



Stability of *-derivations on Lie C*-algebras

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ABSTRACT

In this paper, we investigate the stability and superstability problems for Lie *-derivations associated with the generalized Jensen type functional equation

$$f\left(\frac{\sum_{i=1}^n x_i}{n}\right) + \sum_{j=2}^n f\left(\frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}\right) = f(x_1)$$

on Lie C*-algebras.

KEYWORDS

Stability; Superstability; *-derivation; Generalized Jensen type functional equation; Lie C*-algebras

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INTRODUCTION

The stability theory of functional equations mainly deals with the following equation: Is it true that the solution of a given equation differing slightly from another given one must necessarily be close to the solution of the equation in the equation? A function equation is called stable if any approximate solution to the functional equation is near a true solution of that functional equation and is called superstable if every approximate solution is an exact solution to it. The study of stability problems of functional equations which had been proposed by Ulam [23] concerning the stability of group homomorphisms. The famous Ulam stability problem was partially solved by Hyers [8] for linear functional equation of Banach spaces. Later, the results of Hyers were generalized by Aoki [1], Găvruta [4], Rassias [22] and Gărdarín and Radu [2].

Beginning around the year 1980, the stability problems of many algebraic, differential, integral, operatorial equations have been extensively investigated [9, 10, 13, 14, 22] and the references therein. Among of them, many research papers have been published about the generalized Hyer-Ulam-Rassias stability of homomorphisms and derivations in C^* -algebras and Lie algebras [3, 5, 6, 11, 12, 15, 16, 17, 19, 21].

A Lie algebra A is a linear space over some field F together with a binary operation $[\cdot, \cdot] : A \times A \rightarrow A$ called the Lie bracket, which satisfies the following axioms:

$$\begin{aligned} (L_1)[\alpha x + \beta y, z] &= \alpha[x, z] + \beta[y, z], \quad [z, x + \beta y] = \alpha[z, x] + \beta[z, y], \\ (L_2)[x, x] &= 0, \\ (L_3)[x, [y, z]] + [z, [x, y]] + [y, [z, x]] &= 0 \end{aligned}$$

for all $\alpha, \beta \in F$ and all $x, y, z \in A$.

Note that the bilinearity (L_1) and alternating (L_2) properties imply anticommutativity, i.e., $[x, y] = -[y, x]$ for all $x, y \in A$.

A C^* -algebra A endowed with the Lie bracket $[x, y] = \frac{xy - yx}{2}$ on A is called a Lie C^* -algebra. Let A be a Lie C^* -algebra. A C -linear mapping $D: A \rightarrow A$ is called a Lie derivation of A if

$$D([x, y]) = [D(x), y] + [x, D(y)]$$

for all $x, y \in A$. In addition, if D satisfies the additional condition $D(a^*) = D(a)^*$ for all $a \in A$, then it is called a Lie $*$ -derivations.

Now, we consider a mapping $f: X \rightarrow Y$ satisfying the following functional equation:

$$f\left(\frac{\sum_{i=1}^n x_i}{n}\right) + \sum_{j=2}^n f\left(\frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}\right) = f(x_1) \quad (1.1)$$

for all $x_i, x_j \in X$, where $n \in \mathbb{N}$ is a fixed integer with $n \geq 2$. Park and Rassias [20] proved the stability of homomorphisms and derivations in C^* -algebras of the Jensen functional equation (1.1) for $n = 2$. Gordji et al. [7] establish the stability of n -Lie homomorphisms and Jordan n -Lie homomorphisms on n -Lie algebras associated with the equation (1.1).

Motivated and inspired by the above works, in this paper, we investigate the stability and superstability problems for Lie $*$ -derivations associated with the generalized Jensen type functional equation (1.1) on Lie C^* -algebras. The present theorems generalized and improve many existing results in the Park and Rassias [20].

STABILITY OF LIE $*$ -DERIVATIONS

Throughout this section, we assume that A is a Lie C^* -algebra with the norm $\|\cdot\|$. For convenience, we use the following abbreviation for any mapping $f: A \rightarrow A$,

$$\Delta_\mu f(x_1, \dots, x_n) = \mu f\left(\frac{\sum_{i=1}^n x_i}{n}\right) + \mu \sum_{j=2}^n f\left(\frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}\right) - f(\mu x_1)$$

for all $x_1, \dots, x_n \in X$ ($n \geq 2$) and $\mu \in T^1 = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$.

