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Structure of Superdeformed Rotational Bands in A ~ 150 Mass Region

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ABSTRACT

The structure of superdeformed rotational bands (SDRB's) in A ~ 150 mass region are studied by using the Harris three – parameter expansion and the incremental alignment. The bandhead spins I_o have been determined with best fit procedure in order to obtain a minimum root mean square deviation between the calculated and the experimental dynamical moments of inertia.

The kinematic moment of inertia has been calculated as a function of rotational frequency and compared to the corresponding experimental ones by assuming three spin values $I_o - 2$, I_o , $I_o + 2$. The transition energies and the variation of the moments of inertia as a function of rotational frequency have been calculated. The agreement between theory and experiment are excellent.

The identical bands of SDRB's with ΔI = 2 staggering in ^{148}Gd (SD6) and ^{149}Gd (SD1) are investigated. Also the presence of ΔI = 2 staggering effect in the yrast bands of ^{147}Eu and ^{150}Tb has been examined.

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Introduction

The study of superdeformed rotational bands (SDRB's) is one of the most exciting areas in nuclear structure. More than 335 superdeformed (SD) bands were observed in various mass regions [1,2]. They are associated with extremely large quadrupole deformation. The difference between the SDRB's in various mass regions is manifested through the behavior of the moments of inertia. The exact excitation energies, spins and parities of SDRB's remain unknown. Spin best fit extrapolation procedures were used to predict the spins of these SD states [3-16]. It is noted that all the available approaches gain from the comparison of the calculated transition energies or the dynamical moments of inertia with the experimental results.

One of the most interesting phenomena observed in SD bands is the identical bands (IB's)[17-19] whereby SD bands with very nearly identical energies were observed in different nuclei. The incremental alignment depending only on γ- transition energies was introduced [20-22] to compare the SD bands in neighboring nuclei. The ΔI = 2 energy staggering in gamma – ray transitions [23-26] is one of the few mechanisms which not predicted theoretically and up to now is poorly understood. This phenomenon was first observed in ^{149}Gd [23], where the dynamical moment of inertia of the yrast SD band exhibits a small oscillation when plotted versus the rotational frequency of the nucleus. This effect is commonly called ΔI = 4 bifurcation because the band is consequently divided into two sequences with levels I, I+4, I+8, … and I+2, I+6, I+10, … differing in angular momentum by four units. Despite several theoretical attempts [16, 19, 22, 27-31] search for the physical origin of the staggering, there is no general agreement. Some discussions connect this effect with the presence of a C_4 symmetry of nuclear Hamiltonian [27-29]. Other studies [30, 31] argue that the staggering could be related to band crossing.

The purpose of the present paper is to report results of the structure of SDRB's in mass region A~ 150 by using the three parameters Harris formula and the incremental alignment which has the important advantage that it does not require knowledge of the spins of the states. The paper is arranged as follows: In section 2 , we outline the concept of Harris parameterization for SDRB's. The properties of the incremental alignment of SD bands are discussed in section 3. In section 4 we review the concept of Δ I = 2 staggering. In section 5, we presented a numerical calculations and obtained results and discussions for five SDRB's in Eu / Gd / Tb nuclei in mass region A = 150.

2. Outline of the Theory

A power series expansion with improved convergence properties was first suggested by Harris [32], as an extension of cranked model. In the Harris formulation, the nuclear excitation energy E is given in terms of even powers of the angular frequencies ω. The expansion up to ω^6 is:

$$E = \frac{1}{2}\alpha\omega^2 + \frac{3}{4}\beta\omega^4 + \frac{5}{6}\gamma\omega^6 \quad (1)$$

where the expansion coefficients α, β and γ have the dimensions, $\hbar^2 MeV^{-1}$, $\hbar^4 MeV^{-3}$ and $\hbar^6 MeV^{-5}$ respectively. The angular frequency ω is not directly observed quantity, but is derived from the observed rotational spectrum according to the canonical relation

$$\hbar\omega = \frac{dE}{d\hat{I}} \quad (2)$$

with $\hat{I} = [I(I+1)]^{1/2}$ is the intermediate nuclear spin.

The corresponding expression of dynamical moment of inertia $J^{(2)}$ for the Harris expansion equation (1) is

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{d^2E}{d\hat{I}^2}\right)^{-1} = \frac{1}{\hbar}\frac{d\hat{I}}{d\omega} = \frac{1}{\omega}\frac{dE}{d\omega} = \alpha + 3\beta\omega^2 + 5\gamma\omega^4 \quad (3)$$

which leads to expression for the intermediate nuclear spin I as a function of ω, by integrating equation (3) with respect to ω:

$$\hbar\hat{I} = \int d\omega\, J^{(2)} = \alpha\omega + \beta\omega^3 + \gamma\omega^5 - i_o \quad (4)$$

where i_o is the constant of integration (aligned spin)

