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Collisionless damping of plasma waves as a real physical phenomenon does not exist

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Abstract

Trivial logic of collisionless plasma waves is reduced usually to using non-linear complex exponentially damping/growing wave functions to obtain a complex dispersion equation for their wave number k_1 and the decrement/increment k_2 (for a given real frequency ω and complex wave number $k \equiv k_1 - ik_2$), whose solutions are ghosts k_1, k_2 which do not have anything to do at $k_2 \neq 0$ with the solution of the real dispersion equation for the initial exponentially damping/growing real plasma waves with the physically observable quantities k_1, k_2, for which finding should be added, in this case, the second equation of the energy conservation law. Using a complex dispersion equation with non-linear complex wave functions for the simultaneous determination of k_1 and k_2 violates the law of energy conservation, leads to a number of contradictions, is logical error, and finally also the mathematical error leading to both erroneous statement on the possible existence of exponentially damping/growing harmonic wave solutions and to erroneous values k_1 and k_2. Mathematically correct conclusion about the damping/growing of virtual complex non-linear waves of collisionless plasma is wrongly attributed to the actual real plasma waves. A brief discussion is also on formalism of complex conductivity and dielectric permittivity with the ability to use their real values.

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Key words

collisionless plasma waves; Landau damping; Vlasov non-damping waves; collisionless electron wave damping; dispersion equation of plasma waves.

1. Introduction

Dispersion properties of the ***real*** and ***non-linear complex*** wave functions are not comparable, because are properties of physically observable and abstract mathematical objects of entirely different nature.

Simple calculations to obtain the real dispersion equation for the real exponentially damping/increasing harmonic wave functions (only ***single*** equation with ***two*** unknowns!) lead to the conclusion that a complex solution of the complex dispersion equation for the ***non-linear complex*** wave functions, which reduces to the solution (now of ***two*** equations with ***two*** unknowns for the real and imaginary parts) have completely different physical meaning of procedure, and the use of ***non-linear complex*** wave functions with complex dispersion equation leads to different disparate results and thus is a logical and finally mathematical error. It is shocking, that collisionless Landau damping/growing does not exist as a real physical phenomenon.

But this does not mean that all articles with the usually negligibly small additional terms of "Landau damping" are wrong, if there are used the correct energy balance equations, be it pair collisions of particles or "collisionless" energy exchange of the type of collective collisions, of by radiation, or any quantum-mechanical process, or any other way of energy exchange. In the case of the real dispersion equation with two unknowns there must be added the equation of energy conservation. In the second case, we obtain two ghosts solutions without the energy conservation.

2. Relation of averaged over period exponentially damping/growing plasma waves with the collision term of kinetic equation

It is assumed that the longitudinal in direction x electron wave functions, both real and complex, should satisfy the same equations: (1) the linearized kinetic equation

$$\frac{\partial f_1(v_x,x,t)}{\partial t}+v_x\frac{\partial f_1(v_x,x,t)}{\partial x}+\frac{e}{m_e}\frac{\partial f_0(v)}{\partial v_x}E(x,t)=0 \qquad (1)$$

and Maxwell equation relating the electric field $E(x,t)$ and the charge density

$$\frac{\partial E(x,t)}{\partial x}=4\pi e\int_{-\infty}^{+\infty} f_1(v_x,x,t)dv_x, \qquad (2)$$

and as the standard approach adopted for the full distribution function

$$f = f_0(v)+ f_1(v_x,x,t); \quad |f_1| \; much\ less\ than\ f_0 . \qquad (3)$$

where $e=-|e|$ is electron charge, m_e is electron mass, ω is real frequency, and $v,\ v_e$ are electron velocities. From physical considerations, it is assumed that at the points where the condition (3) may be violated it is necessary to carry out cutting off the function $|f_1|$.