To achieve our main in this section, we used the following lemma.

Lemma 2.1. Let X and Y be complex linear spaces. Suppose that $f: X \rightarrow Y$ is a mapping such that (1.1). Then the mapping f is additive.

Proof. It follows from

$$\begin{aligned} \sum_{j=2}^n f\left(\frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}\right) &= f\left(\frac{x_1 - (n-1)x_2 + \dots + x_n}{n}\right) + f\left(\frac{x_1 + x_2 - (n-1)x_3 + \dots + x_n}{n}\right) + \\ &\dots + f\left(\frac{x_1 + \dots - (n-1)x_n}{n}\right) \end{aligned}$$

that



$$f\left(\sum_{j=1}^n s_j\right) = f(s_1 + \dots + s_n) = f(x_1) = f(s_1) + \sum_{j=2}^n f(s_j) = \sum_{j=1}^n f(s_j) \tag{2.1}$$

for all $s_1, \dots, s_n \in X$, where $s_1 = \frac{\sum_{i=1}^n x_i}{n}$ and $s_j = \frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}$ for $j = 2, 3, \dots, n$. Putting $s_j = 0$ for $j = 3, 4, \dots, n$ in (2.1), we have

$$f(s_1 + s_2) = f(s_1) + f(s_2)$$

for all $s_1, s_2 \in A$. Thus the mapping f is additive. ■

Theorem 2.2. Let $\varphi: A^n \rightarrow [0, \infty)$ and $\psi: A^3 \rightarrow [0, \infty)$ be mappings such that

$$\sum_{m=0}^{\infty} n^m \varphi\left(\frac{x}{n^m}, 0, \dots, 0\right) < \infty, \quad \lim_{m \rightarrow \infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0, \tag{2.2}$$

$$\lim_{m \rightarrow \infty} n^{2m} \psi\left(\frac{a}{n^m}, \frac{b}{n^m}, 0\right) = 0, \quad \lim_{m \rightarrow \infty} n^m \psi\left(0, 0, \frac{c}{n^m}\right) = 0$$

for all $x_1, \dots, x_n, a, b, c \in A$. Suppose that $f: A \rightarrow A$ is a mapping with $f(0) = 0$ satisfying

$$\|\Delta_\mu f(x_1, \dots, x_n)\| \leq \varphi(x_1, \dots, x_n), \tag{2.3}$$

$$\|f([a, b]) - [f(a), b] - [a, f(b)] + f(c^*) - f(c)^*\| \leq \psi(a, b, c) \tag{2.4}$$

for all $x_1, \dots, x_n, a, b, c \in A$ and all $\mu \in T^1$. Then there exists a unique Lie $*$ -derivation $D: A \rightarrow A$ which satisfies the functional equation (1.1) and the inequality

$$\|f(x) - D(x)\| \leq \sum_{m=0}^{\infty} n^m \varphi\left(\frac{x}{n^m}, 0, \dots, 0\right) \tag{2.5}$$

for all $x \in A$.

Proof. Let us assume $x_1 = x, x_2 = \dots = x_n = 0$ and $\mu = 1$ in (2.3). Then we have

$$\left\| n f\left(\frac{x}{n}\right) - f(x) \right\| \leq \varphi(x, 0, \dots, 0), \tag{2.6}$$

for all $x \in A$. Replacing x by $\frac{x}{n^j}$ and multiplying n^j both the sides of (2.6),

$$\left\| n^{j+1} f\left(\frac{x}{n^{j+1}}\right) - n^j f\left(\frac{x}{n^j}\right) \right\| \leq n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right),$$

for all $x \in A$ and all integers $j \in \mathbb{Z}$ with $j = 0, 1, 2, \dots$. Thus we have

$$\left\| n^m f\left(\frac{x}{n^m}\right) - n^k f\left(\frac{x}{n^k}\right) \right\| \leq \sum_{j=k}^{m-1} n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right) \tag{2.7}$$

