



Theoretical Determination of Level Spins of Superdeformed Bands for Nuclei in the Mass Region $A = 80 - 104$

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ABSTRACT

The bandhead spins of seventeen superdeformed bands in $A = 80 - 104$ region ($_{38}\text{Sr}$, $_{39}\text{Y}$, $_{40}\text{Zr}$, $_{41}\text{Nb}$, $_{42}\text{Mo}$, $_{43}\text{Tc}$, $_{46}\text{Pd}$) have assigned by an indirect method. The dynamical moment of inertia $J^{(2)}$ as a function of rotational frequency $\hbar\omega$ are extracted from Harris expansion and fitted to the experimental values by using a computer simulated search program.

The calculated dynamic moment of inertia with the best optimized parameters are integrated to give the spins. The intrinsic aligned angular momentum (the integration constant) is assumed to be zero. The values of the spins resulting from our approach are consistent with all spin assignments of other approaches, and have been used to determine the kinematic moment of inertia $J^{(1)}$. The systematic variation of $J^{(2)}$ and $J^{(1)}$ with rotational frequency $\hbar\omega$ is investigated, which turns out to be helpful in the spin prediction. Most SD bands in this mass region exhibits decreasing in $J^{(1)}$ and $J^{(2)}$ with increasing $\hbar\omega$. The bandhead moment of inertia J_0 which occur at $J^{(2)} = J^{(1)}$ has been sensitive guideline parameter to spin proposition. The relationship between the Harris expansion three parameter model and the four parameter Bohr-Mottelson formula is derived.

KEYWORDS

Superdeformed Bands; Harris Model

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1. Introduction

Study of superdeformed (SD) nuclei has been one of the most exciting fields in nuclear spectroscopy since the discovery of SD states at high spins in ^{152}Dy [1]. More than 330 SD bands have now been observed in the mass regions $A \approx 30, 60, 80, 130, 150$ and 190 [2,3]. Many theoretical and experimental efforts were devoted to explore the nature of SD states in nuclei. The SD mass region $A \approx 80 - 100$ is very interesting region because they exhibit highest rotational frequencies.

In SD bands, gamma ray transition energies are the only spectroscopic information available till now. However, level spins, parities and excitation energies in most of these bands were not determined experimentally because linking transitions between the SD states and the normal deformed (ND) states were not observed. In the past few years several empirical and semiempirical approaches were proposed for the spin assignments in SD bands [4-7]. All these available approaches obtained mainly from the comparison of the calculated gamma transition energies or dynamical moments of inertia with experimental results. In previous papers we have used Harris ω^2 expansion [8-12], Bohr- Mottelson $I(I+1)$ expansion [13], ab expression [14, 15] and variable moment of inertia (VMI) model [16, 17] to assign spin.

The main purpose of the present work is to determine the spins of energy levels of some SD bands in the mass region $80 \leq A \leq 104$ and examine the behaviors of moments of inertia. We will use the Harris expansion and its relationship with the Bohr-Mottelson formula. The paper is arranged as follows: Following this introduction, the Harris and Bohr-Mottelson expansions employed to assign spins are presented and discussed in the next section(2). Numerical calculations and discussion are performed in section (3) for even-even and odd – A SD nuclei in the mass region $A = 80 - 104$. The data set include 17 SD bands in Sr / Y / Zr / Nb / Mo / Tc / Pd nuclei. Conclusion and remarks are given in section (4).

2. Spin assignment in SD bands using Harris and Bohr-Mottelson expansions

For SD bands, γ -ray energies are the only spectroscopic information universally available. There are no direct experimental determinations of the spins in SD bands. Spin assignment is one of the most difficult and unsolved problems in the study of nuclear superdeformation. The spin assignments have received considerable attention. Several theoretical procedures were proposed [4-17].

In this section, we will fit the experimental dynamical moment of inertia values with the Harris power series formula [18]. The expansions parameters obtained from the fitting will be used to assign the spins. In such parameterize the spin may be expressed as an expression in the rotational frequency. Also the relation between Harris expansion which depend on the rotational frequency and the Bohr-Mottelson formula which depends on the spin will be derived.

