ISSN 2347-3487
Volume 11 Number 10
Journal of Advances in Physics

The electron spin and spin magnetic moment in dual-4 d space-time Quantum mechanics

Guoqiu-zhao*
Huazhong university of science and technology-WISCO joint laboratory Wuhan 430074, China,
*Email: zhao66@126.com

Abstract

In dual-four-dimensional space-time quantum mechanics, the spin of micro object is caused by the rotation of field matter sphere itself. In its own frame of reference, the radius is defined as the static Compton momentum $R_0=\hbar/m_0c$, and frequency is $v_0=E_0/h$. In the Dynamic frame of reference, the radius is defined as the dynamic Compton momentum $R_1=\hbar/mc$, and frequency is $v_1=E_1/h$. m_0 and m is the static and dynamic mass of a micro matter sphere, respectively. The electron spin s and spin magnetic moment μ_s both can be calculated by the field matter sphere model. If the frame of reference is in the complex space-time, and we observe the motion in a Minkowshi Space, then there will be a dual four dimensional complex space-time. The fluctuation motion of field matter is De Broglie matter wave.

Keywords

field matter sphere; wave function; spin; spin magnetic moment.

1. Introduction

Spin is the most difficult quantity to be understood in Physics. And it is wrong to treat the electron spin as a mechanical rotation of a sphere. If so, the linear velocity at the edge of electron will larger than the speed of light. The electron spin angular momentum s and spin magnetic moment μ_s are the intrinsic nature of the electron itself. Spin is just an assumption with no real physical process support. Spin is "rotation" of electron and has feature of angular momentum, but can't be understood as the autorotation of electron. What is the intrinsic nature of spin?

In dual-four-dimensional space-time quantum mechanics, electron is the rotating field matter sphere. For the mass of a micro object is changeable in a dynamic reference, the radius will also change with the corresponding motion state. If the rotation frequency increases, the radius will decrease, thus the linear velocity at the edge will not superluminal, which confirms to the theory of relativity. Spin has specific physical model, specific physical concept, angular momentum quantity and physical procession. The electron spin angular momentum s and spin magnetic moment μ_s both can be calculated by the model.[1,2]

Suppose the frame of reference is in the complex space-time, and we observe the motion in a Minkowshi Space, then there will be a dual four dimensional complex space-time. The fluctuation motion of field matter is De Broglie matter wave. Matter wave is physical wave. [1,2]

2. Electron spin and spin wave function hypothesis in quantum mechanics

1) Electron spin, spin wave function assumptions and spin operator in Non-relativistic quantum mechanics

Electron spin and spin magnetic moment in quantum mechanics are the intrinsic nature of electron itself, [3] and are referred as the intrinsic angular momentum and the intrinsic magnetic moment. Their existence show electron has a degree of freedom and it is hidden in the spin wave function.

The electronic spin wave function is also called spinor wave function, denoted as $\Psi(r, s_z)$. s_z represents the projection at z axis of the spin angular momentum s. Spin up is $s_z=\hbar/2$, and spin down is $s_z= -\hbar/2$. In some cases, the spin wave function can use the method of separation of variables $\Psi(r,s_z)=\varphi(r)\chi(s_z)$, in which $\chi(s_z)$ is to describe the spin state wave function of a micro object. The form is

$$\chi(s_z)=\begin{pmatrix} a \\ b \end{pmatrix} \quad (1)$$

a and b are the two components of spin state wave function. For example, the eigenstates $\chi_{\pm 1/2}(s_z)$ of the eigenvalue $s_z=\pm\hbar/2$ are denoted as α and β

$$\alpha=\chi_{1/2}(s_z)=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\beta=\chi_{-1/2}(s_z)=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

α and β constitute a set of complete orthogonal basis in the electron spin state space. Generally, spin state $\chi(s_z)$ can be written as a superposition state

$$\chi(s_z)=\begin{pmatrix} a \\ b \end{pmatrix}=a\alpha+b\beta \quad (2)$$

Or $\psi(r,s_z)=\varphi(r)\chi(s_z)=\psi(r,\hbar/2)\alpha+\psi(r,-\hbar/2)\beta$

For the superposition state

$$\chi(s_z)=a\alpha+b\beta \quad (3)$$

the evolution is reversible, coherent, deterministic and satisfy the U process in quantum measurement and Schrodinger's equation. However, here the spin state wave function is as part of the total wave function of a micro object and is non-relativistic. [3]

Considering spin has the feature of angular momentum, suppose the three components $S_{x,y,z}$ of spin S has commutation relation with the three component of the orbital angular momentum, we will get $\mathbf{S}=(\hbar/2)\sigma$, in which σ is the Pauli operators.

