

Action Flow in Obsessive-Compulsive Disorder Rituals: a model based on Extended Synergetics and a Comment on the 4th Law

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ABSTRACT

The flow of actions in rituals of obsessive individuals is discussed from a nonlinear physics perspective. An amplitude equation model based on extended synergetics is studied. The amplitude dynamics describes both the behavioral actions and the experienced emotions during obsessive-compulsive disorder rituals. The model suggests that in addition to the behavioral and emotional levels that are accessible to external observation and self-reports there are hidden levels captured by parameter dynamics that determine the action flow in obsessive-compulsive disorder rituals. The model can also be used to discuss on the mechanistic and behavioral levels differences between purposeful and purposeless rituals. While purposeful rituals involve a continuous control of the emotional level over the behavioral level, purposeless rituals do not exhibit such a continuous control mechanism. Finally, it is argued that the selection principle determining the action flow in obsessive-compulsive disorder rituals is consistent with the so-called 4th law of non-equilibrium phase transitions in animate and inanimate systems.

Indexing terms/Keywords

Lotka-Volterra model, synergetics, dynamical diseases, obsessive-compulsive disorder

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INTRODUCTION

The concepts of nonlinear physics in general and in particular the theory of nonlinear dynamical systems featuring bifurcations similar to equilibrium phase transitions has been a useful tool to describe phenomena in the life sciences and the animate world in general [1,2]. It has recently been suggested to apply the concepts of nonlinear physics to obtain new insights in the behavior and treatment of patients with obsessive compulsive disorder (OCD) [3-5]. In particular, a dynamical model based on synergetics [6] supplemented with a parameter dynamics has been sketched [5]. However, this model based on extended synergetics has not been worked out in detail and the implications of the model have not been discussed.

OCD patients suffer from obsessive thoughts that are undesired and irrational [7]. For example, a female OCD patient and mother of a child may have repeatedly the thought to hurt her child. To cope with obsessive thoughts of this kind, OCD patients perform rituals, that is, they perform certain sequences of actions. For example, washing rituals are used to cope with obsessive thoughts about contamination and checking rituals are used to cope with thoughts about hurting others [8]. OCD patients are often aware that their thoughts are irrational and that their actions are unreasonable. Nevertheless they can not stop themselves to perform those rituals. A possible reason why these rituals are preformed is that in fact they can help to reduce the feelings of distress, discomfort, and anxiety that are triggered by the obsessive thoughts [9-11].

There is some experimental evidence that the emotional distress states and the behavioral response patterns of OCD patients correspond to certain neurophysiological patterns of brain activity [12-14]. We will consider these patterns as patterns emerging in a self-organizing system via a bifurcation. That is, we will consider the emergence of the disease pattern as a pattern formation phenomenon in a non-equilibrium physical system similar to the formation of a convection roll pattern in a fluid layer that is heated from below (Benar instability). We will describe the emergence of the patterns on the level of their amplitudes. That is, each pattern is assumed to have an amplitude that codes the presence or absence of that pattern [4,5].

As mentioned above, recently, a nonlinear dynamical model has been sketched based on extended synergetics that describes the amplitude dynamics of the emotional and behavioral patterns involved in rituals of OCD patients [5]. However, since the focus of the study was on secondary bifurcations in general, the OCD patient dynamics model has not been worked out and implications of the model have not been addressed. These issues will be covered in the sections below. Moreover, a comment on the relevance of the 4th law of action selection for understanding the behavioral and emotional dynamics of OCD rituals will be given below as well.