The expression for the kinematic moment of inertia $J^{(1)}$ for Harris expansion reads:

$$\frac{J^{(1)}}{\hbar^2} = \frac{\hat{I}}{\hbar\omega} = \alpha + \beta\omega^2 + \gamma\omega^4 \quad (5)$$

One can extract the rotational frequency, kinematic and dynamical moments of inertia by using the experimental interaband E2 transition energies as:

$$\hbar\omega(I) = \frac{E_\gamma(I+2 \to I) + E_\gamma(I \to I-2)}{4} \quad (MeV) \quad (6)$$

$$J^{(2)}(I) = \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)} \quad (\hbar^2 MeV^{-1}) \quad (7)$$

$$J^{(1)}(I) = \frac{2I-1}{E_\gamma(I \to I-2)}\ (\hbar^2 MeV^{-1}) \quad (8)$$

It is seen that, while $J^{(1)}$ depends on the spin proposition, $J^{(2)}$ does not.

3. Transition Energies Based on the Incremental Alignment

The incremental alignment Δi between two bands A and B defined as [20]:

$$\Delta i_{AB} = \frac{\Delta E_\gamma}{\Delta E_\gamma^{ref}} \quad (9)$$

where ΔE_γ is obtained by subtracting the transition energy in a band of interest A from the closest transition energy in the reference SD band B and ΔE_γ^{ref} is calculated as the energy difference between the two closest transitions in the SD band of the reference B. The reference nuclei involves either the same proton $(\pi 6^n)$ or neutron $(\nu 7^n)$ intruder configuration. The incremental alignment has the important advantage that it dose not require knowledge of the spins of the states. To compare directly the transition energies of SD bands in given mass region to reference band one must study the evaluation of Δi with rotational frequency. Δi is linked to the total alignment i through the relation i = Δi + ΔI where ΔI is the difference between the angular momenta associated with the transitions.

If the incremental alignment Δi_{CD} between unknown band D and reference band C has the same value of the incremental alignment Δi_{AB} of another pair A and B, that is

$$\Delta i_{AB} = 2\frac{E_\gamma^A(I+\Delta I) - E_\gamma^B(I)}{E_\gamma^B(I+2) - E_\gamma^B(I)} = \Delta i_{CD} = 2\frac{E_\gamma^D(I+\Delta I) - E_\gamma^C(I)}{E_\gamma^C(I+2) - E_\gamma^C(I)} \quad (10)$$

Then one can calculate the gamma transition energies of band D from the relation

$$E_\gamma^D(I) = E_\gamma^C(I) + \frac{1}{2}\left[E_\gamma^C(I+2) - E_\gamma^C(I)\right]\Delta_{AB} \quad (11)$$

4. ΔI = 2 Staggering in SDRB's

To explore the ΔI = 2 staggering in a band, one must subtract from the difference between two consecutive transitions in the band $\Delta E_\gamma(I) = E_\gamma(I+2) - E_\gamma(I)$ a smooth reference $\Delta E_\gamma^{ref}(I)$ calculated with the help of the finite difference approximation to the n – order derivatives of the transition energies with respect to the spin $d^n E_\gamma(I)/dI^n$. This smooth difference is given by

(i) The Flibotte definition

Flibotte et al [23] described the deviation by means of a function of four consecutive transition energies which is denoted as the 4 point formula

$$\Delta^3 E_\gamma(I) = \frac{1}{4}\left[E_\gamma(I-2) - 3E_\gamma(I) + 3E_\gamma(I+2) - E_\gamma(I+4)\right] \quad (12)$$

(ii) The Cederwall definition

In this case, a function of five consecutive E_γ value is used the 5-point formula [24]

$$\Delta^4 E_\gamma(I) = \frac{1}{16}\left[E_\gamma(I-4) - 4E_\gamma(I-2) + 6E_\gamma(I) - 4E_\gamma(I+2) + E_\gamma(I+4)\right] \quad (13)$$

where $E_\gamma(I)$ is the transition energy from a spin state with I to I-2. It is worth while to point out that $\Delta' E_\gamma(I)$ is proportional to the inverse of the dynamical moment of inertia $J^{(2)}$.

In order to see the variation in the experimental transition energies, we subtract from the 4- point and 5- point formulae the calculated ones. The corresponding staggering parameters are $S^{(3)}(I)$ and $S^{(4)}(I)$ respectively

$$S^{(3)}(I) = 4\left[\Delta^3 E_\gamma(I) - \left(\Delta^3 E_\gamma(I)\right)^{cal}\right] \quad (14)$$

$$S^{(4)}(I) = 16\left[\Delta^4 E_\gamma(I) - \left(\Delta^4 E_\gamma(I)\right)^{cal}\right] \quad (15)$$

5. Numerical Calculations and Discussion

The optimized best parameters α, β, γ of Harris expansion for our selected SDRB's have been calculated by using a computer simulated search program in order to minimize the common definition of the root mean square (rms) deviation χ, given by

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$$\chi = \left[\frac{1}{N}\sum_{i=1}^{N}\left|\frac{J^{(2)}_{exp}(I_i)-J^{(2)}_{cal}(I_i)}{J^{(2)}_{exp}(I_i)}\right|^2\right]^{1/2} \quad (16)$$

where N is the number of experimental data points entering into the fitting procedure. The assigned spin values of the bandhead are extracted from equation (4). It has been argued that at zero frequency the aligned spin i_o is equel to zero or half for our selected SDRB's.

