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Conservative solute transport with periodic velocity and sinusoidal source concentration in semi-infinite porous media: An analytical solution

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Abstract

The present paper has been focused mainly towards understanding of the various parameters affecting the transport of conservative solutes in horizontally semi-infinite porous media. A model is presented for simulating one-dimensional transport of solute considering the porous medium to be homogeneous, isotropic and adsorbing nature under the influence of periodic seepage velocity. Initially the porous domain is not solute free. The solute is initially introduced from a sinusoidal point source. The transport equation is solved analytically by using Laplace Transformation Technique. Alternate as an illustration; solutions for the present problem are illustrated by numerical examples and graphs.

Keywords

Advection; Dispersion, Groundwater; Pollution; Laplace transformation.

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Introduction

Knowledge about groundwater contamination developed approximately parallel to knowledge about the occurrence and movement of groundwater. Changes in groundwater quality may result from direct or indirect anthropogenic activities. Location and delimitation or groundwater contamination is often a difficult and costly .Understanding of the transport and attenuation of pollutants in the subsurface is fundamental to effective management of risks posed by pollutants and their possible impact on groundwater resources. Groundwater flow and transport models are effective tools to assess the present pollution scenario and to predict future situations in order to prevent further contamination of subsurface water. The solute transport in surface is influenced by various physical and biogeochemical processes. Overall transport process, attenuation processes may cause movement of the pollutants to differ from that of the bulk flowing groundwater, for example dispersion, sorption and chemical or biological degradation of the chemical. The natural hydraulic conditions may also affect the behavior of some pollutants. Groundwater velocity is closely related to the hydraulic gradient, which can change temporally and spatially because of changes in the magnitudes and locations of hydraulic stresses imposed on porous domain. Thus the groundwater may, vary with time, a temporal variability in the magnitude and direction of velocity will occur.

Numerous attempts have been made to quantitatively describe the behavior of contaminants in an aquifer through mathematical models. Perhaps, Crank (1956) presented first analytical model in one-dimension for point source. Later Baetsle (1969) extended the model in three-dimension. Fang et al. (1972) simulate the tidal fluctuation of the groundwater table numerically, by using a two-dimensional finite element model. Flow was considered in a simplified domain with a vertical beach face. The cause of the spatial variations in flow velocity is normally attributed to spatial variations in hydraulic conductivity. Kumar (1983) studied with an exponential change in concentration at the source of the dispersion with unsteady one-dimensional flow. Gelhar et al. (1979) and Matheron and de'Marsily (1980) studied solute transport in stratified aquifer of infinite thickness. They calculated dispersion under the assumption that the permeability of each layer is random and the flow is in a direction parallel to the layers. Lowell (1989) illustrates the effects of time-periodic inlet contaminant concentration and groundwater flow velocity on contaminant transport in a single, planar fracture. Watson (1983) determined that solute dispersion effects along a pipe due to steady and oscillatory flow are additive. Oscillatory flow has also been found to influence dispersion in tubes. Molz et al. (1987) have suggested that it may be better to try and incorporate the spatial variations in hydraulic conductivity rather than try to represent the mixing with mechanical dispersion. Latinopoulos et al. (1988) have obtained analytical solution for chemical transport in two-dimensional aquifers.

In most theoretical models considerations are mainly focused on passive pollutants, which mean those, which have no influence on the shape of the flow velocity distribution. Das et al. (2000) have studied solute transport in porous media with first order chemical reaction for various disposal schemes by using numerical method. Goode and Konikow (1990) demonstrated that spatial variations in hydraulic conductivity are not the only cause of spatial variations in groundwater flow velocity. They concluded that dispersion also could be caused by temporal variations in flow velocity. Zi-ting (2001) reported an analytical solution for an exponential-type dispersion process. Mahesh and Babu (2002) have studied the effectiveness of sub-surface barrier on salt-water intrusion for sudden draw down conditions. Cirpka (2005) analyzed the transverse dispersion coefficient considering a spatially uniform flow field of a kinetically sorbing compound under sinusoidal temporal fluctuations. Song et al. (2007) presented a new perturbation solution of the non-linear Boussinesq equation for one-dimensional tidal groundwater flow in a coastal aquifer. Jaiswal et al. (2009, 2011), Kumar et al. (2010) obtained analytical solutions for advection–diffusion equation with variable coefficients for temporally and spatially dependent dispersion problems. Pérez et al. (2009) presented a formal exact solution of the linear advection–diffusion transport equation with constant coefficients for both transient and steady-state regimes.