MODEL

Lotka-Volterra-Haken model

Let us consider the behavior of the aforementioned mother suffering from obsessive thoughts about doing harm to her child. To this end, we consider a checking ritual composed of three actions: checking water taps, checking stove and other kitchen gadgets (e.g. electric coffee maker), checking doors and windows. We assign to the three ritual-related behavioral patterns three amplitudes that evolve in time. In addition, we introduce a catch-all class of behaviors that are not disease related. More precisely, we imagine a coarse-grained pattern representing all those non-disease related patterns and assign a fourth time-dependent amplitude to that pattern. The dynamics of the amplitudes 1,2,3,4 is given by [5]

$$\frac{d}{dt}A_k(t) = A_k \cdot \left(\lambda_k - A_k - g_A \sum_{m \neq k} A_m\right) \tag{1}$$

The amplitudes are positive or zero. The lambda parameters occurring in Equation (1) are growth parameters that are assumed to be positive. They describe the exponential increase of the amplitudes provided all amplitudes are close to zero. The g-parameter is larger than unity and describes the competitive interaction between the behavioral modes. The amplitude equation formally corresponds to a Lotka-Volterra model of four competitive species [15]. Using a variable transformation [16], the model becomes identical to the pattern formation model suggested by Haken (see e.g. [20]) that involves cubic nonlinearities rather than quadratic ones. Therefore, we refer to the model defined by Equation (1) as Lotka-Volterra-Haken model. A linear stability analysis shows that there are winner-takes-all fixed points that describe the four aforementioned behavioral patterns. These fixed points are characterized by the fact that only one of the amplitudes is finite, while all others are equal to zero. More precisely, for the fixed point of the kth behavioral pattern we have

$$A_{st,k} = \lambda_k \qquad \wedge \quad A_{st,j \neq k} = 0 \tag{2}$$

The fixed point is the only stable fixed point if the growth parameter of the kth behavioral pattern is the only growth parameter that is in the so-called "stability band" [5,16-18] such that

$$\forall j \neq k : \lambda_j < \frac{\lambda_k}{g_A} \tag{3}$$



Consequently, if the growth parameter of the kth behavioral mode is increased gradually, while all other growth parameters are fixed, then at a certain critical value of the kth growth parameter a bifurcation happens and the Lotka-Volterra-Haken model will bifurcate towards the fixed point describing the kth behavioral pattern.

In addition to the behavioral dynamics, we consider the emotional dynamics. First of all there is the distress feeling that comes with the obsessive thought (here the thought about hurting the child). Second, we introduce a class of emotions that are not disease related. Consequently, we defined two positive-valued amplitude variables. The first amplitude describes the magnitude of the OCD-related distress feeling. The second amplitude describes all kinds of emotions not related to the disease. The amplitude dynamics on the emotional level is assumed to satisfy again a Lotka-Volterra-Haken model. That is, for the emotional amplitudes 1 and 2 we have

$$\frac{d}{dt}E_k(t) = E_k \cdot \left(L_k - E_k - g_E \sum_{m \neq k} E_m\right) \tag{4}$$

The L-parameters are positive and describe exponential growth parameters again. The g-parameter is the interaction parameter and is assumed to be larger than unity. Again, there are winner-takes-all fixed points given by

$$E_{st,k} = L_k \qquad \wedge \quad E_{st,j \neq k} = 0 \tag{5}$$

The kth fixed point is monostable if

$$L_{j} < \frac{L_{k}}{g_{E}} \tag{6}$$

Extended synergetics

While the amplitude equations of the synergetic model or the Lotka-Volterra-Haken model can describe pattern formation in self-organized systems when control parameters are changed externally (e.g. by the experimenter), they do not account for the self-regulation of system parameters. That is, in the life sciences, system parameters of a self-organized state may change due to the fact that the system under consideration is in that self-organized state. We are dealing with parameter dynamics [19]. The synergetic model has to be generalized. This has been done for example to describe the oscillatory perception of ambivalent figures [20] and to address the phenomenon of negative hysteresis [21]. In line with this idea of a self-regulated parameter dynamics, the Lotka-Volterra-Haken equations for the behavioral dynamics are supplemented by the evolution equation for the growth parameters 1,2,3. These evolution equations can be written in a compact form like [5]

$$\frac{d}{dt}\lambda_k(t) = S \cdot \gamma_k(A_{k-1}) - \beta_k(A_k) \cdot \lambda_k \tag{7}$$

