



Void pattern fluctuation in p-AgBr interactions at 400 GeV/c

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ABSTRACT

The event-by-event fluctuation of hadronic patterns is investigated by finding a measure of the non-hadronic regions, the voids, for the experimental data of p-AgBr interactions at 400 GeV/c considering the anisotropy of phase space. Two moments of the event-to-event fluctuation of voids, $\langle G_q \rangle$ and S_q have been calculated as defined by R. C. Hwa and Q. H. Zhang to quantify the dependence of the voids on the bin sizes. The results suggest that no quark-hadron phase transition of second order have taken place for p-AgBr interactions at 400 GeV/c. The result have been compared with the result of VENUS generated data.

Indexing terms/Keywords

Relativistic Hadron-Nucleus collision, event-to event fluctuation of Voids, second order quark-hadron phase transition.

Academic Discipline And Sub-Disciplines

Physics

SUBJECT CLASSIFICATION

Particle Physics

TYPE (METHOD/APPROACH)

Experimental analysis

1. INTRODUCTION

In high energy heavy ion collision a central fireball comprising very high energy and (or) baryon density can be produced. Depending on the initial conditions, this fireball may achieve the thermal and (or) chemical equilibrium that is necessary for a phase transition from normal nuclear matter to the quark-gluon plasma (QGP) state to take place [1-3]. At a later stage, the fireball expands and cools down, fragmenting into the final-state particles (mostly pions). In many cases, rapidly fluctuating particle densities that are devoid of any apparent regular pattern are observed in the final state [4-5]. In the last few decades, the study of multiplicity distributions in limited regions (bins) has attracted high interest in view of search for local dynamical fluctuations of an underlying self-similar (fractal) structure, the so-called intermittency phenomenon [6-14]. Intermittency is referred to as the power-law dependence of the normalized factorial moments F_q on the phase space bin size [15-16].

Since the pioneering work of Bialas and Peschanski [15-16] on factorial moment much attention has been focused on the possible existence of anomalous scaling in multi-particle final states in various kinds of collisions like hadron-hadron, hadron-nucleus to nucleus-nucleus. Most of the analyses are performed in one-dimension, where the factorial moments tend to saturate at small phase space intervals. However, it is not at all sufficient for extracting the fluctuation patterns of the real three-dimensional process [17-18]. The analysis should be done in higher dimensions. Even in the usual procedure for calculating higher dimensional factorial moments, the corresponding phase-space is subsequently divided into sub-cell by shrinking equally in each dimension. This analysis is known as self-similar analysis. However, the high energy multiparticle phase space is supposed to be anisotropic [19] in nature leading to possible self-affine multiparticle fluctuation [20].

It is considered that when a system undergoing a second order PT, it exhibits large non statistical fluctuation. Such situation is not favourable for smooth hadronization in time and space [21]. Consequently it is expected to generate large fluctuation from event-to-event in the density of the produced hadrons from region to region of the geometrical space into which the hadrons are emitted. Such local hadron density fluctuation leads to the formation of spatial patterns involving clusters of hadrons and regions of no hadrons between clusters. The nonhadronic regions between the clusters are termed as voids. The voids can provide significant insight into the fluctuation phenomenon associated with the critical behaviors of quark-hadron phase transition [21].

The present paper intend to study the signature of quark-hadron second order PT in high energy Hadron-Nucleus collisions in two dimensional anisotropic phase space following the approach proposed by R. C. Hwa and Q. H. Zhang



[21-22] where fluctuation of spatial patterns is analysed with the help of an observable measure of the voids that exhibits scaling properties characteristics of any critical phenomena. Numerical values of the scaling exponents have been found to judge whether hadronization is via second order quark-hadron PT.