for all $x \in A$ and $m > k \geq 0$. It follows from $\sum_{m=0}^{\infty} n^m \varphi\left(\frac{x}{n^m}, 0, \dots, 0\right) < \infty$ of (2.2) that the sequence $\left\{n^m f\left(\frac{x}{n^m}\right)\right\}$ is a Cauchy sequence. Since A is a Lie C^* -algebra, the sequence $\left\{n^m f\left(\frac{x}{n^m}\right)\right\}$ converges. So we can define a mapping $D: X \rightarrow Y$ by

$$D(x) = \lim_{m \rightarrow \infty} n^m f\left(\frac{x}{n^m}\right)$$

for all $x \in A$. Moreover passing the limit as $m \rightarrow \infty$ with $k = 0$ in (2.7), we have

$$\|f(x) - D(x)\| = \left\| n^m f\left(\frac{x}{n^m}\right) - f(x) \right\| \leq \sum_{j=0}^{\infty} n^j \varphi\left(\frac{x}{n^j}, 0, \dots, 0\right)$$

which implies the inequality (2.5) holds for all $x \in A$.

On the other hand, substituting (x_1, \dots, x_n) by $\left(\frac{x_1}{n^j}, \dots, \frac{x_n}{n^j}\right)$ in (2.3), we have

$$\|\Delta_\mu D(x_1, \dots, x_n)\| = \lim_{m \rightarrow \infty} n^m \|\Delta_\mu f\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right)\| \leq \lim_{m \rightarrow \infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0 \tag{2.8}$$

for all $x_1, \dots, x_n \in A$ and all $\mu \in T^1$. Let $\mu = 1$ in (2.8). It follows from $\Delta_1 f(x_1, \dots, x_n) = 0$ that



$$D\left(\frac{\sum_{i=1}^n x_i}{n}\right) + \sum_{j=2}^n D\left(\frac{\sum_{i=1, i \neq j}^n x_i - (n-1)x_j}{n}\right) = D(x_1)$$

for all $x_1, \dots, x_n \in A$. From Lemma 2.1 that the mapping D is additive. Letting $x_1 = x, x_2 = \dots = x_n = 0$ in (2.3), we have $\|\mu f(x) - f(\mu x)\| \leq \varphi(x, 0, \dots, 0)$ for all $x \in A$. Thus

$$\|\mu D(x) - D(\mu x)\| = \lim_{m \rightarrow \infty} n^m \left\| \mu f\left(\frac{x}{n^m}\right) - f\left(\mu \frac{x}{n^m}\right) \right\| \leq \lim_{m \rightarrow \infty} n^m \varphi\left(\frac{x}{n^m}, 0, \dots, 0\right) = 0$$

which implies $\mu D(x) = D(\mu x)$ for all $x \in A$ and all $\mu \in T^1$. By the same reasoning as that the proof of Theorem 2.1 of [18], the mapping D is \mathbb{C} -linear.

Replacing (a, b) by $\left(\frac{a}{n^m}, \frac{b}{n^m}\right)$ and putting $c = 0$ in (2.4), we have

$$\begin{aligned} & D([a, b]) - [D(a), b] - [a, D(b)] \\ &= \lim_{m \rightarrow \infty} n^{2m} \left\| f\left(\frac{[a, b]}{n^{2m}}\right) - \left[f\left(\frac{a}{n^m}, \frac{b}{n^m}\right) - \left[\frac{a}{n^m}, f\left(\frac{b}{n^m}\right)\right]\right\| \\ &\leq \lim_{m \rightarrow \infty} n^{2m} \psi\left(\frac{a}{n^m}, \frac{b}{n^m}, 0\right) = 0 \end{aligned} \tag{2.9}$$

for all $a, b \in A$. Also, if we put $a = b = 0$ and substitute c by $\frac{c}{n^m}$ in (2.4), then

$$D(c^*) - D(c)^* = \lim_{m \rightarrow \infty} n^m \left\| f\left(\frac{c^*}{n^m}\right) - f\left(\frac{c}{n^m}\right)^* \right\| \leq \lim_{m \rightarrow \infty} n^m \psi\left(0, 0, \frac{c}{n^m}\right) = 0 \tag{2.10}$$

for all $c \in A$. Thus it follows from (2.9) and (2.10) that D is a Lie $*$ -derivation on A .