The lambda parameter of the catch-all non-disease related behavioral mode (k=4) is assumed to be constant. The gamma parameters are excitation parameters, whereas the beta parameters are damping parameters. The gamma parameter of the first step of the ritual equals zero. If the first (second) step of the ritual is performed then the excitation parameter gamma of the second (third) step assumes a finite value. Otherwise, there is no excitation of that ritual step.

$$\gamma_{k+1}(A_k) = \begin{cases} \gamma(on) > 0 & A_k & "on" \\ 0 & otherwise \end{cases}$$
(8)

All three ritual steps a self-inhibitory. Accordingly, if a step 1,2, or 3 is performed then the corresponding beta parameter becomes finite like:

$$\beta_{k}(A_{k}) = \begin{cases} \beta(on) > 0 & A_{k} & "on" \\ 0 & otherwise \end{cases}$$
(9)

Note that the S-parameter has been denoted as alpha parameter in previous work [5]. Below, we will use the S-parameter to discuss predictions that can be drawn from the extended synergetics model. As far as the emotional dynamics is concerned, we consider cross-inhibition of the distress emotion by means of the ritual activities. That is, while the ritual is performed the magnitude of the distress emotion decays over time. Consequently, we put





$$\frac{d}{dt}L_1(t) = -B \cdot L_1 \tag{10}$$

with

$$B = \begin{cases} B(on) & A_1 \lor A_2 \lor A_3 & "on" \\ 0 & otherwise \end{cases}$$
 (11)

The L-parameter of the second emotional (catch-all) feeling is a positive constant (i.e., does not vary over time). In the simulations discussed below, an amplitude was considered to be on, when it reached a certain relative and absolute threshold. The relative threshold was defined as 95 percent of the stationary value. The absolute threshold was defined as 0.7. That is, if an amplitude reached at least 95 percent of its stationary (winner-takes-all) value and was larger than 0.7 then the amplitude was considered as being "on". The interpretation of a behavioral amplitude that is "on" is that in this case the computer-simulated OCD patient conducts the corresponding behavioral activity. Likewise, if the emotional amplitude of the distress emotion is "on" the OCD patient is pushed strong enough to take some actions, namely, to initiate the ritual (see below).

RESULTS

In what follows the results of three computer experiments will be presented. In the first experiment only the behavioral dynamics was considered. That is, the evolution equations for the amplitudes of the four behavioral models were solved numerically (Euler forward method with single time step of 0.001), see Figure 1.

At an arbitrary point in time (here at 5 time units) the obsessive thought was experienced and in doing so initiated the ritual. Mathematically speaking, the growth parameter of the first activity (checking the water taps) of the ritual was put to a sufficiently high value

$$\lambda_{_{\mathrm{I}}}(t^*) = D_{_{A}} \tag{12}$$

such that Equation (3) was satisfied and the behavior dynamics became monostable, see Figure 1C (top row). That is, the fixed point representing the not disease-related activities became an unstable fixed point and the fixed point of the first activity of the OCD ritual was the only stable fixed point. As a result, the amplitude of the non-disease related behavioral mode decayed to zero and the amplitude of the water-tap-checking mode became finite, see rows 1 and 4 in Figures 1A and 1B. Note that during this period the growth parameters of the second and third behavioral modes were relatively low. According to the model, the woman was checking the water taps for a certain period.

At this stage, the beta parameter of the active behavioral mode (water-tap-checking mode) was put on the finite positive value defined by Equation (9). In addition, the excitation parameter (gamma parameter) of the behavior of the next step in the ritual (stove-checking mode) was put on the finite value defined by Equation (8). As a consequence, the growth parameter of the active mode (water-tap-checking mode) started to decay monotonically and the growth parameter (lambda parameter) of the follow-up activity (stove-checking) increased monotonically, see Figure 1C (rows 1 and 2). Figure 1A (top row) demonstrates that the fixed point representing the first step of the ritual exhibited a drift in time. The amplitude decayed.