2. EXPERIMENTAL DETAILS

The data were obtained by exposing 400 GeV/c photon beam on Illford G5 emulsion stacks at Fermilab. The plates were scanned by using Leitz Metalloplan microscope with a 10X objective and 10X ocular lens provided with a semi-automatic scanning stage. Scanning efficiency was increase by scanning each plate by two independent observers. For measurement, 100X oil-immersion objective are used. The measuring system fitted with it has 1 μ m resolution along the X and Y axes and 0.5 μ m resolution along the Z axis. Details of events selection criteria and classification of tracks can be found in our earlier communications [23-24].

The emission angle (θ) is measured for each shower track by taking the readings of the coordinate of the interaction point (X_0, Y_0, Z_0), coordinate (X_i, Y_i, Z_i) at a point on each secondary track and coordinate (X_1, Y_1, Z_1) of a point on the incident beam. In case of shower particles the variable used is pseudorapidity (η) which is defined as $\eta = -\ln \tan (\theta/2)$. The accuracy in pseudorapidity in the region of interest is of the order of 0.1 pseudorapidity units.

Nuclear emulsion covers 4π geometry and provides very good accuracy in the measurement of emission angles of pions due to high spatial resolution and thus, is suitable as a detector for the study of fluctuations in the fine resolution of the phase space considered.

3. SELF AFFINE ANALYSIS

We have followed the procedure of self-affine factorial moment analysis, where the size of elementary phase space cell varies continuously. Considering the two-dimensional case and denoting the phase-space variables as η and φ , the factorial moments F_q of order q is defined by Bialas and Peschanski [15-16] as –

$$F_q(\delta\eta\delta\varphi) = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1)\dots(n_m-q+1) \rangle}{\langle n_m \rangle^q} \quad (1)$$

where, $\delta\eta\delta\varphi$ is the size of a two-dimensional cell, n_m is the multiplicity in the m^{th} cell. M is the number of the two-dimensional cells into which the corresponding phase space has been divided, and $\langle \rangle$ denotes averaging over all events.

To determine $\delta\eta$, $\delta\varphi$ and M we can consider to fix a two-dimensional region $\Delta\eta \Delta\varphi$ and divide it into sub-cells with widths

$$\delta\eta = \Delta\eta / M_\eta \quad (2)$$

$$\delta\varphi = \Delta\varphi / M_\varphi \quad (3)$$

in the η and φ directions, respectively. Here M_η and M_φ satisfy the equation

$$M_\eta = M_\varphi^H \quad (4)$$

where the parameter H is called the Hurst exponent [25]. It is the parameter which is characterizing the anisotropy of the system under study.

For $H = 0$, $M_\eta = 1$, the scaling property does not exist in that direction.

For $H = 1$, $M_\eta = M_\varphi$, the self affine transformation reduces to a self similar one.

From Eq. (4) it is clear that the scale factors M_η and M_φ cannot simultaneously be integers, so that the size of elementary phase space cell is able to take continuously varying values.

For performing the analysis with non-integral value of scale factor (M') one can adopt the following method. For simplicity consider one-dimensional space (y) and let

$$M' = N + a \quad (5)$$

where, N is an integer and $0 \leq a < 1$. When the elementary bins of width $\delta y = \Delta y / M'$ is used as 'scale' to 'measure' the region Δy , one can get N of them and a smaller bin of width $a\Delta y / M'$ left. Putting the smaller bin at the last (or first) place of the region and doing average with only the first (or last) N cells we get

$$F_q(\delta\eta) = \frac{1}{N} \sum_{m=1}^N \frac{\langle n_m(n_m-1)\dots(n_m-q+1) \rangle}{\langle n_m \rangle^q} \quad (6)$$

M' determined by Eq. (6) can be any positive real number and so can vary continuously.

The intermittent behavior of multiplicity distribution manifests itself as the power law dependence of factorial moment on the cell size as the cell size $\rightarrow 0$

$$\langle F_q \rangle \propto (M')^{-\alpha_q} \quad (7)$$

where scale factor $M = M_\varphi M_\eta$

The exponent α_q is the slope characterizing linear rise of $\ln \langle F_q \rangle$ with $-\ln(\delta\eta\delta\varphi)$. The strength of intermittency characterized by the exponent α_q , can be obtained from a linear fit of the form

$$\ln \langle F_q \rangle = -\alpha_q \ln(M) + A \quad (8)$$

where A is a constant.