Using these assigned spin values I_o, the kinematic $J^{(1)}(I)$ is plotted versus rotational frequency $\hbar\omega$ and compared to the $J^{(1)}$ value obtained from the experimental transition energies by assuming three different spins I_o-2, I_o, I_o+2 for the lowest SD states. Figure (1) represent an example for ^{148}Gd(SD6) at I_o = 36, 38, 40, we see that the best agreement is obtained for bandhead spin I_o= 38 which is the predicted value from the theory. From the figure one, notice that the absolute value of $J^{(1)}(I)$ and also the slope are sensitive to the spin assignment. Table (1) lists the calculated transition energies $E_\gamma(I)$, the bandhead spin I_o and model parameters α, β, γ resulting from the best fitting procedure for ^{148}Gd(SD1) and ^{149}Gd(SD1). Agreement between theory and experiment are excellent (the experimental data are taken from Refs [1,2]).

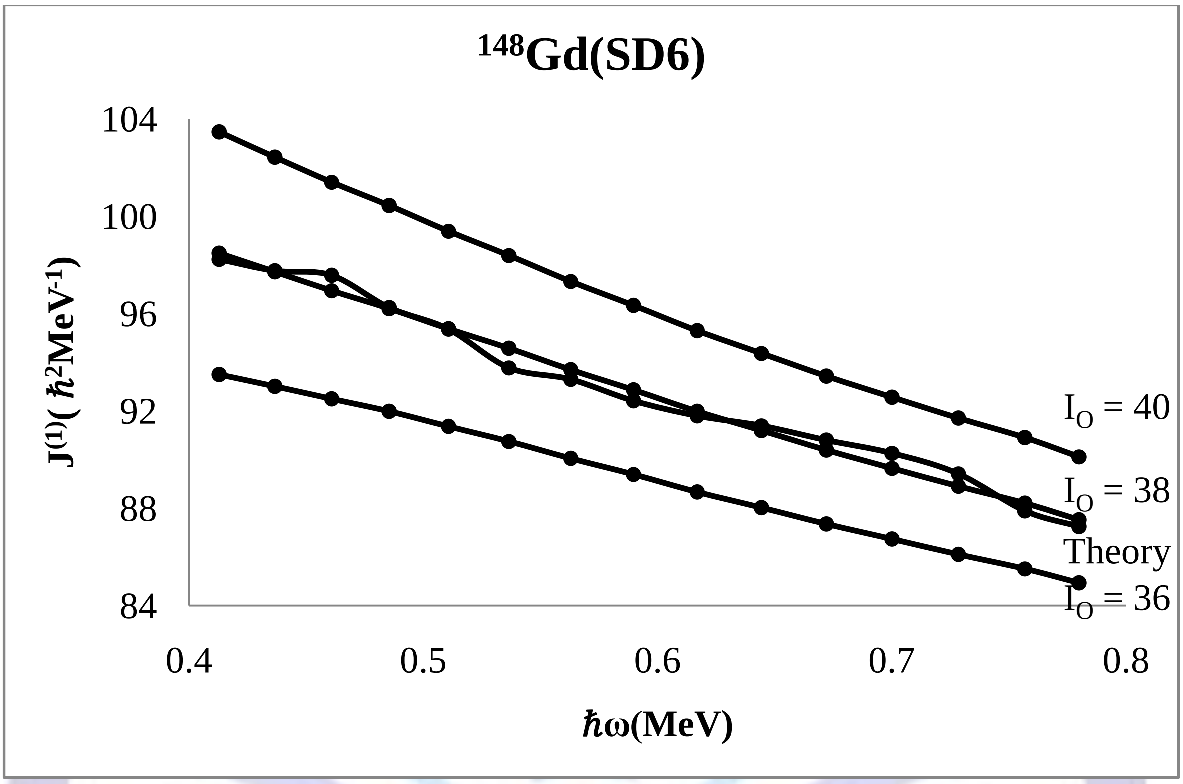

Fig. (1) The calculated kinematic moment of inertia $J^{(1)}$ of ^{148}Gd(SD6) is plotted versus rotational frequency $\hbar\omega$ and compared to the $J^{(1)}$ values obtained from the experimental transition energies assuming three different bandhead spins I_o = 36, 38, 40, for lowest SD state.

Table (1) The calculated transition energies E_γ of the identical SD bands ^{149}Gd(SD1) and ^{148}Gd(SD6) using the Harris expansion and comparison with experimental data, the model parameters α, β, γ and the bandhead spin I_o are listed in the table.