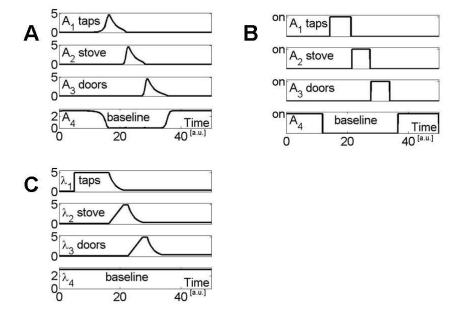


Fig 1: Trajectories of the behavioral dynamics obtained in the first computer experiment. Panels A, B, and C depict the amplitudes as real-valued variables, the amplitudes as categorical variables that are either "on" or not, and the growth parameters of the four modes 1,2,3,4 as functions of time. Time is measured in arbitrary units. See text for details.

At a certain point in time, Equation (3) was satisfied for the second behavioral mode. That is, the behavioral mode 1 became unstable. The behavioral mode 2 was stable and became the only stable fixed point. A bifurcation from the first ritual-related activity to the second ritual-related activity was observed, see rows 1 and 2 in Figures 1A and 1B Accordingly, the computer-simulated female OCD patient engaged herself in checking the stove and other kitchen gadgets. The self-inhibition parameter (beta parameter) of the stove-checking mode became finite, see Equation (9). Likewise, the excitation parameter (gamma parameter) of the follow-up activity (door-checking mode) became finite, see Equation (8). Consequently, the growth parameter of the active mode (stove checking) decayed monotonically, while the growth parameter of the follow-up mode (door checking) increased, see Figure 1C (rows 2 and 3). At a certain point in time, the monostability condition (3) was satisfied and the dynamical system exhibited a bifurcation from the fixed point of the behavioral pattern 2 to the fixed point of the behavioral pattern 3, see rows 2 and 3 of Figures 1A and 1B. According to the model, the women became busy with checking the doors and windows. The self-inhibitory (beta) parameter of the stove-checking pattern became finite and the growth parameter of the stove-checking mode decayed monotonically, see Figure 1C (row 3). At a certain time point, the growth parameter of the door-checking mode dropped below the threshold defined by Equation (3). The fixed point of the door-checking activity became unstable. The dynamical system exhibited a bifurcation towards the fixed point that was stable under these conditions, which was the fixed point of the non-disease related behavioral patterns, see rows 3 and 4 in Figures 1A and 1B. The ritual was completed. Note that in the first computer experiment the following parameter values were used

$$\lambda_4 = 3, \quad \lambda_{m \neq 4}(t = 0) = 0.1, \quad g_A = 1.5, \quad D_A = \lambda_4 \cdot g_A + 0.5,$$

 $\gamma(on) = 1, \quad \beta(on) = 0.5, \quad S = 1$
(13)

In order to study the coupling between emotions and behavior, we conducted a second computer experiment. In this second experiment both the amplitude equations for the behavioral and emotional dynamics, see Equations (1) and (4), were solved numerically (Euler forward scheme with single step 0.001). The trajectories are shown in Figure 2. In the second computer experiment, the obsessive thought triggered the distress feeling. More precisely, the growth parameter of the distress emotion was put to a sufficiently high value such that Equation (6) was satisfied. Consequently, the fixed point of the currently experienced feeling not related to the OCD became unstable and the fixed point related to the OCD distress emotion became stable. The emotional dynamics exhibited a bifurcation, see Figure 2A. The experience of distress then made the growth parameter of the first step of the OCD ritual to jump to a sufficiently high level such that on the behavioral level the OCD ritual was initialized. Mathematically speaking, in the second experiment, the following conditions were used



$$L_{1}(0) = D_{E}$$

$$E_{1}(t^{**}) = "on" \rightarrow \lambda_{1}(t^{**}) = D_{A}$$

$$S = 1$$
(14)

The behavioral ritual followed the dynamics described in the first computer experiment, see Figures 2B,C, D.