In the case of testing complex wave functions, ones use as solution the expression

$$f_1 = f_1(v_x)e^{i(\omega t-kx)}; \quad k=k_1-ik_2 \qquad (4)$$

and necessary condition for the existence of solution

$$E(x,t)=iE_0e^{i(\omega t-kx)}. \qquad (5)$$

Similarly, in the case of real wave functions to receive a real tested solution ones must use the solution in the form

$$f_1 = f_1(v_x)e^{-k_2x}a,\ \ a\equiv\cos(\omega t-k_1x); \quad E(x,t)=-E_0e^{-k_2x}b,\ \ b\equiv\sin(\omega t-k_1x). \qquad (6)$$

Both cases are discussed in [1], so the further results are given only for the case of physically observable real functions and variables.

Substitution of the real expressions (6) into the real wave equations (1) and (2) leads to the real dispersion relation [1]

$$1=\frac{-ab}{ak_1+bk_2}\cdot\omega_0^2\int\frac{\partial f_0(v)/\partial v_x}{(\omega-k_1v_x)b+ak_2v_x}dv_÷ \qquad (7)$$

where ω_0 is Langmuir frequency $\omega_0^2=4\pi e^2/m_e$.

It means that hypothetical solutions of Eq. (7) with constant $k_1;\,k_2\neq 0$ that would not depend on x,t do not exist. However, one can try to obtain some average values $k_1;\ k_2\neq 0$ over the wave period introducing the collision term S on the right hand side of the kinetic equation with the dispersion equation of the form

$$1 = \frac{-ab}{ak_1 + bk_2} \cdot \omega_0^2 \int \frac{\partial f_0(v)/\partial v_x}{(\omega - k_1 v_x)b + ak_2 v_x - A} dv_x \qquad (8)$$

taking

$$A \equiv ak_2 v_x; \;\; S \equiv Af_1(v_x)e^{-k_2 x}. \qquad (9)$$

In this case the dispersion equation takes the form [1], [2]:

$$1 = \frac{-a}{ak_1 + bk_2} \int \frac{\partial f_0(v)/\partial v_x}{(\omega - k_1 v_x)} dv_x \; ; \qquad (10)$$

$$\left(\frac{a}{ak_1 + bk_2} \right)_{av} = \frac{1}{2\pi} \int_0^{2\pi} \frac{dy}{k_1 + k_2 \cdot \tan y} \qquad (11)$$

with the need of correction of Eq. (18) in [2] and

$$f_1(v_x) = \frac{e}{m_e} \frac{\partial f_0(v)/\partial v_x}{(\omega - k_1 v_x)} E_0 \,. \qquad (12)$$

where all integrals can be taken, including improper integrals (10) , in the principal value sense. Besides that , due to singularity of the function $f_1(v_x)$ at the point $v_x = \omega/k_1$ in Eq. (11) near which the kinetic equation does not apply, it is necessary cutting off $f_1(v_x)$ in (11) nearby this point in accordance with the condition of positivity the total distribution function f in (3)

$$\left| f_1(v_x, x, t) \right| < f_0(v) \,. \qquad (13)$$

The required for finding averaged k_1 and k_2 the second integral energy conservation equation can be written, for example, as relation of the type

$$\int f_1(v_x) k_2 v_x \Delta\varepsilon(v_x) dv_x = \int f_1(v_x) \left[n_e^2 \sigma_1(v_x) v_x \Delta\varepsilon_2(v_x) + n_m \sum_{n>1} n_e \sigma_n(v_x) v_x \Delta\varepsilon_{n+1}(v_x) \right] dv_x \qquad (14)$$

(with further integration over v_x) and with possible resonances of k_2 in dependence on ω and arising excited molecules, where $\Delta\varepsilon_1(v_x)$ is energy loss per electron associated with a decrease in the number of electrons with an additional wave propagation velocity v_x; $\Delta\varepsilon_2(v_x), \; \Delta\varepsilon_2(v_x), \; ...$. are portions of transmitted energy; $\sigma_n(v_x)$ are collision cross sections; n_e is electron density; n_m is density of atoms and molecules. In this expression

$$\Delta\varepsilon_1 \sim \frac{m_e}{2} \left[(v_{xM} + v_x)^2 - v_{xM}^2 \right] \qquad (15)$$

where v_{xM} is proper velocity of electron along the axis x in Maxwell distribution $f_0(v)$ independent on the wave speed. To expression (14) one can add also any other actual arbitrary energy exchange process terms.