^{149}Gd(SD1) α = 85.2090 , γ = -0.5726 β = -8.6100 , I_o = 22.5			^{148}Gd(SD1) α = 100.9703 , γ = 20.8539 β = -37.6449 , I_o = 38		
Spin (ħ)	(KeV)	(KeV)	Spin (ħ)	(KeV)	(KeV)
27.5	605.118	617.8			
29.5	642.736	664.2			
31.5	693.621	711.8			
33.5	745.336	759.7			
35.5	797.781	808.1	40	804.192	802.20
37.5	848.099	857.1	42	849.047	849.44
39.5	903.351	906.7	44	891.697	897.40
41.5	965.904	957.1	46	945.545	945.86
43.5	1024.748	1008.7	48	996.198	996.08
45.5	1084.111	1060.7	50	1055.704	1046.83
47.5	1139.423	1113.8	52	1104.063	1099.39
49.5	1198.870	1167.2	54	1157.809	1152.20
51.5	1253.496	1221.8	56	1209.088	1206.26
53.5	1307.262	1276.5	58	1258.264	1261.00
55.5	1354.966	1332.0	60	1310.666	1216.57
57.5	1400.888	1387.6	62	1362.652	1372.10
59.5	1444.776	1444.2	64	1420.210	1428.55
61.5	1481.362	1500.5	66	1490.407	1485.16
63.5	1536.120	1557.8	68	1547.367	1542.40
65.5	1550.705	1615.7			
67.5	1572.227	1672.1			
69.5	1663.264	1729.9			

Now, we will use the incremental alignment as a tool to predict the transition energies of ^{147}Eu(SD1), ^{147}Eu(SD5) and ^{150}Tb(SD1). The incremental alignment Δi_{AB} of the SDRB's in nucleus A relative to nucleus B is calculated and used to predict the transition energies in nucleus D belonging to another pair CD has the same incremental alignment of Δi_{AB}. The nucleus D is an isotope or isotone to the nucleus C which involves either the same proton or neutron intruder configuration. In the A~ 150 mass region, there is a neutron gap at N = 85, this means that for ^{148}Eu$_{85}$ excited neutron configurations can only be created via energetically costly excitations across the gap, while for ^{147}Eu$_{84}$ several low energy neutron excitations are possible. This is analogous to the structure in the isotope ^{148}Gd$_{84}$ and ^{149}Gd$_{85}$. For proton, there is a gap at Z = 64, thus we expect that many of low-lying SD bands in A~ 150 nuclei will be identical to bands in the neighboring Z+1 Gadolinium nuclei. The positive signature (α = + 1/2) 1/2 [301] orbital is also close to Fermi surface, and it may generate identical bands (IB's). The intruder orbital SD band configurations of our reference nuclei B and C are:

$^{148}\mathrm{Eu}(SD1)\ \ \pi\, 6^2\ \otimes \nu\, 7^1$

$^{148}\mathrm{Gd}(SD1)\ \ \pi\, 6^2 \otimes \nu\, 7^1\ \ (1/2[521])^{-1}$

$^{149}\mathrm{Gd}(SD1)\ \ \pi\, 6^2 \otimes \nu\, 7^1$

while the SD configurations in the neigh boring nuclei are:

$^{147}\mathrm{Eu}(SD1)\ \ \pi\, 6^2\ \otimes \nu\, 7^0$

$^{147}\mathrm{Eu}(SD5)\ \ \pi\, 6^2\ \otimes \nu\, 7^0\ \pi\, (1/2\, [301], \alpha = 1/2\,)^{-1}$

$^{148}\mathrm{Eu}(SD2)\ \ \pi\, 6^2 \otimes \nu\, 7^1\ \pi\, (1/2\, [301], \alpha = 1/2\,)^{-1}$

$^{148}\mathrm{Gd}(SD6)\ \ \pi\, 6^2\ \otimes \nu\, 7^1$

$^{149}\mathrm{Tb}(SD1)\ \ \pi\, 6^3\ \otimes \nu\, 7^0$

$^{150}\mathrm{Tb}(SD1)\ \ \pi\, 6^3 \otimes \nu\, 7^1$

The calculated transition energies of ^{147}Eu(SD1), ^{147}Eu(SD5) and ^{150}Tb(SD1) deduced from the incremental alignment and reference SDRB's are shown in Table (2a, 2b, 2c), these values are in good agreement with experimental values[1,2].

The behavior of moments of inertia seems to be very useful to understand the properties of SDRB's, because of $J^{(2)}$ is related to the curvature of the excitation energy as a function of spin and can be derived from the energy difference between two consecutive transitions in the band, therefore, $J^{(2)}$ does not depend on the knowledge of the spin I but only on measured γ- ray energies. The calculated results of the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia as a function of rotational frequency $\hbar\omega$ are plotted in Figure (2) for our selected SDRB's. the general trends of the evolution of $J^{(2)}$, shows considerable variation from one nucleus to another depending on the occupancy of high-N intruder orbitals.