The above scaling of F_q with non vanishing indices α_q is a confirmation for the existence of dynamic fluctuation.

It is known that the single-particle density distribution is generally non-flat. As the shape of this distribution influences the scaling behavior of factorial moments, the cumulant variables X_η and X_φ were introduced by Ochs [17-18] instead of η and φ to reduce the effect of non-flat average distribution. The cumulant variable for η and φ are defined as

$$X_\eta = \int_{\eta_{min}}^{\eta} \rho(\eta) d\eta / \int_{\eta_{min}}^{\eta_{max}} \rho(\eta) d\eta \quad (9)$$

Similar way we can get X_φ from the Eq. (9).

Both the cumulant variables vary from 0 to 1. The average distribution in terms of X_η and X_φ are uniform.

The factorial moments of different orders for different values of Hurst exponent for pions produced in p-AgBr interaction at 400 GeV/c have been calculated to probe the expected anisotropic structure of the phase space. The partition numbers in longitudinal (η) and transverse (φ) directions are chosen as $M_\varphi = 5, 6, 7, \dots, 20$ and M_η satisfy the condition stated in Eq. (4).

The two-dimensional factorial moments of different order ($q = 2, 3, 4$) for the Hurst exponent starting from $H = 0.3$ to 1.0 in steps of 0.1 have been calculated (we have not consider H less than 0.3 because at that values the analysis reduces almost to one dimension). We have plotted $\ln \langle F_q \rangle$ against $\ln M$ for different orders ($q = 2, 3, 4$) and for different Hurst exponents. In order to find the partitioning condition at which the anisotropic scaling behaviour is best revealed, the best linear fits to all the above plots have been performed. We have shown $\ln \langle F_q \rangle$ against $\ln M$ plots only for $H = 0.7$ & 1.0 in figure 1(a)-(b). The χ^2 per degrees of freedom for all the linear fits have been calculated. The values of χ^2 per degrees of freedom are tabulated in table 1. We observe that the χ^2 per degrees of freedom for each order of the linear best fit is minimum with Hurst exponent value 0.7.

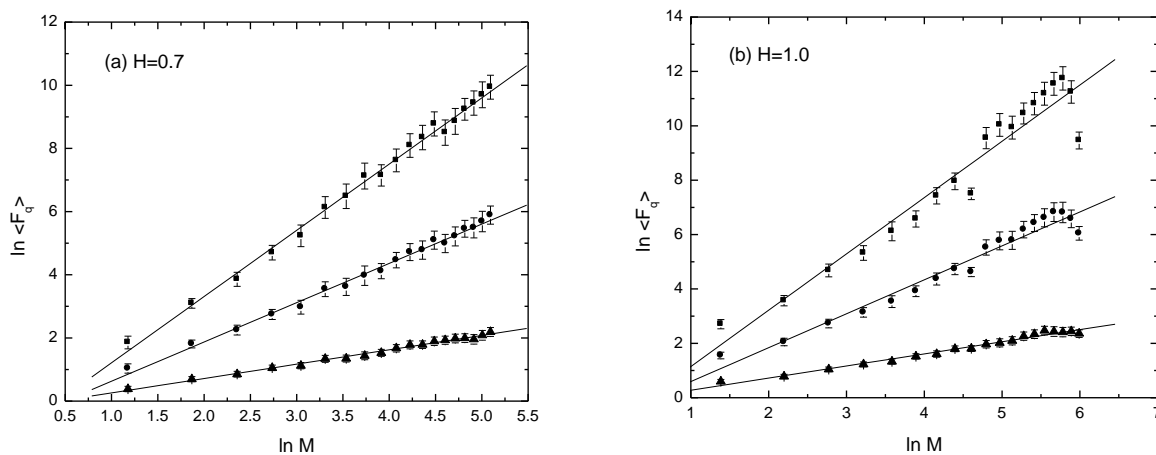


Figure 1. Two dimensional plots of $\ln \langle F_q \rangle$ vs $\ln M$ for p-AgBr interaction at 400 GeV/c for (a) $H = 0.7$, (b) $H = 1.0$

