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The properties of Bessel-type fractional derivatives and integrals

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ABSTRACT

The fractional integrals of Bessel-type Fractional Integrals from left-sided and right-sided integrals of fractional order is established on finite and infinite interval of the real-line, half axis and real axis. The Bessel-type fractional derivatives are also established. The properties of Fractional derivatives and integrals are studied. The fractional derivatives of Bessel-type of fractional order on finite of the real-line are studied by graphical representation. Results are direct output of the computer algebra system coded from MATLAB R2011b.

Indexing terms/Keywords

Fractional derivatives, Bessel type functions, fractional order.

Academic Discipline And Sub-Disciplines

Integral transforms, fractional calculus.

SUBJECT CLASSIFICATION

30, 33, 35

1. INTRODUCTION

In [VI], Fractional integrals were defined for $\phi(x) \in L_1(a,b)$ by

$$\left(I_{a+}^{\alpha}\phi\right)(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}\phi(t)\,dt,\ \ x>a \tag{1.1}$$

$$\left(I_{b-}^{\alpha}\phi\right)(x) = \frac{1}{\Gamma(\alpha)}\int_{-\infty}^b (t-x)^{\alpha-1}\phi(t)\,dt,\ x<b \tag{1.2}$$

where $\alpha > 0$ (α being the order). These integrals are also known as the Riemann-Liouville fractional integrals or the left - sided and right - sided fractional integrals, respectively. The integrals given in (1.1) and (1.2) are extensions to half and (or) whole axis finite interval $[a,b]$. These may be used on the half axis (a,∞) and $(-\infty,\ b),$ respectively, subject to the variable limit of integration.

The Riemann-Liouville fractional derivatives are [IV]

$$\begin{aligned}\left(D_{a+}^{\alpha}y\right)(x) &= \left(\frac{d}{dx}\right)^n \left(I_{a+}^{n-\alpha}y\right)(x) \\ &= \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n \int_a^x \frac{y(t)}{(x-t)^{\alpha-n+1}}\,dt\left(n=\left[\Re(\alpha)+1; x>a\right]\right)\end{aligned} \tag{1.3}$$

and

$$\begin{aligned}\left(D_{b-}^{\alpha}y\right)(x) &= \left(-\frac{d}{dx}\right)^n \left(I_{b-}^{n-\alpha}y\right)(x) \\ &= \frac{1}{\Gamma(n-\alpha)}\left(-\frac{d}{dx}\right)^n \int_x^b \frac{y(t)}{(t-x)^{\alpha-n+1}}\,dt\left(n=\left[\Re(\alpha)+1; x<b\right]\right)\end{aligned} \tag{1.4}$$

In [II] authors introduced the Fractional Calculus and Fractional Differential Equations. In [III], new integral transform and associated distributions were introduced.

2. BESSEL-TYPE OF FRACTIONAL ORDER FUNCTION ON FINITE OR INFINITE INTERVAL OF THE HALF-AXIS

Let $(a,b)(0 \le a < b \le \infty)$ be a finite or infinite interval of the half-axis $\mathbb{R}^+$. Considering the left-sided and right-sided integrals of fractional order $\mathrm{Re}(p)>0; \mathrm{Re}(q)>0$ defined analogous as in [I]

$$\left(I_{a+}^{p} f\right)(x)=\frac{1}{\Gamma(p)}\int_{a}^{x}\left\{(x-t)^{p-1} t^{p} J_{\nu}(t)\right\} f(t)dt. \; (a<x<b) \tag{2.1}$$

$$\left(I_{b-}^{q} f\right)(x)=\frac{1}{\Gamma(q)}\int_{x}^{b}\left\{(t-x)^{q-1} x^{q} J_{\mu}(t)\right\} f(t)dt. \; (a<x<b) \tag{2.2}$$

respectively. When $a=0$ and $b=\infty$, equation (1.1) and (1.2) are given by

$$\left(I_{0+}^{p} f\right)(x)=\frac{1}{\Gamma(p)}\int_{0}^{x}\left\{(x-t)^{p-1} t^{p} J_{\nu}(t)\right\} f(t)dt. \; (x>0) \tag{2.3}$$

$$\left(I_{0-}^{q} f\right)(x)=\frac{1}{\Gamma(q)}\int_{x}^{\infty}\left\{(t-x)^{q-1} x^{q} J_{\mu}(t)\right\} f(t)dt. \; (x>0) \tag{2.4}$$

The fractional integrals in more general than (1.3) and (1.4) can be written as

$$\left(I_{0+,\alpha}^{p} f\right)(x)=\frac{1}{\Gamma(p)}\int_{0}^{x}\left\{(x-t)^{p+\alpha-1} t^{p+\alpha} J_{\nu}(t)\right\} f(t)dt. \; (x>0) \tag{2.5}$$

$$\left(I_{0-,\beta}^{q} f\right)(x)=\frac{1}{\Gamma(q)}\int_{x}^{\infty}\left\{(t-x)^{q+\beta-1} x^{q+\beta} J_{\mu}(t)\right\} f(t)dt. \; (x>0) \tag{2.6}$$