The nuclear energy E of the nucleus can be expanded in powers of angular velocity ω by Harris expansion [18] as an extension of cranking model:

$$E = \frac{1}{2}\alpha\omega^2 + \frac{3}{4}\beta\omega^4 + \frac{5}{6}\gamma\omega^6 + \frac{7}{8}\delta\omega^8 \quad (1)$$

Where: only even powers of ω are present in systems invariant with respect to time reversal.

In general, the above Harris expansion converges faster than the Bohr-Mottelson expansion [19] in powers of $I(I+1)$:

$$E(I) = A [I(I + 1)] + B[I(I + 1)]^2 + C[I(I + 1)]^3 + D[I(I + 1)]^4 \quad (2)$$

where A is the rotational constant parameter and B, C and D are the corresponding higher order constant parameters.

In framework of nuclear collective rotational model, two types of moments of inertia are usually discussed, which are related to the first and second order derivatives of the excitation energy with respect to the angular momentum. We define the second order derivative dynamical moment of inertia by:

$$\frac{J^{(2)}}{\hbar^2} = \left[\frac{d^2E}{d(\sqrt{I(I+1)})^2} \right]^{-1} = \frac{1}{\omega} \frac{dE}{d\omega} = \frac{1}{\hbar} \frac{d\sqrt{I(I+1)}}{d\omega} \quad (3)$$

The use of $\sqrt{I(I + 1)}$ rather than angular momentum I provide the proper limiting case for an ideal rotor with energy proportional to the quintal square $I(I+1)$ rather than I^2 .

The corresponding expression for formulae (1) and (2) are:

$$\begin{aligned} \frac{J^{(2)}}{\hbar^2} &= \alpha + 3\beta\omega^2 + 5\gamma\omega^4 + 7\delta\omega^6 \\ &= [2A + 12B[I(I + 1)] + 30C[I(I + 1)]^2 + 56D[I(I + 1)]^3]^{-1} \end{aligned} \quad (4)$$

The parameter α corresponds to the bandhead moment of inertia.

Integrating $J^{(2)}$ yields the intermediate level spin:



$$\hbar\sqrt{I(I+1)} = \int d\omega J^{(2)} = \alpha\omega + \beta\omega^3 + \gamma\omega^5 + \delta\omega^7 + i_0 \tag{5}$$

Where: the intrinsic alignment i_0 appears as a constant of integration.

The first order derivative, kinematic moment of inertia is defined as:

$$\frac{J^{(1)}}{\hbar^2} = \sqrt{I(I+1)} \left(\frac{dE}{d\sqrt{I(I+1)}} \right)^{-1} = \frac{\sqrt{I(I+1)}}{\hbar\omega} \tag{6}$$

The corresponding expression for formulae (1) and (2) are

$$\begin{aligned} \frac{J^{(1)}}{\hbar^2} &= \alpha + \beta\omega^2 + \gamma\omega^4 + \delta\omega^6 \\ &= [2A + 4B[I(I+1)]^{3/2} + 6C[I(I+1)]^{5/2} + 8D[I(I+1)]^{7/2}]^{-1} \end{aligned} \tag{7}$$

Now, $J^{(1)}$ is equal to the inverse of the slope of the curve of energy E versus $I(I+1)$ times $\hbar^2/2$, while $J^{(2)}$ is related to the curvature in the curve E versus $\sqrt{I(I+1)}$. In case of a rigid rotor where the energy is directly proportional to $I(I+1)$, both definitions for $J^{(1)}$ and $J^{(2)}$ coincide. That is, $J^{(2)}$ is a quantity defined locally; while $J^{(1)}$ is a more global quantity since the spin I itself is not a local quantity.

