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Homomorphisms and derivations on C*-ternary algebras assocoated with a generalized Cauchy-Jensen type additive functional equation

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ABSTRACT

In this paper, we prove the generalized Hyers-Ulam stability of C*-ternary homomorphisms and C*-ternary derivations onC*-ternary algebras associated with the generalized Cauchy-Jensen type additive functional equation

$$\sum_{1\le i<j\le n} f\left(\frac{x_i+x_j}{2}+\sum_{l=1,k_l\neq i,j}^{n-2} x_{k_l}\right)=\frac{(n-1)^2}{2}\sum_{i=1}^{n} f(x_i)$$

for allx_i, $x_j \in X$ where $n \in \mathbb{Z}^+$ is a fixed integer with $n \geq 3$.

KEYWORDS

C*-ternary homomorphism, C*-ternary derivation, Generalized Cauchy-Jensen type equation, Generalized Hyers-Ulam stability, C*-ternary algebras.

SUBJECT CLASSIFICATION

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INTRODUCTION

Ternary algebraic operations were considered in the 19th century by several mathematicans and physicists such that as Cayley [3] who introduced the notion of a cubic matrix, which in turn was generalized by Kapranov et al. [9]. Tenary structures and their generalization, the so-called n-ary structures, raise certain hopes in view of their applications in physics [10]. As it is extensively discussed in [24], the full description of a physical system implies the knowledge of three basic ingredients: the set of the observable, the set of the states, and the dynamics that describes the time evolution of the system by means of the time dependence of the expectation value of a given observable on a given status. Originally the set of the observable was considered to be a C*-algebra [6]. In many applications, however, this was shown not to be the most convenient choice, and so the C*-algebra was replaced by a Von Neumann algebra. This is because the role of the representation turns out to be crucial, mainly when long range interactions are involved. Here we used a different algebraic structure.

A C*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x,y,z) \longmapsto [x,y,z]$ of A^3 into A, which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linear in the middle variable, and associative in the sence that $[x,y,[z,w,v]] = [x,[w,z,y],v] = [[x,y,z],w,v]$, and satisfies $\|[x,y,z]\| \leq \|x\|\|y\|\|z\|$ and $[x,x,x] = \|x^3\|$. If a C*-ternary algebra $(A,[\cdot,\cdot,\cdot])$ has an unit element $e \in A$ such that $x = [x,e,e] = [e,e,x]$ for all $x \in A$, then it is routine to verify that A, endowed with $x \circ y := [x,e,y]$ and $x^* := [e,x,e]$, is a unital C*-ternary algebra. Conversely, if $(A,\circ)$ is a unital C*-ternary algebra, then $[x,y,z] := x \circ y^* \circ z$ makes A into a C*-ternary algebra.

Let A and B be C*-ternary algebras. A $\mathbb{C}$-linear mapping $H: A \to B$ is called a C*-ternary homomorphism if

$$H([x,y,z]) = [H(x),H(y),H(z)]$$

for all $x,y,z \in A$. If, in addition, the mapping H is bijevtive, then the mapping $H: A \to B$ is called a C*-ternary isomorphism. A $\mathbb{C}$-linear mapping $D: A \to A$ is called a C*-ternary derivation if

$$D([x,y,z]) = [D(x),y,z] + [x,D(y),z] + [x,y,D(z)]$$

for all $x,y,z \in A$.

The stability problem of functional equations originated from a question of Ulam [25] concerning the stability of group homomorphisms. In 1941, the famous Ulam stability problem was partially solved by Hyers [7] for linear functional equation of Banach spaces. In 1950, Aoki [1] was the second author to treat this problem for additive mappings. In 1978, Rassias [23] provided a generalization of Hyers' theorem which allows the Cauchy difference to be unbounded. A generalization of the Rassias' theorem was obtained by Găvruta [4] by replacing the unbounded Cauchy difference by a general control function in the sprite of the Rassias approach. On the other hand, Rassias [20] generalized the Hyers stability result by presenting a weak condition controlled by a product of different powers of norms.

Bourgin [2] is the first mathematican dealing with stability of ring homomorphism. Later, the topic of approximate homomorphism and approximate derivations on algebras have been extensively investigated by a number of mathematician ([5],[11]-[14],[16],[18],[19],[22], and references therein).