Property 2.1 If $0<a<b<\infty$ and $\mathrm{Re}(\nu)>-1$, then from (2.3)

$$\left(I_{0+}^{p}\left((x-t)^{1-p}\right)\right)(x)=\frac{x^{\nu+1}\,{}_1F_2\left[\frac{\nu+1}{2},\left[\nu+1,\frac{\nu+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu}\Gamma(\nu+2)}. \tag{2.7}$$

$$\left(D_{0+}^{p}\left((x-t)^{1-p}\right)\right)(x)=\frac{\nu J_{\nu}(t)}{t}-J_{\nu+1}(t). \tag{2.8}$$

For $x>0$, from (2.4)

$$\left(I_{0-}^{q}(t-x)^{1-q}\right)(x)=1-\frac{x^{\mu+1}\,{}_1F_2\left[\frac{\mu+1}{2},\left[\frac{\mu+3}{2},\mu+1\right],\frac{-x^2}{4}\right]}{2^{\mu}\Gamma(\mu+2)}. \tag{2.9}$$

$$\left(D_{0-}^{q}\left((t-x)^{1-q}\right)\right)(x)=-\left[J_{\mu+1}(t)-\frac{\mu J_{\mu}(t)}{t}\right]. \tag{2.10}$$

Property 2.2 Let $\mathrm{Re}(\nu)\le -1$, then

$$\left(I_{0+}^{p}\left((x-t)^{1-p}\right)\right)(x)=\int_{0}^{x} J_{\nu}(t)dt. \tag{2.11}$$

Property 2.3 For $\left(\mathrm{Re}(\mu)\le 0 \text{ and } x>0 \text{ or } x<0\right)$ and $x\le 0$

$$\left(I_{0-}^{q}\left(t-x\right)^{1-q}\right)\left(x\right)=\int_{x}^{\infty}J_{\mu}\left(t\right)dt. \quad (2.12)$$

Property 2.4 If $0<a<b<\infty$ and $\mathrm{Re}\left(\mu\right)+\mathrm{Re}\left(p\right)>-1$, then from (1.5)

$$\left(I_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{x^{\nu+p+1}{}_{1}F_{2}\left[\frac{\nu+p+1}{2},\left[\nu+1,\frac{\nu+p+3}{2}\right],\frac{-x^{2}}{4}\right]}{2^{\nu}\left(\nu+p+1\right)\Gamma\left(p\right)\Gamma\left(\nu+1\right)}. \quad (2.13)$$

$$\left(D_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{t^{p}\left(\nu+p\right)J_{\nu}\left(t\right)}{t}-t^{p}J_{\nu+1}\left(t\right). \quad (2.14)$$

For $x>0$, from (1.6)

$$\left(I_{0-}^{q}\left(t-x\right)^{1-q}x^{q}\right)\left(x\right)=x^{q}-\frac{x^{q+\mu+1}{}_{1}F_{2}\left[\frac{\mu+1}{2},\left[\frac{\mu+3}{2},\mu+1\right],\frac{-x^{2}}{4}\right]}{2^{\mu}\Gamma\left(q\right)\Gamma\left(\mu+2\right)}. \quad (2.15)$$

$$\left(D_{0-}\left(\left(t-x\right)^{1-q}x^{q}\right)\right)\left(x\right)=-x^{p}\left[J_{\mu+1}\left(t\right)-\frac{\mu J_{\mu}\left(t\right)}{t}\right]. \quad (2.16)$$

Property 2.5 Let $\mathrm{Re}\left(\nu\right)+\mathrm{Re}\left(p\right)\le-1$, then

$$\left(I_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{1}{\Gamma\left(p\right)}\int_{0}^{x}t^{p}J_{\nu}\left(t\right)dt. \quad (2.17)$$

Property 2.6 For $\left(\mathrm{Re}\left(\nu\right)\le0 \text{ and } x>0 \text{ or } x<0\right)$ and $x\le0$

$$\left(I_{0-}^{q}\left(t-x\right)^{1-q}x^{q}\right)\left(x\right)=\frac{1}{\Gamma\left(q\right)}\int_{x}^{\infty}x^{q}J_{\mu}\left(t\right)dt. \quad (2.18)$$

