)}+\tau_{(IJ)}\right)\beta_K - 2\left(\gamma_{(IJ)}+4\tau_{(IJ)}\right)\rho_K\right)w\phi^J\phi^K$$
$$+i\left(2\tau_{(IJ)}\beta_K - 2\beta_I\gamma_{(JK)} + \gamma_{(IK)}\beta_J + 2\left(\gamma_{(IJ)}+4\tau_{(IJ)}\right)\rho_K\right)\phi^J\vec{k}\cdot\vec{G}^K$$
$$+i\left(\gamma_{(IJ)}\beta_K - 2\rho_I\gamma_{(JK)}\right)w\vec{G}^J\cdot\vec{G}^K + \left(\gamma_{(IJ)}+\tau_{(IJ)}\right)\gamma_{(KL)}\phi^J\phi^K\phi^L$$
$$-\left(\gamma_{(IK)}\gamma_{(JL)}+\tau_{(IJ)}\gamma_{(KL)}\right)\phi^J\vec{G}^K\cdot\vec{G}^L$$

and

$$\vec{J}_I^S(G) = \vec{J}_I^{S-K}(G) + \vec{J}_I^{S-I}(G)$$

$$\vec{J}_I^{S-K}(G) = \left(2(\beta_I+\rho_I)\beta_J + 2(\beta_I+4\rho_I)\rho_J + \frac{1}{2}\xi_{IJ}\right)\left(w\vec{k}\phi^J - \vec{k}\vec{k}\cdot\vec{G}^J\right)$$

$$\vec{J}_I^{S-I}(G) = i\left(-\gamma_{(IJ)}\beta_K + 2\beta_I\gamma_{(JK)} - 2\tau_{(IK)}\beta_J - 2\left(\gamma_{(IK)}+4\tau_{(IK)}\right)\rho_J\right)w\phi^J\vec{G}^K$$
$$+i\left(2\tau_{(IJ)}\beta_K + \gamma_{(IK)}\beta_J - 2\beta_I\gamma_{(JK)} + 2\left(\gamma_{(IJ)}+4\tau_{(IJ)}\right)\rho_K\right)\vec{G}^J\vec{k}\cdot\vec{G}^K$$
$$+i\left(-\gamma_{(IJ)}\beta_K + 2\rho_I\gamma_{(JK)}\right)\vec{k}\phi^J\phi^K + i\left(\gamma_{(IJ)}\beta_K - 2\rho_I\gamma_{(KL)}\right)\vec{k}\vec{G}^J\cdot\vec{G}^K$$
$$+\left(\gamma_{(IJ)}\gamma_{(KL)} + \tau_{(IL)}\gamma_{(KJ)}\right)\phi^J\phi^K\vec{G}^L - \left(\tau_{(IJ)}\gamma_{(KL)} + \gamma_{(IK)}\gamma_{(JL)}\right)\vec{G}^J\vec{G}^K\cdot\vec{G}^L$$

$$\rho_I^{ST}(G) = -2\eta\gamma_{[IJ]}b_K\vec{G}^J\cdot\vec{B}^K - 2\eta\gamma_{[IJ]}\vec{G}^J\cdot\vec{b} + 4i\eta b_I\vec{k}\cdot\vec{b}$$

$$\vec{J}_I^{ST}(G) = -2\eta\gamma_{[IJ]}b_K\phi^J\vec{B}^K - 2\eta\gamma_{[IJ]}\phi^J\vec{b} + 2\eta\gamma_{[IJ]}b_K\vec{G}^J\times\vec{E}^K + 2\eta\gamma_{[IJ]}\vec{G}^J\times\vec{e} +$$
$$+4i\eta b_I w\vec{b} - 4i\eta b_I\vec{k}\times\vec{e} \quad . \tag{189}$$

Thus for the particular case without source and massless we obtain the following orthogonality relationships

$\vec{E}_I \perp \vec{B}_I$, $\vec{e} \perp \vec{b}$, $\{\vec{E}_I, \vec{B}_I, \vec{e}, \vec{b}\} \perp \vec{k}$, $\vec{G}_I \perp \vec{B}_I$,

which show that such generic E-M fields are in the same plane. Defining $\alpha \equiv arc(\vec{E}_I, \vec{b})$, $\beta \equiv arc(\vec{B}_I, \vec{e})$, one gets that

$$\alpha + \beta = \pi \quad and \quad |\vec{E}_I||\vec{b}| = |\vec{B}_I||\vec{e}|. \tag{190}$$

The massive case only differs from Eq. (53) in the condition

$$\vec{k} \perp \vec{E}_I \ . \tag{191}$$

Notice that these transversality conditions are basically lost when sources are included.

10 MACROELECTROMAGNETISM

There is a macroelectromagnetism to be understood. Eqs. (4) are fundamental ones for they are described in terms of the most elementary observables of the theory. However, for a magnetometer, it can be observed not in terms of its microscopic variables. Depending on type of measurement, it yields that, one has also to explore cases where Eqs. (4) can be reinterpreted through a superposition of observables.

A first superposition is to define total fields $\{\vec{E}_T, \vec{B}_T\}$. They represent the resulting field from a medium. It gives $\vec{E}_T = b_I \vec{E}^I + \vec{e}$, $\vec{B}_T = b_I \vec{B}^I + \vec{b}$. Then, for $b_I b^I = 1$, Eqs. (4) can be reduced in terms of four equations:

$$\vec{\nabla} \cdot \vec{E}_T = b_I \rho^I (G, \vec{E}_T, \vec{B}_T) - \frac{1}{2} b_I m_I^2 \phi^I ,$$

$$\vec{\nabla} \times \vec{B}_T = \frac{\partial}{\partial t} \vec{E}_T + b_I \vec{J}^I (G, \vec{E}_T, \vec{B}_T) - \frac{1}{2} b_I m_I^2 \vec{G}^I ,$$

$$\vec{\nabla} \times \vec{E}_T + \frac{\partial}{\partial t} \vec{B}_T = \gamma_{[IJ]} \left(\vec{G}^I \times \vec{E}^J - \phi^I \vec{B}^J \right),$$

$$\vec{\nabla} \cdot \vec{B}_T = \gamma_{[IJ]} \vec{G}^I \cdot \vec{B}^J . \tag{192}$$

Notice that $\rho_I^A(G)$ and $\vec{J}_I^A(G)$ are written in terms of $\vec{E}_T, \vec{B}_T$. At large distances, as in cosmology, these total fields are expected to be those which are experimentally detected. The corresponding wave equations are

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \vec{E}_T = \gamma_{[IJ]} \vec{\nabla} \times \left(\vec{G}^I \times \vec{E}^J - \phi^I \vec{B}^J \right) - b_I \left(\vec{\nabla} \rho^I (G) - \frac{\partial}{\partial t} \vec{J}^I (G) - \frac{1}{2} m_I^2 \vec{E}^I \right),$$

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \vec{B}_T = \gamma_{[IJ]} \frac{\partial}{\partial t} \left(\vec{G}^I \times \vec{E}^J - \phi^I \vec{B}^J \right) - \gamma_{[IJ]} \vec{\nabla} \left(\vec{G}^I \cdot \vec{B}^J \right)$$

$$+ b_I \left(\vec{\nabla} \times \vec{J}^I (G) - \frac{1}{2} m_I^2 \vec{B}^I \right). \tag{193}$$

A second case of observables superposition in Eqs. (4) means the nearest reproduction to Maxwell. Multiplying Eq. (54) by $\Omega^{-1}{}_{I1}$ and $\Omega^{-1}{}_{Ii}$, one rotates for

$$\partial_\nu \left(\Omega^{-1}{}_{I1} G^{\nu\mu I} \right) + d \partial_\nu z^{[\nu\mu]} = J^\mu{}_1, \qquad J^\mu{}_1 = \eta d E^{\mu\nu\rho\sigma} \partial_\nu z_{[\rho\sigma]} \tag{194}$$

and

$$\partial_\nu \left(\Omega^{-1}{}_{Ii} G^{\nu\mu I} \right) + \alpha_i \partial_\nu z^{[\nu\mu]} + \frac{1}{2} m_i^2 G^\mu{}_i = J^\mu{}_i ,$$

$$J^\mu{}_i = \gamma_{[iJ]} G_\nu{}^J Z^{[\nu\mu]} - \beta_i \partial_\nu Z^{(\nu\mu)} - \rho_i \partial^\mu Z_{(\nu}{}^{\nu)} + \gamma_{(iJ)} G_\nu{}^J Z^{(\nu\mu)}$$

$$+ \tau_{(iJ)} G^{\mu J} Z_{(\nu}{}^{\nu)} + \eta E^{\mu\nu\rho\sigma} \gamma_{[iJ]} G_\nu{}^J Z_{[\rho\sigma]} + \eta \alpha_i E^{\mu\nu\rho\sigma} \partial_\nu z_{[\rho\sigma]} \tag{195}$$

which yields the corresponding vector form

$$\vec{\nabla} \cdot \left(\vec{E} + d \vec{e} \right) = \rho_1 (G),$$

$$\vec{\nabla} \times \left(\vec{B} + d \vec{b} \right) = \frac{\partial}{\partial t} \left(\vec{E} + d \vec{e} \right) + \vec{J}_1 (G) \tag{196}$$

and

$$\vec{\nabla} \cdot \left(\vec{E}_i + \alpha_i \vec{e} \right) + \frac{1}{2} m_i^2 \phi_i = \rho_i (G),$$

$$\vec{\nabla} \times \left(\vec{B}_i + \alpha_i \vec{b} \right) - \frac{\partial}{\partial t} \left(\vec{E}_i + \alpha_i \vec{e} \right) + \frac{1}{2} m_i^2 \vec{G}_i = \vec{J}_i (G) \tag{197}$$

where $\vec{E} = \Omega^{-1}{}_{I1}\vec{E}^{I}$, and so on. Eq. (8) reproduces formally Maxwell equations for $\eta = 0$. These equations are showing that Maxwell and Global Maxwell are connected through a macroscopic electromagnetism to be investigated.

Another aspect to be included on this macroscopic view is about a collective physics. Eq. (4) is not associated only to individual quanta but also develop a macroscopic regime made of by composite fields $\vec{e}$, $\vec{b}$, s, $\vec{s}$, s_{ij}, r, r_{ij}. They are new elements for describing an fields environment, where while $\vec{e}$ and $\vec{b}$ are entities with space-time evolution, the others are sources of a given input. As a many body system they develop a local definition for an environment without quanta. Being gauge invariant they are candidates for measurement, defined similarly to usual electric and magnetic fields they are local fields representing excitations derived from many particles. Thus as an Eq. (4) limit, there is a macroscopic dynamical theory based on collective fields appear by taking $G_{\mu I} = \partial_{\mu}\varphi_{I}$.