For the projection of S at any direction is always $\pm\hbar/2$, so the projection of σ at any direction is ±1.

$$S_z\chi_{\hbar/2}=(\hbar/2)\chi_{\hbar/2}$$

$$S_z\chi_{-\hbar/2}=(-\hbar/2)\chi_{-\hbar/2}$$

In quantum mechanics, spin wave function of the micro object is just a kind of mathematical assumptions. Spin angular momentum S has no real physical procession correspondence. What on earth is spin? Is it simply a feature of mass point? We think not. But in non-realistic quantum mechanics, the discussion on electron spin, spin wave function and spin operator doesn't refer to own reference system. So it is difficult to further understand spin.

In dual four dimensional space-time quantum mechanics, electron is field matter sphere, spin has corresponding specific physical model, specific physical concept, angular momentum quantity and physical procession. In its own reference system, spin is easy to understand.

2) **Electron spin and spin wave function hypothesis in relativistic quantum mechanics**

a. The spin operator S_j and spin wave function B_α

To calculate the spin value of Dirac electron, we must know its spin operator. Considering the spin angular momentum **S** and the orbital angular momentum **l** of Dirac particle are not constant, there is non-commutation relationship between them [**l, H**]≠0. But the total angular momentum **j=l+S** is constant and [**j, H**]=0, we can get the equation [4]:

$$[\boldsymbol{S}, \beta mc^2]=0$$

Therefore, we can get the spin operator S and its three projection S_j (j=x,y,z)

$$\boldsymbol{S} = (\hbar/2)\,\sigma'$$

The three eigenvalue S_j (j=x,y,z) of Dirac particle S are all ±ħ/2, and

$$\sigma' = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$$

Because the spin operator S is a 4×4 metric, the number of spin wave function should be 4. Denoted as $B_\alpha(\alpha = 1,2,3,4)$, then

$$S_z B_1 = (+\hbar/2)B_1 \quad S_z B_2 = (+\hbar/2)B_2$$

$$S_z B_3 = (-\hbar/2)B_3 \quad S_z B_4 = (-\hbar/2)B_4$$

The spin wave function must be orthonormal, so

$$B_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad B_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad B_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

The spin operator S_j only has relationship with Pauli metric σ_j, and has no relationship with Coordinate and momentum in space-time. [4]

b. Further discussion on spin wave function B_α in its own reference system

We firstly calculate the solution of Dirac equation and then analysis the spin wave function in its own reference system.

Dirac equation has the following four-dimensional form:

$$(c\gamma_{\mu}\partial_{\mu}+m_0c^2)\Psi(x)=0$$

$$\boldsymbol{\gamma_{\mu}=1,2,3,4,\quad \gamma_{j}=-ic\beta\alpha_{j},\quad j=1,2,3,\quad \gamma_{4}=\beta}$$

$$\Psi(x)=\begin{pmatrix}\psi_1(x)\\ \psi_2(x)\\ \psi_3(x)\\ \psi_4(x)\end{pmatrix}$$

In Dirac equation, there are 4 γ_{μ} metrics and 16 independent 4×4 metrics. So the wave function $\Psi(x)$ is a 4×1 metric.

The spin operator S_j only has relationship with Pauli metric σ_j, and has no relationship with Coordinate and momentum in space-time. So spin is defined in its own reference system. Hamilton operator is H=βm_0c^2 and the Dirac equation should be

$$i\hbar\partial\psi/\partial t=\beta m_0c^2\psi \qquad (5)$$

For $i\hbar\partial\psi/\partial t=E\psi$, so $\beta\psi=(E/m_0c^2)\psi$.

In the own reference system: $E=\pm m_0c^2$, so we have

$$\beta\psi(x)=\pm\psi(x) \qquad (6)$$

$\psi(x)$ is the eigenfunction of β. And the eigenvalue is ± 1. β is a 4×4 metric, so the number of eigenfunction of the spinor wave function $\psi(x)$ is 4:

$$\beta\Psi^{(1)}=\Psi^{(1)}\qquad \beta\Psi^{(2)}=\Psi^{(} \qquad (7)$$

$$\beta\Psi^{(3)}=-\Psi^{(3)}\quad \beta\Psi^{(4)}=-\Psi^{(4)} \qquad (8)$$

Take equation (7) and (8) into equation (5), considering the spinor wave function $\Psi(x)$ has no relationship with coordinate, namely $\partial/\partial t\to d/dt$, we can get