Note that the complex dispersion equation with the substitution of complex wave functions (3), (4) into (1), (2) at enough large $k_2 \neq 0$ has nothing to do with the real dispersion equation of the real Eq. (7) and leads to collisionless damping in violation of the law of energy conservation [1].

3. Inadequacy of the direct application of the concepts of complex electric conductivity/dielectric permittivity and other complex parameters

The laws of nature are determined by relations between the physically observable quantities, but in the case of complex values with their mathematical transformations, besides simple linear relationships, to extract the true relationships between physically observable quantities may be impossible, since the complex non-linearity arises even in the simplest cases, for example, in differentiating $d\left(\exp^{ikx}\right)/dx$, as at real and all the more complex k.

This is confirmed by results obtained above with completely different meaning of the dispersion equations for the real and non-linear complex wave functions, with the “discovery” of the collisionless damping of the real plasma waves, which is attributed them from entirely mathematically correct collisionless damping of non-linear complex waves.
This raises the question of obtaining the true relationship between real physically observed values from commonly used quantities such as the complex conductivity, that is not reducible to the simple use of its real or imaginary parts.

For example, in the case of the complex tensor conductivity $\sigma_{ij}\left(\omega,\vec{k}\right)$ (see [2]), to obtain the real current density $j_i\left(\omega,\vec{k}\right)$ without a separate computation of the real and imaginary parts of constituent complex terms one must use compound expressions in (16), but not the real (or imaginary) parts separately of incoming expressions.

$$\mathrm{Re}\int_{-\infty}^{\infty}\vec{j}_i\left(\omega,\vec{k}\right)e^{i\left(\omega t-\vec{k}\vec{r}\right)}d\omega=\left[\frac{1}{2}\int_{-\infty}^{\infty}\sigma_{ij}\left(\omega,\vec{k}\right)E_j\left(\omega,\vec{k}\right)e^{i\left(\omega t-\vec{k}\vec{r}\right)}d\omega\right]+\left[compl.\,conjug.\right] \quad (16)$$

with the integration $\sigma_{ij}\left(\omega,\vec{k}\right)E_j\left(\omega,\vec{k}\right)\exp\left[i\left(\omega t-\vec{k}\vec{r}\right)\right]$ over ω in accordance with the Fourier transform at $0\le t\le\infty$ and inverse Fourier transform at $-\infty<\omega<\infty$ with analytical continuation to complex ω in the lower and upper complex half–plane ω [6].
For real $\vec{j}\left(\vec{r},t\right)$ and its Fourier transforms with real ω, using relation (16) results in identically the same result with and without complex conjugation. The difference arises only in the case of the Fourier transforms of *non-linear complex* $\vec{j}\left(\vec{r},t\right)$ that may occur, as shown in Sec. 2, at using "wild" non-linear wave functions with complex $\omega,\vec{k}$ in the function of the complex conductivity. This leads to a collisionless damping of plasma waves or to elicitation the real part with neglecting imaginary part of the type discussed in ***Appendix 1***, pp 6, 8 of [5] which also leads to erroneous values k_1,k_2 in the expression $k=k_1-ik_2$.

Collisionless damping is a consequence of this particular, local error of “wild” non-linear complex wave functions, with correctness of the general linear theory of complex plasma conductivity and complex dielectric permittivity only when using Fourier transforms of real boundary and initial conditions including boundary electric field and real non-damping wave functions.