Property 2.7 If $\mathrm{Re}\left(p\right)>0,\mathrm{Re}\left(q\right)>0,0<a<b<\infty;$ then

$$\begin{aligned}D_{0+}^{q}\left(D_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)&=D_{0+}^{p}\left(D_{0+}^{q}\left(\left(x-t\right)^{-q}t^{q}\right)\right)\left(x\right)\\&=t^{p+q}J_{\nu+1}\left(t\right)J_{\mu+1}\left(t\right)-t^{p+q-1}\left(\mu+2p+q-1\right)J_{\mu}\left(t\right)J_{\nu+1}\left(t\right)\\&\quad-t^{p+q-1}\left(\nu+p\right)J_{\nu}\left(t\right)J_{\mu+1}\left(t\right)\\&\quad+t^{p+q-2}J_{\mu}\left(t\right)J_{\nu}\left(t\right)\left(\mu\nu-p-\nu+\mu p+2\nu p+\nu q+pq+\nu^{2}+p^{2}-t^{2}\right).\end{aligned} \quad (2.19)$$

$$\begin{aligned}D_{0-}^{q}\left(D_{0-}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)&=D_{0-}^{p}\left(D_{0-}^{q}\left(\left(t-x\right)^{-q}t^{q}\right)\right)\left(x\right)\\&=x^{p+q}\left[\begin{array}{l}J_{\mu+1}\left(t\right)J_{\nu+1}\left(t\right)-\dfrac{\nu J_{\nu}\left(t\right)J_{\mu+1}\left(t\right)+\left(\mu-1\right)J_{\nu+1}\left(t\right)J_{\mu}\left(t\right)}{t}\\-\dfrac{\left(\nu-\nu^{2}-\nu\mu+t^{2}\right)J_{\mu}\left(t\right)J_{\nu}\left(t\right)}{t^{2}}\end{array}\right].\end{aligned} \quad (2.20)$$

If $\mathrm{Re}\left(\mu\right)+\mathrm{Re}\left(p\right)>-1$, and $x>0$ or $x<0$ then

$$I_{0+}^{q}\left(I_{0+}^{p}\left((x-t)^{1-p}t^{p}\right)\right)(x)$$
$$=\frac{1}{\Gamma(p)\Gamma(q)}\int_0^x t^p J_\nu(t)\left(\int_0^x t^q J_\mu(t)dt\right)dt+$$
$$\frac{x^{\mu+\nu+p+q+2}\,{}_1F_2\left[\frac{\nu+q+1}{2},\left[\nu+1,\frac{\nu+q+3}{2}\right],\frac{-x^2}{4}\right]{}_1F_2\left[\frac{\mu+p+1}{2},\left[\mu+1,\frac{\mu+p+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\mu+\nu}(\mu+p+1)(\nu+q+1)\Gamma(p)\Gamma(q)\Gamma(\mu+1)\Gamma(\nu+1)}. \quad (2.21)$$

Property 2.8 If $\mathrm{Re}(\mu)+\mathrm{Re}(p)\le -1,$ and $x>0$ or $x<0$ then

$$I_{0+}^{q}\left(I_{0+}^{p}\left((x-t)^{1-p}t^{p}\right)\right)(x)$$
$$=\frac{1}{\Gamma(p)\Gamma(q)}\int_0^x t^p J_\nu(t)\left(\int_0^x t^q J_\mu(t)dt\right)dt \quad (2.22)$$
$$+\int_0^x t^p J_\nu(t)x^{\mu+q+1}\frac{{}_1F_2\left[\frac{\mu+q+1}{2},\left[\mu+1,\frac{\mu+q+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\mu}(\mu+q+1)\Gamma(\mu+1)\Gamma(p)\Gamma(q)}dt.$$

Property 2.9 If $x=0$; then

$$I_{0+}^{q}\left(I_{0+}^{p}\left((x-t)^{1-p}t^{p}\right)\right)(x)=0 \quad (2.23)$$

Property 2.10. For $x>0$;

$$I_{0-}^{q}\left(I_{0-}^{p}\left((t-x)^{1-p}t^{p}\right)\right)(x)$$
$$=\frac{x^{p+q}\left[\begin{array}{l}\left\{2^{\mu}\Gamma(\mu+2)-x^{\mu+1}{}_1F_2\left[\frac{\mu+1}{2},\left[\mu+1,\frac{\mu+3}{2}\right],\frac{-x^2}{4}\right]\right\}\\ -\left\{2^{\nu}\Gamma(\nu+2)-x^{\nu+1}{}_1F_2\left[\frac{\nu+1}{2},\left[\nu+1,\frac{\nu+3}{2}\right],\frac{-x^2}{4}\right]\right\}\end{array}\right]}{2^{\mu+\nu}\Gamma(\mu+1)\Gamma(\nu+2)\Gamma(p)\Gamma(q)}. \quad (2.24)$$

Property 2.11 And for $\mathrm{Re}(\mu)>0$ and $x=0$

$$I_{0-}^{q}\left(I_{0-}^{p}\left((t-x)^{1-p}t^{p}\right)\right)(x)=0 \quad (2.25)$$