$$id\psi^{(1)}/dt=m_0c^2\psi^{(1)}\qquad id\psi^{(2)}/dt=m_0c^2\psi^{(2)}$$

$$id\psi^{(3)}/dt=-m_0c^2\psi^{(3)} \quad id\psi^{(4)}/dt=-m_0c^2\psi^{(4)}$$

The solutions of the above equations is

$$\psi^{(1)}=B_1e^{-i\omega_0t_0},\ \psi^{(2)}=B_2e^{-i\omega_0t_0},\ \psi^{(3)}=B_3e^{i\omega_0t_0},\ \psi^{(4)}=B_4e^{i\omega_0t_0} \tag{9}$$

In the solution, $\omega_0=m_0c^2/\hbar$. Take equation (9) into equation (7) and (8), then we will get the eigenvalue equation for B_1, B_2, B_3, B_4:

$$\beta B_1=B_1,\ \beta B_2=B_2\ \ (E=m_0c^2)$$

$$\beta B_3=-B_3,\ \beta B_4=-B_4\ \ (E=-m_0c^2)$$

$$B_1=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\quad B_2=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\quad B_3=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\quad B_4=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

So

$$\psi^{(1)}=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}e^{-i\omega_0t_0},\ \psi^{(2)}=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}e^{-i\omega_0t_0},\ \psi^{(3)}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}e^{i\omega_0t_0},\ \psi^{(4)}=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}e^{i\omega_0t_0} \tag{10}$$

This is the microscopic spinor wave function in the micro object itself reference. Specifically, the spin state of a micro object is presented by the amplitude B_α of the spinor wave function.

There are two different parts in the microscopic spinor wave function in the micro object itself reference: one is the phase factor $e^{\pm i\omega_0t_0}$ to represent the dynamic motion, and the other one is the amplitude of the spinor value B_α which has no relationship with the motion in time and space. The later one is related to the inner degree of freedom of the micro object, so we call B_α the spinor wave function.

In the micro object its own reference system, the spinor wave function can be differed as two types: positive energy solution $\psi^{(1)}$ and $\psi^{(2)}$, and negative energy solution $\psi^{(3)}$ and $\psi^{(4)}$. However, no matter the positive energy solution or the negative energy solution, they both can decompose into two spin states: $\psi^{(1)}$ and $\psi^{(3)}$ represent the +ħ/2 state which means the projection of spin angular momentum at the z axis is +ħ/2; $\psi^{(2)}$ and $\psi^{(4)}$ represent the -ħ/2 state. Spinor amplitude B_α becomes a dual spinor four dimensional space. And it indicates the four components of spinor wave function have relationship with the four types of Dirac equation solution that have different energy signal and spin value. There should be a comprehensible quantum graph. But it is a pity that we can not see now. But in the dual four dimensional space-time quantum mechanics, combining with the positive/negative electron, positive/negative energy and negative Compton momentum, the meaning of the four components of spinor wave function seems ready to come out.

3. wave function ψ_0 , ψ and Dirac wave function B_α , ψ in dual four dimensional space-time quantum mechanics.

1) Comparison of spin wave function ψ_0 and Dirac spin wave function B_α in dual four dimensional space-time quantum mechanics

In dual four dimensional space-time quantum **mechanics**, micro object is rotating field matter sphere, spin is defined as the rotation in complex space-time. The spin wave function[1,2] is

$$\psi_0 = w = 1/z^* = ke^{i\omega_o t_o} = Ae^{ik_0 x_0}$$

Dirac positive and negative electrons correspond to two field matter spheres with opposite directions of rotation. In dual four-dimensional space-time quantum mechanics , the relativistic energy is

$$(mc^2)^2 = (Pc)^2 + (m_0 c_2)^2 \text{ or } P_I^{\ 2} = P^2 + P_0^{\ 2}$$

In the micro object its own reference system, p=0, so we have

$$P_I = \pm P_0 \tag{11}$$

This means that there are two opposite directions of rotation for a micro object in the micro object its own reference system. If one represents the negative electron $\psi_0 = ke^{ik_0 x_0}$, then the other one is the positive electron $\psi_0' = ke^{ik_0 x_0}$. If the own reference system is in complex space-time, they two will constitute four dimensional space (sum of the two rotating two dimensional space) namely a dour dimensional double spinal space. This is the physical graph of Dirac relativistic quantum mechanics. Here, we translate the positive/negative energy to be positive/negative momentum. [1,2] Momentum can be positive or negative, thus eliminating the negative energy crisis.