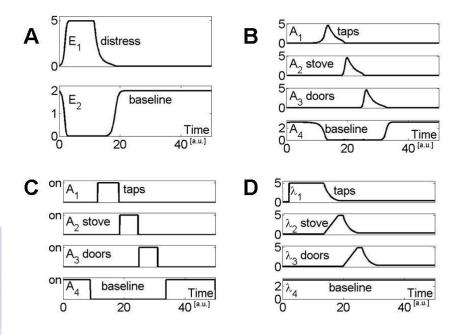


Fig 2: Trajectories of the behavioral-emotional dynamics of a purposeless OCD ritual as obtained in the second computer experiment. The panels depict the two emotional amplitudes as real-valued variables (A), the behavioral amplitudes as real-valued variables (B), the behavioral amplitudes as categorical variables (C), and the growth parameters of the four behavioral modes 1,2,3,4 as functions of time (D). See text for details.

While the behavioral activities of the OCD ritual were performed, the B-parameter was on a high level defined by Equation (10). Consequently, the growth parameter (L-parameter) of the distress emotional mode decayed monotonically (data not shown). At a certain point in time, the growth parameter of the distress emotion dropped below its critical value, see Equation (6), and the fixed point of the distress emotion became unstable. At that point in time, the fixed point of the catchall class of emotions not related to OCD was stable. Consequently, the emotional dynamics exhibited a bifurcation, see Figure 2A. The computer-animated OCD patient stopped experiencing the distress feeling.

For the parameters chosen in the second computer experiment, this emotional bifurcation happened before the ritual on the behavioral level was completed. In the second computer experiment, we put the S-parameter equal to unity. This corresponds to the situation where there is no feedback from the emotional level to the behavioral level with regard when to terminate the ritual. For this reason, the ritual was completed even in the absence of a distress feeling. We refer to this situation as a purposeless OCD ritual. Once the ritual is initiated it is completed irrespective of the state of the emotional dynamics. Note that in the second computer experiment the parameters listed in Equations (13) were used. In addition, we used

$$L_2 = 2$$
, $g_E = 1.5$, $D_E = L_2 \cdot g_E + 2.0$, $B(on) = 1.0$ (15)

While the second computer experiment addressed the case of purposeless OCD rituals, in the third computer experiment we considered the case of purposeful OCD rituals. That is, we considered the situation in which the behavioral ritual is terminated once the distress emotion disappears. In other words, the purpose of the ritual is to make the distress feeling going away. As soon as this purpose is fulfilled, the ritual is stopped even if it is not completed. In order to model purposeful OCD rituals, we replaced Equation (14) by



$$L_{1}(0) = D_{E}$$

$$E_{1}(t^{*}) = "on" \rightarrow \lambda_{1}(t^{*}) = D_{A}$$

$$S = \begin{cases} 1 & E_{1} & "on" \\ 0 & otheriwise \end{cases}$$
(16)

Accordingly, the S-parameter is only unity as long as the distress emotion is experienced. In the third computer experiment the amplitude equations of the emotional and behavioral dynamics were solved numerically (Euler-forward method with single time step 0.001) using Equation (16) rather than Equation (14). The results of this computer experiment are shown in Figure 3.

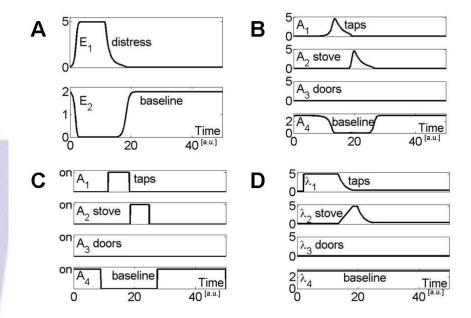


Fig 3: Trajectories of the behavioral-emotional dynamics of a purposeful OCD ritual as obtained in the third computer experiment. Panels as in Figure 2. See text for details.