Table 1. χ^2 /DoF for linear best fits for $H = 0.3$ to 1.0 with data from p-AgBr interaction at 400 GeV/c

H	q	χ^2	χ^2/dof
0.3	2	6.46	0.4
	3	17.40	1.0
	4	26.12	1.5
0.4	2	5.56	0.3
	3	14.09	0.8
	4	20.99	1.2
0.5	2	5.86	0.3

	3	16.56	1.0
	4	21.80	1.3
0.6	2	6.43	0.4
	3	10.33	0.6
	4	13.49	0.8
0.7	2	3.05	0.2
	3	5.03	0.3
	4	5.77	0.3
0.8	2	10.89	0.6
	3	27.97	1.6
	4	54.50	3.2
0.9	2	8.28	0.5
	3	39.48	2.3
	4	77.57	4.6
1.0	2	9.44	0.6
	3	44.97	2.6
	4	114.78	6.8

4. VOID ANALYSIS

Void in multiparticle production have been defined and analysed following the proposal of R. C. Hwa and Q. H. Zhang [21-22]. As advocated by R. C. Hwa and Q. H. Zhang [21-22] the two-dimensional pseudorapidity (X_η) – azimuthal angle space (X_ϕ) is divided into M equal blocks or bins. Bins with very low hadron density are regarded as empty. This particular bin or combination of the entire bins which are connected with the empty bin with at least one side is considered as a void region. figure 2 illustrates a pattern of voids in a configuration generated for $M=5 \times 3$. An open square indicates an empty bin, while a black square contains particle. In that configuration there are 2 no of voids, the sizes of which are 1 & 4.



Figure 2. A void pattern

Let V_k be the sum of the empty bins that are connected to one another by at least one side; k simply labels a particular void. One can then define x_k to be the fraction of bins on the lattice that the k^{th} void occupies

$$x_k = V_k / M \quad (10)$$

For every event we thus have a set $S_e = \{x_1, x_2, \dots\}$ of void fractions that characterizes the spatial pattern. Since the pattern fluctuates from event-to-event, S_e cannot be used to compare patterns in an efficient way. For a good measure to facilitate the comparison, the moments g_q is define by R. C. Hwa and Q. H. Zhang [21] for each event

$$g_q = \frac{1}{m} \sum_{k=1}^m x_k^q \quad (11)$$

where the sum is over all voids in the event, and m denotes the total number of voids and q is the order. Hence the normalized G moments are define as

$$G_q = g_q / g_1^q \quad (12)$$

which depends not only on the order q , but also on the total number of bins M . Thus by definition $G_0 = G_1 = 1$. This G_q is defined in the same spirit as that in Ref. [26] for rapidity gaps, but they are not identical because the x_k here for voids do not satisfy any sum rule. Now, G_q as defined in Eq. (12) is a number for every event for chosen values of q and M . With q and M fixed, G_q fluctuates from event-to-event and it is the quantitative measure of the void patterns, which in turn is the characteristic features of phase transition.



The event-to-event fluctuation of G_q can be described by a probability distribution. While many moments of that distribution can be studied, R. C. Hwa and Q. H. Zhang [21] have proposed the two lowest moments

$$\langle G_q \rangle = \frac{1}{N} \sum_{e=1}^N G_q^{(e)} \quad (13)$$

$$\text{and } S_q = \langle G_q \ln G_q \rangle \quad (14)$$

where the superscript e denotes the e^{th} event and N is the total number of events.