Squaring equation (5) four times, yield

$$[I(I+1)] = \alpha^2\omega^2 + 2\alpha\beta\omega^4 + (2\alpha\gamma + \beta^2)\omega^6 + (2\alpha\delta + 2\beta\gamma)\omega^8 + \dots \tag{8}$$

$$[I(I+1)]^2 = \alpha^4\omega^4 + 4\alpha^3\beta\omega^6 + (4\alpha^3\gamma + 6\alpha^2\beta^2)\omega^8 + \dots \tag{9}$$

$$[I(I+1)]^3 = \alpha^6\omega^6 + 6\alpha^5\beta\omega^8 + \dots \tag{10}$$

$$[I(I+1)]^4 = \alpha^8\omega^8 + \dots \tag{11}$$

Substituting from equations (8-11) into equation (2), yield

$$\begin{aligned} E(I) &= [A\alpha^2]\omega^2 + [2A\alpha\beta + B\alpha^4]\omega^4 + [A(2\alpha\gamma + \beta^2) + 4B\alpha^3\beta + C\alpha^6]\omega^6 \\ &+ [A(2\alpha\delta + 2\beta\gamma) + B(4\alpha^3\delta + 6\alpha^2\beta^2) + 6C\alpha^5\beta + D\alpha^8]\omega^8 \end{aligned} \tag{12}$$

Comparing equation (12) with equation (1), yield the relations:

$$A = \hbar^2 \left(\frac{1}{2\alpha} \right) \tag{13}$$

$$B = -\hbar^4 \left(\frac{\beta}{4\alpha^4} \right) \tag{14}$$

$$C = \hbar^6 \left(\frac{\beta^2}{2\alpha^7} - \frac{\gamma}{6\alpha^6} \right) \tag{15}$$

$$D = \hbar^8 \left(\frac{\beta\gamma}{\alpha^9} - \frac{3\beta^3}{2\alpha^{10}} - \frac{\delta}{8\alpha^8} \right) \tag{16}$$

If we truncate the expressions (1) and (2) at the second term only, we obtain:

$$E = \frac{1}{2}\alpha\omega^2 + \frac{3}{4}\beta\omega^4 = A[I(I+1)] + B[I(I+1)]^2 \tag{17}$$

$$\frac{J^{(2)}}{\hbar^2} = \alpha + 3\beta\omega^2 = [2A + 12BI(I+1)]^{-1} \tag{18}$$

$$\hbar\sqrt{I(I+1)} = \omega(\alpha + \beta\omega^2) \tag{19}$$

$$\frac{J^{(1)}}{\hbar^2} = \alpha + \beta\omega^2 = \frac{1}{2A} \left[1 + \frac{2B}{A}I(I+1) \right]^{-1} \tag{20}$$



Eliminating ω from the two equations (17) and (18), we get a cubic equation for the energy

$$e^3 + 2e^2 + (1 + 36d)e - 4(1 + 27d)d = 0 \tag{21}$$

where

$$d = \frac{\beta}{3\alpha^3} I(I + 1) \tag{22}$$

$$e = \frac{4\beta}{\alpha^2} E \tag{23}$$

Putting $X = 2d$ we obtain:

$$E(I) = \frac{\hbar^2}{2\alpha} I(I + 1)[1 - X + 4X^2 - 24X^3 + \dots] \tag{24}$$

Equation (24) is an expression for the energy levels in terms of α , β and I .

For SD bands experimentally, the rotational frequency $\hbar\omega$, the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia are usually extracted from the observed transition energies E_γ between two consecutive quadruple transition within a band from the following finite difference approximations,

$$\hbar\omega = \frac{1}{4} [E_\gamma(I + 2 \rightarrow I) + E_\gamma(I \rightarrow I - 2)] \tag{25}$$

$$\frac{J^{(2)}(I)}{\hbar^2} = \frac{4}{E_\gamma(I+2 \rightarrow I) - E_\gamma(I \rightarrow I-2)} \tag{26}$$

$$\frac{J^{(1)}(I)}{\hbar^2} = \frac{2I-1}{E_\gamma(I \rightarrow I-2)} \tag{27}$$

where, the experimental γ - transition energies of the SD band is in MeV.