Different authors have taken over this issue in different aspects or hypothesis. Solute concentrations can be obtained with a wide variety of techniques. In order to understand hydrodynamic dispersion, how groundwater flows in porous media is an essential idea. Groundwater flow varied vertically and laterally with time. Variation of solute concentration is strongly controlled by the dynamic of flow process In this paper a theoretical model has developed for the advection-dispersion problem in porous media in which the flow is one- dimensional and periodic. The analytical solution is derived for semi-infinite porous domain with appropriate initial and boundary conditions involving continuous sinusoidal function of time. The analytical solution to the governing partial differential Eq. (1), which is written in next section, can be derived by a variety of methods, including the conventional method of separation of variables, as well as integral transform methods. However, in the present study, Laplace transform techniques are employed to get the solution.

Methodology

In this study, we assume that solute transport is in horizontal direction and described by one-dimensional advection-dispersion equation. Laplace transformation technique is used to get the analytical solutions. The Laplace transformation may be defined as,

If $K(x,T)$ is a any function and $T>0$, then its Laplace transform with respect to T is denoted by $L\{K(x,T)\}=\bar{K}(x,p)$ and is defined by;

$$L\{K(x,T)\}=\bar{K}(x,p)=\int_0^\infty K(x,T)\,exp(-pT)\,dT \qquad \text{(M1)}$$

where $p>0$ is Laplace parameter.

The procedure used to invert the Laplace transform is to evaluate the closed contour and used the residue theorem. Branch point must be excluded from inside the contour and the original solution. $c(x,T)$, can be obtained by finding the solution to

$$c(x,T)=\frac{1}{2\pi i}\int_{\Gamma-i\infty}^{\Gamma+i\infty}\bar{K}(x,p)\,exp(pT)\,dp \qquad \text{(M2)}$$

or $$c(x,T)=\sum_i Res\,(i)+\frac{1}{2\pi i}\int_{\Omega}\bar{K}(x,p)\,exp(pT)\,dp \qquad \text{(M3)}$$

where *Res* (*i*) are the residue at the poles which lie to the left of the line $p=\Gamma$ and outside of contour Ω .

Mathematical Model and Solution of the Problem

The mathematical formulation and analysis starts by investigating one-dimensional advection-dispersion to find the concentration as a function of time 't' and position 'x' as the solute flow through porous media in a longitudinally semi-infinite domain. First, we wish to outline the basic mathematical components of the model that are included in the simulation. Mass conservation of conservative solutes transported in porous media is described by a partial differential equation known as advection-dispersion equation. Mathematically the advection-dispersion equation is a second order parabolic partial differential equation. When transport processes in the subsurface are to be considered, water movement has a major impact on the spreading of the solutes. The hydrodynamic processes of advection and dispersion are chief concerns in assessing the time required to renovate/rehabilitate an aquifer. Several other factors may also affect the time required to renovate/rehabilitate a contaminated aquifer. The governing equation of solute concentration can be expressed by the one-dimensional advection-dispersion equation as follows (Bear, 1972)

$$\frac{\partial c}{\partial t}+\frac{1-n_p}{n_p}\frac{\partial(K_1c+K_2)}{\partial t}=\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial c}{\partial x}-u(x,t)c\right) \qquad (1)$$

The advection-dispersion equation has served as the main theoretical framework for modeling the fate and transport of solutes in soil / porous media, and for addressing critical environmental issues stemming from agricultural practices or waste disposal operations during the last few decades (Jury and Flühler, 1992).