11 LIGHT INVARIANCE AND LIGHT

The essence of this work is to understand light as the beginning. Maxwell stands EM fields being emanated from an electric charge. Its first two equations already show charge conservation and implicitly light invariance. Our viewpoint is that there is a primordial light interpretation to be grasped. A perspective supported by the expression $E = mc^2$, which says that any matter can be converted into energy, and so, indicating that radiation is not uniquely originated from the electric charge, and inversely, one should ask whether matter cannot be created from ordinary photons. However, the main difficulty for a photon physics overture is due to Maxwell's equation, it localizes photons as a consequence. They are derived just as a Larmor electromagnetic radiation.

Thus, we have been looking for an EM interpretation beyond electric charge. It starts by noticing that Maxwell introduces a passive light. In the Maxwell radiation field one gets light not as a source but as a lake transmitting EM fields. This can be seen through the D'Alembertian expression $A_{\mu}(t;\vec{x}) = \int dt'\, d^3\vec{x}'\, G_{\mu\nu}(t-t';\vec{x}-\vec{x}')\, J^{\nu}(t',\vec{x}')$ requiring an external source. An active light urges for being considered. We are challenged to work out this hypothesis. Introduce a step forward, first with respect to the original Maxwell prescription when Gauss and Ampère wrote their equations by observing electronic charges and currents, and second, to the abelian gauge theory that reinterpreted this process through gauge symmetry. We have to look for a new EM foundation. Light is a cause or consequence? EM phenomena should start with electric charge or self-interacting photons?

Light invariance is an interrogative principle, probably we cannot fully understand it, but at least one should relate it to the photon field. As the singular porter of this physics dogma, light should be a distinguishable particle. But just with Lorentz symmetry we cannot do that. Probably some symmetry is missing. We are looking for an electromagnetism not only based on the light metric but that one which takes the photon so singular as light invariance is. Then, after physics be pushed to a global approach, a next step is to differentiate the photon as a special particle between that others inserted in the set.

Light should be as beginning as light invariance is. There is a missing connection between light invariance and light. It is still a physical challenge to pursue on such correlation. Something is hidden for understanding the photon as origin. In the previous sections, a whole electromagnetism was introduced obeying the light metric and associated to conserved charges different than electric charge, but now, we wish to stipulate light as primordial. After the performances given by electric charge, the constant of light velocity in vacuum and Lorentz group algebra, a further configuration becomes to explore a relationship between light invariance and light through the wholeness principle. Perhaps through a new information, as to consider Lorentz and antireductionist gauge symmetries working together one is able to distinguish the photon field. Enlight light.

There is something more for the photon. Given these four interpretations, one derives a fields association based on a whole gauge symmetry where the $(\frac{1}{2},\frac{1}{2})$ set develops two special properties: two underlines symmetries (directive and circumstantial) and quanta diversity [57]. Then, the next question is over which quanta the directive symmetry will be manifested. Beyond that light speed is an invariant, there is another information coming through the set gauge symmetry. Analyzing the mass sector, Eqs. (7) and (36) show that the $(\frac{1}{2},\frac{1}{2})$ system contains as directive the presence of one massless quanta. This fact is crucial. It reveals that the photon field is that one to be identified as the system directive. Knowing that, one gets that given a potential fields set the photon field works as guideline. Let us there be light.

There is a wholeness principle for introducing the photon as primordial. Work as source of a whole. The novelty is that there is an antireductionist photon. Given the fields set $\{G_{\mu I}\}$, it is driven by light, where we must consider the photon field $G_{\mu 1}$ as origin. Nevertheless, one has to assign that this photon field is made of by vector and scalar photons. This means that both particles must be considered as basic. Therefore, although through the vector photon the Michelson-Morley experiment brings a physics based in the sense of absolute, we should not discard on scalar photon meaning. For physics, today, its existence is still a mystery. Our observation, is that this model naturally contains on its massless existence without requiring mass as in supersymmetry [111]. However, in order rewrite Maxwell as limit one has to avoid it, be decoupled as through Eq. (23) or Eq. (56). For nomenclature, we will call vector photon as photon.

The introduction of the antireductionist gauge symmetry offers four possibilities for the photon to be selected in the fields set $\{G_{\mu I}\}$. They are to understand it as the genuine gauge field, associated to two degrees of freedom controlled by the canonical momenta and the gauge fixing term, related to the directive symmetry and coupled to the Noether current. Firstly, one notices that the fields basis $\{D, X_i\}$ is showing the presence of just one genuine gauge field inserted between another potential fields. So, although the photon was introduced together with other potential fields in a set, as Eq. (7) denotes, one can distinguish it between different fields transforming under a same U(1)-antireductionistic symmetry by assuming as being that one directly connected to the gauge parameter. Consequently, Eq. (1) contains only one field directly associated to the gauge parameter, and so, this one, $G_{\mu 1}$, should be interpreted as the massless photon field.

Secondly, one should analyze on degrees of freedom. The corresponding canonical momenta at basis $\{D, X_i\}$ is $\Pi_D^{\mu} = \Pi_D^{\mu A} + \Pi_D^{\mu S} + \tilde{\Pi}_D^{\mu}$,where the antisymmetric contribution is $\Pi_D^{\mu A} = dZ^{[0\mu]}$, the symmetric $\Pi_D^{\mu S} = 0$ and the semitopological $\tilde{\Pi}_D^{\mu} = \eta d\gamma_{(ij)}\varepsilon^{\mu}{}_{\alpha\beta 0}X^{\alpha i}X^{\beta j}$, are saying that dynamically the theory suppress one degree of freedom from D_{μ}-field. Meanwhile the corresponding canonical momenta at physical basis is $\Pi_{G_I}^{\mu} = 2b_I Z^{[0\mu]} + \beta_I Z^{(0\mu)} + \rho_I Z^{\sigma}{}_{\sigma}\delta^{0\mu} + \eta b_I \varepsilon^{\mu}{}_{\alpha\beta 0}(z^{[\alpha\beta]} + z^{(\alpha\beta)})$, which relates a $\Pi_{G_I}^{0}$ expression not necessarily zero. Given that the number of d. f. is independent on the fields reference system [35], one expects at $\{G_I\}$ basis to cancel 1 d. f. through the circumstantial symmetry. This can be done by taking $\Pi_{G_1}^{0} = 0$ at physical basis through a convenient choice of the free parameters [112]. The second d. f. is taken from gauge fixing. Observe that, being the gauge fixing solution, as in the Lorentz gauge with $\alpha(x) = -\int d^4 y G(x-y)\partial^{\mu}\cdot G_{\mu 1}(y)$ where $G(x-y)$ is the Green's function of W. At this way, one gets that $G_{\mu 1}$ becomes the massless Maxwell field with two degrees of freedom. Notice that these 2 d. f. are taken circumstantially, while as directive the model incorporates the scalar photon. This is physics, a subject where any solution is valid since the basic symmetries are not violated.

The third photon characterization comes out from the set determinism. As studied at section 3, this whole approach drives directive and circumstantial symmetries, then, the photon field works as that one directly associated to the antireductionist gauge parameter. It yields a systemic physical solution where the photon field becomes the fields set directive while the other potential fields are under circumstances as studied at Appendices. While the photon field assumes the symmetry, other fields do their services. Finally, one makes a decision where the photon field is physically selected as that one for coupling to the conserved charge generated by the entire fields set. From the U(1)-whole symmetry one derives a Noether current to be coupled to the photon.

Therefore before the displacement current there is a correlation between light and light invariance to be explored. And so, by introducing an antireductionist gauge symmetry together with Lorentz invariance, one states a distinguished photon. Where, Eq. (8) is rewritten as

$$L(G) = Z_{\mu\nu}Z^{\mu\nu} - m_I^2 G_{\mu I}G^{\mu I} + \frac{1}{2\xi}(\partial_{\mu}G_1^{\mu})^2 + g_w j_N^{\mu}G_{\mu 1}. \tag{198}$$

Eq. (198) defines a Physics of Light [113]. It relates a global photon determinism $\gamma \cdot \{G_{\mu I}\}$. It makes the photon as the quanta responsible for carrying the physics conducting the space-time Lorentz symmetry and the antireductionistic gauge symmetry. A configuration characterizing a light electromagnetism where the electric charge is not more on the origin but anteceded by a photon simultaneous interaction with all other fields through the Noether conserved current, $j_N^{\mu} = \frac{\partial L}{\partial\partial_{\mu}G_{\nu I}}\delta G_{\nu I}$, and the whole coupling constant g_w .

Eq. (198) addresses to an electromagnetic behaviour where light becomes primordial. It proposes a model where besides light metric, it yields an antireductionist photon field. This fourth interpretation reveals that the photon can work as the symmetry manager and the whole maker with the following properties: singular (distinguished), selfinteracting, active, directive, global. Its singularity means be associated to light invariance and to the antireductionist gauge parameter; selfinteractivity comes from abelian non-linearity; activeness from being able to generate its own EM fields; directivity because it becomes compulsory for connecting $I-$EM layers and four regimes; and, global due to its corresponding connectivities. At this way, the former Planck-Einstein photon turns into a globalink particle responsible for connecting the pieces of the electromagnetic mosaic at Eq. (4). Conceptually, it is ubiquous, because it shall be everywhere where light invariance is. There is an ubiquous lux to be understood. Consequence from the correlation between light invariance and light from joining the Lorentz and the antireductionist gauge symmetries it generates an antireductionist messenger photon (singular-selfinteracting-active-directive-global). Light become a root acting everywhere through an $(\frac{1}{2},\frac{1}{2})$ physics that enlarges the light metric with four EM regimes entitled as photonic, massive, neutral, electric charged. Therefore, while light radiation is the appropriated term for electric charge, absolute for relativity, global light will be the terminology for this whole physics. There is an EM network physics being driven by such global photon acting as the light invariance promotor. It says that, the $2N+7$ classical equations are interdependencies coordinated through a global photon participating directly and indirectly in all physical processes involving the set $\{G_{\mu I}\}$.

These four non-Maxwellian regimes are the new aspects to be considered. They are meaning that light interacts more with matter and fields than one suspects. The first one is a photonic physics. We consider as photonic physics any property derived from photons. Similarly to the electron, the photon is expected to be an EM source. Evidently the 20th-century was dominated by the electron but the expectation for this millennium is on the photonic physics. Nowadays, a related fact is the presence of a Photonic Engineering. However, it still needs be supported by a Photonic Physics, as defining from first principles a photonic current to replace the electric current. Maxwell is not able to express this fact mathematically, it just provokes a misunderstanding by associating the Poynting vector as a photonic current. The origin for this photonic world must be expected from the photon as physical agent. Given the abelian non-linearity, as Eqs. (166)-(167) show, Eq. (198) become candidate for producing self-interacting photons, photonic current, non-linear electromagnetic waves, new dispersion relation, force depending on photon field. These features is that will be opening a view for Photonic Engineering [114] where light is more than an electromagnetic radiation.