If $\langle G_q \rangle$ and S_q obey power law behaviour with M as follows

$$\langle G_q \rangle \propto M^{\gamma_q} \quad (15)$$

$$S_q \propto M^{\sigma_q} \quad (16)$$

One can conclude that voids of all sizes occur. Since the moments at different q are highly correlated, one expects the scaling exponents, γ_q and σ_q to depend on q in some simple way as,

$$\gamma_q = c_0 + cq \quad (17)$$

$$\sigma_q = s_0 + sq \quad (18)$$

Thus the values c and s (which are the slopes of slopes) are concise characterizations of the fluctuation near the critical point.

Two-dimensional analysis of fluctuation of voids have been performed by dividing the whole azimuthal angle space (X_ϕ) into M_ϕ equal divisions of width $1/M_\phi$ and the whole pseudorapidity (X_η) space into M_η divisions of width $1/M_\eta$, where M_η is related to M_ϕ as $M_\eta = M_\phi^{0.7}$ [from table 1]. Thus the two-dimensional pseudorapidity (X_η) - azimuthal angle space (X_ϕ) is divided into M equal bins, where $M = M_\phi \times M_\eta$. We divide the $X\phi$ phase space region into M_ϕ bins where M_ϕ varies from 4 to 9 in steps of one.

We have calculated the number of voids using connecting bin approach following Ref. [21-22]. If one side of an empty bin is in contact with other empty bins then we cumulatively add the bins and consider it as a single void. Whereas, if an empty bin is connected by corner, we consider it as a separate void.

To confine the fluctuation of voids we have calculated the normalized G_q moments for each M by calculating void fractions x_k and the moments g_q using Eqs. (10-12). The normalized G_q moment is a number which corresponds to each event for a chosen value of q and M . With q and M fixed, G_q fluctuates from event-to-event and is a quantitative measure of the void patterns, which in turn are the characteristic features of phase transition. Figure 3(a)-(b) show the probability distribution $P(G_q)$ of second order G moment for two extreme values of M_ϕ i.e. 4 & 9 for p-AgBr interactions at 400 GeV/c. It is evident from figure 3 that G_q fluctuates from event-to-event.

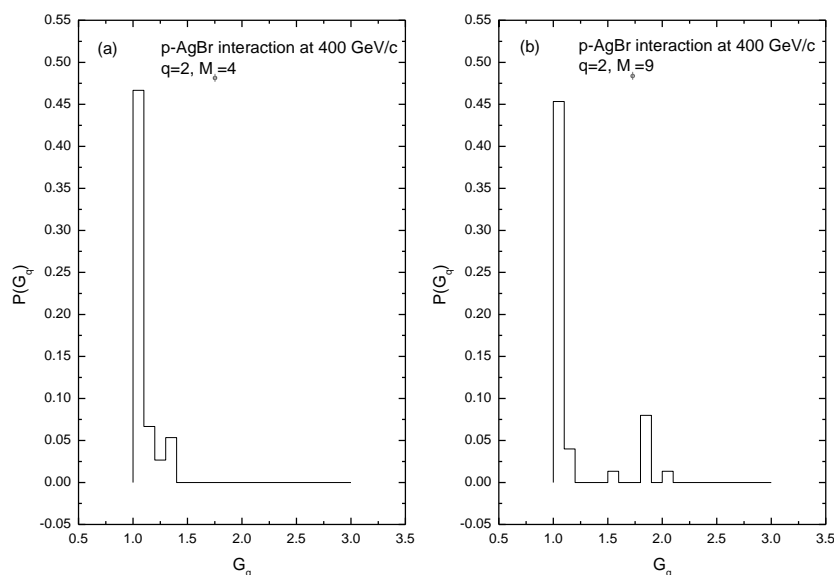


Figure 3. The probability distribution of G_q of order $q=2$ for (a) $M_\phi=4$ and (b) $M_\phi=9$ for experimental data.