Table (2a) The calculated transition energies E_γ of the band 1 in ^{147}Eu using the incremental alignment Δi of (^{148}Gd(SD6), ^{149}Gd(SD1)) and the transition energies of ^{148}Eu(SD1).

^{148}Gd(SD1) E_γ(I)(KeV)	^{149}Gd(SD1) E_γ(I)(KeV)	^{148}Eu(SD1) E_γ(I)(KeV)	^{147}Eu(SD1) E_γ(I)(KeV)	^{147}Eu(SD1) E_γ^{cal}(I)(KeV)	Δi
795.8	759.7	747.7	790.6	785.141	1.4917
846.7	808.1	797.9	842.3	837.602	1.5755
897.9	857.1	848.3	892.3	890.414	1.6451
950.3	906.7	899.5	946.8	943.357	1.7233
1003.9	957.1	950.4	1001.3	998.831	1.8139
1058.7	1008.7	1003.8	1056.3	1055.047	1.9230
1114.2	1060.7	1057.1	1112.5	1111.102	2.0150
1170.6	1113.8	1110.7	1169.4	1168.775	2.1273
1227.8	1167.2	1165.5	1226.6	1226.230	2.2197
1285.6	1221.8	1220.1	1284.2	1284.949	2.3327
1344.0	1276.5	1275.7	1342.7	1342.834	2.4324
1402.5	1332.0	1330.9	1401.6	1402.660	2.5357
1461.4	1387.6	1387.5	1460.5	1460.254	2.6077
1520.5	1444.2	1443.3	1519.3	1518.649	2.7104
1580.5	1500.5	1498.9	1578.5	1577.363	2.7923

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Table (2b) The calculated transition energies E_γ of the band 5 in ^{147}Eu using the incremental alignment Δi of (^{148}Eu(SD2), ^{149}Gd(SD1)) and the transition energies of ^{148}Gd(SD1).

^{148}Eu(SD2) $E_\gamma(I)$(KeV)	^{149}Gd(SD1) $E_\gamma(I)$(KeV)	^{148}Gd(SD1) $E_\gamma(I)$(KeV)	^{147}Eu(SD5) $E_\gamma(I)$(KeV)	^{147}Eu(SD5) $E_\gamma^{cal}(I)$(KeV)	Δi
844.2	808.1	795.8	835.9	833.298	1.4734
894.8	857.1	846.7	889.0	885.614	1.5201
946.1	906.7	897.9	940.5	938.861	1.5634
998.1	957.1	950.3	994.7	992.887	1.5891
1050.9	1008.7	1003.9	1048.6	1048.370	1.6230
1104.2	1060.7	1058.7	1103.7	1104.165	1.6384
1157.9	1113.8	1114.2	1155.4	1160.775	1.6516
1212.4	1167.2	1170.6	1222.8	1217.950	1.6556
1268.7	1221.8	1227.8	1276.0	1277.357	1.7148
1322.0	1276.5	1285.6	1331.8	1333.476	1.6396
1377.8	1332.0	1344.0	1388.3	1392.186	1.6474
1434.0	1387.6	1402.5	1447.6	1450.783	1.6395
1489.2	1444.2	1461.4	1506.9	1508.635	1.5985
1544.1	1500.5	1520.5	1561.1	1566.165	1.5218