This photonic world should be found through astrophysics and laboratory. For astrophysics view, looking at Hubble telescope photographs, we have a clue that there is a subtle dynamics where light precedes electric charge. There is a dynamical cosmological world entirely governed by light. Interstellar gases of photons interacting without electrons and positrons are showing a physics beyond QED. We expect this fact to be common in the stars formation. There, self-interacting photons may be responsible for increasing photon's energy. The creation of various cosmic structures as stellar explosions, rounded city of stars, interstellar clouds of gas are expected to be depending more on self-interacting photons than on gravitational effects and electronic charge. Antennae galaxies collisions (NGC 4038/4039), γ-rays bursts [115]-[116], observation of a photonic magnetoresistance, transmission bits, and so on, are yielding a photonic data [117]. The behavior of light in a strong magnetic field environment as magnetars [118] is another motivation, and, on a more speculative vein, superconducting cosmic strings [119]. For laboratory view, SLAC has demonstrated creation of matter out of light [120] and the PVLAS experiment on non-linearity [121]. QED should work as a boundary condition for a new photon physics: the electron-positron reaction offers a first range of energy, it belongs to a region where photons are carrying bundles of energy bellow to 1 MeV. Inelastic light-by-light scattering involving only real photons is another candidate for this lab determination [122]. There is a new physics which means to explore a kind of light where one can conjure matter from light and vice-versa.

A second regime concerns to associate EM fields with mass. There is a mass physics to be understood. Given that nearly 96% of the universe is dark [123], i.e. unknown, this assumption gets another opportunity to be investigated. However, fundamental physics experiments often refer to gravitation or particle physics and not electromagnetism. There is no relation between mass and electromagnetism. Here, differently from General Relativity where mass works as fields source through the energy momentum tensor, its participation is through field equations, conservation laws and force. Eqs. (4) and (160) are showing on possibilities for fields relationships with mass. They bring a contribution for the physical processes different from inertia and gravity.

A third regime is on neutral electromagnetism: in the realm of Atomic, Condensed Matter Physics, Astrophysics there are systems that present very strong magnetic fields, as neutron stars, without any electric charge be presented [124]. There is a clear Maxwell difficulty for describing neutral interactions as a neutron under an external magnetic field. There are different cases in the literature supporting a neutral electromagnetism. A first example is the electric dipole, quadrupole interactions with a globally neutral charge; a second is on neutral particles which exhibit magnetic dipole moment and couple to external magnetic fields $\mu\vec{S}\cdot\vec{B}$, despite their vanishing electric charge; a third one the Aharonov-Casher term $\vec{\mu}\times\vec{E}$; finally, on EM fields coupled with constants that are not the electric charge. The last frontier of this neutral electromagnetism is to detect the interaction $gG_{\mu 1}$ with a photon coupling beyond electric charge. For this, here one is able to introduce a gauge invariant neutral coupling as $a_{AI}\, j_{\mu}^{A} G^{\mu I}$ where j_{μ}^{A} can be any conserved charge $\{j_{\mu}^{A}\}$ developed at section 6. Concluding, we could say that Maxwell's theory is not so clear in situations involving spin-electromagnetism and completely absent when the electromagnetic fields are not coupled to electric charges.

The fourth regime comes back to electric charge. As we know electromagnetism is an interaction based on electric charge as quantum number. The extension here with respect to Maxwell and QED is that it introduces new possibilities for the electric charge be exchanged. Consider particles reactions associated to electric charge conservation but beyond photon intermediation. Explore an electromagnetism that introduces charged currents with $\Delta Q \neq 0$. Besides

Maxwell displacement current introduce a field set to support the electric charge conservation. Primarily to the usual electroweak intermediation $\{\gamma, Z^0, W^\pm\}$, there is an EM manifestation with an electric charge transmission through a potential fields set like $\{\gamma, \gamma_m \equiv \text{massive photon}, Y_\mu^\pm \equiv \text{massive charged bosons}\}$ without electric charge violation.

These interdependent four regimes will be explored in further works. Under this expectative, Eq. (7) is proposing a totality of potential fields guided by photons. Different physical regions are revealed. And so, a reason for photons fields existence is to connect different physical processes.

12 CONCLUSION

The number (31) and variety of non-Maxwellian electromagnetisms is relevant [79], [125], [126]. Since the 1920s with the works of de Broglie [80] that various attempts have been done, both in classical and quantum domains. They contain many hitherto independent lines of thought. This work selects 12 tasks beyond Maxwell. They are on polarization and magnetization vectors, massive photon, non-linearity, new EM fields, physical potential fields, new conserved charges, EM photonic, EM neutral, EM spin, EM mass, EM charged, new dispersion relation. As challenge, there is no model in literature providing all these features together. A fact which interrogates on existence of a new principle for the electromagnetism comprehension.

Our effort on this work is that there is still a fourth interpretation of light invariance to investigate on an EM formulation that generalizes that of Maxwell. Our observation is that nature works as a group. This implies that a new relationship between the parts must be introduced. Fundamental physics relates parts through symmetry, and as discussed, there are two possibilities for doing that, which are through multiplets and LG irreducible representations. However, there is a qualitative difference between these two ways of grouping, while through multiplets usual gauge theory ends up at the QED reductionism, based on LG one builds up a mechanism where the EM works under an antireductionist order.

Light invariance may be a strange experimental principle but we do not have another option than to follow it. It leads us to understand on physical laws. But, there is a contradiction, which is that its carrier, the photon, is being understood not as cause but as electric charge consequence. Something is missing. Everything should start with a deeper comprehension into the photon nature. So, after Maxwell, Relativity, Lorentz group, one introduces a fourth interpretation where fields are originated as Lorentz group niches, and discovers that, what is missing for a more complete understanding on the EM phenomena is the presence of an antireductionist photon. There is an ubiquous lux to be understood. It is in everywhere where light invariance is and works for associating fields located in any LG irreducible representation. It comes up as a consequence from two experimental symmetries. They are the photon space-time symmetry originated from the Michelson-Morley experiment and the antireductionist gauge symmetry from that nature works as a group. And so, light becomes a root that controls the fields systemic dynamics. The photon becomes a singular-active-directive-global messenger and its action precedes the electric charge behaviour. It appears a light universalism not ruled by the electric charge as coupling constant.

Three structures are derived from this fourth interpretation to the light invariance dogma. They are a singular photon, Global Maxwell state equation and Global Lorentz force. They extend Maxwell and pushes the limits of our EM knowledge to new frontiers expressed through Eq. (198). First, being the particle that porters the presence of light invariance the photon should receive a special distinction. Be more than a passive radiation or just a tensor transforming equally to others LG associated fields. Through the antireductionistic gauge parameter there are four opportunities for privileging the photon in a fields set. They are to associate it as the genuine gauge field, degrees of freedom fixed from the gauge-fixing term, directive in a set determinism, coupled to the system conserved currents. They distinguish the photon and it becomes singular, as one expects from the light invariance messenger. This fact is fundamental, encountered such light primordial view, the fine relationship between light invariance and light is realized. Then, with a singular photon in the physics scenario one derives, based on the correlation between light and light invariance, a model called Physics of Light [113]. As follows, it derives a Global Maxwell equation with laws articulated by a global photon and new possibilities for exchanging momentum from relationships between fields with fields and masses through a Global Lorentz force.

The Maxwell frontier is crossed. Considering the principles of light invariance, wholeness and gauge symmetry one has moved from Maxwell EM fields, QED photon field, to an Physics of Light with four regimes (electric charge, photonic, massive, neutral) interlaced by a global photon. The first regime nearest to Maxwell is the electric charge sector. It adds new possibilities for electric charge exchange. Given that electromagnetism standard definition is an interaction based on the electric charge quantum number conservation, there is room for the EM phenomena be extended through mediators beyond the photon, as massive photon, charged vector bosons. While at Maxwell-QED such exchange is just based on the photon intermediation, it is still possible to associate three carriers to the electric charge manifestation. They will provide an EM charged preserving the electric charge conservation, but beyond displacement current and photon field, it is possible to predict a new EM exchange through photon, massive photon, massive charged bosons. The wholeness principle introduces a fields set like $\{\gamma, \gamma_m, Y_\mu^\pm\}$.

The second regime is the photonic. QED does not support a photon age with photonic Hall-effect, photonic magnetoresistance and there is a new light behaviour to be understood. It is derived from light and not depending on electric charge. It discovers another EM region based on selfinteracting photons, photonic charge, new dispersion relation, Lorentz photon force. Phenomenologically, this photonic regime with photons generating photons also brings new phenomenological possibilities as photon-jets, photon-balls, frequency variations. This photonic territory sustains a Photonic Engineering based on a Physics of Light. For innovations, the novelty is on the possibilities to develop electromagnetic apparatus based on the photon new features and propose a new photonic telecommunication. The last two regimes are opportunities to investigate on EM phenomena through spin, mass and neutral charges. They provide another motivations to our understanding of the universe as largely based on EM observations.

A new EM building arises. It designs a non-Maxwellian model, an $(\frac{1}{2},\frac{1}{2})$ EM coordinated through an antireductionist photon with an universality which extends the electric charge presence. Its infrastructure is based on a new principle (wholeness), two symmetries (Lorentz and whole gauge abelian), Lagrangian (as Eq. (198)) and consistencies (local, positive Hamiltonian, renormalizable, unitary), where unitarity still has to be proved. These foundations will be arising Maxwell plus the 12 new phenomenological floors stated before. This “12 integral building” associated through a photon networking will provide opportunities for the next EM technologies [127].

As conclusion, we should pursue on light as the next energy shape to be explored. Given the photon antireductionist features (singular, selfinteracting, own EM fields, directive, global), light becomes the original energy. The $(\frac{1}{2},\frac{1}{2})$ EM extends the meaning of light conversion, it states that, besides $E = mc^2$, Larmor expression, particles-antiparticles collisions there are other relationships matter-fields-light due to the presence of a global photon interconnecting different matters and fields. As consequence, all forms of energy derived from these I -layers and four EM regimes will be depending on light primordial manifestation. Besides oil and gas, eolian and so on, light becomes the next energy to be produced and converted into another ones.

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APPENDIX A. VOLUME OF CIRCUMSTANCES

A consequence from Eq. (1) is the appearance of the so-called free and global coefficients. They are coefficients that can take any value without violating gauge invariance. The constructor basis, $\{D, X_i\}$, is the natural place to observe the free coefficients. And at physical basis, $\{G_I\}$, is where the global coefficients are identified.