To have a measure of these fluctuations the authors of Ref. [21] have suggested to study the two lowest moments $\langle G_q \rangle$ (Eq. (13)) and S_q (Eq. (14)) of $P(G_q)$. We have calculated and plotted $\ln \langle G_q \rangle$ against $\ln M$ in figure 4 for p-AgBr



interactions. Again, the variation of $\ln S_q$ with $\ln M$ have been depicted in figure 5. All the plots show a good linear behaviour suggesting that power law behaviour of the form of Eq. (15) and Eq. (16) are obeyed by the data. Such scaling behavior implies that voids of all sizes occur [21].

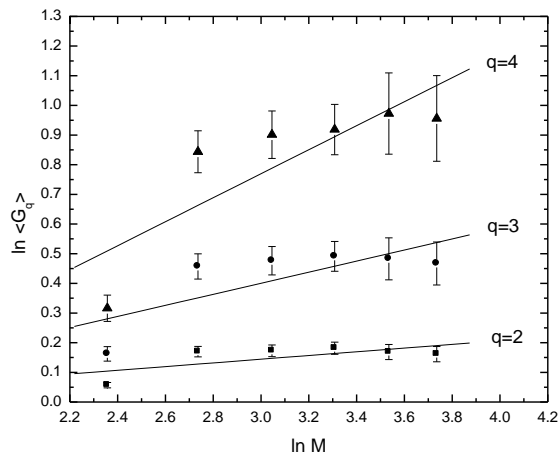


Figure 4. Dependence of the logarithm of $\langle G_q \rangle$ on the logarithm M for experimental data.

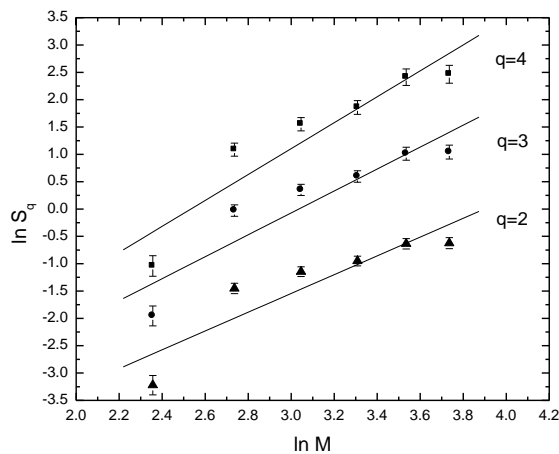


Figure 5. Dependence of the logarithm of S_q on the logarithm of M for experimental data.

We have performed the best linear fits to the plots of figures 4-5 and plotted the respective slope values against the order q in figure 6. As expected due to the correlation among G_q moments of different order the scaling exponents γ_q and σ_q are observed to show linear dependence on the order q for the interactions (Eq. (17) and Eq. (18)).

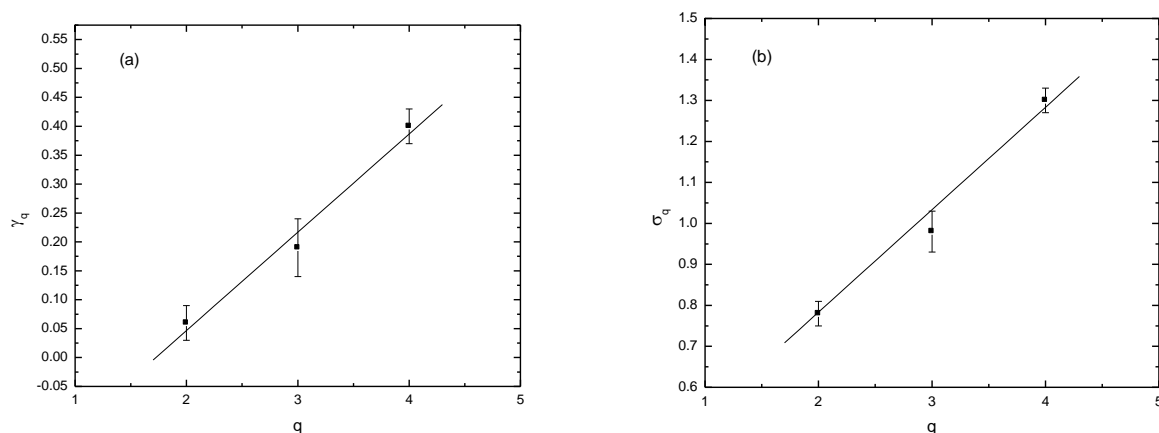


Figure 6. Dependence of (a) γ_q and (b) σ_q on q for experimental data.