In dual four-dimensional space-time quantum mechanics, though the amplitude of the matter wave function and the probability wave is different, they have the same mathematic forms. [1,2] And there is no obstacle to calculate. There are two wave functions with two opposite rotation directions for a field matter sphere in complex space-time:

$$\psi_0 = w = 1/z^* = u + iv = ke^{i\omega_o t_o} = Ae^{ik_0 x_0}$$

$$\psi_0' = w = 1/z^* = u - iv = ke^{-i\omega_o t_o} = Ae^{-ik_0 x_0}$$

The spinor wave function of Dirac equation $i\hbar\partial\psi/\partial t=\beta m_0 c^2\psi$ is

$$\psi = B_\alpha e^{\pm i\omega_0 t_0} \tag{12}$$

They are the same if $k = A = B_\alpha$. So the original methods in quantum mechanics are totally valid. One is considering the corresponding vector rotation in real space-time for a mass point in real space-time, and we get a probability wave with a complex nature in Minkowshi space. The other one is considering the corresponding vector rotation in complex space-time for a field matter sphere in real space-time, then we get a matter wave with a complex nature in dual four-dimensional space-time, and $|\Psi|^2$ is the density distribution of field matter. But the later one has a clear physical graph and is easy to understand the spin. The spinor wave function has 4 corresponding components, because both negative and positive electrons have two components. Relation $k = A = B_\alpha$ gets a reasonable explanation.

Obviously, compared with the entity $R_0=\hbar/m_0c$ in the own reference system, spin of micro object represents the intrinsic nature of noumenon and has the invariance.

2) Dirac plane wave function and the dual-four-dimensional plane wave function

a. Dirac plane wave function

A Dirac particle in freedom of movement with momentum **p**, the plane wave function is [3]

$$\Psi(x)=u(\boldsymbol{p},E)e^{i(\boldsymbol{p}.\boldsymbol{x}-Et)/\hbar} \qquad (13)$$

$u(\mathbf{p}, E)$ is amplitude of the spinal wave function and $E = \pm(p^2c^2 + E_0{}^2)^{1/2}$

In the own reference system, $p = 0, E = \pm E_0 = \pm m_0c^2$

$$\psi(x) = u(\pm E_0)\, e^{\mp iE_0t/\hbar} = B_\alpha\, e^{\mp i\omega_0 t_0}$$

And this is the spinal wave function (6.10).

To calculate the specific form of the amplitude $u(\mathbf{p}, E)$, we can take the wave function (13) into the Dirac equation, so

$$(c\alpha\cdot p + \beta m_0c^2)u = Eu \qquad (14)$$

Figure out the above equation, and then set the direction **p** as the positive direction.

For $E = E_+ = +(p_z{}^2c^2 + E_0{}^2)^{1/2}$

the first solution $u_1=1,\ u_2=0,\ u_3=cp_z/(m_0c^2+E_+), u_4=0$

the second solution $u_1=0\quad u_2=1\quad u_3=0\quad u_4=-cp_z/(m_0c^2+E_+)$;

For $E = E_- = -(p_z{}^2c^2 + E_0{}^2)^{1/2}$

the first solution $u_1=-cp_z/(m_0c^2-E_-),\ u_2=0,\ u_3=1,\ u_4=0$

the second solution $u_1=0,\ u_2=cp_z/(m_0c^2-E_-),\ u_3=0,\ u_4=1$.

There are 4 linear independent solutions for the equation (14)

$$u^{(1)} = N\begin{pmatrix} 1 \\ 0 \\ cp_z/(m_0c^2+E_+) \\ 0 \end{pmatrix} \qquad u^{(2)} = N\begin{pmatrix} 0 \\ 1 \\ 0 \\ -cp_z/(m_0c^2+E_+) \end{pmatrix} \tag{15-1}$$

$$u^{(3)} = N\begin{pmatrix} -cp_z/(m_0c^2-E_-) \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u^{(4)} = N\begin{pmatrix} 0 \\ cp_z/(m_0c^2-E_-) \\ 0 \\ 1 \end{pmatrix} \tag{15-2}$$

in which N is the normalization factor. $N=[1+c^2p^2/(m_0c^2+E)^2]^{-1/2}$.