3. BESSEL-TYPE OF FRACTIONAL ORDER FUNCTION ON FINITE OR INFINITE INTERVAL OF THE REAL-AXIS

Let $[a,b]$ be a finite or infinite interval of the real-axis $\mathbb{R}$. Considering the left-sided and right-sided integrals of fractional order $\mathrm{Re}(p)>0;\mathrm{Re}(q)>0$.

$$\left(I_{a+}^{p}f\right)(x)=\frac{1}{\Gamma(p)}\int_a^x\left\{(x-t)^{p-1}t^pJ_\nu(t)\right\}f(t)dt.\ (a<x<b) \quad (3.1)$$

$$\left(I_{b-}^{q} f\right)(x)=\frac{1}{\Gamma(q)}\int_{x}^{b}\left\{(t-x)^{q-1}x^{q}J_{\mu}(t)\right\}f(t)dt.\ (a<x<b) \tag{3.2}$$

respectively. When $a=0$ and $b=1$, equation (3.1) and (3.2) are given by

$$\left(I_{0+}^{p} f\right)(x)=\frac{1}{\Gamma(p)}\int_{0}^{x}\left\{(x-t)^{p-1}t^{p}J_{\nu}(t)\right\}f(t)dt.\ (x>0) \tag{3.3}$$

$$\left(I_{0-}^{q} f\right)(x)=\frac{1}{\Gamma(q)}\int_{x}^{1}\left\{(t-x)^{q-1}t^{q}J_{\mu}(t)\right\}f(t)dt.\ (x>0) \tag{3.4}$$

The function which is considered in the study in this paper is $f=(x-a)^{p}J_{\nu}(x-a)$, where p is initially arbitrary and $J_{\nu}(x-a)$ is Bessel function of order ν, with variable $'x'$ and constant $'a'$. It is observed that p must exceed -1 for differintegration to have the properties of the operator as in [V].

Property 3.1 If $[a,b]$ and $\mathrm{Re}(\nu)>-1$, then from (3.3)

$$\left(I_{0+}^{0}\left((x-t)\right)\right)(x)=\frac{x^{\nu+1}\,{}_1F_2\left[\frac{\nu+1}{2},\left[\nu+1,\frac{\nu+3}{2}\right],\frac{-x^{2}}{4}\right]}{2^{\nu}\Gamma(\nu+2)}. \tag{3.5}$$

$$\left(D_{0+}^{0}\left((x-t)\right)\right)(x)=\frac{x^{\nu+1}(\nu+1)\,{}_1F_2\left[\frac{\nu+1}{2},\left[\nu+1,\frac{\nu+3}{2}\right],\frac{-x^{2}}{4}\right]}{2^{\nu}\Gamma(\nu+2)}-\frac{x^{\nu+2}\,{}_1F_2\left[\frac{\nu+3}{2},\left[\nu+2,\frac{\nu+5}{2}\right],\frac{-x^{2}}{4}\right]}{2^{\nu+2}(\nu+3)\Gamma(\nu+2)} \tag{3.6}$$

Property 3.2 For $x>0$, equation (3.4) gives

$$\left(I_{0-}^{0}(t-x)\right)(x)=\int_{x}^{1}J_{\mu}(t)dt. \tag{3.7}$$

$$\left(D_{0-}^{0}\left((t-x)\right)\right)(x)=-J_{\mu}(x). \tag{3.8}$$

Property 3.3 Let $\mathrm{Re}(\nu)\le -1$, then

$$\left(I_{0+}^{p}\left((x-t)^{1-p}\right)\right)(x)=\int_{0}^{x}J_{\nu}(t)dt. \tag{3.9}$$

Property 3.4 For $\left(\mathrm{Re}(\mu)\le 0 \text{ and } x>0 \ or \ x<0\right)$ and $x\le 0$

$$\left(I_{0-}^{q}(t-x)^{1-q}\right)(x)=\int_{x}^{1}J_{\mu}(t)dt. \tag{3.10}$$

Property 3.5 If $0<a<b<\infty$ and $\mathrm{Re}(\mu)+\mathrm{Re}(p)>-1$, then from (3.3)

$$\left(I_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{x^{\nu+p+1}{}_1F_2\left[\frac{\nu+p+1}{2},\left[\nu+1,\frac{\nu+p+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu}\left(\nu+p+1\right)\Gamma\left(p\right)\Gamma\left(\nu+1\right)}. \quad (3.11)$$

$$\left(D_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{x^{\nu+p}\left(\nu+1\right){}_1F_2\left[\frac{\nu+p+1}{2},\left[\nu+1,\frac{\nu+p+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu}\Gamma\left(\nu+2\right)\Gamma\left(p\right)} -\frac{x^{\nu+p+2}{}_1F_2\left[\frac{\nu+p+3}{2},\left[\nu+2,\frac{\nu+p+5}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu+1}\left(\nu+p+3\right)\Gamma\left(\nu+2\right)\Gamma\left(1-p\right)} \quad (3.12)$$