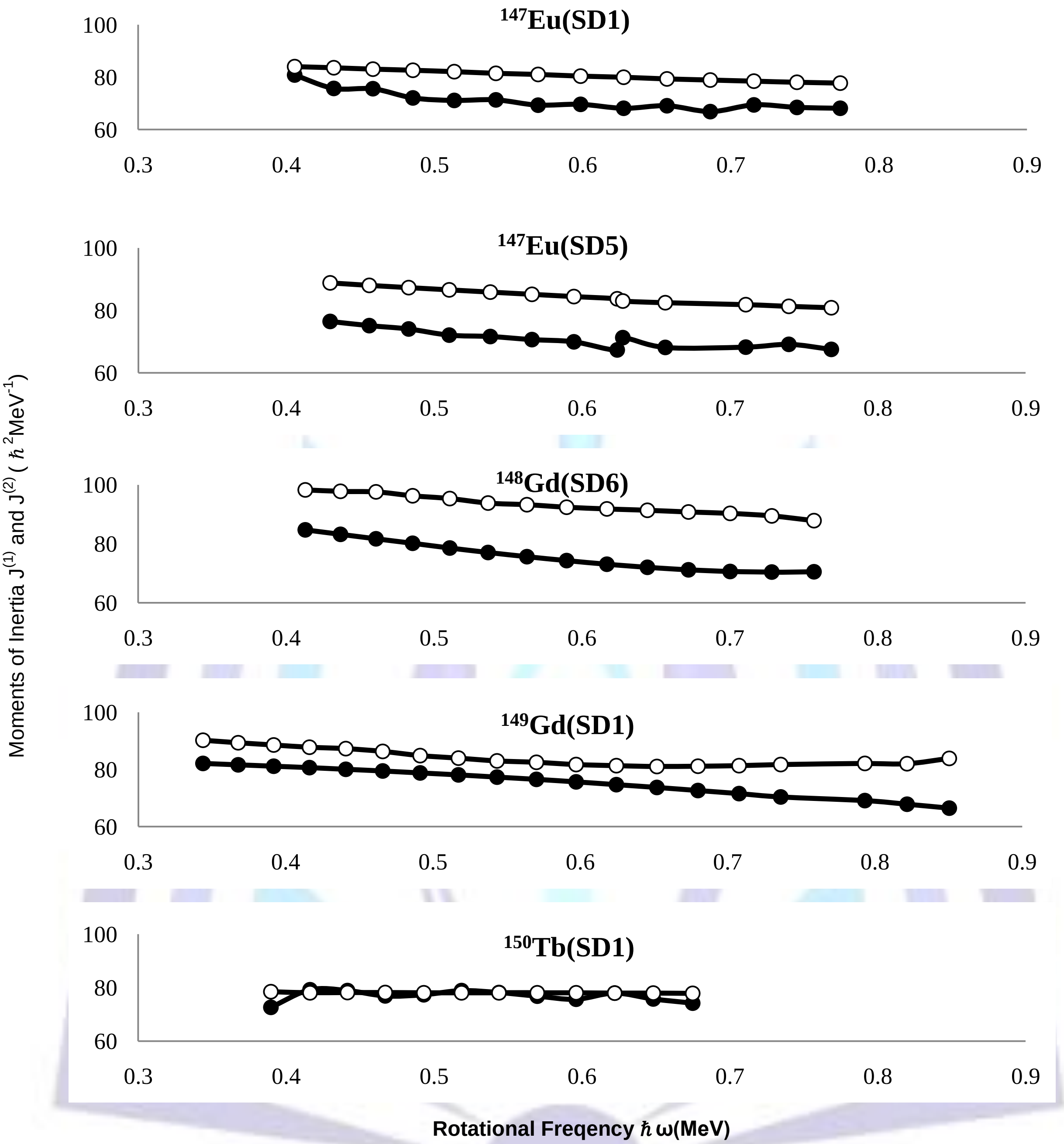

Fig. (2) calculated results of the dynamic moment of inertia $J^{(2)}$ (closed circle) and kinematic moment of inertia $J^{(1)}$ (open circles) as a function of rotational frequency ℏω of the SD bands in ^{147}Eu(SD1) , ^{147}Eu(SD5), ^{148}Gd(SD6), ^{149}Gd(SD1) and ^{150}Tb(Sd1).

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Table (2c) The calculated transition energies E_γ of the band 1 in ^{150}Tb using the incremental alignment Δi of (^{149}Tb(SD1), ^{148}Gd(SD1)) and the transition energies of ^{149}Gd(SD1).

^{149}Tb(SD1) $E_\gamma(I)$(KeV)	^{148}Gd(SD1) $E_\gamma(I)$(KeV)	^{149}Gd(SD1) $E_\gamma(I)$(KeV)	^{150}Tb(SD1) $E_\gamma(I)$(KeV)	^{150}Tb(SD1) $E_\gamma^{cal}(I)$(KeV)	Δi
740.1	699.9	711.8	748.2	751.916	1.6750
794.7	747.9	759.7	799.2	806.906	1.9540
847.1	795.8	808.1	850.5	857.484	2.0157
899.4	846.7	857.1	902.1	908.150	2.0585
953.5	897.9	906.7	954.1	960.176	2.1221
1007.2	950.3	957.1	1006.9	1011.875	2.1231
1060.7	1003.9	1008.7	1059.6	1062.595	2.0729
1114.2	1058.7	1060.7	1112.4	1113.800	2.0000
1169.2	1114.2	1113.8	1165.5	1165.873	1.9503
1224.6	1170.6	1167.2	1218.8	1218.745	1.8881
1278.8	1227.8	1221.8	1272.3	1270.064	1.7647
1334.4	1285.6	1276.5	1326.4	1322.875	1.6712
1391.1	1344.0	1332.0	1380.3	1376.763	1.6102

Another result of the present work is the existence of ΔI= 2 staggering in the transition energies of ^{147}Eu(SD1), ^{148}Gd(SD6), ^{149}Gd(SD1) and ^{150}Tb(SD1). Figure (3) show the calculated results of staggering parameters $S^{(3)}$ (I) (definition of Flibotte [23]) and $S^{(4)}$ (I) (definition of Cederwall [24]) as a function of rotational frequency $\hbar\omega$. The numerical values of $J^{(1)}$ (I), $J^{(2)}$ (I), $S^{(3)}$ (I) and $S^{(4)}$ (I) are listed in Table (3a-3e).