As it follows from Sec. 2, exact damping solution can exist only as an integrated set of waves ω,k with different dispersion relations. It requires an entirely new approach to the calculation of the complex conductivity and dielectric permittivity in the presence of collisional energy exchange and to the calculation of real wave functions.
In the case of the component of Fourier expansion $\sim e^{ikx}$ with real k ($k_2=0$ with the single root $k=f\left(\omega\right)$ of dispersion equation (7)), the dispersion equation coincides with the real dispersion equation for the real wave functions, moreover, it contains only one parameter to be finding: the wave number k of non-damping waves. Thus the need for the energy conservation equation is eliminated (non-damping waves).
If the direct and inverse Fourier transforms are possible, both dispersion relations in the input variables $\vec{r},t,$ as well as variables $\omega,\vec{k}$ in correspondingly direct and inverse Fourier transforms must be real in order to avoid the appearance of two-parameter complex roots. Reality of both equations is the criterion of correctness of carried out mathematical transformations. However, there may be severe error, in addition to using the dispersion equation for the non-linear complex wave functions of the type (4), (5), when the approximate equality of the product of the Fourier transforms of the real functions may differ significantly from the product of the Fourier transforms of these functions in violation of the generally accepted theory of the complex conductivity and dielectrical permittivity.
It is necessary to exclude the possibility of relationships with complex $\sigma\left(\omega,\vec{k}\right)$ that are specific for using “wild” wave functions and lead to erroneous relations between physically observable parameters of the problem of the type discussed above in Sec. 2. Eq. (16) is equivalent to using only real part of the complex dispersion equation in the case of complex $\vec{k}$.
The wave damping corresponds to the Fourier transform in which for each real frequency ω corresponds the spectrum of waves with different real $\vec{k}$ ’s. This can significantly hinder or prevent the application of the theory of complex electrical conductivity and permittivity with the need to consider the totality of waves with different dispersion dependences $k=f_n\left(\omega\right)$. This is clearly demonstrated by Eq. (10) in Sec. 2, when the collisional kinetic equation with fixed ω is followed by a *set o*f dispersion equations in (10) with a variety of k_1 in the terms $a,\, b$ and equation of energy conservation.

4. Expansion of the wave solutions in the case of the collisionless kinetic equation with a nonlinear term, which is due to the perturbation of distribution function, in overtones

When substituting the electron distribution function in the form $f = f_0(v) + f_1(v_x, x, t)$ in the kinetic equation of longitudinal waves in a collisionless plasma (1) it is usually neglected nonlinear term of second order of smallness

$$\frac{|e| E_x(x,t)}{m_e} \cdot \frac{\partial f_1}{\partial v_x} \qquad (17)$$

in the right hand side of the kinetic equation. It may seem that this "collision" term will result in a loss of waves energy, respectively, to their damping. However, attempts were made to obtain non-damping solutions in the form of expansions in the overtones of the form

$$f_1(\vec{v}, v_x, v_z, x, t) = \sum_{n=1} F_n(\vec{v}, v_x, v_z) \cos\left[n(\omega t - kx + \varphi_1)\right], \qquad (18)$$

$$E(x,t) = \sum_{n=1} E_n(x,t) \cos\left[n(\omega t - kx + \varphi_1) - \pi/2\right]; \qquad (19)$$

with recurrent relations between F_n and preceding parameters f_n, E_n with indices $n \le 3$.

Amplitudes E_n, F_n are proportional to $(E_1)^n$, what provides the convergence of expansions at any enough small E_1 or more precisely, convergence parameter

$$\eta \approx \left| \frac{e}{m_e} \frac{E_1}{\sqrt{\overline{v_x^2}}} \frac{\pi}{\omega} \right| \textit{much less than } 1. \qquad (20)$$

It should be noted that the rather cumbersome transformations given in [3] were made without the use of complex variables and are brought to $n \le 3$.