Property 3.6 For $x>0$, equation (3.4) becomes

$$\left(I_{0-}^{q}\left(t-x\right)^{1-q}x^{q}\right)\left(x\right)=\frac{1}{\Gamma\left(q\right)}\int_x^1 t^{q}J_{\mu}\left(t\right)dt. \quad (3.13)$$

$$\left(D_{0-}^{q}\left(\left(t-x\right)^{1-q}x^{q}\right)\right)\left(x\right)=-\frac{x^{q}J_{\mu}\left(x\right)}{\Gamma\left(1-q\right)}. \quad (3.14)$$

Property 3.7 Let $\mathrm{Re}\left(\nu\right)+\mathrm{Re}\left(p\right)\leq -1,$ then

$$\left(I_{0+}^{p}\left(\left(x-t\right)^{1-p}t^{p}\right)\right)\left(x\right)=\frac{1}{\Gamma\left(p\right)}\int_0^x t^{p}J_{\nu}\left(t\right)dt. \quad (3.15)$$

Property 3.8 For $\mathrm{Re}\left(\nu\right)+1\leq \mathrm{Im}\left(p\right);$

$$\left(I_{0+}^{ip}\left(\left(x-t\right)^{1-ip}t^{ip}\right)\right)\left(x\right)=\frac{1}{\Gamma\left(ip\right)}\int_0^x t^{ip}J_{\nu}\left(t\right)dt. \quad (3.16)$$

$$\left(D_{0+}^{ip}\left(\left(t-x\right)^{1-ip}t^{ip}\right)\right)\left(x\right)=-\frac{x^{ip}J_{\nu}\left(x\right)}{\Gamma\left(1-ip\right)}.$$

Property 3.9 Let $\mathrm{Re}\left(\nu\right)+1>\mathrm{Im}\left(p\right)$; then

$$\left(I_{0+}^{ip}\left(\left(x-t\right)^{1-ip}t^{ip}\right)\right)\left(x\right)=\frac{x^{\nu+ip+1}{}_1F_2\left[\frac{\nu+ip+1}{2},\left[\nu+1,\frac{\nu+ip+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu}\left(\nu+ip+1\right)\Gamma\left(ip\right)\Gamma\left(\nu+1\right)}. \quad (3.17)$$

$$\left(D_{0+}^{ip}\left(\left(x-t\right)^{1-ip}t^{ip}\right)\right)\left(x\right)=\frac{x^{\nu+ip}\left(\nu+1\right){}_1F_2\left[\frac{\nu+ip+1}{2},\left[\nu+1,\frac{\nu+ip+3}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu}\Gamma\left(\nu+2\right)\Gamma\left(ip\right)} -\frac{x^{\nu+ip+2}{}_1F_2\left[\frac{\nu+ip+3}{2},\left[\nu+2,\frac{\nu+ip+5}{2}\right],\frac{-x^2}{4}\right]}{2^{\nu+1}\left(\nu+ip+3\right)\Gamma\left(\nu+2\right)\Gamma\left(1-ip\right)} \quad (3.18)$$

$$\left(I_{0-}^{q}\left(t-x\right)^{1-q}x^{q}\right)\left(x\right)=\frac{1}{\Gamma\left(qi\right)}\int_x^1 t^{qi}J_{\mu}\left(t\right)dt. \quad (3.19)$$

$$\left(D_{0-}^{iq}\left((t-x)^{1-iq}t^{iq}\right)\right)(x)=\frac{J_\mu(x)}{\Gamma(1-qi)}. \quad (3.20)$$

4. GRAPHICAL REPRESENTATIONS OF DERIVATIVES OF BESSEL-TYPE OF FRACTIONAL ORDER FUNCTION ON FINITE OR INFINITE INTERVAL

In [VI], the authors have published Fractional Integrals and Derivatives, Theory and Applications. In [VII], authors have developed theory and applications of fractional differential equations. The graphical representation studied in this paper are from the developments of fractional integrals and derivatives of Bessel-type and may be used in developing the study in applications of fractional differential equations. The wider applications are found in the Mathematical Physics which also helps researchers for the future scope and developments from this study.