The slopes of the linear fits (denoted by c and s) corresponding to the plots of figure 6 are tabulated in table 2.

Table 2. The values of c and s for experimental data and VENUS generated data.

Interactions		c	s
p-AgBr interaction at 400 GeV/c	Experimental	0.17 ± 0.02	0.25 ± 0.03
	Venus	0.40 ± 0.03	0.33 ± 0.02
^{32}S – AgBr interaction at 200 AGeV [28]	Experimental	0.56 ± 0.01	0.43 ± 0.02
	Venus	0.66 ± 0.03	0.44 ± 0.03
π^- -AgBr interaction at 350 GeV/c [28]	Experimental	0.15 ± 0.02	0.22 ± 0.02
	Venus	0.17 ± 0.01	0.16 ± 0.01

For comparison we have simulated 10000 p-AgBr collisions at 400 GeV/c using VENUS generator [27]. We have calculated the normalized G_q moments for each M by calculating void fractions x_k and the moments g_q using Eqs. (10-12). We have calculated and plotted $\ln \langle G_q \rangle$ against $\ln M$ in figure 7 and $\ln S$ against $\ln M$ in figure 8. Here also all the plots show a good linear behaviour.

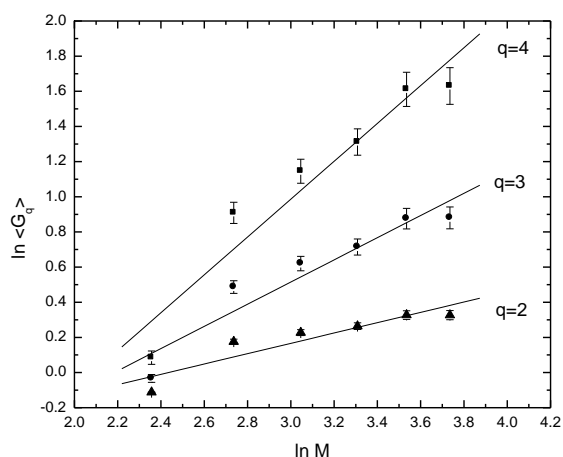


Figure 7. Dependence of the logarithm of $\langle G_q \rangle$ on the logarithm M for VENUS generated data.

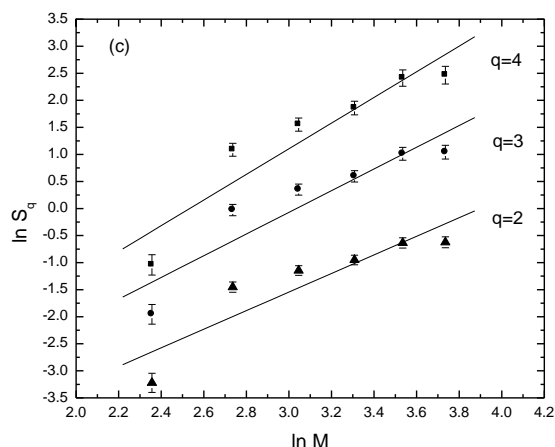


Figure 8. Dependence of the logarithm of S_q on the logarithm of M for VENUS generated data.

Again the best linear fits to the plots of figures 7-8 are performed and plotted the respective slope values against the order q in figure 9. The slopes of the linear fits corresponding to the plots of figure 9 are tabulated in table 2.