The parameters used in the third computer experiments were those listed in Equations (13) and (15). As it is clear from Figure 3, the emotional dynamics evolved just as in the second computer experiment. However, on the behavioral level the third part of the ritual was not performed -- as expected. To repeat, the reason for this was that when the distress emotion disappeared the switch-like S-parameter was put equal to zero, which implied that the growth terms in the ritual-related activities 2 and 3 became zero as well. In particular, the overall growth term of the third activity (door checking) became zero, which is why the activity was not performed.

DISCUSSION AND COMMENT ON THE 4TH LAW

We presented a nonlinear physics model for OCD rituals in terms of amplitude equations that address both the behavioral and the emotional dynamics of such rituals. The model is based on the notion of self-organizing systems in which patterns emerge via bifurcations and is motivated in particular by synergetics. In order to model the sequence of actions that compose OCD rituals we extended the synergetic approach and supplemented the amplitude equations with evolution equations that describe how system parameters change over time. Importantly, we considered the case in which the system parameters depend on the amplitude dynamics. That is, the model does not depend explicitly on time. The model-based approach allowed us to distinguish between purposeless and purposeful OCD rituals. More precisely, rituals may be completed irrespective of the emotional state (purposeless rituals) or they may be terminated once the distress emotion goes away (purposeful rituals). In our model, the difference between these two scenarios is captured by the S-parameters. In the case of the purposeful rituals there is continuous control of the emotional level over the behavioral level, which is modeled by S-parameters that depend on the amplitudes of the emotional dynamics. In the case of the purposeless rituals there is no continuous control of the emotion level over the behavioral level only triggers the ritual on the behavioral level. After that initialization, the emotional level is affected by the behavioral level (distress is reduced due to behavioral activities related to the ritual) but the emotional level does not affect the behavioral level any more.



The notion of behavioral-emotional patterns emerging via bifurcations is consistent with the concept of dynamical diseases [22]. Accordingly, a disease state emerges via a bifurcation when certain disease-relevant parameters increase or decrease beyond certain threshold values. In our context, bifurcations on the behavioral and emotional level describing the emergence of the disease state of OCD patients occur when the growth parameters of the non-disease related activities or emotions drop below the thresholds defined by Equations (3) and (6).

These critical values defined by Equations (3) and (6) do not only allow us to conduct quantitative modeling of OCD rituals (see above) and to estimate model parameters [23], they also help us to speculate about a possible connection between disease dynamics and the so-called 4th law of non-equilibrium systems. The 4th law states that non-equilibrium phase transitions occur in such a way that the rate of entropy production is maximized [24]. That is, the 4th law defines a selection principle. However, Equations (3) and (6) define a similar selection principle. Accordingly, when considering OCD patients from the perspective of self-organized systems, the behaviors or the emotional states of the patients exhibit bifurcations from a self-organized state (activity, feeling) A to a self-organized state B such that the growth parameter of the emerging state B is larger than the growth parameter of the previous state A. This selection principle does not only apply for the OCD model discussed above but holds in general for Haken's amplitude equations of pattern formation systems [16-18] and for competitive Lotka-Volterra systems [5]. Most importantly, it has been advocated that the two selection principles are related to each other. For a large class of systems it has been shown that the growth rate parameters can be regarded as measures for the speed of entropy production [25,26]. Consequently, for such systems and in particular for the behavioral and emotional dynamics observed in OCD patients we are inclined to say that the 4th law applies and that transitions (bifurcations) happen in such a way to increase the rate of entropy production.

On the one hand, the modeling efforts discussed above go beyond earlier work presented in this regard [5]. On the other hand, the scenarios considered above are limited in their scope and future work may be conducted to obtain a more comprehensive picture about predictions made by the extended synergetic approach for the emotional and behavioral dynamics observed in OCD patients. For example, we used a bang-bang excitation (hard excitation) of the lambda parameter of the first ritual-related activity (k=1). That is, we considered the case when the corresponding growth rate parameter is put from zero to a finite value, see Equation (12). Likewise, the emotional level involved a bang-bang component, see Equations (11) and (12) for the first L-parameter. This bang-bang excitation may be replaced by an alternative excitation mechanics, for example by a ramp-like (soft) excitation.

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