All the details can also be found in ***Appendix 6*** of the easily accessible work [5]. Calculations are made for both the longitudinal and transverse plasma waves, although in the process of calculation it should be made successive cutting off, according to the condition of positivity the function f, that involves some uncertainty in the result.

An interesting fact would be the experimental discovery of multiple overtones, that does not seem an insurmountable task.

5. Polarization estimation hypothesis

The distribution function of charged particles in the plasma is characterized by the collective distribution of particle swarm on the coordinates and velocities in the created by them (or external) electric field, neglecting the individual interactions of each particle with its nearest neighbors, therefore, as a measure of the applicability of the collisionless approximation one can take comparison impact force $eE(x,t)$ of electric field $E(x,t)$ on the particle and the Coulomb force of interaction between two nearest particles. Conventionally, one can assume that the collisional interaction can be neglected, starting with the value $|eE(x,t)| > or \approx e^2/r_{av}^2$ where $r_{av} \sim n_e^{-1/3}$ is average distance between charged particles in plasma, i.e. actual interacting of $E(x,t)$ with the charged particles is equivalent to the replacing

$$E(x,t) \to E_{eff}(x,t) = \gamma E(x,t), \quad \gamma \sim E_0 / |e| n_e^{2/3} \sim \text{const}; \quad 0 < \gamma < or \approx 1; \quad \text{if } \gamma > 1, \text{ then } \gamma \sim 1 \qquad (21)$$

where E_0 is amplitude of the $E(x,t)$ and n_e is average electron density.

The polarization hypothesis allows to estimate the parameters of electron waves in collisional plasmas with $\gamma < 1$, and can be a useful tool for the study of plasma waves.

The polarization hypothesis underlies the estimates of the parameters in the dipole-dynamic model of ball lightning (DDM BL) in [4].

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6. Conclusion

There is proposed the fundamentally new approach to basic concepts of electron waves in a low-temperature plasma including:

- discussion of the possible existence of exponentially damping/growing sinusoidal waves in collisionless and weakly collisional plasma, non existence of damping/growing waves in collisionless plasma [1];
- inadmissibility of using complex dispersion equations obtained for the non-linear complex wave functions, instead of the real dispersion equations obtained for the real wave functions [1] or real ω, k ;
- non-linearity of kinetic equations due to the perturbation of the electron distribution function $f_0(v)$ by perturbation $f_1(v_x, x, t)$ in collisionless plasma, leads to the appearance of multiple (integer n, $\sin[n(\omega t - kx)]$, $\cos[n(\omega t - kx)]$) overtones without showing any effects of increasing-damping of waves [3];
- polarization hypothesis with the possibility of simple estimates with transport the parameters of plasma waves in weakly collisional plasma to a more collisional plasma by introducing such a parameter as the effective electric field strength $E_{eff}(x,t)$ [4].

The polarization hypothesis was actively used when creating a dipole dynamic model of ball lightning [4], but it was entirely distinct from the parameter estimates of the plasma waves with $\gamma = \text{const}$ and variable $E(x,t)$, wherever in DDM BL the constant was the atmospheric electric field E_{env} with variable γ.

Dispersion properties of actual real and virtual non-linear complex wave functions are not comparable, because are properties of physically observable and abstract mathematical objects of entirely different nature.

The foregoing results are presented collectively and discussed in detail in the cumulative work [5].

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Biography

I am scientist 82, now referent-freelance VINITI (All-Russian Institute of Scientific and Technical Information), Department of Physics (Sec. Astrophysics). My past theoretical activities: contribution of the radiation of diatomic molecules and NO2 in the radiation of the hot (up to 10 000 K) air and planetary atmospheres, inductive discharge in air at 1 at, the transition layer in the solar atmosphere, explosive gas dynamics of hot air, the dipole-dynamical model of ball lightning and other problems of low temperature plasma.