Now let $D' : A \rightarrow A$ be another additive mapping satisfying (2.5). Then we have

$$\|D(x) - D'(x)\| = \lim_{m \rightarrow \infty} n^m \left\| D\left(\frac{x}{n^m}\right) - D'\left(\frac{x}{n^m}\right) \right\| \leq \lim_{m \rightarrow \infty} n^{j+m} \sum_{j=0}^{\infty} n^{j+m} \varphi\left(\frac{x}{n^{j+m}}, 0, \dots, 0\right)$$

for all $x \in A$. So we can conclude that $D(x) = D'(x)$ for all $x \in A$. Therefore the mapping D is a unique Lie $*$ -derivation on A satisfying (2.5), as desired. This complete the proof. ■

Corollary 2.3. Let $r (> 1)$ and θ be positive real numbers. Suppose that a mapping $f : A \rightarrow A$ satisfies

$$\begin{aligned} & \|\Delta_\mu f(x_1, \dots, x_n)\| \leq \theta(\|x_1\|^r + \dots + \|x_n\|^r), \\ & \|f([a, b]) - [f(a), b] - [a, f(b)] + f(c^*) - f(c)^*\| \leq \theta(\|a\|^r + \|b\|^r + \|c\|^r) \end{aligned}$$

for all $x_1, \dots, x_n, a, b, c \in A$ and all $\mu \in T^1$. Then there exists a unique Lie $*$ -derivation $D : A \rightarrow A$ which satisfies the inequality

$$\|f(x) - D(x)\| \leq \frac{n^r \theta \|x\|^r}{n^r - n}$$

for all $x \in A$.

Proof. The proof follows Theorem 2.2 by taking

$$\varphi(x_1, \dots, x_n) = \theta(\|x_1\|^r + \dots + \|x_n\|^r) \text{ and } \psi(a, b, c) = \theta(\|a\|^r + \|b\|^r + \|c\|^r)$$

for all $x_1, \dots, x_n, a, b, c \in A$. This completes the proof. ■

In the followings corollary, we show that when f is an additive mapping, the superstability for the inequalities (2.3) and (2.4) holds.

Corollary 2.4. Let A, φ, ψ be as in Theorem 2.2. If $f : A \rightarrow A$ is an additive mapping with (2.5), then f is a Lie $*$ -derivation.

Proof. It follows immediately from additivity of f that $f(0) = 0$. Thus $n^m f(x) = f(x^m x)$ for all $x \in A$ and $m \in \mathbb{N}$ and so

$$f(x) = n^m f\left(\frac{x}{n^m}\right)$$

for all $x \in A$ and $m \in \mathbb{N}$. Now it follows from Theorem 2.2 that f is a Lie $*$ -derivation on A . This completes the proof. ■

Next we prove another theorem in superstability of a Lie $*$ -derivation on A for the functional equation (1.1).

Theorem 2.5. Suppose that there exist mappings $\varphi : A^n \rightarrow [0, \infty), \psi : A^3 \rightarrow [0, \infty)$ and a constant $0 < L < 1$ such that

$$\varphi\left(\frac{x_1}{n}, \dots, \frac{x_n}{n}\right) \leq \frac{L}{n} \varphi(x_1, \dots, x_n), \quad \psi\left(\frac{a}{n}, \frac{b}{n}, 0\right) \leq \frac{L}{n^2} \varphi(a, b, 0), \quad \psi\left(0, 0, \frac{c}{n}\right) \leq \frac{L}{n} \varphi(0, 0, c), \tag{2.11}$$

for all $x_1, \dots, x_n, a, b, c \in A$. If $f : A \rightarrow A$ is a mapping satisfying (2.3) and (2.4), then there exists a unique Lie $*$ -derivation f on A .