Thus one should systematize the presence of these coefficients in the model. The free coefficients are associated to scalar terms. There are two origins: from gauge scalars given by fields strengths, and from gauge and Lorentz scalars given by the Lagrangian terms. So from Eq. (5) one notices that d , α_i , $\gamma_{[ij]}$, ..., $\tau_{(ij)}$ represent the gauge free coefficients. Observe that d and α_i coefficients can be absorbed through fields redefinition. At table A the free coefficients are listed:

TABLE A		
Scalars	*Free Coefficient*	*N º of Free Coefficients*
$D_{\mu\nu}D^{\mu\nu}$	d^2	1
$D_{\mu\nu}X^{\mu\nu}{}_i$	$2d\alpha_i$	$N-1$
$D_{\mu\nu}X^{\mu}{}_i X^{\nu}{}_j$	$2d\gamma_{[ij]}$	$(N-1)(N-2)/2$
$X_{\mu\nu i}X^{\mu\nu}{}_j$	$\alpha_i\alpha_j$	$(N-1)^2$
$X_{\mu\nu i}X^{\mu}{}_k X^{\nu}{}_j$	$2\alpha_i\gamma_{[kj]}$	$(N-1)^2(N-2)/2$
$X^{\mu}{}_i X^{\nu}_j X_{\mu k} X_{\nu l}$	$\gamma_{[ij]}\gamma_{[kl]} + \gamma_{(ij)}\gamma_{(kl)}$	$(N-1)^2[(N-1)^2+1]/2$
$\Sigma_{\mu\nu i}\Sigma^{\mu\nu}{}_j$	$\beta_i\beta_j$	$(N-1)^2$

$g^{\mu\nu}\Sigma_{\mu\nu i}\Sigma^{\alpha}{}_{\alpha j}$	$2\beta_i\rho_j$	$(N-1)^2$
$\Sigma_{\mu\nu i}X^{\mu}{}_k X^{\nu}{}_j$	$2\beta_i\gamma_{(kj)}$	$N(N-1)/2$
$g^{\mu\nu}\Sigma_{\mu\nu i}X_{\alpha k}X^{\alpha}{}_j$	$2\beta_i\tau_{(kj)}$	$N(N-1)^2/2$
$\Sigma^{\alpha}{}_{\alpha i}\Sigma^{\beta}{}_{\beta j}$	$4\rho_i\rho_j$	$(N-1)^2$
$g^{\mu\nu}\Sigma^{\alpha}{}_{\alpha i}X_{\mu k}X_{\nu j}$	$2\rho_i\gamma_{(kj)}$	$N(N-1)^2/2$
$\Sigma^{\alpha}{}_{\alpha i}X_{\beta k}X^{\beta}{}_j$	$8\rho_i\tau_{(kj)}$	$N(N-1)^2/2$
$X_{\alpha i}X^{\alpha}{}_j X_{\beta k}X^{\beta}{}_l$	$2\left[\gamma_{(ij)}+2\tau_{(ij)}\right]\tau_{(kl)}$	$[N(N-1)]^2/2$
$X_{\mu i}X^{\mu}{}_j$	$\frac{1}{2}m_{ij}^2$	$N(N-1)/2$

Considering the contributions from antisymmetric, symmetric and mass sectors the number of free coefficients is $1+(N-1)\left[N^3+2N-4\right]$. Adding the semitopological term we have more $\frac{1}{4}\left[N^4-2N^3+N^2-4N+4\right]$. Thus the total number of free coefficients carried by (11) Lagrangian is $\frac{1}{4}\left[3N^4-8N^3+13N^2-12N+8\right]$ where just $\frac{1}{2}\left[3N^2-3N+4\right]$ involves D_μ field. This means the volume of circumstances propitiated by this abelian global model. Given these initial coefficients plus the Ω matrix elements (which are also determined by free coefficients) one derives the global coefficients. They appear on the physical basis, $\{G_I\}$.

APPENDIX B. CIRCUMSTANTIAL SYMMETRIES

Two consequences from the circumstancial symmetry property developed by Eq. (1) global gauge model will be considered. They are the longitudinal decoupling and the new symmetries. Here circumstance on symmetry means to determine the coefficients b_I, β_I, ρ_I, $\gamma_{[IJ]}$, $\gamma_{(IJ)}$, $\tau_{(IJ)}$ for a certain objective without breaking gauge invariance. However being a physical problem one has first to establish what would be the physical precription. Thus we define by order as the model intention to have the following characteristics: antissymetric quanta (b_I), fields environment ($\gamma_{(IJ)}$), self-interaction photons ($\gamma_{(IJ)}$, $\tau_{(IJ)}$), symmetric quanta (β_I, ρ_I).

Let us start by studying the circumstances for conserved currents (c.c.). In order to avoid undesired models the classical prescription is conserved currents. For this every field in this global model must be associated to a corresponding conserved current (c.c.). Noether and Ward identities already inform on the existence of only one natural conservation law. In this appendice we will explore such circumstancial symmetry on conserved currents, for cases involving just two and three fields, $I=2,3$.

CASE 1: I=2.

Let us choose

$$\partial.J_2=0. \tag{B.1}$$

For being studied. There are two possibilities:

1.1 Usual case (no $E_{\mu\nu\rho\sigma}$ term).

There are 12 equations for the c.c. to be studied. Two possible solutions are

(a)

$$b_I = \frac{\beta_1 \gamma_{(I2)}}{\gamma_{[12]}}, \quad \rho_1 = \frac{\beta_1 RootOf\left(4\_Z^2 + 2\gamma_{(22)}\_Z + \gamma_{(22)}{}^2\right)}{\gamma_{(22)}}, \quad \rho_2 = \beta_2 = 0$$

$$\gamma_{(11)} = \frac{\gamma_{(12)}{}^2 + \gamma_{[12]}{}^2}{\gamma_{(22)}}, \quad \tau_{(11)} = \frac{\left(\gamma_{(12)}{}^2 + \gamma_{[12]}{}^2\right) RootOf\left(4\_Z^2 + 2\gamma_{(22)}\_Z + \gamma_{(22)}{}^2\right)}{\gamma_{(22)}{}^2}$$

$$\tau_{(12)} = \frac{\gamma_{(12)} RootOf\left(4\_Z^2 + 2\gamma_{(22)}\_Z + \gamma_{(22)}{}^2\right)}{\gamma_{(22)}}$$

$$\tau_{(22)} = RootOf\,(4\_Z^2 + 2\gamma_{(22)}\_Z + \gamma_{(22)}{}^2 \qquad \text{(B.2)}$$

where $\_Z$ is the any variable.

(b)

$$\tau_{(11)} = \frac{(2\beta_1\beta_2 + 4\beta_1\rho_2 + 8\rho_1\rho_2) RootOf\left(4\_Z^2 + 2RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)\_Z - \gamma_{[12]}{}^2\right)}{\beta_2{}^2 + 2\beta_2\rho_2 + 4\rho_2{}^2}$$
$$+ \frac{2(\beta_1\rho_2 - \beta_2\rho_1) RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)}{\beta_2{}^2 + 2\beta_2\rho_2 + 4\rho_2{}^2}$$

$$\gamma_{(22)} = \tau_{(22)} = 0$$

$$b_2 = -\frac{(2\beta_2 + 8\rho_2) RootOf\left(4\_Z^2 + 2RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)\_Z - \gamma_{[12]}{}^2\right)}{\gamma_{[12]}}$$
$$+ \frac{(\beta_2 - 2\rho_2) RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)}{\gamma_{[12]}}$$

$$\gamma_{(12)} = RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)$$

$$\tau_{(12)} = RootOf\left(4\_Z^2 + 2RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)\_Z - \gamma_{[12]}{}^2\right)$$

$$\gamma_{(11)} = \frac{8\beta_1(\rho_1 - \rho_2) RootOf\left(4\_Z^2 + 2RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)\_Z - \gamma_{[12]}{}^2\right)}{\beta_2{}^2 + 2\beta_2\rho_2 + 4\rho_2{}^2}$$
$$+ \frac{(2\beta_1\beta_2 + 4\beta_2\rho_1 + 8\rho_1\rho_2) RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)}{\beta_2{}^2 + 2\beta_2\rho_2 + 4\rho_2{}^2}$$

$$b_1 = -\frac{2\beta_1\beta_2{}^2 + 12\beta_1\beta_2\rho_2 + 8\beta_2\rho_2{}^2 + 16\beta_2\rho_1\rho_2 + 32\rho_1\rho_2{}^2}{\gamma_{[12]}\left(\beta_2{}^2 + 2\beta_2\rho_2 + 4\rho_2{}^2\right)} *$$
$$* RootOf\left(4\_Z^2 + 2RootOf\left(\_Z^2 + \gamma_{[12]}{}^2\right)\_Z - \gamma_{[12]}{}^2\right) +$$

$$+\frac{\left(\beta_1\beta_2{}^2-2\beta_1\beta_2\rho_2-4\beta_1\rho_2{}^2+2\beta_2{}^2\rho_1+4\beta_2\rho_1\rho_2-8\rho_1\rho_2{}^2\right)RootOf\left(\_Z^2+\gamma_{[12]}{}^2\right)}{\gamma_{[12]}\left(\beta_2{}^2+2\beta_2\rho_2+4\rho_2{}^2\right)} \quad \text{(B.3)}$$

1.2 Including semi-topological term

Analysing this sector isolatedly, the necessary and sufficient condition is $\gamma_{[12]}=0$. Considering all theory sectors we can choose the following possibilities:

(a) $\gamma_{[IJ]}=\gamma_{(IJ)}=\tau_{(IJ)}=0$ (B.4)

(b) $\gamma_{[IJ]}=\beta_I=\rho_I=0$ (B.5)

(c) $b_I=\gamma_{(IJ)}=\tau_{(IJ)}=0$ (B.6)

(d) $b_I=\beta_I=\rho_I=0$ (B.7)

CASE 2: I=3.

We have to study the case $\partial\cdot J_2=\partial\cdot J_3=0$ (B.8)

For without/with semi-topological term three general solutions are

1. $\gamma_{[IJ]}=\gamma_{(IJ)}=\tau_{(IJ)}=0$, exceto $\tau_{(11)}$ (B.9)
2. $\gamma_{[IJ]}=\beta_2=\beta_3=\gamma_{(IJ)}=\tau_{(IJ)}=0$, exceto $\gamma_{(11)}$ e $\tau_{(11)}$ (B.10)
3. $b_I=\beta_I=\rho_I=0$ (B.11)

Concluding, one notices in cases 1 and 2 the existence of solutions where undesired model can be decoupled without the considered physical prescription be violated.