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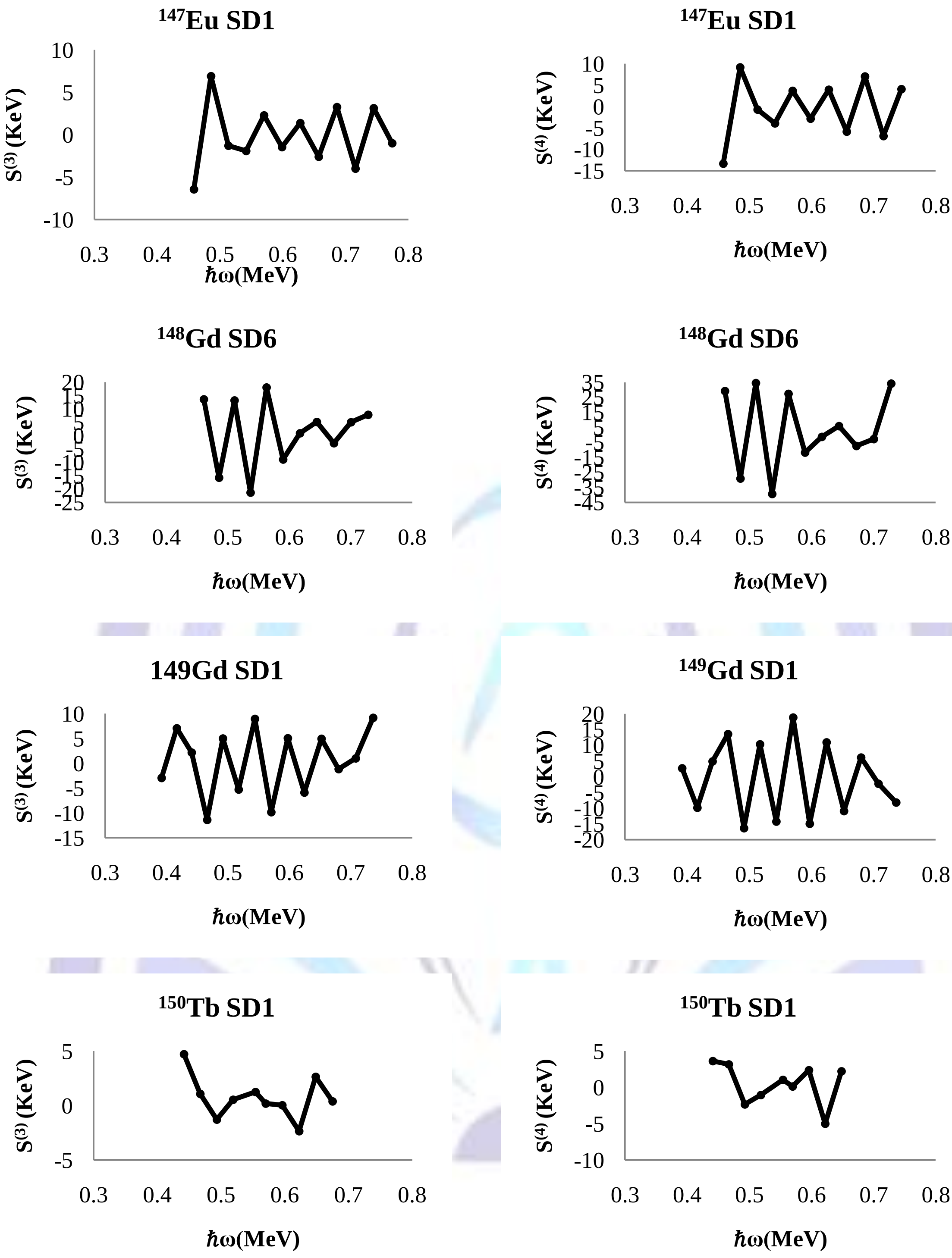

Fig. (3) Calculated results of staggering parameters $S^{(3)}(I)$ obtained by Flibotte definition[23](4-point formula) and $S^{(4)}(I)$ obtained by Cederwall definition [24] (5-point formula) as a function of rotational frequency $\hbar\omega$ of SDRB's in ^{147}Eu(SD1), ^{148}Gd(SD6), ^{149}Eu(SD1) and ^{150}Tb(SD1).

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Table (3a) Calculated dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia and the staggering parameters $S^{(3)}(I)$ obtained by Flibotte definition[23](4-point formula) and $S^{(4)}(I)$ obtained by Cederwall definition[24](5-point formula) for ^{147}Eu(SD1).