In (15-1) and (15-2), $u^{(1)}$ and $u^{(2)}$ are the eigenstate of $E=E_+$, corresponding to electron. $u^{(3)}$ and $u^{(4)}$ are the eigenstate of $E=E_-$, corresponding to positive electron. When p=0, the amplitude u of the spinor wave function will back to spin wave function B_α, namely $u_{1,2,3,4}=B_{1,2,3,4}$. In $u^{(1)}$ and $u^{(3)}$ state, the component Z of the spin angular momentum is $\hbar/2$. And in $u^{(2)}$ and $u^{(4)}$ state, the component Z of the spin angular momentum is $-\hbar/2$. It must be pointed that spin is defined in its own reference system, and it has nothing to do with the coordinate of the micro object in dynamic system.

b. The dual-four-dimensional space-time plane wave function

In dual four dimensional space-time quantum mechanics, micro object is a rotating field matter sphere. And in the curvature of the complex space, it can be represented as $w=1/z*=u+iv=ke^{i\omega t}$, in which ω is the angular frequency. [1,2]

Set the micro object its own reference system is $K_0(x_0,t_0)$, the dynamic rotation motion equation of "the inner of the static micro object" is

$$\Psi_0=A_0 exp(i\omega_0 t_0) \tag{16}$$

in which $\omega_0=m_0c^2/\hbar$. Ψ_0 is the spinor wave function defined from the micro object its own reference system in quantum mechanics. $A_0=B_\alpha$ is just the spin wave function.

In observer system, using the Lorentz transformation

$t_0=(t-vx/c^2)/(1-v^2/c^2)^{1/2}$ and insert into equation (16), we can get[1,2]

$$\begin{aligned}\psi &= A exp\{i\omega_0 (t-vx/c^2)/(1-v^2/c^2)^{1/2}\} \\ &= A exp\{i\omega(t-vx/c^2)\} \\ &= A exp\{-i(\mathbf{p}\cdot\mathbf{x}-Et)/\hbar\}\end{aligned} \tag{17}$$

in which $\omega=\omega_0/(1-v^2/c^2)^{1/2}=mc^2/\hbar$, $\omega t=Et/\hbar$, $E=mc^2$, $p=mv$, $A=u(p,E)$. For a free micro object, A is the constant normalization factor.

Equation (17) represents the fluctuation of field matter, namely the spinal wave function. This is a De Broglie matter wave and is a physical wave. The intrinsic physical nature of matter wave is the fluctuation of rotating field matter.

There are two cognition methods to understand matter wave (17):

(a) Equation (17) is the complex representation of matter wave in Minkowshi space-time. The amplitude is A＝u(p,E) and is similar to the alternating current. This method is limited by the physical meaning. And $|\Psi|^2$ is not simply the density distribution of probability, but the density distribution of the field matter. Only in the quantum measurement, can density distribution of field matter be reflected as the distribution of probability. The original mathematic methods are valid.

(b) To further understand the intrinsic physical nature of matter wave, combining with the field matter sphere model and curvature coordinates K, we find the matter wave is porpagating in dual four dimensional complex space-time W(x,k). Dual four dimensional complex space-time is an expand of Minkowshi complex space-time, and exhibits its performance in the phase factor of the wave function, [1,2] which is similar to phase integration in Feynman integration. The spinor wave function in dual four-dimensional complex space-time is

$$\psi = Aexp\{-i(\mathbf{k}\cdot\mathbf{x} - Et)/\hbar\}$$

$$c = Aexp\{-i(k_\mu x_\mu)\} \qquad (18)$$

μ＝1,2,3,4, $k_1 = mc/\hbar$, $x_1 = ct$, $k_2 = mv_2/\hbar$, $k_3 = mv_3/\hbar$, $k_4 = mv_4/\hbar$ and $A = u(x,k)$. m is the dynamic mass. Positive energy **$E=E_+$** corresponds to k＝**+**k, and negative **$E=E_-$** corresponds to k＝**-**k. They two represent two opposite directions of rotation in complex space-time. They are electron and positive electron. [1,2] The advantage of dual four dimensional space-time quantum mechanics is it can translate the energy and momentum in equation to be a variate k, and translate time to be x_1 (x_1=ct). The differential quotient of time is $\partial t = (1/c)\partial x_1$. For steady wave function, $A = u(x_i, k_i)$，i＝2,3,4, thus coming back to the discussion of steady wave function in dual four dimensional space-time quantum mechanics.

When $k_{234} = +p_{234}/\hbar = 0$，$k_1 = k_0 = m_0 c/\hbar$，$x_0 = ct_0$，equation (18) is

$$\Psi = Aexp\{-i(k_0 x_0)\}$$

$$= B_\alpha exp\{-i(k_0 x_0)\}$$

$$= B_\alpha exp\{-i(\omega_0 t_0)\} \qquad (19)$$

Equation (19) is also the spinor wave function (10) because of $A = B_\alpha$ and Ψ is the solution of $i\hbar\partial\psi/\partial t = i\hbar c\partial\psi/\partial x_1 = \mathbf{E}\psi$.

