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Approximation of derivations on proper*JCQ**-algebras

Ga Ya Kim[1],SeongSik Kim[2] andSoo Hwan Kim[3]
[1]Department of Urban Engineering, Dong-eui University, Busan 614-714, Repubic of Korea
kimgya@deu.ac.kr
[2, 3]Department of Mathematics, Dong-eui University, Busan 614-714, Repubic of Korea
sskim@deu.ac.kr, sh-kim@deu.ac.kr

ABSTRACT

In this paper, we prove the generalized Hyers-Ulam stability of proper *JCQ**-derivations on proper*JCQ**-triples associated to the general *(m,n)*-Cauchy-Jensen additive functional equation:

$$\sum_{\substack{1\le i_1<\cdots<i_m\le n,\ 1\le k_l\le n\\ k_l\neq i_j,\ \forall j\in\{1,\cdots,m\}}} f\left(\frac{1}{m}\sum_{j=1}^{m} x_{i_j} + \sum_{l=1}^{n-m} x_{k_l}\right) = \frac{n-m+1}{n}\binom{n}{m}\sum_{i=1}^{n} f(x_i)$$

KEYWORDS

Proper*JCQ**-triples; proper*JCQ**-derivations; *(m,n)*-Cauchy-Jensen additive mappings; generalized Hyers-Ulam stability; contractivelysubadditive mappings;k-contractiuelysubhomogeneous mappings

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INTRODUCTION AND PRELIMINARIES

Ternary algebraic operations were considered in the 19th century by several mathematicians such as Cayley [2] who introduced the notion of cubic matrix which in turn was generaliaed by Kapranov, Gelfand and Zelevinskil et al. [14]. Ternary structures and their generalization, the so-called *n*-ary structures, raise certain hopes in view of their possible applications in physics. Some significant physical applications are described in [15, 16].

The study of stability problems of functional equations which had been proposed by Ulam [29], concerned the stability of group homomorphisms. The famous Ulam stability problem was partially solved by Hyers [8] for a linear functional equation in Banach spaces. Later, the results of Hyers was generalized by Rassias [26] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Rassias [26] is called the generalized Hyers-Ulam stability. Since then, the stability problems of many algebraic, differential, integral, operatorial equations have been extensively investigated [9, 12, 13]. Several mathematician have contributed works of approximate homomorphisms and their stability theory in the field of functional equations on *C**-algebras, *JB** -algebras, *CQ**-algebras, *JCQ**-algebras [4, 6, 10, 11, 18, 19, 21 - 24, 27, 28].

In the sequel, we use the definitions and notations of a proper *CQ**-algebra as in [3].

Let A be a linear space and A_0 is a $*$-algebra contained in A as a subspace. $A$is called a *quasi $*$-algebra* over A_0 if the following three conditions hold:

(i)the right and left multiplications of an element of A and an element of A_0 are defined and bilinear;

(ii) $x_1(x_2a) = (x_1x_2)a$and$x_1(ax_2) = (x_1a)x_2$ for all $x_1, x_2 \in A_0,\ a \in A$;

(iii) an involution $*$, which extends the involution of A_0, is defined in a linear space A with the property that $(ax)^* = x^*a^*$ for all $x \in A_0$, $a \in A$, whenever the multiplication is defined.

Many authors ([3], [4], [28]) have considered a special class of quasi $*$-algebras, called proper*CQ**-algebra,which arise as completions of *C**--algebras.

Definition 1.1. Let A be a Banach module over the *C** -algebra A_0 with involution $*$ and *C**-norm $||\cdot||_{A_0}$ such that $A_0 \subset A$. Then (A, A_0) is called a *proper CQ*-algebra* if the following three conditions hold:

(i) A_0 is dense in A with respect to its norm $||\cdot||$;

(ii) $(ab)^* = b^*a^*$for all $a, b \in A_0$, whenever the multiplication is defined;

(iii) $||y||_{A_0} = \sup_{a\in A, ||a||\leq 1} ||ay||$for all $y \in A_0$.