In Eq. (1), c is the solute concentration in the liquid phase. The dispersion coefficient, D presumably includes the effects of both molecular diffusion and mixing in the axial direction, however molecular diffusion is negligible due to very low seepage velocity. D and u may be constant or function of time or space. The dimensions of D and u are L^2T^{-1} and LT^{-1} respectively. n_p is the porosity, K_1 and K_2 are empirical constants. The term on the left side of the equal sign indicate the retardation factor and change of concentration in time, the first two terms on the right side describe hydrodynamic dispersion and groundwater velocity. If both the parameters are independent to independent variables x and t, then these are called constant dispersion and uniform flow velocity respectively. The coefficient of dispersion is considered directly proportional to seepage velocity (Yim and Mohsen, 1992), i.e.,

$$D(x,t)\propto|u(x,t)| \qquad (2)$$

Let us write $u(x,t)=u_0|cos(mt)|$, so that $D(x,t)=D_0|cos(mt)|$, where D_0, u_0 are constant. $D(x,t)=D_0$ and $u(x,t)=u_0$, when $mt=0$. But $D(x,t)=0=u(x,t)$ at $mt=\pi/2$. So Eq. (1) is valid in $t\geq 0$ domain. m is unsteady parameter whose dimension is inverse to the time variable t.

Eq. (1) becomes,

$$R\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_0|cos(mt)|\,\frac{\partial c}{\partial x}-u_0|cos(mt)|\,c\right) \qquad (3)$$

where D_0 , u_0 are constants along the respective direction and $R=(1+\frac{1-n_p}{n_p}K_1)$ is a retardation factor describing solute sorption. The retardation factor accounts of transport processes occurring both in liquid and in solid phases unlike contaminant transport and *R* is the dimension less quantity. Let us introduce a new time variable using the following transformation (Crank, 1975) as,

$$T=\int_0^t|cos(mt)|dt=\int_0^t cos(mt)dt$$

as $|cos(mt)|=cos(mt)$ in $mt\geq 0$ domain (4)

or $$mT=sin(mt) \qquad (5)$$

Therefore differential Eq. (3) reduces into constant coefficients as

$$R\frac{\partial c}{\partial T}=\frac{\partial}{\partial x}\left(D_0\,\frac{\partial c}{\partial x}-u_0c\right) \qquad (6)$$

As per hypothesis the porous domain is semi-infinite and initially not solute free. Let us assume it is exponentially decreasing function of space variable. Sinusoidal input source is considered at origin of the domain. The input concentration may not always uniform in the presence of source of pollution. It may fluctuate due to human and other natural activities. As the pollutant solute reaches the groundwater domain due to the infiltration from the earth surface, so the input concentration will fluctuate (if seepage velocity fluctuates) with time after the elimination of the source, instead of becoming zero at once. On other, end of the domain flux type boundary condition is assumed. Physically the boundary condition (9) indicate that the concentration gradient is zero, when $x \to \infty$. Mathematically, initial and boundary conditions may be written are,

$$c(x,t) = C_i \exp(-\alpha x),\ \ 0 \le x < \infty\ ,\ t = 0 \qquad (7)$$

$$-D(x,t)\frac{\partial c}{\partial x} + u(x,t)c = u_0 C_0\{1 - \sin(mt)\},\ \ x = 0\ ,\ t > 0 \qquad (8)$$

$$\frac{\partial c}{\partial x} = 0\ ,\quad x \to \infty\ ,\quad t \ge 0 \qquad (9)$$

where C_i is the resident concentration. To keep the initial concentration in feasible range α is taken less than one and its dimension is inverse of space variable. The solution of advection-dispersion solute transport for sinusoidal input condition may be written in terms of $c(x,T)$as,

$$c(x,T) = C_i H_1(x,T) + C_0 H_2(x,T) - C_0 H_3(x,T)$$

Details of mathematical calculation are given in Appendix.