It is seen that while the extracted $J^{(1)}$ depends on the spin I proposition, $J^{(2)}$ does not (see equations 27, 26). Thus, if the dynamic moments of inertia $J^{(2)}$ were a constant, the transition energy difference would be the same for all values of spin. Often this is not the true and $J^{(2)}$ is found to change with increasing spin. The two moments of inertia can related as follows:

$$\frac{J^{(2)}}{\hbar^2} = \frac{1}{\hbar} \frac{d\sqrt{I(I+1)}}{d\omega} = \frac{1}{\hbar} \frac{d}{d\omega} \left(\frac{1}{\hbar} J^{(1)} \omega \right) = \frac{1}{\hbar^2} \left(J^{(1)} + \omega \frac{dJ^{(1)}}{d\omega} \right) \tag{28}$$

Solving for $J^{(1)}$, yield:

$$J^{(1)} = J^{(2)} + \frac{const.}{\omega} \tag{29}$$

3. Numerical Calculations and Discussion

For SD bands, gamma-transition energies E_γ are the only spectroscopic information universally available. The information about E_γ are commonly translated into values of rotational frequency $\hbar\omega$ equation (25) and dynamical moment of inertia $J^{(2)}$ equation (26). One of the most supervising characteristic of data on SD bands is the different behavior of $J^{(2)}$ as a function of $\hbar\omega$.

The optimized expansion parameters α , β , γ of $J^{(2)}$ values in the Harris parameterization for each SD band have been calculated from best fit method [12] to the experimental $J^{(2)}$ values extracted from E_γ . The quality of the fit is indicated by the common χ quantity

$$\chi = \left[\frac{1}{N} \sum_{i=1}^N \left(J_{exp}^{(2)}(i) - J_{cal}^{(2)}(i) \right)^2 \right]^{1/2}$$

in order to obtain a minimum root – mean square (rms) deviation. N is the total number of experimental points entering into the fitting procedure. It was found that the rms deviation of the calculated results with experiments, χ , depends on the number of transitions involved, and in some cases χ is insensitive to the suggested spin, that is the rms deviations may be close to each other for two or more spin propositions in this case, it is difficult to make a unique spin proposition.

The best adopted optimized parameters α , β and γ obtained from the fitting procedure have been used to determine the spins with the help of equation (5). The constant of integration i_0 which represent the aligned angular momentum at zero frequency has been taken to be zero. The resulting best parameters α , β and γ and values of the lowest bandhead spin I_0 and the bandhead moment of inertia $J_0 = \alpha$ are listed in Table (1). The data set include 17 SD bands in $A = 80- 104$ mass region for Strontium ($_{38}\text{Sr}$), Yttrium ($_{39}\text{Y}$), Zirconium ($_{40}\text{Zr}$), Niobium ($_{41}\text{Nb}$), Molybdenum ($_{42}\text{Mo}$), Technetium ($_{43}\text{Tc}$) and



Palladium ($_{46}\text{Pd}$). The experimental data of transition energies are taken from references [1, 2]. Table (2) lists the optimized parameters A,B,C and D of the Bohr – Mottelson four expansion.

Using our assigned spin values, the Kinematic moment of inertia $J^{(1)}$ of the SD bands can be consequently determined. The evolution of dynamic $J^{(2)}$ moments of inertia as a function of rotational frequency $\hbar\omega$ are illustrated in Figure(1). It is seen that most SD bands exhibits decreasing $J^{(2)}$ with increasing $\hbar\omega$.

Table (1): The bandhead spin proposition I_0 and the best adopted Harris expansion, three parameters α , β and γ for the SD bands in the A= 80 – 104 mass region. N_γ denote the number of observed γ - ray transition energies included in fits.

Nuclide	I_0 (\hbar)	α ($\hbar^2 \text{MeV}^{-1}$)	β ($\hbar^4 \text{MeV}^{-3}$)	γ ($\hbar^6 \text{MeV}^{-5}$)	N_γ
$^{80}\text{Sr}(\text{SD1})$	20	32.1730	-6.1996	1.9108	10
$^{81}\text{Sr}(\text{SD2})$	34	47.8842	-13.4868	3.60224	6
(SD3)	31.5	40.0343	-7.6329	1.6390	7
(SD4)	37.5	51.9122	-16.9938	4.7869	5
$^{82}\text{Sr}(\text{SD1})$	11	13.9105	7.5991	-2.1220	9
$^{83}\text{Sr}(\text{SD1})$	13.5	21.3843	3.7615	-1.0107	10
$^{82}\text{Y}(\text{SD1})$	17	25.5827	-0.1437	-0.0613	9
$^{83}\text{Zr}(\text{SD2})$	16	24.0201	-0.8493	0.4037	8
$^{84}\text{Zr}(\text{SD1})$	25	37.2858	-5.5342	1.0711	9
$^{86}\text{Zr}(\text{SD1})$	27	40.4403	-6.7342	1.3607	9
$^{87}\text{Nb}(\text{SD4})$	26.5	38.0647	-9.5649	2.6942	6
$^{88}\text{Mo}(\text{SD1})$	31	61.1494	-26.3033	7.9670	8
(SD2)	20	30.5872	0.0903	-1.2453	6
(SD3)	23	42.7575	-10.6713	2.8096	8
$^{89}\text{Tc}(\text{SD1})$	22	43.4085	-10.3209	2.4376	11
$^{91}\text{Tc}(\text{SD1})$	23.5	37.6403	-1.9334	1.0853	11
$^{104}\text{Pd}(\text{SD1})$	22	34.4727	0.8903	-2.5341	7