Definition 1.2. A proper *CQ** -algebra (A, A_0), endowed with the triple product $A_0 \times A \times A_0 \ni (w_0, w, w_1) \to [w_0, w^*, w_1] \in A$ which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linerar in the middle variable and satisfies that $[w_0, w, w_1] \in A_0$ for all $w_0, w_1 \in A_0$ and all $w \in A$, is called a *proper CQ* -ternary algebra* and denoted by $(A, A_0, [\cdot,\cdot,\cdot])$.

Note that if (A, A_0) is a proper *CQ**-algebra and $[z, x, w] = zx^*w$ for all $x \in A$ and all $z, w \in A_0$, then $(A, A_0, [\cdot,\cdot,\cdot])$ is a proper *CQ** -ternary algebra.

Definition 1.3. A proper *CQ**-algebra (A, A_0), endowed with Jordan triple product

$$\{z, x, w\} = \frac{zx^*w + wx^*z}{2}$$

for all $x \in A$ and all $z, w \in A_0$, is called a *proper JCQ*-triple* and denoted by $(A, A_0, \{\cdot,\cdot,\cdot\})$.

Let A be a proper *CQ**-algebra with respect to the Jordan product $x \circ y = \frac{xy+yx}{2}$. Then we get the Jordan triple product

$$\{z, x, w\} = (z \circ x^*) \circ w + (w \circ x^*) \circ z - (z \circ w) \circ x^*$$

for all $x \in A$ and all $z, w \in A_0$.

Deflnition 1.4. Let $(A, A_0, \{\cdot,\cdot,\cdot\})$ be a proper *JCQ**-triple. A $\mathbb{C}$-linear mapping $\delta: A_0 \to A$ is called a *proper JCQ*-triple derivation* if

$$\delta(\{w_0, w_1, w_2\}) = \{\delta(w_0), w_1, w_2\} + \{w_0, \delta(w_1), w_2\} + \{w_0, w_1, \delta(w_2)\}$$

for all $w_0, w_1, w_2 \in A_0$.

We recall that a mapping $\rho: A \to B$ having a domain A and a codomain $(B, \leq)$ that are both closed under addition. A mapping $\rho: A \to B$is *contractivelysubadditive* if ther exists a constant L with $0 < L < 1$ such that $\rho(x + y) \leq L(\rho(x) + \rho(y))$ for all $x, y \in A$. A mapping ρ is *expansively superadditive*if there exists a constant L with $0 < L < 1$ such that $\rho(x + y) \geq \frac{1}{L}(\rho(x) + \rho(y))$ for all $x, y \in A$. Therefore, if a mapping ρ is contractively subadditive $(l = 1)$ and expansively superadditive $(l = -1)$, then ρ satisfies the properties $\rho(\lambda^{nl}\, x) \leq (\lambda\, L)^{nl}\, \rho(x)$, respectively.

Let $k \in \mathbb{Z}^+$ be fixed. A mapping ρ is a *k-contractiuelysubhomogeneous* if there exists a constant L with $0 < L < 1$ such that a mapping $\rho(\lambda\, x) \leq \lambda^k\, L\, \rho(x)$, and ρ is an *k-expansively superhomogeneous* if there exists a constant L with $0 < L < 1$ such that a mapping $\rho(\lambda x) \leq \frac{\lambda^k}{L}\, \rho(x)$ for all $x \in A$ and $\lambda \in \mathbb{Z}^+$.

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Now, we consider a mapping $f: X \to Y$ satisfying the following functional equation:

$$\sum_{\substack{1\le i_1<\cdots<i_m\le n,\ 1\le k_l\le n \\ k_l\neq i_j,\ \forall j\in\{1,\cdots,m\}}} f\left(\frac{1}{m}\sum_{j=1}^{m} x_{i_j} + \sum_{l=1}^{n-m} x_{k_l}\right) = \frac{n-m+1}{n}\binom{n}{m}\sum_{i=1}^{n} f(x_i) \quad (1.1)$$

for all $x_1, x_2, \cdots, x_n \in X$, where n, m are fixed integers with $n \ge 2$ and $n \ge m \ge 1$. In case $m = 1$, the functional equation (1.1) yields the Cauchy additive functional equation

$$f\left(\sum_{l=1}^{n} x_{k_l}\right) = n\sum_{i=1}^{n} f(x_i).$$

Also, in case $m = n$, the functional equation (1.1) yields the Jensen additive functional equation

$$f\left(\frac{1}{n}\sum_{j=1}^{n} x_j\right) = \frac{1}{n}\sum_{i=1}^{n} f(x_i).$$