$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$S^{(3)}(I)$ (KeV)	$S^{(4)}(I)$ (KeV)
0.4056	80.871	84.061		
0.4320	75.740	83.571		
0.4584	75.552	83.107	-6.420	-13.320
0.4855	72.105	82.683	6.900	9.187
0.5134	71.154	82.095	-1.289	-0.686
0.5415	71.358	81.512	-1.903	-3.882
0.5699	69.356	81.000	2.279	3.715
0.5987	69.619	80.426	-1.436	-2.818
0.6277	68.121	79.919	1.382	3.980
0.6569	69.102	79.380	-2.598	-5.873
0.6863	66.86	78.937	3.275	7.048
0.7157	69.451	78.422	-3.973	-6.906
0.7447	68.499	78.068	3.133	4.115
0.7740	68.126	77.700	-0.982	

Table (3b) The Calculated dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia for ^{147}Eu(SD5).

$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)
0.4297	76.458	88.803
0.4561	75.121	88.074
0.4829	74.038	87.339
0.5103	72.094	86.616
0.5381	71.691	85.847
0.5662	70.658	85.132
0.5946	69.960	84.426
0.6238	67.332	83.747
0.6277	71.277	82.983
0.6564	68.131	82.491
0.7107	68.262	81.885
0.7398	69.141	81.335
0.7687	67.528	80.867

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Table (3c) The same as Table (3a) but for ^{148}Gd(SD6).

$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$S^{(3)}(I)$ (KeV)	$S^{(4)}(I)$ (KeV)
0.4129	84.746	98.235		
0.4367	83.224	97.756		
0.4608	81.690	97.566	13.623	29.276
0.4855	80.144	96.240	-15.653	-28.931
0.5107	78.606	95.362	13.278	34.554
0.5366	77.099	93.776	-21.280	-39.374
0.5629	75.654	93.291	18.094	27.448
0.5897	74.304	92.415	-8.854	-11.788
0.6169	73.091	91.804	0.934	-1.245
0.6444	72.053	91.395	5.179	5.951
0.6722	71.230	90.793	-2.772	-7.300
0.7002	70.665	90.265	5.028	-2.809
0.7284	70.402	89.423	7.827	34.213
0.7569	70.492	87.895		

Table (3d) The same as Table (3a) but for ^{149}Gd(SD1).

$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$S^{(3)}(I)$ (KeV)	$S^{(4)}(I)$ (KeV)
0.3440	82.112	90.239		
0.3678	81.658	89.385	-0.300	
0.3919	81.172	88.550	-2.957	2.657
0.4163	80.653	87.743	7.061	-9.818
0.4409	80.077	87.253	2.167	4.894
0.4659	79.465	86.345	-11.410	13.577
0.4914	78.789	84.894	5.028	-16.348
0.5173	78.100	83.923	-5.270	10.298
0.5436	77.314	83.017	8.986	-14.255
0.5702	76.514	82.497	-9.856	18.842
0.5972	75.639	81.743	5.061	-14.917
0.6245	74.717	81.372	-5.902	10.963
0.6521	73.711	81.085	4.980	-10.882
0.6799	72.653	81.182	-1.152	6.132
0.7079	71.542	81.376	1.032	-2.184
0.7361	70.377	81.673	9.174	-8.142
0.7933	69.112	82.079		
0.8219	67.833	82.024		
0.8505	66.449	83.832		

Table (3e) The same as Table (3a) but for ^{150}Tb(SD1).

^{150}Tb SD1				
$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$S^{(3)}(I)$ (KeV)	$S^{(4)}(I)$ (KeV)
0.3897	72.634	78.466		
0.4161	79.211	78.068		
0.4414	78.948	78.135	4.740	3.648
0.4670	76.884	78.180	1.092	3.179
0.4930	77.370	78.110	-1.287	-2.335
0.5186	78.864	78.072	0.548	-1.016
0.5440	78.117	78.110	1.264	1.081
0.5699	76.815	78.110	0.183	0.152
0.5961	75.654	78.053	0.031	2.383
0.6222	77.943	77.949	-2.352	-4.997
0.6482	75.741	77.948	2.645	2.260
0.6749	74.228	77.860	0.385	

Conclusion

The three parameters Harris formula for energy levels is proposed in this paper to parameterize the E2 transition γ- ray energies and the dynamical moment of inertia in five SD bands in the A = 150 mass region. The incremental alignment which depends on the occupation of specific single particle orbitals and not depends on the knowledge of the spin has been also used to predict the transition energies of SD bands of ^{147}Eu and ^{150}Tb. The role of occupation of high – j intruder orbitals in the structure of the SD bands has been investigated. Our results indicate that the N = 80 gap is considerable more stable than that Z = 64 gap. In all our selected SDRB's the calculated results agree with experimental data very well, this indicate that Harris formula and incremental alignment can describe both the yrast and the excited SD bands. Finally our results suggest that the identical bands ^{148}Gd (SD6), ^{149}Gd (SD1) and also the yrast SD bands of ^{147}Eu and ^{150}Tb exhibits ΔI = 2 staggering pattern.

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