4. Calculation of electron spin in dual four dimensional space-time

1) Radius of electron in field matter sphere model

Spin radius for static electron

$$R_0 = \lambda_0/2\pi = \hbar/m_0 c \qquad (20)$$

for kinetic electron

$$R_1 = \lambda_1/2\pi = \hbar/mc \qquad (21)$$

Spin frequency for static electron

$$\nu_0 = m_0 c^2 / h \tag{22}$$

for kinetic electron

$$\nu_1 = mc^2 / h \tag{23}$$

2) Calculation of electron spin

If we treat the electron spin as electronic mechanical sphere rotation, the linear velocity at the edge of the movement of electronic is more than the speed of light. But in dual four dimensional space-time quantum mechanics, electron is not classical mechanical sphere but field matter sphere. The morphology is changeable. If the velocity is increasing, the mass will decrease and frequency will increase. According to equation (21), the radius will decrease but the linear velocity will be light speed in vacuum, thus comply with theory of relativity. Real electrons obey equation (20) and (21). In dual four-dimensional space-time, the electron spin is the rotation of electron field matter sphere.

The electron spin can be calculated as follows:

In dual four dimensional space-time quantum mechanics, R_0 is the radius of field matter sphere, the position uncertainty of the micro object is the diameter of the sphere: $\Delta x = 2R_0$. And the momentum uncertainty is the momentum of the micro object: $\Delta p = P_0$. According to the uncertainty principle[1,2] : $\Delta x \Delta P$=ħ Therefore. $R_0 P_0$=ħ/2.

$R_0 P_0 = S$, where S is the rotation angular momentum of the field matter sphere. The autorotation angular momentum of electron, namely spin value S =ħ/2 can be calculated by the field matter sphere model directly. Considering the rotation of electron or positive electron has two opposite value at any direction, the spin value should be

$S_z = R_0 P_0 = \pm ħ/2.$

It can be seen that spin is an intrinsic nature for mass point model and corresponds no physical procession. However for field matter wave sphere model, there is a comprehensible physical procession. R_0, ħ, P_0=$m_0 c$ are all constant values. So we say spin has no relationship with the dynamic reference system and is defined in its own reference system.

3) Meaning of spin in complex space-time field matter sphere model.

We have known a rotation in three dimensional correspond to two unimodular rotation [U+(-U)] in complex space. This means there is 2:1 homomorphism relation between SU(2) group and S0(3) group. And isomorphism is a one-to-one homomorphism relationship. U(1) is isomorphism with S0(2), which means the rotation (U) of a particle with spin value of 1 corresponds to a rotation (0) in group S0(2). The spin value of photons is 1, so the rotation of U(1) can be used to describe the rotation of photon's spin.

In physics, element in group SU(2) corresponds to a rotation of particle with spin value of 1/2. The 2:1 homomorphism relationship between this an rotation group S0(3) means if the rotation of an 1/2 spin corresponds to one rotation (0) in group S0(3), then the rotation transformation (-U) will also correspond to on rotation (0). [5] The positive/negative Dirac energy corresponds to positive/negative electron. We believe the rotation (U) and (-U) is just the unification of electron rotation and positive electron rotation.

It is easier to understand the electron spin in complex space-time using field matter sphere model than in real space-time using mass point model. Because of the 2:1 homomorphism relation between SU(2) group and S0(3) group. A single complete cycle of such rotation in SU(2) is 360^o, while in S0(3) is two cycles and 720^o. The electron spin value is 1/2, which is result of one rotation (U) in group SU(2). For example, clockwise rotation 180^o corresponds to opposite direction and 360^o means reset. So the spin has 1/2 as well as -1/2. The positive electron spin value is also 1/2, which means the rotation (-U) in group SU(2). For example, counterclockwise rotation 180^o corresponds to opposite direction and 360^o means reset. So the spin has ±1/2 as well. We know the "rotating field matter sphere electron" is a rotation in complex space-time. In the past, no one has differ the difference of the complex number between complex space-time and real space-time. And the space feature of SU(2) and S0(3) is blurring during the transformation, which increases the difficulty to understand spin.

Half integer of spin value is feature of fermion. Saying the rotation in spin space is just half of that in real space, this is a misunderstanding and a fuzzy conclusion without considering its antiparticle. In traditional quantum mechanics, spin is thought to be the intrinsic nature of a micro object and has no corresponding things in classical world. Now we can say this is also not the truth.