Results and discussion

The parameters governing the solute transport through porous domain vary significantly depending upon the nature of any particular site of the pollutant. Thus, to illustrate the significant factors arising from the use of this formulation, consideration will be given to the hypothetical case of porous domain. Hypothetical examples are chosen to demonstrate how periodic flow and sinusoidal boundary conditions influence the concentration distribution behavior. Result have been illustated by considering the different aspect of the problem in the form of graphs. The numerical values of majority of the parameters used for model simulations presented here are taken directly from the literature or determined using existing empirical relationships. To illustrate the solution (21), defined in appendix, the common input parameters are considered as $D_0 = 1.16$ (m^2/day), $u_0 = 0.64$ (m/day), $C_0 = 1.0$ (kgm^{-3}), $C_i = 0.1$ (kgm^{-3}), $m = 0.1$ (day^{-1}) and $\alpha = 0.024$ (m^{-1}). The concentration values are evaluated in a finite longitudinal space domain, $0 \le x$(m) ≤ 10.0 of the semi-infinite medium.

Fig. (1) shows dimensionless concentration profiles (0.976, 0.454, 0.836) computed for different times t (day) = 2 ,14 ,26, retardation $R = 1.30$ and unsteady parameter $m = 0.1$. The figure shows the concentration behavior at source boundary is periodically changes with time. Thus the concentration levels are highly depend on time.

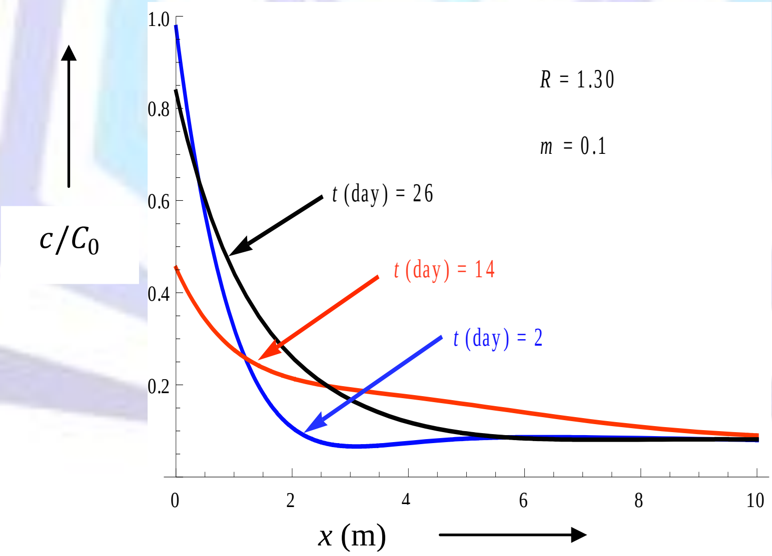

Fig. 1 Dimensionless concentration distribution for various times for solution (21).

Fig. (2) illustrates the influence of unsteady parameter on dimensionless concentration profiles (0.662, 0.454, 0.616) computed for various $m = 0.05$, 0.1, 0.15 , $R = 1.30$ and t (day) = 14. It reveals that the concentration levels vary with unsteady parameter. Figs. (1, 2) are drawn when the source pollution being entering in the domain at the boundary $x = 0$ and disperse along the longitudinal direction of the flow. Physically, the solution is expected to become periodic as t increases/decreases.