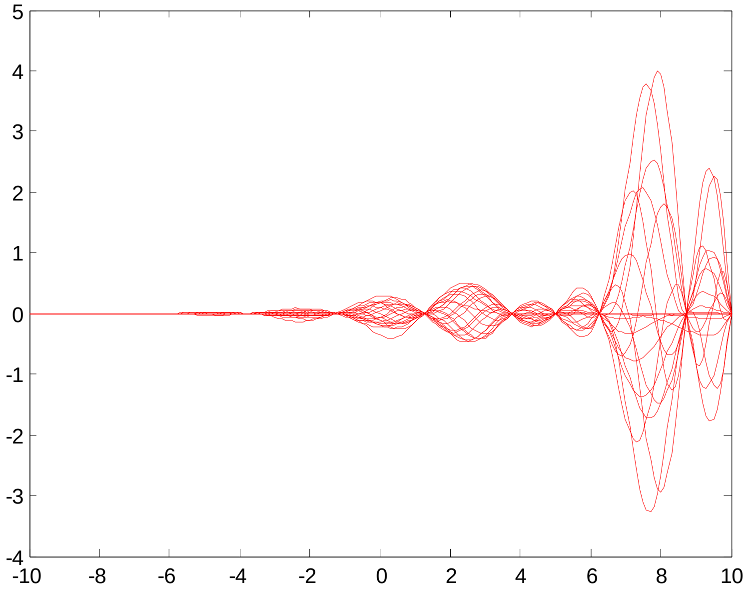

Figure 1 plot of (x,y); y= (besselj(nu, x - a).*(x - a).^p)./gamma(1 - p); nu,p<201;x=[-10,10]; for Re(nu)+ Re(p)>-1; a<25.

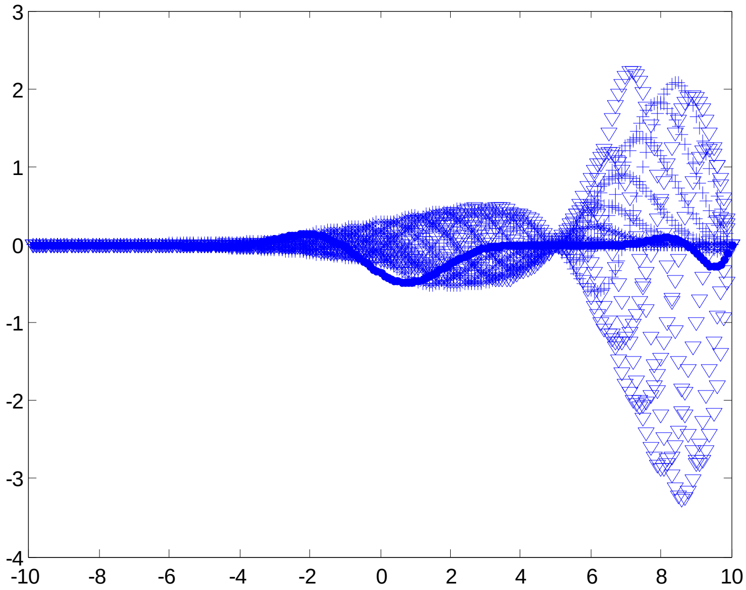

Figure 2 plot (x,y); y=- ((besselj(mu, a - x).*(a - x).^q)./gamma(1 - q)); nu,p<201;x=[-10,10];a<20.

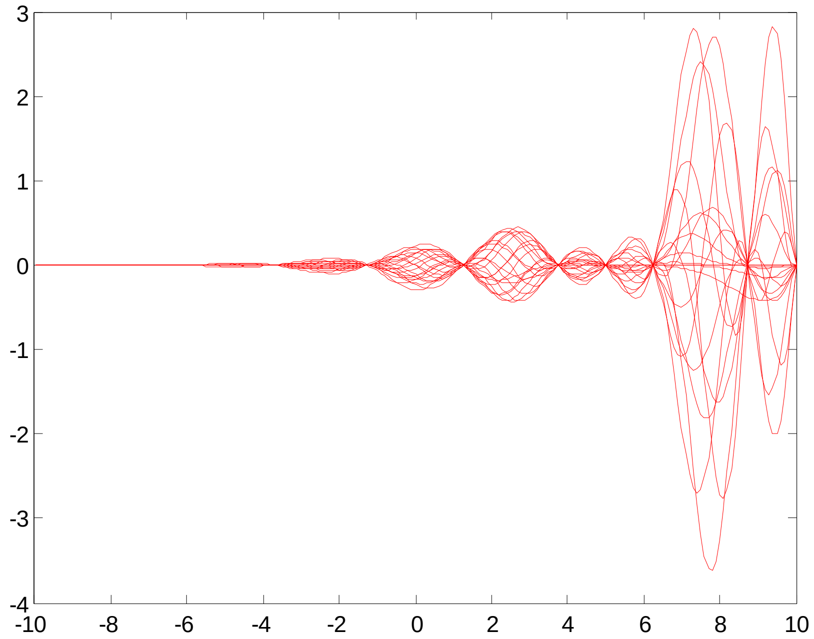

Figure 3 plot (x,y) nu,p<201;x=[-10,10]; for Re(nu)+ Re(p)>-1; a<25

In Figure 3 the derivative is obtained from (1/gamma(1-p))*diff(int((x-t)^((p-1))*((x-t)^(-p+1))*((t-a)^(p))*besselj(nu,t-a),'t',a,x),'x',2). y=- ((besselj(nu+1, x-a) + (nu.*besselj(nu, x-a))./(a-x)).*(x-a).^p - p.*besselj(nu, x-a).*(x-a).^(p-1))./gamma(1-p);