5. spin magnetic moment μ_s

1) Spin magnetic moment μ_s of electron in quantum mechanics

The Hamilton quantity of a freedom of a non-relativistic micro object usually is written as

$$H = P^2 / 2m \tag{24}$$

Considering the spin of an electron, the Hamilton quantity is

$$H = (\sigma \cdot P)^2 / 2m \tag{25}$$

in which σ the Pauli metic.

If there is no outer field, (25) will back to (6.24), and

$$(\sigma \cdot P)^2 = P^2 \tag{26}$$

It indicates that if there is no outer magnetic field, spin will not affect the motion state of a micro object, or will be negligible.

In a magnetic field $B = \nabla \times A$, the Hamilton quantity H should be

$$H = \frac{1}{2m}[\sigma \cdot (p + \frac{e}{c}A)]^2 = \frac{1}{2m}(p + \frac{e}{c}A)^2 + \frac{1}{2m} i\sigma \cdot [(p + \frac{e}{c}A) \times (p + \frac{e}{c}A)] \tag{27}$$

(27) Simplified as：

$$(ie/mc)\,\sigma \cdot (\mathbf{p}\times\mathbf{A}+\mathbf{A}\times\mathbf{p}) = (ie/2mc)\,\sigma \cdot (-i\hbar\nabla\times\mathbf{A})$$

$$= (e\hbar/2mc)\,\sigma \cdot \mathbf{B} = -\mu_s \cdot \mathbf{B} \qquad (28)$$

$$\mu_s = (e\hbar/2mc)\,\sigma = (e/mc)\, S \qquad (29)$$

In equation (28), μ_s can be understood as the magnetic moment corresponding to spin value of S, and is called as intrinsic magnetic moment. μ_s represents the interaction of intrinsic magnetic moment and outer magnetic field B. The intrinsic magnetic moment value is called Bohr magneton μ_B =eħ/2mc.

In quantum mechanics we get the value of an intrinsic magnetic moment is Bohr magneton μ_B =eħ/2mc is totally from the mathematic calculation by using the Pauli metric σ . The physical model is mass point model, and spin is the result of that a feature of rotation has something to do with an outer magnetic field B. There is no physical graph and mechanism in this interaction and is totally artificial speculation with abstract mathematical operation and recognition. Such a difficult quantum **mechanics** !

2) Calculation of spin magnetic moment in quantum mechanics

(1) Static electron:

$$\mu_s = P_{0m} = IdS = I\pi R_0^2 = em_0c^2h^2/4\pi hm_0^2c^2 = e\hbar/2m_0 \qquad (30)$$

The spin magnetic moment of electron is equal to Bohr Magnetron.

(2) Kinetic electron:

$$P_m = IdS = I\pi R_1^2 = eh/4\pi m = P_{0m}(1-v^2/c^2)^{1/2} \qquad (31)$$

in which P_{0m}==eħ/2m_0. So the kinetic electron magnetic moment will decrease as the velocity increase.

(3) Nuclear magneton

For nuclear magneton: $R_p=\lambda_p/2\pi = h/2\pi m_pc$ $v_p=m_pc^2/h$ $I=e/T_p=em_pc^2/h$

The definition of nuclear magneton is

$$P_{NM}=IdS=I\pi R_p^2=em_pc^2h^2/4\pi hm_p^2c^2=e\hbar/2m_p \qquad (32)$$

(4) Proton magneton

Set P_{NM} =eħ/2m_p. For proton magneton, we know that $m_p = 3m_u = 3m_d$,

where m_u and m_d is the mass of up quark and down quark, respectively.

The spin magnetic moment of up quark is

$$P_u=I_udS_u=I_u\pi R_u^2=(2/3)em_uc^2h^2/4\pi hm_u^2c^2$$
$$=(2/3)e\hbar/2m_u=2e\hbar/2m_p \qquad (33)$$

The spin magnetic moment of down quark is

$$P_d=I_ddS_d=I_d\pi R_d^2=-(1/3)em_dc^2h^2/4\pi hm_d^2c^2$$
$$=-(1/3)e\hbar/2m_d=-e\hbar/2m_p \qquad (34)$$

A rough calculation of the proton magneton is

μ_s $=P_p=2P_u+P_d=2(2/3)e\hbar/2m_u-(1/3)e\hbar/2m_d$

$=3e\hbar/2m_p=3P_{NM}$.