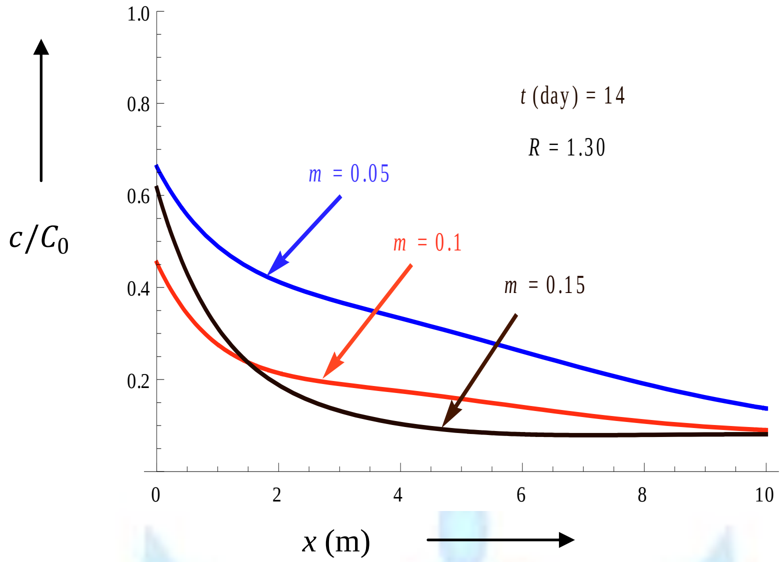

Fig. 2 Dimensionless concentration distribution for unsteady parameter for solution (21).

The concentration profile at various positions $x\,(\mathrm{m}) = 0,1,2$ is shown in Fig. (3). It is observed that the concentration value is least at time $t\,(\mathrm{day}) = 16$ and repeat this position after interval $t\,(\mathrm{day}) = 16$ for all position. The analytical solutions derived in this work can be readily employed to simulate transport of a variety of contaminants, including viruses and bacteria.

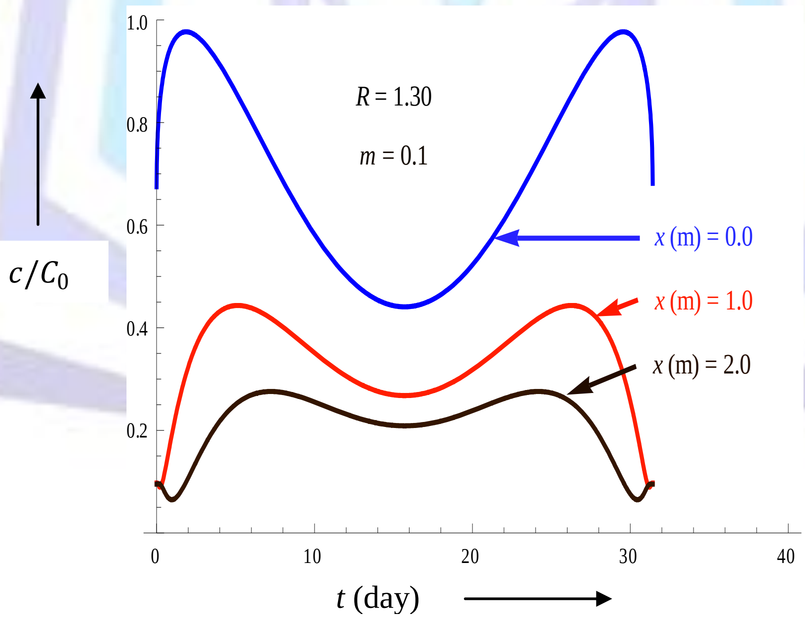

Fig. 3 Periodic concentration profile at various positions for solution (21).

Conclusion

We have focused our attention to the dispersion process of the non-reactive solute in one-dimensional semi-infinite porous domain with sinusoidal input boundary and periodic flow velocity. In the present work, a one-dimensional analytical model is developed to simulate groundwater transport of a solute undergoing advection, dispersion and retardation. Due to the effects of the boundary condition and flow velocity, the amount of solute retained decreases with time and position. The solutions are obtained for sinusoidal form of velocity which represent the seasonal pattern in tropical regions An analytical solution for this idealized scenario, which is based on the assumption that the contaminant is distributed exponentially decreasing function of position throughout the domain, can be used as a benchmark tool for analytical analyses. The governing transport equation is solved analytically by employing Laplace Transformation Technique. The development and use of mathematical models provides a better understanding of the important, biological, chemical and hydraulic process relevant to contaminant transport.