Table (2): The same as in table (1) but for Bohr – Mottelson four parameters A, B, C and D expansion (in KeV)

Nuclide	I_0 (\hbar)	A	B	C	D
$^{80}\text{Sr}(\text{SD1})$	20	15.540	1.446×10^{-3}	2.514×10^{-7}	-1.995×10^{-11}
$^{81}\text{Sr}(\text{SD2})$	34	10.442	6.414×10^{-4}	1.078×10^{-7}	2.137×10^{-11}
(SD3)	31.5	12.489	7.525×10^{-4}	1.150×10^{-7}	1.760×10^{-11}
(SD4)	37.5	9.630	5.847×10^{-4}	1.012×10^{-7}	2.205×10^{-11}
$^{82}\text{Sr}(\text{SD1})$	11	35.944	-5.073×10^{-2}	3.352×10^{-4}	-3.253×10^{-6}
$^{83}\text{Sr}(\text{SD1})$	13.5	23.413	-4.517×10^{-3}	5.241×10^{-6}	-8.099×10^{-9}
$^{82}\text{Y}(\text{SD1})$	17	19.544	8.392×10^{-5}	3.792×10^{-8}	1.917×10^{-12}
$^{83}\text{Zr}(\text{SD2})$	16	20.816	6.378×10^{-4}	2.7124×10^{-7}	-1.444×10^{-10}
$^{84}\text{Zr}(\text{SD1})$	25	13.409	7.158×10^{-4}	8.641×10^{-8}	6.399×10^{-12}
$^{86}\text{Zr}(\text{SD1})$	27	12.363	6.294×10^{-4}	7.634×10^{-8}	7.482×10^{-12}
$^{87}\text{Nb}(\text{SD4})$	26.5	13.135	1.139×10^{-3}	2.474×10^{-7}	5.193×10^{-11}
$^{88}\text{Mo}(\text{SD1})$	31	8.176	4.703×10^{-4}	8.280×10^{-8}	1.981×10^{-11}
(SD2)	21	16.346	-2.580×10^{-5}	2.536×10^{-7}	-4.802×10^{-12}



(SD3)	23	11.693	7.981×10^{-4}	1.412×10^{-7}	2.648×10^{-11}
^{89}Tc (SD1)	22	11.518	7.267×10^{-4}	1.226×10^{-7}	2.344×10^{-11}
^{91}Tc (SD1)	23.5	13.292	2.385×10^{-4}	-4.628×10^{-8}	-1.181×10^{-11}
^{104}Pd (SD1)	20	14.504	-1.576×10^{-4}	2.585×10^{-7}	-3.326×10^{-11}