As new symmetries and circumstances we understand possibilities on A_{IJ} matrix given by Eq. (14). For $I=2$, the conditions for a conserved charge (15) are

$$A_{11}=-\frac{\beta_2 A_{21}}{\beta_1},\quad A_{12}=-\frac{\beta_2{}^2 A_{21}}{\beta_1^2},\quad A_{22}=\frac{\beta_2 A_{21}}{\beta_1}$$

$$b_1=\frac{\beta_1 b_2}{\beta_2},\quad \rho_2=\frac{\rho_1\beta_2}{\beta_1},\quad \gamma_{(11)}=\frac{\beta_1\gamma_{(12)}}{\beta_2}$$

$$\gamma_{(22)}=\frac{\beta_2\gamma_{(12)}}{\beta_1},\ \tau_{(12)}=\frac{\beta_2\ \tau_{(11)}}{\beta_1},\ \tau_{(22)}=\frac{\beta_2{}^2\tau_{(11)}}{\beta_1{}^2}, \quad \text{(B.12)}$$

For N-conservation laws (17), we get

$$A_{I11}=-\frac{\beta_2 A_{I21}}{\beta_1},\quad A_{I12}=-\frac{\beta_2{}^2 A_{I21}}{\beta_1^2},\quad A_{I22}=\frac{\beta_2 A_{I21}}{\beta_1}$$

$$b_1=\frac{\beta_1 b_2}{\beta_2},\quad \rho_2=\frac{\rho_1\beta_2}{\beta_1},\quad \gamma_{(11)}=\frac{\beta_1\gamma_{(12)}}{\beta_2}$$

$$\gamma_{(22)}=\frac{\beta_2\gamma_{(12)}}{\beta_1},\ \tau_{(12)}=\frac{\beta_2\ \tau_{(11)}}{\beta_1},\ \tau_{(22)}=\frac{\beta_2{}^2\tau_{(11)}}{\beta_1{}^2}. \quad \text{(B.13)}$$

Notice that (B.12) and (B.13) are obtained for massless case.

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Appendix C. LONGITUDINAL DECOUPLING

We should investigate on the consequences from the relationship $\partial_\mu G^{\mu I}=0$. For charges and currents, it yields

$$\rho_I^{S-K}(G)=0 \tag{C.1}$$

$$\rho_I^{S-I}(G)=\gamma_{(IJ)}\vec{G}^J\cdot(\vec{s}+\vec{\sigma})+\gamma_{(IJ)}\phi^J(s+\sigma+r)+\tau_{(IJ)}\phi^J\left(s_j{}^j+s+4r\right)+$$

$$+\beta_I\vec{\nabla}\cdot\vec{s}-\rho_I\frac{\partial}{\partial t}s_j{}^j-(\beta_I+\rho_I)\frac{\partial}{\partial t}s-(\beta_I+4\rho_I)\frac{\partial}{\partial t}r$$

$$\vec{J}_I^{S-K}(G)=0 \tag{C.2}$$

$$\vec{J}_I^{S-I}(G)=-\gamma_{(IJ)}\phi^J(\vec{s}+\vec{\sigma})+\tau_{(IJ)}\vec{G}^J\left(s_j{}^j+s+4r\right)+\gamma_{(IJ)}\vec{G}^J r+\gamma_{(IJ)}G_j{}^J\left(s^{ij}+\sigma^{ij}\right)+$$

$$+\beta_I\frac{\partial}{\partial t}\vec{s}-\beta_I\nabla_j s^{ij}+\rho_I\vec{\nabla}s_j{}^j+\rho_I\vec{\nabla}s+(\beta_I+4\rho_I)\vec{\nabla}r$$

For the tensor energy momentum components:

$U_L=0$ (C.3)

$$U_{I-S}=2\sigma_{kl}s^{kl}+4\vec{\sigma}\cdot\vec{s}-6\sigma s+2\sigma_{kl}r^{kl}-2\sigma r-4\tilde{\sigma}s_k{}^k-4\tilde{\sigma}s+$$

$$-4\tilde{\sigma}r_k{}^k-4\tilde{\sigma}r+s_{kl}s^{kl}+2\vec{s}\cdot\vec{s}-3s^2+2s_{kl}r^{kl}-2sr+$$

$$-4s_k{}^k w-4sw+r_{kl}r^{kl}+r^2-4r_k{}^k w-4rw+$$

$$+4\frac{\partial}{\partial t}\left(\rho_I s_k{}^k\phi^I+(\beta_I+\rho_I)s\phi^I+\rho_I r_k{}^k\phi^I+(\beta_I+\rho_I)r\phi^I\right)+$$

$$-4\vec{\nabla}\cdot\left(\rho_I s_k{}^k\vec{G}^I+4\beta_I\vec{s}\phi^I+(\beta_I+\rho_I)s\vec{G}^I+\rho_I r_k{}^k\vec{G}^I+(\beta_I+\rho_I)r\vec{G}^I\right),$$

$U_{gf}=0$,

$\vec{S}_L=0,$ (C.4)

$$\vec{S}_{I-S}=4\sigma_{ik}s^k+4\vec{\sigma}s+4\sigma_k s_i{}^k+4\sigma\vec{s}+4\sigma_k r_i{}^k+4\tilde{\vec{\sigma}}s_k{}^k+4\tilde{\vec{\sigma}}s+4\tilde{\vec{\sigma}}r+$$

$$+4\tilde{\vec{\sigma}}r_k{}^k+4s_{ik}s^k+4s\vec{s}+4s_k r_i{}^k+4s_k{}^k\vec{w}+4s\vec{w}+4r_k{}^k\vec{w}+4r\vec{w}+$$

$$+4\frac{\partial}{\partial t}\left(\rho_I s_k{}^k\vec{G}^I+(\beta_I+\rho_I)s\vec{G}^I+\rho_I r_k{}^k\vec{G}^I+(\beta_I+\rho_I)r\vec{G}^I\right)+$$

$$-4\rho_I\vec{\nabla}\left(r_k{}^k\phi^I+r\phi^I+s_k{}^k\phi^I+s\phi^I\right)+4\beta_I\nabla_k\left(\vec{s}G^{kI}+s^k\vec{G}^I-r_i{}^k\phi^I-s_i{}^k\phi^I\right),$$

$\vec{S}_{gf}=0$,

$$T_L^{ij}=0, \tag{C.5}$$

$$T_{I-S}^{ij}=4\sigma^i{}_k s^{jk}+4\sigma^j{}_k s^{ik}+4\sigma^i s^j+4\sigma^j s^i+4\sigma^j{}_k r^{ik}+4\tilde{\sigma}^{ij}s_k{}^k+4\tilde{\sigma}^{ij}s+4\tilde{\sigma}^{ij}r_k{}^k$$

$$+4\tilde{\sigma}^{ij}r+4s^i{}_k s^{jk}+4s^i s^j+4s^j{}_k r^{ik}+4s_k{}^k w^{ij}+4sw^{ij}+4r_k{}^k w^{ij}+4rw^{ij}+$$

$$+4\beta_I \frac{\partial}{\partial t}\left(s^{ij}\phi^I - s^i G^{jI} - s^j G^{iI} + r^{ij}\phi^I\right)+$$

$$-4\rho_I \nabla^i\left(s_k{}^k G^{jI} + sG^{jI} + r_k{}^k G^{jI} + rG^{jI}\right)+$$

$$-4\rho_I \nabla^j\left(s_k{}^k G^{iI} + sG^{iI} + r_k{}^k G^{iI} + rG^{iI}\right)+$$

$$+4\beta_I \nabla_k\left(s^{ij}G^{kI} - s^{ik}G^{jI} - s^{jk}G^{iI} + r^{ij}G^{kI} - r^{ik}G^{jI} - r^{jk}G^{iI}\right)+$$

$$+\eta^{ij}\left(-2\sigma_{kl}s^{kl} + 4\vec{\sigma}\cdot\vec{s} - 2\sigma s - 2\sigma_{kl}r^{kl} - 2\sigma r + \right.$$

$$-2\theta r - s_{kl}s^{kl} + 2\vec{s}\cdot\vec{s} - s^2 - 2s_{kl}r^{kl} - 2sr - r_{kl}r^{kl} - r^2 +$$

$$\left. +4\rho_I \frac{\partial}{\partial t}\left(s_k{}^k\phi^I + s\phi^I + r_k{}^k\phi^I + r\phi^I\right) + 4\rho_I \vec{\nabla}\cdot\left(s_k{}^k\vec{G}^I + s\vec{G}^I + r_k{}^k\vec{G}^I + r\vec{G}^I\right)\right),$$

$$T_{gf}^{ij} = 0$$

The above equations are yielding that the condition $\partial_\mu G^{\mu I} = 0$ cancel the L-sector. They show that only physical entities associated to the symmetric interaction is different from zero. However, being an interaction without propagation, it does not have physical implication.

APPENDIX D. GLOBAL LORENTZ FORCE AT $\{D, X_i\}$ BASIS

For simplicity, we are going first to derive the force expression at $\{D, X_i\}$ basis. Following the procedure given by [78], one couples the current j_μ and $j_{\mu i}$ to fields D_μ and $j_{\mu i}$ respectively. It gives

$$\mathrm{L} = \mathrm{L}_0 + j_\mu D^\mu + j_{\mu i}X^{\mu i} \tag{D.1}$$

where L_0 is expressed at Eq. (2). It yields the following expression for the energy momentum tensor

$$\tilde{\theta}^{\mu\nu} = \theta^{\mu\nu}[\mathrm{L}_0] + j^\mu D^\nu + \frac{\alpha_i}{d}(j^\mu X^{\nu i} + j^\nu X^{\mu i}) - j_i^\nu X^{\mu i} - g^{\mu\nu}(j_\alpha D^\alpha + j_{\alpha i}X^{\alpha i}) \tag{D.2}$$

where $\theta^{\mu\nu}[\mathrm{L}_0]$ is the energy momentum, Eq. (105), written at $\{D, X_i\}$ basis. It gives

$$\begin{aligned}
\theta_{\mu\nu}[\mathrm{L}_0] = {} & 4Z_{[\mu\rho]}Z_{[\nu}{}^{\rho]} + 4Z_{(\mu\rho)}(\beta_i\Sigma_\nu{}^{\rho i} + \gamma_{(ij)}X_\nu{}^i X^{\rho j}) + 4Z_{(\rho}{}^{\rho)}(\rho_i\Sigma_{\mu\nu}{}^i + \tau_{(ij)}X_\mu{}^i X_\nu{}^j) \\
& -4\beta_i\partial_\rho(Z_{(\mu}{}^{\rho)}X_\nu{}^i) - 4\beta_i\partial_\rho(Z_{(\nu}{}^{\rho)}X_\mu{}^i) + 4\beta_i\partial_\rho(Z_{(\mu\nu)}X^{\rho i}) \\
& -4\rho_i\partial_\mu(Z_{(\rho}{}^{\rho)}X_\nu{}^i) - 4\rho_i\partial_\nu(Z_{(\rho}{}^{\rho)}X_\mu{}^i) + 4\rho_i\eta_{\mu\nu}\partial_\rho(Z_{(\sigma}{}^{\sigma)}X^{\rho i}) \\
& +4\eta\varepsilon_{\mu\alpha\rho\sigma}dD_\nu{}^\alpha z^{[\rho\sigma]} + 4\eta\varepsilon_{\mu\alpha\rho\sigma}\alpha_i X_\nu{}^{\alpha i}z^{[\rho\sigma]} + 4\eta\varepsilon_{\mu\alpha\rho\sigma}z_{[\nu}{}^{\alpha]}Z^{[\rho\sigma]} \\
& -2m_i^2 X_{\mu i}X_\nu{}^i - \eta_{\mu\nu}\mathrm{L}
\end{aligned} \tag{D.3}$$