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APPENDIX

The initial and boundary value problem (6-9) may be written interms of $c(x,T)$ as,

$$R\frac{\partial c}{\partial T} = \frac{\partial}{\partial x}\left(D_0 \frac{\partial c}{\partial x} - u_0 c\right) \tag{10}$$

$$c(x,T) = C_i \exp(-\alpha x),\ 0 \le x < \infty\ ,\ T = 0 \tag{11}$$

$$-D_0 \frac{\partial c}{\partial x} + u_0 c = \frac{u_0 C_0 (1-mT)}{\sqrt{1-m^2T^2}},\ x = 0\ ,\ t > 0 \tag{12a}$$

$$\frac{\partial c}{\partial x} = 0\ ,\quad x \to \infty\ ,\ t \ge 0 \tag{12b}$$

Since, $mT < 1$, so neglecting higher order term from binomial expansions of $\left(1 - m^2T^2\right)^{-1/2}$.

Now introducing a transformation,

$$c(x,T) = K(x,T)\exp\left[\frac{u_0}{2D_0}x - \frac{u_0^2}{4RD_0}T\right] \tag{13}$$

Eqs. (10) – (12) reduced into

$$R\frac{\partial K}{\partial T} = D_0 \frac{\partial^2 K}{\partial x^2} \tag{14}$$

$$K(x,T) = C_i \exp(-\alpha x - \beta x),\quad 0 \le x < \infty,\quad T = 0 \tag{15}$$

$$-D_0 \frac{\partial K}{\partial x} + \frac{u_0}{2}K = u_0 C_0 (1 - mT)\exp\left(\gamma^2 T\right),\ x = 0\ ,\ t > 0 \tag{16a}$$

$$\frac{\partial K}{\partial x} = -\frac{u_0}{2D_0}K\ ,\ x \to \infty,\quad t \ge 0 \tag{16b}$$

where $\beta = \frac{u_0}{2D_0}$, $\gamma^2 = \frac{u_0^2}{4RD_0}$.

Applying Laplace transformation on Eqs. (14) – (16), we get,

$$Rp\bar{K} - C_i R \exp(-\alpha x - \beta x) = D_0 \frac{d^2\bar{K}}{dx^2} \tag{17}$$

$$-D_0 \frac{d\bar{K}}{dx} + \frac{u_0}{2}\bar{K} = u_0 C_0 \left[\frac{1}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2}\right],\ x = 0,\ t > 0 \tag{18a}$$

$$\frac{d\bar{K}}{dx} = -\frac{u_0}{2D_0}\bar{K},\ x \to \infty\ ,\ t \ge 0 \tag{18b}$$

where $\bar{K}(x,p) = \int_0^\infty K(x,T)\,e^{-pT}\,dT$ and p is the Laplace transformation parameter.

Thus the general solution of Eq. (17) may be written as

$$\bar{K}(X,p) = C_1 \exp(-xM) + C_2 \exp(xM) + \frac{C_i \exp(-\alpha x - \beta x)}{(p-\delta)}; \tag{19}$$

$$M = \sqrt{\frac{pR}{D_0}}\ ,\ \delta = \frac{D_0(\alpha-\beta)^2}{R}$$

Using conditions (18a, b) on the above solution, we get

$$C_1 = \frac{u_0 C_0}{\sqrt{D_0 R}(\sqrt{p}+\beta)}\left[\frac{1}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2}\right] + \frac{C_i(-\alpha-2\beta)}{\sqrt{\frac{R}{D_0}}(p-\delta)(\sqrt{p}+\beta)} \quad \text{and} \quad C_2 = 0$$