Therefore, the functional equation (1,1) is a generalized form of the Cauchy-Jensen additive equation and every solution of the functional equation (1.1) may be analogously called the general (m, n)-Cauchy-Jensen additive functional equation. Recently, the generalized Hyers-Ulam stability of homomorphisms and derivations in several Banachalgebras associated to the functional equation (1.1) have investigated by[1],[7],[17],[25].

Let X, Y be linear spaces. For each $m \in \mathbb{Z}^+$with $1 \le m \le n$, a mapping $f: X \to Y$ satisfies the functional equation (1.1) for all $n \ge 2$ if and only if $f(x) - f(0) = A(x)$ is Canuchy additive, where $f(0) = 0$ if $m < n$. In particular, $f(n-m+1)x) = (n-m+1)f(x)$ and $f(mx) = m\, f(x)$ for all $x \in X$.

Throughout this paper, let A be a unital proper *JCQ**-triple, $\lambda = n - m + 1$ be a fixed positive integer with $n \ge 2$, $n \ge m \ge 1$and $T^1 = \{\mu \in \mathbb{C} : |\mu| = 1\}$. For any mapping $f: A \to A$, we define

$$\Delta_\mu f(x_1,\cdots,x_n) = \sum_{\substack{1\le i_1<\cdots<i_m\le n,\ 1\le k_l\le n \\ k_l\neq i_j,\ \forall j\in\{1,\cdots,m\}}} f\left(\frac{1}{m}\sum_{j=1}^{m} \mu x_{i_j} + \sum_{l=1}^{n-m} \mu x_{k_l}\right) - \frac{n-m+1}{n}\binom{n}{m}\sum_{i=1}^{n} f(\mu x_i) \quad (1.2)$$

for all $\mu \in T^1$ and all $x_1, \cdots, x_n \in A$.

STABILITY OF PROPER JCQ*-TRIPLES DERIVATIONS

In this section, we investigate the generalized Hyers-Ulam stability results for proper *JCQ**-triple derivations associated to the functional equation (1.2) in proper *JCQ**-triples.

Theorem 2.1. Assume that there exist a contractively subadditive mapping $\varphi: A_0^n \to [0,\infty)$ and a 3-contractively subhomogeneous mapping $\psi: A_0^3 \to [0,\infty)$with a constant $L < 1$ such that a mapping $f: A_0 \to A$ satisfies

$$\left\|\Delta_\mu f(x_1,\cdots,x_n)\right\|_A \le \varphi(x_1,\cdots,x_n), \quad (2.1)$$

$$\|f(\{w_0,w_1,w_2\}) - \{f(w_0),w_1,w_2\} - \{w_0,f(w_1),w_2\} - \{w_0,w_1,f(w_2)\}\|_A \le \psi(w_0,w_1,w_2) \quad (2.2)$$

for all $\mu \in T^1$ and all $x_1, \cdots, x_n, w_0, w_1, w_2 \in A_0$. Then there exists a unique proper *JCQ**-triples derivation $\delta: A_0 \to A$ such that

$$\|f(x) - \delta(x)\|_A \le \frac{1}{\binom{n}{m}(n-m+1)(1-L)}\varphi(x,\cdots,x) \quad (2.3)$$

for all $x \in A_0$.