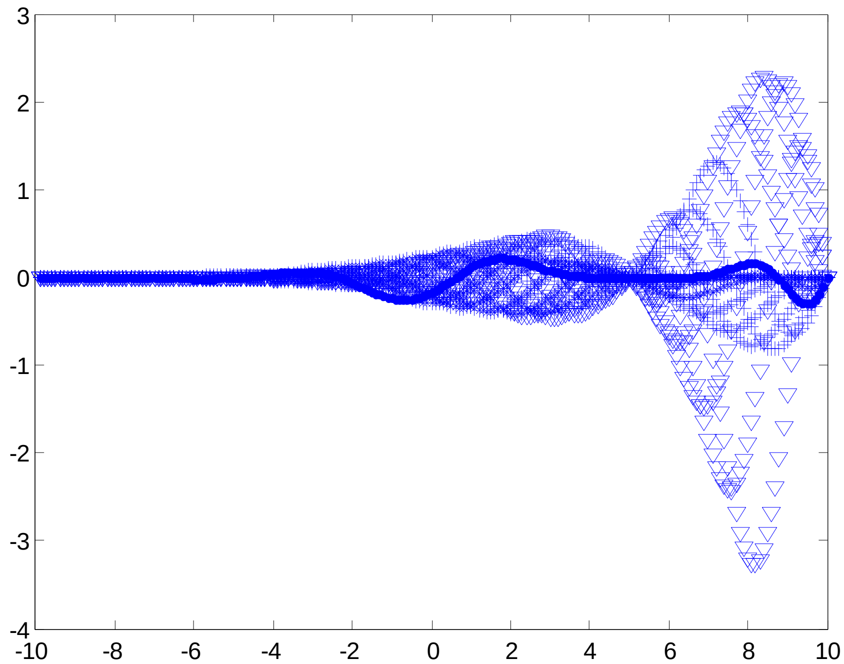

Figure 4 plot (x,y) where nu,p<201;x=[-10,10];a<20.

The derivative of Dbminus2=(1/gamma(1-q))*diff(int((t-x)^((q-1))*((t-x)^(-q+1))*((a-t)^(q))*besselj(mu,a-t),'t',x,1),'x',2) where

y=-(((besselj(mu + 1, a - x) - (mu.*besselj(mu, a - x))./(a - x)).*(a - x).^q - q.*besselj(mu, a - x).*(a - x).^(q - 1))./gamma(1 - q)) in figure 4.

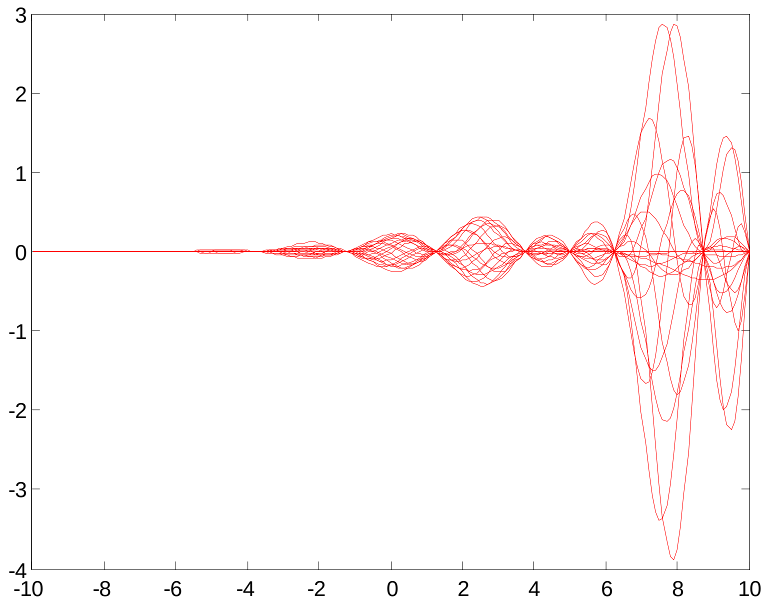

Figure 5 plot (x,y), nu,p<201;x=[-10,10]; for Re(nu)+ Re(p)>-1; a<25.

Figure 5 is obtained from the calculation of Daplus3=(1/gamma(1-p))*diff(int((x-t)^((p-1))*((x-t)^(-p+1))*((t-a)^(p))*besselj(nu,t-a),'t',a,x),'x',3) considering nu,p<201;x=[-10,10]; for Re(nu)+ Re(p)>-1; a<25.