But this theory result is quite different from the experiment result.

A refined calculation of the proton magnetic moment is

$$P_p = P_u + P_u + P_d = P_{\mathrm{NM}}(\cos\alpha \ +2 \qquad (35)$$

where α is the angle between the two up quark.

When $\alpha = 37^\circ$, $P_p = 2.793P_{\mathrm{NM}}$. The results were in agreement with experimental data well.

It must be pointed that, quark cannot be directly observed. So the magnetic angle assumption has no contradiction with the fact. And it is the result after we have considered all the interactions in micro field. In spite of the inequality of the mass between the up quark and the down quark, we can adjust the angle to make sense and make it equal to the experiment results. The magnetic angle assumption is general applicable. The correspondence between the angle and spin magnetic moment in experiment is just like the correspondence between eigenstate and eigenvalue.

(5) Neutron magneton.

If the up quark is anti-parallel to the two down quarks, the neutron magnetic moment is

$$\mu_s = P_n = P_u + 2P_d = 0$$

While in fact the magnetic moment of neutron is not 0, so the above assumption is wrong and the magnetic inclination is not 180°. Adjusting the angle α

$$P_n = P_u + P_d + P_d$$

$$\mu_s = P_p = P_u + P_d + P_d = P_{\mathrm{NM}}(2\cos\alpha - 1)$$

So the neutron magnetic moment value is negative. If the mass of up and down quark is not equal, by adjusting the angle, we can make the theory value and the experiment value exactly match. We are looking forward to the experimental physicists' work.

Dual four-dimensional complex space-time offers a reasonable physical model, and clearly shows the physical graph and mechanism which are not available in traditional quantum **mechanics**, thus replenishing the cognition of foundations of quantum **mechanics**. It helps people go out from the abstract and fine mathematic calculation. But it does not reject the original methods, and all the original mathematic methods can be well used here, as well.

6. Conclusion and discussion

1) In dual four-dimensional space-time quantum **mechanics** , spin is the rotation of field matter sphere and has specific physical concept and process.

2) The spin value s and spin magneton μ_s of Electron, proton and neutron, all can be calculated by the field matter

sphere model.

3) Positive energy $E=E_+$ corresponds to k＝+k, and negative $E=E_-$ corresponds to k＝-k. They two represent two opposite directions of rotation in complex space-time. They are electron and positive electron. They two will constitute four dimensional space (sum of the two rotating two dimensional space) namely a dour dimensional double spinor space. This is the physical graph of Dirac relativistic quantum mechanics. Here, we translate the positive/negative energy to be positive/negative momentum. Momentum can be positive or negative, thus eliminating the negative energy crisis.

4) Spin value $S=R_0P_0=\hbar/2$ R_0, $\hbar$, $P_0=m_0c$ are all constant values. So we say spin has no relationship with the coordinate of the micro object in dynamic system.

References

I. G.Q.Zhao (2014) *Describe quantum mechanics in Dual-4d complex space-time and the ontological basis of wave function*, Journal of Modern Physics.Vol.5,NO.16,p1684,October 2014.

II. Guoqiu Zhao*, Mingqiang Huang（2016）,Gauge transformation in dual-four-dimensional space-time and the new internal relationship between Quantum Mechanics and Theory of Relativity，Global Journal of Physics,Vol.3,NO2, January 28,2016.www.gpcpublishing.com

III. J.Y. Zeng (1992) *An introduction to quantum mechanics* (M), Beijing: Peking University press, 1992.196 206.

IV. H.J. Li (1989) *An introduction to quantum field theory* (M), kunming: yunnan science and technology press, 1989.19, 24-86.

V. Blog of T.R. Zhang (2015)：（**1**）Gelfand,I.M.; Minlos, R.A.; Shapiro, Z.Ya. (1963), Representations of the Rotation andLorentz Groups and their Applications, New York: Pergamon Press. （**2**）R. P.Feynman: Elementary Particles and the Laws of Physics (1986 Dirac memoriallecture). （**3**）Choquet-Bruhat, Yvonne; Dewitt-Morette, Cécile;Dillard-Bleick, Margaret (1982), Analysis, manifolds and physics. Part I:Basics, North Holland, page150-190. （**4**）Wolfgang Pauli，Sources in the History of Mathematics and Physical Sciences，Volume 18 2005 ，Editors: ProfessorDr. Karl von Meyenn. （**5**）W. Pauli,"On the Connexion between the Completion of Electron Groups in an Atomwith the Complex Structure of Spectra", Z. Physik 31, 765ff, 1925.