Proof. Letting $\mu = 1$ and $x_1 = \cdots = x_n = x$ in (2.1), we get

$$\left\|f(x) - \frac{1}{\lambda}f(\lambda x)\right\|_A \le \frac{1}{\binom{n}{m}\lambda}\varphi(x,\cdots,x) \quad (2.4)$$

for all $x \in A_0$, where $\lambda = n - m + 1$. Using the induction method, we get

$$\left\|\frac{f(\lambda^k x)}{\lambda^k} - \frac{f(\lambda^j x)}{\lambda^j}\right\|_A = \sum_{i=k}^{j-1}\left\|\frac{f(\lambda^i x)}{\lambda^i} - \frac{f(\lambda^{i+1} x)}{\lambda^{i+1}}\right\|_A$$

$$\le \frac{1}{\binom{n}{m}\lambda}\sum_{i=k}^{j-1}\frac{1}{\lambda^i}\varphi(\lambda^i x,\cdots,\lambda^i x) \le \frac{1}{\binom{n}{m}\lambda}\sum_{i=k}^{\infty} L^i\,\varphi(x,\cdots,x) \quad (2.5)$$

for all $x \in A_0$ and all integers j,k with $j > k \geq 0$. Then, the sequence $\left\{\frac{f(\lambda^j x)}{\lambda^j}\right\}$ is a Cauchy sequence in A for all $x \in A_0$. Since A is complete, it converges in A. So, we can define a mapping $\delta: A_0 \to A$ by

$$\delta(x) = \lim_{j\to\infty} \frac{f(\lambda^j x)}{\lambda^j} \tag{2.6}$$

for all $x \in A_0$. Passing the limit $j \to \infty$ in (2.5) with $k = 0$, we get

$$\|f(x) - \delta(x)\|_A \leq \frac{1}{\binom{n}{m}\lambda(1-L)}\varphi(x,\cdots,x) = \frac{1}{\binom{n}{m}(n-m+1)(1-L)}\varphi(x,\cdots,x)$$

for all $x \in A_0$. Now, we show that δ is $\mathbb{C}$-linear mapping. It follows from (2.1) and (2.6) that

$$\left\|\Delta_\mu \delta(x_1,\cdots,x_n)\right\|_A \leq \lim_{j\to\infty}\frac{1}{\lambda^j}\left\|\Delta_\mu f(\lambda^j x_1,\cdots,\lambda^j x_n)\right\|_A \leq \lim_{j\to\infty} L^j\ \varphi(x_1,\cdots,x_n) = 0 \tag{2.7}$$

forall$x_1, x_2,\cdots,x_n \in A_0$. Then, letting $\mu = 1$, the mapping δ satisfies (1.1). So, $\delta: A_0 \to A$ is Cauchy additive. Also, taking $x_1 = x$ and $x_2 = \cdots = x_n = 0$ in (2.1), we get $\delta(\mu x) = \mu\delta(x)$ for all $x \in A_0$. By the same reasoning as that the proof of Theorem 2.1 of [20], the mapping $\delta: A_0 \to A$ is $\mathbb{C}$-linear. Since 3-contractively subhomogeneityof ψ, (2.2) and (2.6), we obtain that

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$

$$= \lim_{j\to\infty}\frac{1}{\lambda^{3j}}\left\|f(\{\lambda^j w_0,\lambda^j w_1,\lambda^j w_2\}) - \{f(\lambda^j w_0),\lambda^j w_1,\lambda^j w_2\} - \{\lambda^j w_0, f(\lambda^j w_1),\lambda^j w_2\} - \{\lambda^j w_0,\lambda^j w_1, f(\lambda^j w_2)\}\right\|_A$$

$$\leq \lim_{j\to\infty}\frac{1}{\lambda^{3j}}\ \psi(\lambda^j w_0,\lambda^j w_1,\lambda^j w_2) \leq \lim_{j\to\infty} L^j\ \psi(w_0,w_1,w_2) = 0$$

for all $w_0,w_1,w_2 \in A_0$. So, we have

$$\delta(\{w_0,w_1,w_2\}) = \{\delta(w_0),w_1,w_2\} + \{w_0,\delta(w_1),w_2\} + \{w_0,w_1,\delta(w_2)\}$$

for all $w_0,w_1,w_2 \in A_0$. Thus, the mapping δ is a proper *JCQ**-triples derivation on A_0.