Considering currents as explicitly depending on coordinates, one gets by definition [78]

$$\partial_\mu\tilde{\theta}^{\mu\nu} = -\frac{\partial\mathrm{L}}{\partial x_\nu}\big|_{expl} = -D^\mu\partial^\nu j_\mu - X^{\mu i}\partial^\nu j_{\mu i} \tag{D.4}$$

where $\frac{\partial L}{\partial x_\nu}|_{expl}$ means a derivative acting on the sector explicitly depending on x^μ. On the other hand, calculating $\partial_\mu \tilde{\theta}^{\mu\nu}$ from (2), one derives,

$$\partial_\mu \tilde{\theta}^{\mu\nu} = \partial_\mu \theta^{\mu\nu} + j_\mu D^{\mu\nu} + \frac{\alpha_i}{d} j_\mu \partial^\mu X^{\nu i} - j_{\mu i}\partial^\nu X^{\mu i} + \partial_\mu[(\frac{\alpha_i}{d} - j_i^\nu)X^{\mu i}]$$

$$- D^\mu \partial^\nu j_\mu - X^{\mu i}\partial^\nu j_{\mu i} \quad \text{(D.5)}$$

Comparing (4) and (5), we have

$$\partial_\mu \theta^{\mu\nu} = j_\mu D^{\nu\mu} - \frac{\alpha_i}{d} j_\mu \partial^\mu X^{\nu i} + j_{\mu i}\partial^\nu X^{\mu i} - \partial_\mu[(\frac{\alpha_i}{d} j^\nu - j_i^\nu)X^{\mu\nu}] \equiv f^\nu \quad \text{(D.6)}$$

where f^ν is the force density.

Considering the Euler-Lagrange relationships:

$$\partial_\mu(\frac{\partial L_0}{\partial\partial_\mu D_\nu}) = j^\nu \quad \text{(D.7)}$$

$$\partial_\mu(\frac{\partial L_0}{\partial\partial_\mu X_\nu{}^i}) = \frac{\partial L_0}{\partial X_\nu{}^i} - \frac{\alpha_i}{d} j^\nu + j_i^\nu \quad \text{(D.8)}$$

one gets,

$$f^\nu = j_\mu D^{\nu\mu} + j_{\mu i} X^{\nu\mu i} + K_i^\mu \partial_\mu X^{\nu i} + \partial_\mu(K_i^\nu X^{\mu i}) \quad \text{(D.9)}$$

where

$$K_i^\mu = \partial_\nu(\frac{\partial L_0}{\partial\partial_\nu X_\mu{}^i}) - \frac{\partial L_0}{\partial X_\mu{}^i} = 2m_{ij}^2 X^{\mu j} - 4J_1{}^\mu \quad \text{(D.10)}$$

Substituting in (D.3),

$$f^\nu = j_\mu D^{\nu\mu} + j_{\mu i} X^{\nu\mu i} - 2m_{ij}^2 X^{j\nu}\partial_\mu X^{\mu i} + 4X^{\nu i}\partial_\mu J_i{}^\mu$$

$$+\partial_\mu(4m_{ij}^2 X^{\mu j} X^{\nu i} - J_i{}^\mu X^{\nu i} - J_i{}^\nu X^{\mu i}) \quad \text{(D.11)}$$

Finally, not considering the total derivative

$$f^\nu = j_\mu D^{\nu\mu} + j_{\mu i} X^{\nu\mu i} - 2m_{ij}^2 X^{\nu j}\partial_\mu X^{\mu i} + 4X^{\nu i}\partial_\mu J_i{}^\mu \quad \text{(D.12)}$$

Writing in terms of physical basis $\{G_{\mu I}\}$,

$$f^\nu = j_{\mu I} G^{\nu\mu I} - 2m_I^2 G_I^\nu \partial_\mu G^{\mu I} + 4G^{\nu I}\partial^\mu J_{\mu I}(G) \quad \text{(D.13)}$$

where

$$j_{\mu I} = j_\mu \Omega_{1I} + j_{\mu i}\Omega^i{}_I, \quad m_{IJ}^2 = m_{ij}^2 \Omega^i{}_I \Omega^j{}_J, \quad J_{\mu I}(G) = J_{\mu i}(D, X)\Omega^i{}_I \quad \text{(D.14)}$$

APPENDIX E. DISPERSION RELATION FOR $N = 2$

Considering Eq. (36) involving two potential fields one gets the following expression for the poles in the corresponding dispersion relation matrix.

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$$\det(M) = A_1 w^{16} + A_2 w^{14} + A_3 w^{12} + A_4 w^{10} + A_5 w^8 + A_6 w^6 + A_7 w^4 + A_8 w^2 + A_9 \quad \text{(E.1)}$$

$$A_1 = -n_{12}^{(1)^2} + n_{11}^{(1)} n_{22}^{(1)}$$

$$A_2 = (-2n_{22}^{(1)} n_{11}^{(1)^2} + (2n_{12}^{(1)^2} - 2n_{22}^{(1)^2} - 12n_{22}^{(1)})n_{11}^{(1)} + (2n_{22}^{(1)} + 12)n_{12}^{(1)^2})k^2$$

$$+((\frac{1}{2} - \frac{3}{2}n_{22}^{(1)})n_{11}^{(1)} + \frac{3}{2}n_{12}^{(1)^2})m_2{}^2$$

$$A_3 = ((16n_{22}^{(1)} + 4n_{22}^{(1)^2})n_{11}^{(1)^2} + ((-8n_{22}^{(1)} - 16)n_{12}^{(1)^2} + 16n_{22}^{(1)^2} + 52n_{22}^{(1)} - 2)n_{11}^{(1)} + 4n_{12}^{(1)^4}$$

$$+(-52 - 16n_{22}^{(1)})n_{12}^{(1)^2} - 2n_{22}^{(1)})k^4 + ((3n_{22}^{(1)} - 1)n_{11}^{(1)^2} + (-\frac{11}{2} + 2n_{22}^{(1)^2} - 3n_{12}^{(1)^2}$$

$$+\frac{29}{2}n_{22}^{(1)})n_{11}^{(1)} + (-2n_{22}^{(1)} - \frac{31}{2})n_{12}^{(1)^2})m_2{}^2 k^2 + ((-\frac{3}{4} + \frac{3}{4}n_{22}^{(1)})n_{11}^{(1)} - \frac{3}{4}n_{12}^{(1)^2})m_2{}^4$$

$$A_4 = ((-16n_{22}^{(1)^2} - 42n_{22}^{(1)} + 4)n_{11}^{(1)^2} + ((32n_{22}^{(1)} + 42)n_{12}^{(1)^2} + 16 - 42n_{22}^{(1)^2} - 100n_{22}^{(1)})n_{11}^{(1)}$$

$$-16n_{12}^{(1)^4} + (108 + 42n_{22}^{(1)})n_{12}^{(1)^2} + 16n_{22}^{(1)} + 4n_{22}^{(1)^2})k^6 + ((-17n_{22}^{(1)} - 4n_{22}^{(1)^2} + 7)n_{11}^{(1)^2}$$

$$+((17 + 8n_{22}^{(1)})n_{12}^{(1)^2} - 14n_{22}^{(1)^2} + \frac{49}{2} - \frac{95}{2}n_{22}^{(1)})n_{11}^{(1)} - 4n_{12}^{(1)^4} + (14n_{22}^{(1)} + \frac{109}{2})n_{12}^{(1)^2}$$

$$+3n_{22}^{(1)} - 1)m_2{}^2 k^4 + ((-\frac{3}{2}n_{22}^{(1)} + \frac{3}{2})n_{11}^{(1)^2} + (\frac{3}{2}n_{12}^{(1)^2} - \frac{1}{2}n_{22}^{(1)^2} + 7 - \frac{11}{2}n_{22}^{(1)})n_{11}^{(1)}$$

$$+(\frac{1}{2}n_{22}^{(1)} + \frac{13}{2})n_{12}^{(1)^2})m_2{}^4 k^2 + ((-\frac{1}{8}n_{22}^{(1)} + \frac{3}{8})n_{11}^{(1)} + \frac{1}{8}n_{12}^{(1)^2})m_2{}^6$$

$$A_5 = ((-16 + 48n_{22}^{(1)} + 24n_{22}^{(1)^2})n_{11}^{(1)^2}$$

$$+((-48n_{22}^{(1)} - 48)n_{12}^{(1)^2} + 48n_{22}^{(1)^2} - 42 + 86n_{22}^{(1)})n_{11}^{(1)} + 24n_{12}^{(1)^4} + (-48n_{22}^{(1)} - 118)n_{12}^{(1)^2}$$

$$-16n_{22}^{(1)^2} - 42n_{22}^{(1)} + 4)k^8 + ((12n_{22}^{(1)^2} - 22 + 30n_{22}^{(1)})n_{11}^{(1)^2}$$

$$+((-30 - 24n_{22}^{(1)})n_{12}^{(1)^2} + \frac{121}{2}n_{22}^{(1)} - \frac{115}{2} + 28n_{22}^{(1)^2})n_{11}^{(1)} + 12n_{12}^{(1)^4}$$

$$+(-\frac{173}{2} - 28n_{22}^{(1)})n_{12}^{(1)^2} - 17n_{22}^{(1)} - 4n_{22}^{(1)^2} + 7)m_2{}^2 k^6 + ((-8 + 5n_{22}^{(1)} + n_{22}^{(1)^2})n_{11}^{(1)^2}$$

$$+((-2n_{22}^{(1)} - 5)n_{12}^{(1)^2} + 3n_{22}^{(1)^2} + \frac{45}{4}n_{22}^{(1)} - \frac{93}{4})n_{11}^{(1)} + n_{12}^{(1)^4} + (-\frac{69}{4} - 3n_{22}^{(1)})n_{12}^{(1)^2} - \frac{3}{2}n_{22}^{(1)}$$

$$+\frac{3}{2})m_2{}^4 k^4 + ((\frac{1}{4}n_{22}^{(1)} - \frac{3}{4})n_{11}^{(1)^2} + (-\frac{1}{4}n_{12}^{(1)^2} - \frac{23}{8} + \frac{5}{8}n_{22}^{(1)})n_{11}^{(1)} - \frac{7}{8}n_{12}^{(1)^2})m2^6 k^2$$

$$-\frac{1}{16}m_2{}^8 n_{11}^{(1)}$$

$$A_6 = ((-16n_{22}^{(1)^2} + 24 - 22n_{22}^{(1)})n_{11}^{(1)^2} + ((22 + 32n_{22}^{(1)})n_{12}^{(1)^2} - 22n_{22}^{(1)^2} - 20n_{22}^{(1)} + 48)n_{11}^{(1)}$$

$$-16n_{12}^{(1)^4} + (68 + 22n_{22}^{(1)})n_{12}^{(1)^2} - 16 + 48n_{22}^{(1)} + 24n_{22}^{(1)^2})k^{10} + ($$