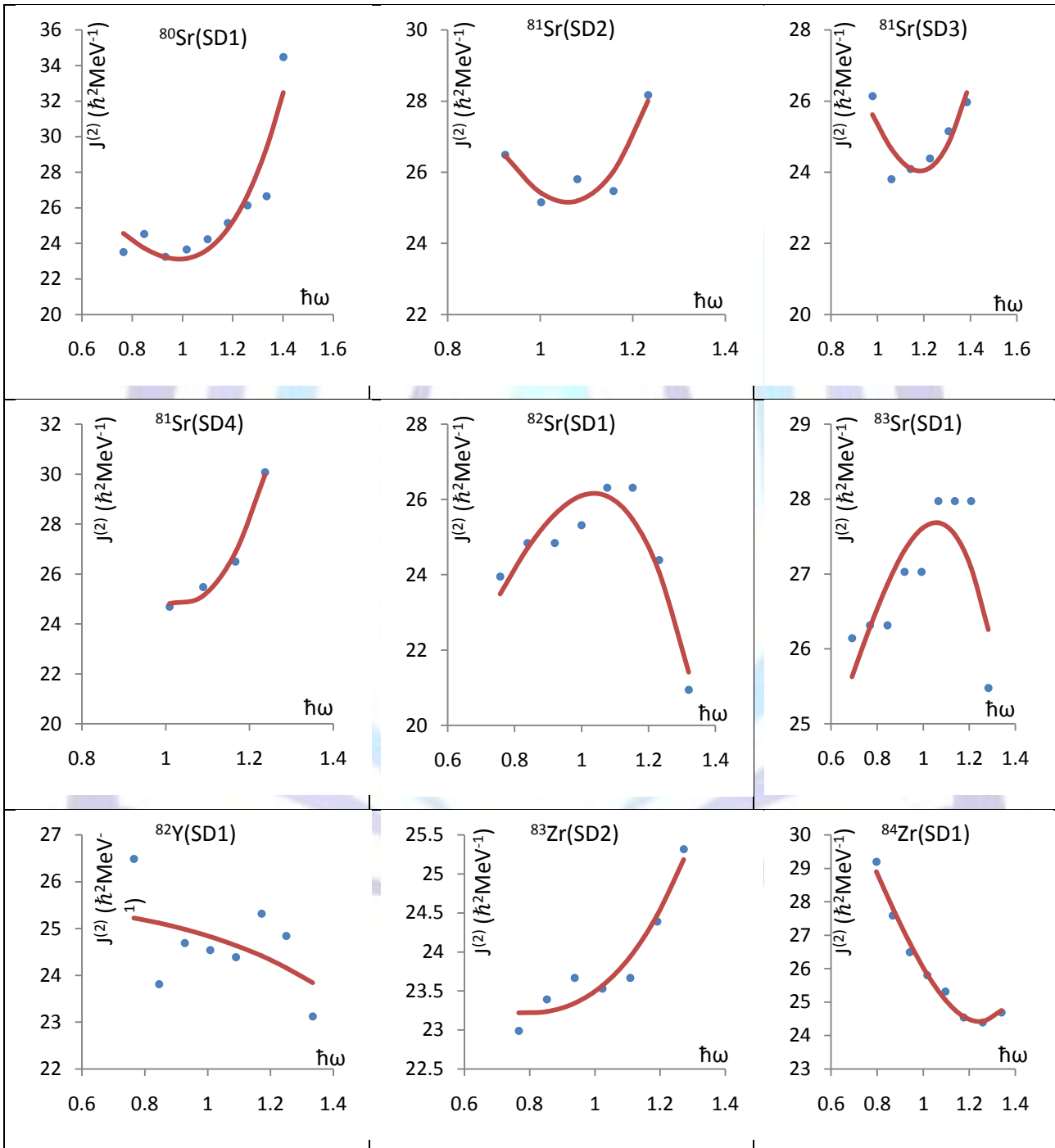


Figure (1) The calculated dynamic $J^{(2)}$ moments of inertia are plotted as a function of rotational frequency $\hbar\omega$. The experimental $J^{(2)}$ are labeled by closed circles.