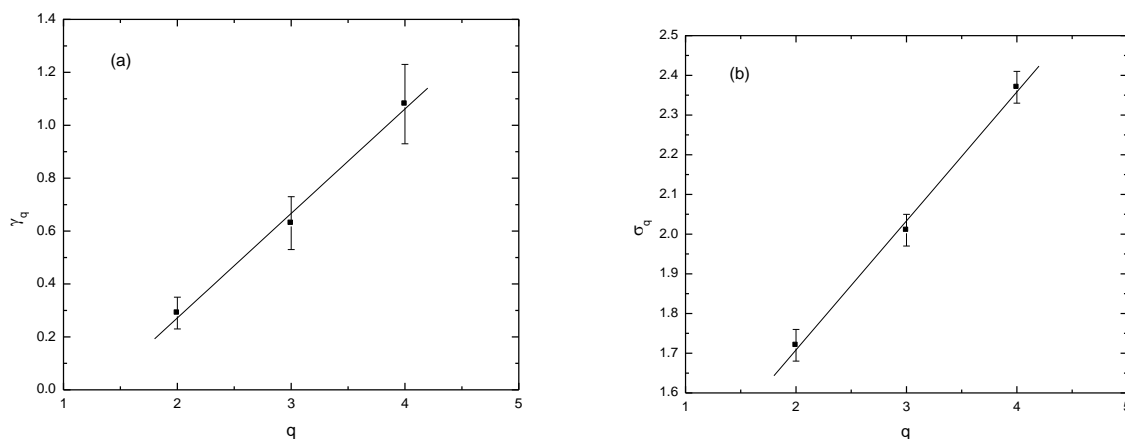


Figure 9. Dependence of (a) γ_q and (b) σ_q on q for VENUS generated data.

5. CONCLUSIONS

The phase space in p-AgBr interaction at 400 GeV/c is anisotropic in nature as table 1 shows that the scaling behaviour is best revealed for Hurst exponent $H=0.7$. It indicates that the produced pion density fluctuation pattern is self-affine in nature.

Two moments of the event-to-event fluctuation of voids, $\langle G_q \rangle$ and S_q exhibit scaling behaviour with the bin sizes which indicates that voids of all sizes occur. The scaling exponents, γ_q and σ_q depend on q in some simple way as in Eq. (17) & (18) respectively. The values of c & s for experimental data of p-AgBr interaction at 400 GeV/c are much less than the proposed critical values (the value of c ranging between 0.75 and 0.96 & the value of s ranging between 0.7 and 0.9 [22]) for exhibiting the signature of quark-hadron phase transition. Therefore, our result suggests that no quark-hadron phase transition of second order have taken place for the p-AgBr interaction at 400 GeV/c.

For comparison we have tabulated the values of c & s for the ^{32}S -AgBr interaction at 200 AGeV & π^- -AgBr interaction at 350 GeV/c [28].

It is observed from table 2 that for π^- -AgBr interactions at 350 GeV/c and p-AgBr interaction at 400 GeV/c, where quark-hadron phase transition is not likely, both c and s values are much less than the proposed critical values. On the other hand, for ^{32}S -AgBr interaction at 200 AGeV both c and s values are greater than those of the π^- -AgBr interaction at 350 GeV/c and p-AgBr interaction at 400 GeV/c but again these values are much lesser than the predicted critical values.



Further, the value of c obtained for the VENUS events are greater than the experimental values for all the interactions and the VENUS generated value of s is slightly greater for $^{32}\text{S-AgBr}$ interaction at 200 AGeV & p-AgBr interactions at 400 GeV/c and is lesser for $\pi\text{-AgBr}$ interaction at 350 GeV/c.

It may be noted that in case of NA 49 data of 400 GeV/c Pb-Pb collisions, the values of c and s are also significantly lower than the suggested value for second order quark-hadron Phase Transition to occur as mentioned in Ref. [22].

It is worthwhile to mention here that scaling behaviour is not the property of a second order phase transition alone. The scaling behaviour may also be revealed in self organised criticality [29-31]. Further research with different type of data may be useful for better understanding of such scaling properties of the voids.

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