$$(-18n_{22}^{(1)}-12n_{22}^{(1)^2}+34)n_{11}^{(1)^2}+((24n_{22}^{(1)}+18)n_{12}^{(1)^2}+\frac{139}{2}-20n_{22}^{(1)^2}-\frac{41}{2}n_{22}^{(1)})n_{11}^{(1)}$$

$$-12n_{12}^{(1)^4}+(20n_{22}^{(1)}+\frac{133}{2})n_{12}^{(1)^2}+12n_{22}^{(1)^2}-22+30n_{22}^{(1)})m_2{}^2k^8+($$

$$(16-2n_{22}^{(1)^2}-4n_{22}^{(1)})n_{11}^{(1)^2}+((4+4n_{22}^{(1)})n_{12}^{(1)^2}-4n_{22}^{(1)^2}+35-5n_{22}^{(1)})n_{11}^{(1)}-2n_{12}^{(1)^4}$$

$$+(19+4n_{22}^{(1)})n_{12}^{(1)^2}-8+5n_{22}^{(1)}+n_{22}^{(1)^2})m_2{}^4k^6$$

$$+((\frac{11}{4}-\frac{1}{4}n_{22}^{(1)})n_{11}^{(1)^2}+(\frac{1}{4}n_{12}^{(1)^2}+\frac{27}{4}-\frac{1}{4}n_{22}^{(1)})n_{11}^{(1)}-\frac{3}{4}+\frac{1}{4}n_{22}^{(1)}+\frac{3}{2}n_{12}^{(1)^2})m2^6k^4$$

$$+(\frac{3}{8}n_{11}^{(1)}+\frac{1}{8}n_{11}^{(1)^2})m_2{}^8k^2$$

$$A_7=((-16+4n_{22}^{(1)^2})n_{11}^{(1)^2}+(-22-8n_{12}^{(1)^2}n_{22}^{(1)}-12n_{22}^{(1)})n_{11}^{(1)}+24+4n_{12}^{(1)^4}-20n_{12}^{(1)^2}$$

$$-16n_{22}^{(1)^2}-22n_{22}^{(1)})k^{12}+((4n_{22}^{(1)^2}-25-n_{22}^{(1)})n_{11}^{(1)^2}$$

$$+((1-8n_{22}^{(1)})n_{12}^{(1)^2}-\frac{73}{2}-\frac{25}{2}n_{22}^{(1)}+2n_{22}^{(1)^2})n_{11}^{(1)}+4n_{12}^{(1)^4}+(-\frac{49}{2}-2n_{22}^{(1)})n_{12}^{(1)^2}-18n_{22}^{(1)}$$

$$-12n_{22}^{(1)^2}+34)m_2{}^2k^{10}+((n_{22}^{(1)^2}-14-n_{22}^{(1)})n_{11}^{(1)^2}$$

$$+((-2n_{22}^{(1)}+1)n_{12}^{(1)^2}+n_{22}^{(1)^2}-\frac{89}{4}-\frac{19}{4}n_{22}^{(1)})n_{11}^{(1)}+n_{12}^{(1)^4}+(-\frac{37}{4}-n_{22}^{(1)})n_{12}^{(1)^2}+16-2n_{22}^{(1)^2}$$

$$-4n_{22}^{(1)})m_2{}^4k^8$$

$$+((-\frac{1}{4}n_{22}^{(1)}-\frac{13}{4})n_{11}^{(1)^2}+(-\frac{23}{4}-\frac{3}{4}n_{22}^{(1)}+\frac{1}{4}n_{12}^{(1)^2})n_{11}^{(1)}+\frac{11}{4}-n_{12}^{(1)^2}-\frac{1}{4}n_{22}^{(1)})m_2{}^6k^6$$

$$+(\frac{1}{8}-\frac{1}{4}n_{11}^{(1)^2}-\frac{1}{2}n_{11}^{(1)})m_2{}^8k^4$$

$$A_8=((2n_{22}^{(1)}+4)n_{11}^{(1)^2}+(2n_{22}^{(1)^2}-2n_{12}^{(1)^2}+4n_{22}^{(1)})n_{11}^{(1)}+(4-2n_{22}^{(1)})n_{12}^{(1)^2}-16+4n_{22}^{(1)^2})k^{14}+$$

$$((7+3n_{22}^{(1)})n_{11}^{(1)^2}+(2n_{22}^{(1)^2}+\frac{11}{2}n_{22}^{(1)}+\frac{3}{2}-3n_{12}^{(1)^2})n_{11}^{(1)}+(-2n_{22}^{(1)}+\frac{11}{2})n_{12}^{(1)^2}+4n_{22}^{(1)^2}$$

$$-25-n_{22}^{(1)})m_2{}^2k^{12}+((\frac{9}{2}+\frac{3}{2}n_{22}^{(1)})n_{11}^{(1)^2}+(\frac{1}{2}n_{22}^{(1)^2}-\frac{3}{2}n_{12}^{(1)^2}+\frac{5}{2}n_{22}^{(1)}+2)n_{11}^{(1)}$$

$$+(-\frac{1}{2}n_{22}^{(1)}+\frac{5}{2})n_{12}^{(1)^2}+n_{22}^{(1)^2}-14-n_{22}^{(1)})m_2{}^4k^{10}$$

$$+((\frac{1}{4}n_{22}^{(1)}+\frac{5}{4})n_{11}^{(1)^2}+(\frac{7}{8}+\frac{3}{8}n_{22}^{(1)}-\frac{1}{4}n_{12}^{(1)^2})n_{11}^{(1)}-\frac{1}{4}n_{22}^{(1)}+\frac{3}{8}n_{12}^{(1)^2}-\frac{13}{4})m2^6k^8$$

$$+(\frac{1}{8}n_{11}^{(1)}-\frac{1}{4}+\frac{1}{8}n_{11}^{(1)^2})m_2{}^8k^6$$

$$A_9=((2+n_{22}^{(1)})n_{11}^{(1)}+4+2n_{22}^{(1)}-n_{12}^{(1)^2})k^{16}$$

$$+((\frac{3}{2}n_{22}^{(1)}+\frac{7}{2})n_{11}^{(1)}+3n_{22}^{(1)}-\frac{3}{2}n_{12}^{(1)^2}+7)m_2{}^2k^{14}$$

$$+((\frac{9}{4}+\frac{3}{4}n_{22}^{(1)})n_{11}^{(1)}-\frac{3}{4}n_{12}^{(1)^2}+\frac{9}{2}+\frac{3}{2}n_{22}^{(1)})m_2{}^4k^{12}$$

$$+((\frac{5}{8}+\frac{1}{8}n_{22}^{(1)})n_{11}^{(1)}+\frac{1}{4}n_{22}^{(1)}-\frac{1}{8}n_{12}^{(1)^2}+\frac{5}{4})m_2{}^6k^{10}+(\frac{1}{8}+\frac{1}{16}n_{11}^{(1)})m_2{}^8k^8$$

with the following solutions:

$$\omega^2 = k^2 \tag{E.2}$$

$$\omega^2 = k^2 + \frac{1}{2}m_2^2 \tag{E.3}$$

and

$$\omega^2 = k^2 - \frac{1}{2}\frac{n_{11}^{(1)}}{n_{11}^{(1)}n_{22}^{(1)} - n_{12}^{(1)}n_{12}^{(1)}}m_2^2 \tag{E.4}$$

Thus we have to investigate on the possibilities for having a speed greater than c. The first two cases are forbidden. However Eq. (D.4) shows possibilities for the case:

$$\frac{n_{11}^{(1)}}{n_{11}^{(1)}n_{22}^{(1)} - n_{12}^{(1)}n_{12}^{(1)}} > 0. \tag{E.5}$$

References

[1] J. C. Maxwell, London Edinburgh Dublin Phys. Mag and J. Sc. Series IV, **21**, 161 (1861)

[2] A. Einstein, Ann. Phys. **17**, 831 (1905)

[3] E. Wigner, *Gruppe Theory Und Three Anwendung Auf Die Quantum Mechanik Der Atomspektren*. Vieng & Sohn, Braunshweig (1931)

[4] G. t'Hooft. *50 years of Yang-Mills theory*. World Scientific (2005)

[5] I. Prigogine. *From being to becoming: time and complexity in the physical sciences*. W. H. Freeman (1980)

[6] P. M. Zerwas and H. A. Kastrup (Eds.). *QCD: 20 years later*. World Scientific (1993)

[7] W. Marciano and H. Pagels, Phys. Rep. **36C**, 137 (1978)

[8] M. Gell-Mann. *The Quark and the Jaguar: Adventures in the Simple and the Complex*. W. H. Freeman (1994)

[9] K. Mainzer. *Thinking in Complexity: the complex dynamics of matter, mind, and mankind*. Springer-Verlag (1994)

[10] D. Bohm. *Unfolding Meaning*. Routledge (1985)

[11] S. Carroll. *From ethernity to here: the quest for the ultimate theory of time*. Dutton Adult (2010)

[12] M. Gell-Man. *The Eightfold Way: A Theory of Strong Interaction Symmetry*, California Institute of Technology Synchroton Laboratory Report CTSL-20 (1961) unpublished

[13] C. Kittel, *Introduction to Solid State Physics*, John Wiley, New York (1986)

[14] H. Weyl, Zeit. f. Phys. **56**, 330 (1929)

[15] J. Chauca, PhD thesis, Centro Brasileiro de Pesquisas Físicas, (2012) unpublished

[16] H. Georgi and S. Glashow, Phys. Rev. Lett. **28**, 1494 (1972)

[17] J. Pati and A. Salam, Phys. Rev. **D8**, 1240 (1973)

[18] L. Fonda and G. C. Ghirardi. *Symmetry Principles in Quantum Physics*. Marcel Decker, Inc. (1970)

[19] J. C. Taylor. *Gauge theories of weak interactions*, Cambridge University Press (1976)

[20] R. M. Doria, J. A. Helayël-Neto, C. Pombo, Il Nuovo Cim. **38A** (1987)

[21] A. P. Lima, J. Chauca and R. Doria. *Systemic Gauge Theory*. PoS ISFTG (2009) 023

[22] R. M. Doria and C. Pombo, I1 Nuovo Cim., **96B**, 2 (1986)

[23] C. M. Doria, R. M. Doria, J. A. Helayël-Neto, Rev. Bras. Fis., Vol 17, (1987) 3.