Finally, let$\delta': A_0 \to A$ be another proper *JCQ**-triples derivation satisfying (2.3). Then, we have

$$\begin{aligned}\|\delta(x) - \delta'(x)\|_A &= \frac{1}{\lambda^j}\left\|\delta(\lambda^j x) - \delta'(\lambda^j x)\right\|_A \\ &\leq \frac{1}{\lambda^j}\left(\left\|\delta(\lambda^j x) - f(\lambda^j x)\right\|_A + \left\|\delta'(\lambda^j x) - f(\lambda^j x)\right\|_A\right) \\ &\leq \frac{2\varphi(x,\cdots,x)L^j}{\binom{n}{m}(n-m+1)},\end{aligned}$$

which tends to zero as $j \to \infty$ for all $x \in A_0$. Thus, we can conclude that $\delta(x) = \delta'(x)$ for all $x \in A_0$. This completes the proof.

Theorem 2.2. Assume that there exists an expansively superadditive mapping $\varphi: A_0^n \to [0,\infty)$ and a 3-expansively superhomogenus mapping $\psi: A_0^3 \to [0,\infty)$ with a constant $L < 1$ such that a mapping $f: A_0 \to A$ satisfies (2.1) and (2.2). Then there exists a unique perper*JCQ**-triples derivation $\delta: A_0 \to A$ such that

$$\|f(x) - \delta(x)\|_A \leq \frac{L}{\binom{n}{m}(1-L)}\varphi(x,\cdots,x) \tag{2.8}$$

for all $x \in A_0$.

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique $\mathbb{C}$-linear mapping $\delta: A_0 \to A$ such that (2.8). The mapping $\delta: A_0 \to A$ is given by

$$\delta(x) = \lim_{j\to\infty} \lambda^j f(\frac{x}{\lambda^j}) \tag{2.9}$$

for all $x \in A_0$. Since a 3-expansively superhomogeneity of ψ, (2.2) and (2.9), we get

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$

$$= \lim_{j\to\infty}\ \lambda^{3j}\left\|f\left(\{\frac{w_0}{\lambda^j},\frac{w_1}{\lambda^j},\frac{w_2}{\lambda^j}\}\right) - \{f\left(\frac{w_0}{\lambda^j}\right),\frac{w_1}{\lambda^j},\frac{w_2}{\lambda^j}\} - \{\frac{w_0}{\lambda^j}, f(\frac{w_1}{\lambda^j}),\frac{w_2}{\lambda^j}\} - \{\frac{w_0}{\lambda^j},\frac{w_1}{\lambda^j}, f(\frac{w_2}{\lambda^j})\}\right\|_A$$

$$\leq \lim_{j\to\infty}\ L^j\ \psi(w_0,w_1,w_2) = 0$$

for all $w_0,w_1,w_2 \in A_0$. The rest of proof is the similar way to the proof of Theorem 2.1. This completes the proof. ■

Corollary 2.3. Let s,θ be nonnegative real numbers with $s < 3$. Suppose that a mapping $f: A_0 \to A$ satisfies

$$\|\Delta_1 f(x_1,\cdots,x_n)\|_A \le \theta, \quad (2.10)$$

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$
$$\le \theta\left(\|w_0\|_{A_0}^s + \|w_1\|_{A_0}^s + \|w_2\|_{A_0}^s\right) (2.11)$$

for all $x_1,\cdots,x_n,w_0,w_1,w_2 \in A_0$. Then there exists a unique proper proper *JCQ**-triple derivation $\delta: A_0 \to A$ such that

$$\|f(x)-\delta(x)\|_A \le \frac{\theta}{\binom{n}{\mathrm{m}}(n-m)} \quad (2.12)$$

for all $x \in A_0$.

Corollary 2.4. Let $r,s \in \mathbb{R}$ and θ be nonnegative real numbers with $r \neq 1, s \neq 3$. Suppose that a mapping $f: A_0 \to A$ satisfies

$$\left\|\Delta_\mu f(x_1,\cdots,x_n)\right\|_A \le \theta \sum_{i=1}^{n} \|x_i\|_{A_0}^r, \quad (2.13)$$

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$
$$\le \theta\left(\|w_0\|_{A_0}^s + \|w_1\|_{A_0}^s + \|w_2\|_{A_0}^s\right) (2.14)$$

for all $x_1,\cdots,x_n,w_0,w_1,w_2 \in A_0$. Then there exists a unique proper *JCQ**-triple derivation $\delta: A_0 \to A$ such that

$$\|f(x)-\delta(x)\|_A \le \begin{cases} \dfrac{n\theta\|x\|_{A_0}^r}{\binom{n}{\mathrm{m}}((n-m+1)-(n-m+1)^r)}, & r<1, s<3 \\ \dfrac{n\theta\|x\|_{A_0}^r}{\binom{n}{\mathrm{m}}((n-m+1)^r-(n-m+1))}, & r>1, s>3 \end{cases} \quad (2.15)$$

for all $x \in A_0$.