The value of y=-(((x - a).^p.*(besselj(nu, x - a) - (nu.*(besselj(nu + 1, x - a) + (nu.*besselj(nu, x - a))./(a - x)))./(a - x) + (nu.*besselj(nu, x - a))./(a - x).^2 + (besselj(nu + 1, x - a).*(nu + 1))./(a - x)) + 2.*p.*(besselj(nu + 1, x - a) + (nu.*besselj(nu, x - a))./(a - x)).*(x - a).^(p - 1) - p.*besselj(nu, x - a).*(x - a).^(p - 2).*(p - 1))./gamma(1 - p));

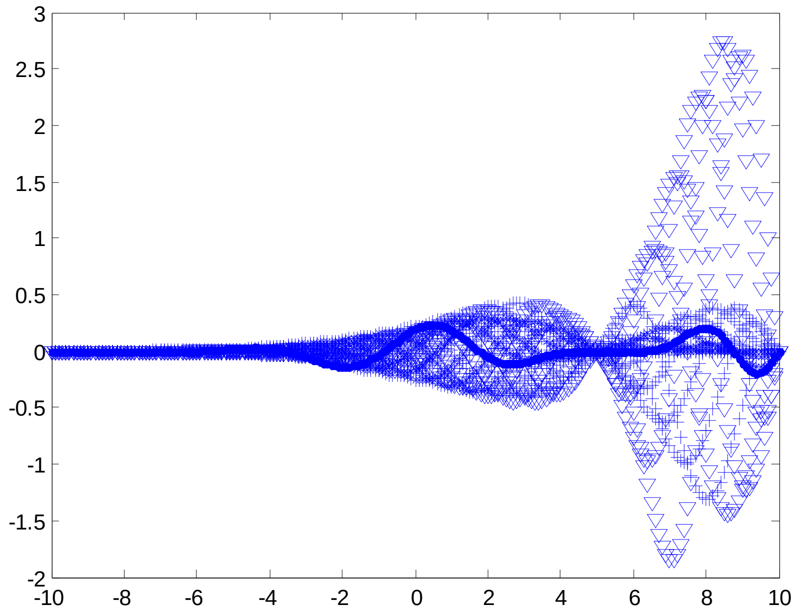

Figure 6 plot (x,y) where nu,p<201;x=[-10,10];a<20.

In Figure 6 Dbminus3=(1/gamma(1-q))*diff(int((t-x)^((q-1))*((t-x)^(-q+1))*((a-t)^(q))*besselj(mu,a-t),'t',x,1),'x',3); where

y= (((a - x).^q.*(besselj(mu, a - x) + (mu.*(besselj(mu + 1, a - x) - (mu.*besselj(mu, a - x))./(a - x)))./(a - x) + (mu.*besselj(mu, a - x))./(a - x).^2 - (besselj(mu + 1, a - x).*(mu + 1))./(a - x)) + 2.*q.*(besselj(mu + 1, a - x) - (mu.*besselj(mu, a - x))./(a - x)).*(a - x).^(q - 1) - q.*besselj(mu, a - x).*(a - x).^(q - 2).*(q - 1))./gamma(1 - q)).

5. CONCLUSION

The properties of fractional integrals of Bessel-type Fractional Integrals from left-sided and right-sided integrals of fractional order are established on finite and infinite interval of the real-line, half axis and real axis. The fractional derivatives of Bessel-type of fractional order on finite of the real-line are studied by graphical representation. Results are comprising of results from MATLAB R2011b.

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REFERENCES

I. Erdelyi, A. 1972. Fractional integrals of generalized functions, J. Australian Math. Soc. Vol. 14, pp. 30-37.

II. Miller, K. S. and Ross, Bertam 1993. An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley & Sons, Inc., New York.

III. Eltayeb, H., Kilicman, A. and Fisher, B. 2010. A new integral transform and associated distributions, Integral Transforms and Special Functions, Vol. 21, No. 5, pp. 367-379.

IV. Podlubny, Igor 1999. Fractional Differential Equations, An Introduction to Fractional Derivatives, Fractional Differential Equations and some of their Applications, Academic Press, New York, London.

V. Rubin, B. 1995. On fractional integration of generalized functions on a half line, Proc. Edinburgh Math. Soc. Vol. 38, pp. 387-396.

VI. Samko, S. G., Kilbas, A. A. and Marichev, O. I. 1993. Fractional Integrals and Derivatives, Theory and Applications, Gordon and Breach Science Publishers.

VII. Anatoly A. Kilbas, Hari M. Srivastava and Juan J. Trujillo 2006. Theory and Applications of Fractional Differential Equations, Elsevier Publishers.

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