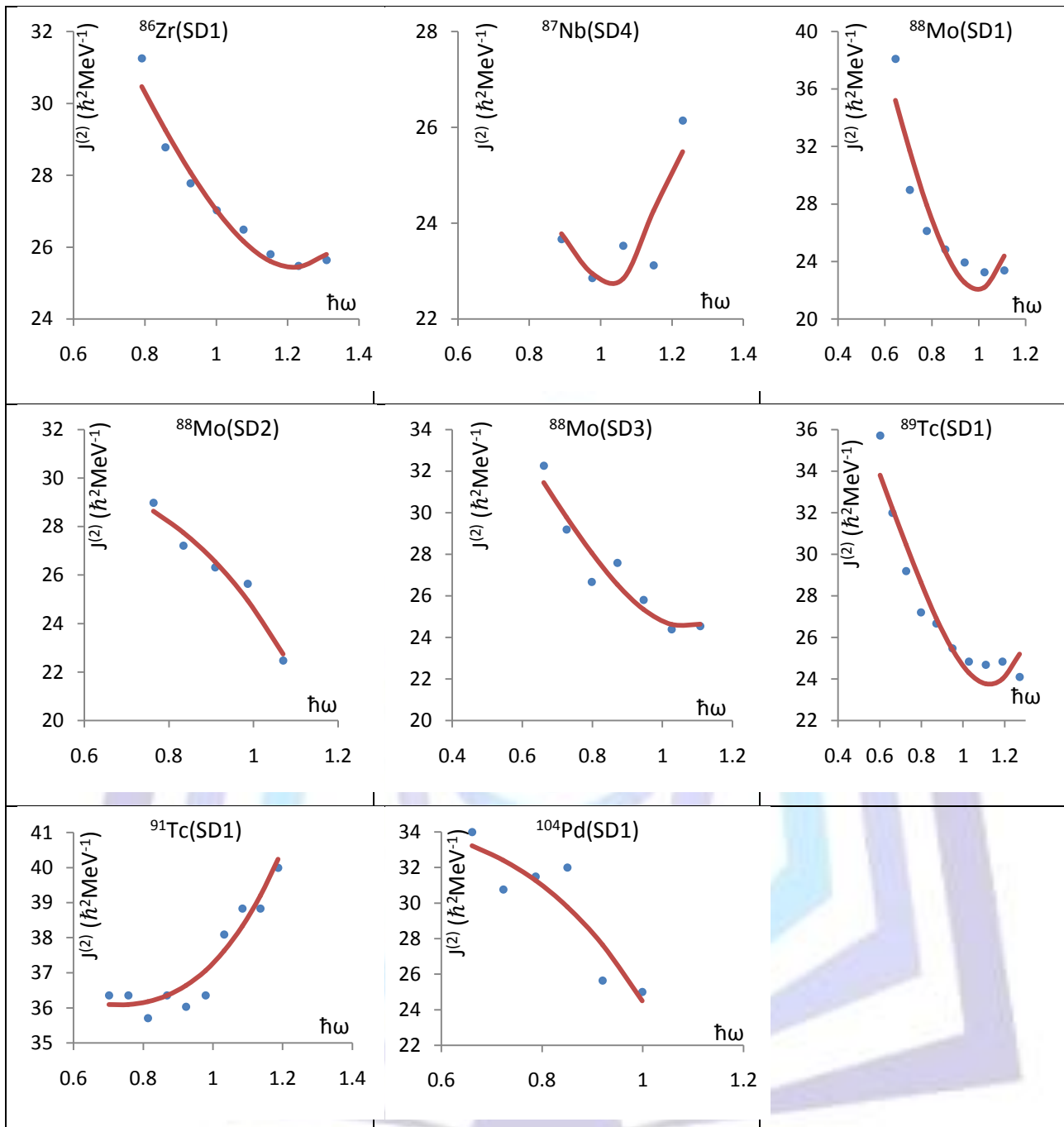


Figure (1) Continued

4. Conclusion

The main conclusion of the present work can be summarized as follows: The transition energies of SD nuclei in the mass region $A = 80 - 104$ can be quantitatively described excellently by Harris expansion to third term. The dynamical moment of inertia $J^{(2)}$ has been derived in terms of Harris parameters. The optimized parameters have been adjusted by using a computer simulated search program to fit the calculated theoretical $J^{(2)}$ with the corresponding experimental values. The bandhead spins have been assigned by integrating $J^{(2)}$ and using the best optimized parameters. The bandhead spins of our selected SD bands from the present study are excellent consistent with all spin assignments of other approaches. The calculated transition energies, level spins, rotational frequencies, kinematic and dynamic moments of inertia and bandhead moments of inertia are analyzed as a function of rotational frequency. It was found that the bandhead moments of inertia are helpful guide line in the spin prediction.