Proof. Let $\varphi(x_1,\cdots,x_n) = \theta\sum_{i=1}^n \|x_i\|_{A_0}^r$ and $\psi(w_0,w_1,w_2) = \theta\left(\|w_0\|_{A_0}^s + \|w_1\|_{A_0}^s + \|w_2\|_{A_0}^s\right)$ for all $x_1,\cdots,x_n,w_0,w_1,w_2 \in A_0$. If we can choose $L=(m-m+1)^{r-1}$ if $r<1, s<3$ and $L=(m-m+1)^{1-r}$ if $r>1, s>3$, respectively and by applying Theorem 2.1 and 2.2, then we obtain the desired results. This completes the proof. ■

Corollary 2.5. Let r_i, s, θ be nonnegative real numbers with $0 \le \sum_{i=1}^n r_i < 1$and $s<1$. Suppose that a mapping $f: A_0 \to A$ satisfies

$$\|\Delta_1 f(x_1,\cdots,x_n)\|_A \le \theta \prod_{i=1}^{n} \|x_i\|_{A_0}^r, \quad (2.16)$$

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$
$$\le \theta\left(\|w_0\|_{A_0}^s \cdot \|w_1\|_{A_0}^s \cdot \|w_2\|_{A_0}^s\right) (2.17)$$

for all $x_1,\cdots,x_n,w_0,w_1,w_2 \in A_0$. Then f is a proper *JCQ**-triple derivation A_0.

Proof. Putting $x_1 = \cdots = x_n = 0$ in (2.16), we obtain $f(0)=0$. Replacing $\mu = 1$ and $x_1 = x$, $x_2 = \cdots = x_n = 0$ in (2.16), we get $f(x) = \frac{f((n-m+1)x)}{(n-m+1)}$. By induction, we get

$$f(x) = \frac{f((n-m+1)^j x)}{(n-m+1)^j}$$

for all $x \in A_0$ and all $j \in \mathbb{Z}^+$. It follows from Theorem 2.1 that f is a proper *JCQ**-triple derivation A_0. This completes the proof. ■

Corollary 2.6. Let r, r_i, s, θ be nonnegative real numbers with $r<1, 0 \le \sum_{i=1}^n r_i < 1$and $s<1$. If a mapping $f: A_0 \to A$ satisfies

$$\|\Delta_1 f(x_1,\cdots,x_n)\|_A \le \theta\left[\sum_{i=1}^{n} \|x_i\|_{A_0}^r + \prod_{i=1}^{n} \|x_i\|_{A_0}^r\right] \quad (2.18)$$

$$\|\delta(\{w_0,w_1,w_2\}) - \{\delta(w_0),w_1,w_2\} - \{w_0,\delta(w_1),w_2\} - \{w_0,w_1,\delta(w_2)\}\|_A$$
$$\le \theta\left(\|w_0\|_{A_0}^{3s} + \|w_1\|_{A_0}^{3s} + \|w_2\|_{A_0}^{3s} + \|w_0\|_{A_0}^s \cdot \|w_1\|_{A_0}^s \cdot \|w_2\|_{A_0}^s\right) (2.19)$$

for all $x_1,\cdots,x_n,w_0,w_1,w_2 \in A_0$, then there exists a unique a proper *JCQ**-triple derivation $\delta: A_0 \to A$ such that

$$\|f(x)-\delta(x)\|_A \le \frac{n\theta\|x\|_{A_0}^r}{\binom{n}{\mathrm{m}}\left((n-m+1)-(n-m+1)^r\right)} \quad (2.20)$$

for all $x \in A_0$.

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