Thus the solution in the Laplace domain may be written as

$$\bar{K}(x,p) = \frac{u_0 C_0}{\sqrt{D_0 R}(\sqrt{p}+\beta)}\left[\frac{1}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2}\right]\exp(-xM)$$

$$+\frac{C_i(-\alpha-2\beta)exp\ (-xM)}{\sqrt{\frac{R}{D_0}}(p-\delta)(\sqrt{p}+\beta)}+\frac{C_i\ exp\ (-\alpha x-\beta x)}{(p-\delta)} \quad (20)$$

Taking inverse Laplace transform of Eq. (20) and using the table of van Genuchten and Alves (1982), the solution of advection-dispersion solute transport for sinusoidal input condition in terms of $c(x,T)$, as

$$c(x,T)=C_i H_1(x,T)+C_0H_2(x,T)-C_0H_3(x,T) \quad (21)$$

Where

$$H_1(x,T)=\frac{(-\alpha-2\beta)}{\sqrt{\frac{R}{D_0}}}\left[\frac{1}{2(\sqrt{\delta}+\beta)}exp\ (-\alpha x+\delta T-\gamma^2T)\ erfc\ \left\{\frac{Rx+2D_0(-\alpha-\beta)T}{2\sqrt{D_0RT}}\right\}\right.$$

$$\frac{1}{2(\sqrt{\delta}-\beta)}exp\ (2\beta x+\alpha x+\delta T-\gamma^2T)erfc\ \left\{\frac{Rx-2D_0(-\alpha-\beta)T}{2\sqrt{D_0RT}}\right\}$$

$$\left.-\frac{\beta}{(\delta-\beta)}exp\ \left(\frac{u_0}{D_0}x\right)erfc\ \left\{\frac{Rx-u_0T}{2\sqrt{D_0RT}}\right\}\right]+exp\ (-\alpha x+\delta T-\gamma^2T),$$

$$H_2(x,T)=\frac{u_0}{\sqrt{D_0R}}\left[\sqrt{\frac{T}{\pi}}exp\ \left\{-\frac{x^2R}{4D_0T}\right\}exp\ (\beta x-\gamma^2T)+\frac{D_0}{2u_0}erfc\ \left\{\frac{Rx-u_0T}{2\sqrt{D_0RT}}\right\}\right.$$

$$\left.-\frac{D_0}{2u_0}\left(1+\frac{u_0}{D_0}x+4\gamma^2T\right)exp\ \left(\frac{u_0}{D_0}x\right)erfc\ \left\{\frac{Rx-u_0T}{2\sqrt{D_0RT}}\right\}\right],$$

$$H_3(x,T)=\frac{u_0m}{\sqrt{D_0R}}\left[\frac{1}{4\gamma^2}\sqrt{\frac{T}{\pi}}\left(1+\frac{u_0}{2D_0}x+2\gamma^2T\right)exp\ \left\{-\frac{x^2R}{4D_0T}\right\}exp\ (\beta x-\gamma^2T)\right.$$

$$+\frac{1}{4\gamma^{3/2}}\left(4\gamma^2T-1-\frac{u_0}{D_0}x\right)erfc\ \left\{\frac{Rx-u_0T}{2\sqrt{D_0RT}}\right\}$$

$$\left.-\frac{1}{4\gamma^{3/2}}\left\{4\gamma^2T-1+2\gamma^2\left(x\sqrt{\frac{R}{D_0}}+2\beta T\right)^2\right\}exp\ \left(\frac{u_0}{D_0}x\right)erfc\ \left\{\frac{Rx+u_0T}{2\sqrt{D_0RT}}\right\}\right],$$

$$\delta=\frac{D_0(-\alpha-\beta)^2}{R},\ \beta=\frac{u_0}{2D_0},\ \gamma^2=\frac{u_0^2}{4RD_0}\ \text{and}\ T=\frac{sin\ (mt)}{m}.$$

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