References

- [1] Twin P.J., Nyak B.M., Observation of a Discrete-Line Superdeformed Band up to $60\hbar$ in ^{152}Dy . *Physical Review Letters*, 1986, v. 57, 811–814.
- [2] Sing B., Zywina R. and Firestone R.B. , *Nuclear Data Sheets, Table of Superdeformed Nuclear Bands and Fission Isomers, Third Edition (July 2002)*.
- [3] National Nuclear Data Center NNDC, Brookhaven National Laboratory (Cited on July 2012) [http. // www.nndc.bnl.gov/chart/](http://www.nndc.bnl.gov/chart/).
- [4] Becker J.A. et al., Level spin and moments of inertia in superdeformed nuclei near $A = 194$. *Nuclear Physics*, 1990, v. A520, C187–C194.
- [5] Draper J.E., et al., Spins in superdeformed bands in the mass 190 region. *Physical Review*, 1994, v. c42 R1791–R1795.
- [6] Muntain I. and Sabiczowski A., Superdeformed ground state of superheavy nuclei. *Physics Letters*, 2004, v. B586, 254–257.
- [7] He X.T. et al., The intruder orbitals in superdeformed bands and alignment additively of odd-odd nuclei in the $A \approx 190$ region. *Nuclear Physics*, 2005, v. A760, 263–273.
- [8] Khalaf A.M. et al., Band Head of the Superdeformed Bands in the $A \approx 150$ Mass region Nuclei. *Egypt Journal of Physics*, 2002, v. 33 (1), 67– 87.
- [9] Khalaf A.M. et al. Spin Prediction and Systematics of Moments of inertia of superdeformed Nuclear Rotational Band in the Mass Region $A \approx 190$. *Egypt Journal of Physics*, 2002, v. 33 (3), 585–602.
- [10] Khalaf A.M. et al., Description of Rotational Bands in Superdeformed Nuclei by Using Two-parameter Empirical Formula. *Egypt Journal of Physics*, 2003, v. 34 (2), 159–177.
- [11] Khalaf A.M. and Sirag M.M. Analysis of $\Delta I = 2$ Staggering in Nuclear Superdeformed Rotational Bands. *Egyptian Journal of Physics*, 2006, v. 35, 359–375.
- [12] Khalaf A.M., Allam M.A., Saber E., Signature Partners in Odd Superdeformed Nuclei in Mass Region $A \approx 190$. *Egypt Journal of Physics*, 2008, v. 39 (1), 41–65.
- [13] Khalaf A.M., Sirag M. and Taha M., Spin assignment and behavior of superdeformed bands in $A=150$ mass region, *Turk J. Phys.* 37 (2013) 49.
- [14] Khalaf A.M. and Sirag M.M. Analysis of $\Delta I = 2$ Staggering in Nuclear Superdeformed Rotational Bands. *Egypt Journal of Physics*, 2004, v. 35 (2), 359–375.
- [15] Khalaf A.M. et al., Description of Rotational Bands in Superdeformed Nuclei by Using Two-parameter Empirical Formula. *Egypt Journal of Physics*, 2003, v. 34 (2), 159–177.
- [16] Khalaf A.M., Taha M.M. and Kotb M., Identical Bands and $\Delta I = 2$ Staggering in superdeformed Nuclei in $A \approx 150$ Mass Region using Three Parameters Rotational Model. *Progress in Physics*, 2012, v. 4, 39–43.
- [17] Khalaf A.M., Taha M.M. and Kotb M., Studies of Superdeformation in Gadolinium Nuclei using Three Parameters Rotational Formula. *Arab Journal of Nuclear Science and Applications* 46(2013)234
- [18] Khalaf A.M., Allam M.A., Saber E., Rotational Bands of Superdeformed Nuclei in Framework of Variable Moment of Inertia Model. *Egypt Journal of Physics*, 2006, v. 73 (3), 195.
- [19] Khalaf A.M., Allam M.A. and Sirag M.M., Bandhead Spin Determination and Moments of inertia of Superdeformed Nuclei in Mass Region 60-90 Using Variable Moment of inertia Model. *Egypt Journal of Physics*, 2010, v. 41 (2), 13–27.
- [20] Harris S. M., Higher Order Corrections to the Cranking Model, *Physical Review*, 1965, v. 138, B509-B513.
- [21] A.Bohr and B. R. Mottelson, "Nuclear Structure" Volume II Nuclear Deformation (W.A.Benjamin,INC.)1975.