[24] N. Chair, J. A. Helayël-Neto and A. William Smith, Phys. Lett. **B 233**, (1989) 173

[25] S. A. Dias, R. M. Doria, J. L. Matheus Valle, Rev. Bras. Fis., **1**, Vol. 21 (1991)

[26] C. A. S. Almeida and R. M. Doria, Rev. Bras. Fis., **21**, Vol 13 (1991)

[27] C. M. Doria, R. M. Doria, F. A. B. Rabelo de Carvalho, Acta Physica Hungarica**73** (1), (1993) 51-58

[28] C. M. Doria, R. M. Doria, J. A. Helayël-Neto, Comm. Theor. Phys. (1992)

[29] R. M. Doria, J. A. Helayël-Neto, S. Mokhtari, Europhysics Letters **16** Vol. 1 (1991)

[30] R. M. Doria, F. A. B. Rabelo de Carvalho, G. Oliveira Neto, Brazilian Journal of Physics, **3** Vol. 24 (1994)

[31] H. J. Borchers, IL Nuovo Cim. **15** 784 (1960)

[32] R. M. Doria and M. Werneck de Oliveira, Rev. Bras. Fis., **1**, Vol. 20 (1990)

[33] F. A. B. Rabelo de Carvalho, R. Doria, S. P. Sorella, Rev. Bras. Fis., **4**, Vol. 20, (1990) 316-321

[34] R. M. Doria, H. P. Gersdorff, E. Silva, CBPF preprint NF032/84, unpublished

[35] R. M. Doria, J. A. Helayël-Neto, Acta Physica Hungarica **73**(2-4), 243-256 (1993)

[36] J. Domingos, R. M. Doria, R. Portugal, Acta Physica Hungarica, 73(1-2) (1993)

[37] J. Chauca, R. Doria, W. Soares, *Non-linear Abelian Gauge Model*, AIP Conf. Proc. **1483**, 432-351 (2012)

[38] E. C. G. Stueckelberg, Helv. Phys. Acta, **11**, 225 (1938)

[39] F. London, Zeit. f. Phys. **42**, 375 (1927)

[40] V. Ginzburg and L. Landau, JETP **20**, 1064-1082 (1950)

[41] B. Podolsky, Phys. Rev. **62**, 68 (1942)

[42] C. K. Kuchi, Phys. Rev. **65**, 228 (1944)

[43] P. Schwed, Rev. Mod. Phys. **20**, 40 (1945)

[44] Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960)

[45] J. Goldstone, Nuovo Cimento **13**, 154 (1961)

[46] S. Weinberg, Phys. Rev. **D13**, 974 (1976)

[47] L. Susskind, Phys. Rev. **D20**, 2619 (1979)

[48] S. Chern, J. Simons, Annals. Math. **99**, 48 (1974)

[49] P. W. Higgs, Phys. Lett. **12**, 132 (1964)

[50] F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321-3 (1964)

[51] J. Chauca, R. Doria, W. Soares, *Gauge Invariance for a Whole Abelian Model*, AIP Conf. Proc. **1483**, 400-406 (2012)

[52] J. Chauca, R. Doria, J. L. M. Valle, Rev. Mexicana de Física **58**, 152-159 (2012)

[53] J. Chauca, R. Doria. *Renormalization issues for a whole abelian model*, preprint (2013)

[54] F. Wilczek. *Origins of Mass*, arXiv 1206.7114v1 (2012)

[55] J. Chauca, R. Doria, *On Whole Abelian Model Dynamics*, AIP Conf. Proc. **1483**, 352-370 (2012)

[56] J. Chauca, R. Doria, *On Vectorial Fields as Lorentz Specie*, AIP Conf. Proc. **1483**, 371-376 (2012)

[57] J. Chauca, R. Doria, *On Spectroscopy for a Whole Abelian Model*, AIP Conf. Proc. **1483**, 407-418 (2012)

[58] N. Cabbibo and E. Ferrari, Nuovo Cimento **23**, 1147 (1962)

[59] J. Chauca, R. Doria, *Charged Fields in a Whole Abelian Model*, submitted paper (2013)

[60] J. Chauca, R. Doria, W. Soares, *Lorentz Transformations for Whole Electromagnetic Fields*, AIP Conf. Proc. **1483**, 383-387 (2012)

[61] C. F. Gauss, *Resultate aus den Beobachtungen des magnetischen Vereins in Jahre 1839*, 1 (1840); reprinted in Kön.

Ges. Wiss. Götingen, **5**, 195 (1867)

[62] A. M. Ampère, *Mémoire sur l'action mutuelle d'un conducteur Voltaïque et d'un aimant* (M. Heinz, Bruxelles, 1826)

[63] M. Faraday, *On the Physical Character of the Lines of Magnetic Force*, Philosophical Magazine, pag. 401, June 1852

[64] M. Born, I. Infeld, Proc. Roy. Soc. London **A147**, 522-546 (1934)

[65] J. J. Thomson, Phil. Magazine, 44-293 (1897)

[66] B. Franklin, *Experiments and Observations on Electricity, made at Philadelphia in America*. Printed and fold by E. Cave, at St. John's Gate (1751)

[67] A. A. Michelson and E. W. Morley, Am. J. Sci. (3), **34**, 333 (1887)

[68] J. Schwinger. *Selected Papers on Quantum Electrodynamics*. Dover (1956)

[69] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959)

[70] Y. Aharonov and A. Casher, Phys. Rev. Lett., **53**, 319 (1984)

[71] B. Pelle, H. Bitard, G. Bailly, C. Robilliard, Phys. Rev. Lett., **106**, 133003, 1-4 (2011)

[72] J. Klaers, J. Schmitt, F. Vewinger and M. Weitz, Nature **468**, 545 (2010)

[73] J. D. Jackson. *Classical Electrodynamics*. Wiley (1962)

[74] E. Noether, Königliche Gesellschaft der Wüssenschaften zu Göttingen. Mathematisch-Physikalische Klasse. Nachrichten, 235 (1918)

[75] H. Georgi. *Lie Algebras in Particle Physics*, Addisson-Wesley (1982)

[76] G. Scharf, *From Electrostatics to Optics*, Springer-Verlag (1994)

[77] L. H. Ryder, *Quantum Field Theory*, Cambridge University Press (1996)

[78] A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles*, Macmillan (1964)

[79] J. A. Helayël-Neto, Professor Global Class Notes, www.professorglobal.cbpf.br, 2013

[80] L. de Broglie, Phys. et Radium, **6**, 422 (1922)

[81] A. Proca, Comptes Rendus Acad. Sc. Paris, **202**, 1366 (1936)

[82] W. Heisenberg, H. Euler, Z. Phys., **98**, 714 (1936)

[83] P. A. M. Dirac, Proc. Roy. Soc. **A33**, 60-72 (1931)

[84] M. Kalb and P. Rammond, Phys. Rev. **D9**, 2273 (1974)

[85] E. Cremmer and J. Scherk, Phys. Lett. **B69**, 97 (1977)

[86] F. Wilczek, Phys. Rev. Lett. **58**, 1799 (1987)

[87] S. Carrol, G. Field, R. Jackiw, Phys. Rev. D, **41**, 1231 (1990)

[88] R. Gambini, J. Pullin, Phys. Rev. D, **59**, 124021 (1999)

[89] R. C. Myers, M. Pospelov, Phys. Rev. Lett., **90**, 211601-1 (2003)

[90] T. Kinoshita, *History of Original Ideas and Basic Discoveries in Particle Physics*, Plenum Press, New York, ed. H. Newman and T. Ypsilantis (1995)

[91] J. Chauca, R. Doria, W. Soares, *Electromagnetic Fields from Two Potential Fields*, AIP Conf. Proc. **1483**, 377-382 (2012)

[92] A. Retinó, A. Spallicci, in preparation (2013)

[93] S. G. Nibbelink, M. Pospelov, Phys. Rev. Lett. **94**, 081601 (2005)

[94] P. A. Bolokhov, M. Pospelov, Phys. Rev. D, **77**, 025022 (2008)

[95] E. Schrödinger, Proc. Royal Irish Ac. A, **47**, 1 (1941)

[96] A. S. Goldhaber, M. M. Nieto, Rev. Mod. Phys., **82**, 939 (2010)

[97] J. Chauca et al., work in preparation

[98] G. Mie, Ann. Physik **37**, 511 (1912)

[99] E. Goulart, S. E. Perez Berglialla, Phys. Rev. D, **63**, 064006 (2011)

[100] T. J. M. Boyd, J. J. Sanderson. *The Physics of Plasmas*. Cambridge University Press (2003)

[101] P. Goddard and D. Olive, Rep. Prog. Phys. **41**, 1357 (1978)

[102] R. W. Blois and C. D. Graham, J. Appl. Phys. **29**, 931 (1959)

[103] C. Castelnovo, R. Moessner, S. L. Sondhi, Nature **451**, 42 (2008)

[104] G. t'Hooft, Nucl. Phys. **B 79**, 276 (1974)

[105] A. M. Polyakov, JETP Letters 20 (1974)

[106] C. Montonen and D. Olive, Phys. Lett. **B 72**, 117 (1977)

[107] S. Verkovic, J. Schidrivov, Y. Kivshor, Appl. Phys. Lett. **95**, 041902 (2009)

[108] L. Tu, J. Luo, G. T Gillies, Rep. Progr. Phys. **68**,77 (2005)

[109] J. Alfaro and L. Urrutia, Phys. Rev. **D 81**, 025007 (2010)

[110] L. Shao and Bu-Quiang Ma, Phys. Rev. **D 83**, 127702 (2011)

[111] J. Scherk, *Extended Supersymmetry and Extended Supergravity Theories, in Recent Development in Gravitation*, edited by Maria Lévy and S. Deser, Cargìse (1978)

[112] R. M. Doria, F. A. B. Rabelo de Carvalho, Rev. Bras. Fís., Vol 18, **4**, 453-476 (1988)

[113] R. Doria, *Physics of Light*, IP Conf. Proc. **1483**, 95-117 (2012)

[114] M. Zhang, Phys. Rev. Lett. **109**, 233 (2012)

[115] A. A. Abdo, et al., Nature **462**, 331 (2009)

[116] G. Amelino-Camelia, Nature **462**, 291 (2009)

[117] D. Ehrenstein, *Conjuring Matter From Light*, Science, vol. 277 (1997),

[118] L. Gualtieri et al., Clas. Q. Grav., **28**, 114014 (2011)

[119] B. Hartmann, B. Carter, Phys. Rev. D **77**, 103516 (2008)

[120] D. L. Burke et al., Phys. Rev. Lett. *79*, 9 (1997)

[121] M. Bregant et al., Phys. Rev D, **78**, 032006 (2008)

[122] F. Abe, Phys. Rev. Lett. **73**, 2662 (1994)

[123] V. Trimble, Annual Review of Astronomy and Astrophysics, **25**, 425 (1987)

[124] S. L. Shapiro, S. A. Teukolsky. *Black Holes, White Dwarfs and Neutron Stars*, Wiley (1983)

[125] L. Tu, J. Luo, G. T. Gillies, Rep. Progr. Phys. **68**, 77 (2005)

[126] G. Spavieri, J. Quintero, J. T. Gillies, M. Rodrigues, Eur. Phys. J. D, **61**, 531 (2011)

[127] X. Qi, S. -C. Zhang, Rev. Mod. Phys., **83**, 1057 (2011)