Proof. It follows from (2.11) that

$$\lim_{m \rightarrow \infty} n^m \varphi\left(\frac{x_1}{n^m}, \dots, \frac{x_n}{n^m}\right) = 0, \quad \lim_{m \rightarrow \infty} n^{2m} \psi\left(\frac{a}{n^m}, \frac{b}{n^m}, 0\right) = 0, \quad \lim_{m \rightarrow \infty} n^m \psi\left(0, 0, \frac{c}{n^m}\right) = 0 \quad (2.12)$$

for all $x_1, \dots, x_n, a, b, c \in A$. Putting $x_1 = \dots = x_n = 0$ and $\mu = 1$ in (2.3), we obtain $f(0) = 0$. Replacing $x_1 = x, x_2 = \dots = x_n = 0$ and $\mu = 1$ in (2.3), we have

$$f(x) = n^m f\left(\frac{x}{n^m}\right) \quad (2.13)$$

for all $x \in A$ and $m \in \mathbb{N}$. It follows from (2.4) and (2.13) that

$$\begin{aligned} \|f([a, b]) - [f(a), b] - [a, f(b)]\| &= n^{2m} \left\| f\left(\frac{[a, b]}{n^{2m}}\right) - \left[f\left(\frac{a}{n^m}, \frac{b}{n^m}\right) - \left[\frac{a}{n^m}, f\left(\frac{b}{n^m}\right) \right] \right\| \\ &\leq n^{2m} \psi\left(\frac{a}{n^m}, \frac{b}{n^m}, 0\right) = 0 \end{aligned} \quad (2.14)$$

for all $a, b \in A$. Taking the limit as $m \rightarrow \infty$ in (2.14) and using (2.12), we obtain

$$f([a, b]) = [f(a), b] + [a, f(b)]$$

for all $a, b \in A$. Also, if we put $a = b = 0$ and substitute c by $\frac{c}{n^m}$ in (2.4), then

$$\|f(c^*) - f(c)^*\| = n^m \left\| f\left(\frac{c^*}{n^m}\right) - f\left(\frac{c}{n^m}\right)^* \right\| \leq n^m \psi\left(0, 0, \frac{c}{n^m}\right) \quad (2.15)$$

for all $c \in A$. Passing the limit as $m \rightarrow \infty$ in (2.15), we conclude that $f(c^*) = f(c)^*$ for all $c \in A$. Therefore f is a Lie $*$ -derivation on A . This complete the proof. ■

Corollary 2.6. Let r, r_j ($j = 1, 2, \dots, n$) and θ be nonnegative real numbers such that $0 < \sum_{j=1}^n r_j \neq 1$. Suppose that $f: A \rightarrow A$

is a mapping such that

$$\|\Delta_\mu f(x_1, \dots, x_n)\| \leq \theta \prod_{j=1}^n \|x_j\|^{r_j} \quad (2.16)$$

$$\|f([a, b]) - [f(a), b] - [a, f(b)] + f(c^*) - f(c)^*\| \leq \theta (\|a\|^r \|b\|^r \|c\|^r)$$

for all $x_1, \dots, x_n, a, b, c \in A$ and all $\mu \in T^1$. Then f is a Lie $*$ -derivation on A .

Proof. Putting $x_1 = \dots = x_n = 0$ and $\mu = 1$ in (2.16), we obtain $f(0) = 0$. Replacing x_1 by x and setting $x_2 = \dots = x_n = 0$ and $\mu = 1$ in (2.16), we have

$$f(x) = n^m f\left(\frac{x}{n^m}\right)$$

for all $x \in A$ and $m \in \mathbb{N}$. The remaining assertion goes through by the similar method to be the proof of Theorem 2.5. This completes the proof. ■

Remark. Suppose that $f: A \rightarrow A$ is a mapping with $f(0) = 0$ such that there exist mappings $\varphi: A^n \rightarrow [0, \infty), \psi: A^3 \rightarrow [0, \infty)$ satisfying (2.3) and (2.4). Let $0 < L < 1$ be a constant such that

$$\varphi\left(\frac{x_1}{n}, \dots, \frac{x_n}{n}\right) \leq \frac{L}{n} \varphi(x_1, \dots, x_n), \quad \psi\left(\frac{a}{n}, \frac{b}{n}, 0\right) \leq \frac{L}{n^2} \varphi(a, b, 0), \quad \psi\left(0, 0, \frac{c}{n}\right) \leq \frac{L}{n} \varphi(0, 0, c),$$

for all $x_1, \dots, x_n, a, b, c \in A$. By the similar method as in the Theorem 2.5, we can show that there exists a unique Lie $*$ -derivation $D: A \rightarrow A$ satisfying

$$\|f(x) - D(x)\| \leq \frac{1}{1-L} \varphi(x, 0, \dots, 0)$$

for all $x \in A$.

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