Now, we consider a mapping $f: X \to Y$ satisfying the following functional equation, which was introduced by Rassias and Kim [21], in 2009, as follows:

$$\sum_{1 \leq i < j \leq n} f\left(\frac{x_i + x_j}{2} + \sum_{l=1, k_l \neq i,j}^{n-2} x_{k_l}\right) = \frac{(n-1)^2}{2} \sum_{i=1}^{n} f(x_i) \quad (1.1)$$

forall $x_i, x_j \in X$ where $n \in \mathbb{Z}^+$ is a fixed with $n \geq 3$ in quasi-β-normed spaces and proved the generalized Hyers-Ulam stability for the functional equation (1.1). The case $n = 2$ and $n = 3$ of the functional equation (1.1) yields the Cauchy-Jensen additive functional equation and many interesting results concerning the stability problems of the Cauchy-Jensen equation are given by Jun et al. [8], Najati et al. [15], respectively. Therefore, the functional equation (1.1) is called the generalized Cauchy-Jensen type additive functional equation. Note that (1) if we put $x_1 = \cdots = x_n = 0$ in the functional equation (1.1), then $f(0) = 0$, (2) $f(-x) = -f(x)$ and (3) $f(2x) = 2f(x)$ for all $x \in X$.

STABILITY

Throughout this section, let A and B be C*-ternary algebras, $\lambda = n - 1$ be a fixed positive integer with $n \geq 3$ and $T^1 = \{\mu \in \mathbb{C} : |\mu| = 1\}$. For a given mapping $f: A \to B$, we define

$$D_\mu f(x_1, \cdots, x_n) = \sum_{1 \leq i < j \leq n} f\left(\frac{\mu x_i + \mu x_j}{2} + \sum_{l=1, k_l \neq i,j}^{n-2} \mu x_{k_l}\right) - \frac{(n-1)^2}{2} \sum_{i=1}^{n} \mu f(x_i) \quad (2.1)$$

for all $x_1, \cdots, x_n \in X$ and $\mu \in T^1$.

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Stability of C*-ternary homomorphism

We prove the generalized Hyers-Ulam stability of C*-ternary homomorphisms on C*-ternary algebra for the functional equation $D_\mu f(x_1,\cdots,x_n)=0$ in the sprite of Hyers, Ulam and Rassias. We need the following lemma in the main theorems.

Lemma 2.1. [21] Let X and Y be linear spaces and $n(\geq 3)$ be a fixed positive integer. A mapping $f: X \to Y$ satisfies the following equation

$$\sum_{1\leq i<j\leq n} f\left(\frac{x_i+x_j}{2}+\sum_{l=1,k_l\neq i,j}^{n-2} x_{k_l}\right)=\frac{(n-1)^2}{2}\sum_{i=1}^{n} f(x_i)$$

If and only if f is additive.

Theorem 2.2. Let $p<1$, $s<3$ and θ be positive real numbers. If a mapping $f: A \to B$ satisfies

$$\|D_\mu f(x_1,\cdots,x_n)\| \leq \theta \sum_{j=1}^{n} \|x_n\|^p, \qquad (2.2)$$

$$\|f([x,y,z])-[f(x),f(y),f(z)]\| \leq \theta\,(\|x\|^s+\|y\|^s+\|z\|^s) \qquad (2.3)$$

for all $x_1,\cdots,x_n,x,y,z \in A$ and $\mu \in T^1$, then there exists a unique C*-ternary homomorphism $H: A \to B$ such that

$$\|f(x)-H(x)\| \leq \frac{2\theta\,\|x_n\|^p}{\lambda^2-\lambda^{p+1}} \qquad (2.4)$$

for all $x \in A$ and where $\lambda=n-1$ with $n \geq 3$.

Proof. Substituting $x_1=\cdots=x_n=x$ and $\mu=1$ in (2.2), we have

$$\left\|\binom{n}{2} f((n-1)x)-\frac{n(n-1)^2}{2} f(x)\right\| \leq n\theta\,\|x\|^p, \qquad (2.5)$$

which gives

$$\left\|f(x)-\frac{f(\lambda x)}{\lambda}\right\| \leq \frac{2\theta}{\lambda^2}\|x\|^p \qquad (2.6)$$

for all $x \in A$. If we replace x by $\lambda^j x$ and divide λ^j both sides of (2.6), then we have

$$\left\|\frac{f(\lambda^j x)}{\lambda^j}-\frac{f(\lambda^{j+1}x)}{\lambda^{j+1}}\right\| \leq \frac{2\theta\|x\|^p}{\lambda^2}(\lambda^{p-1})^j$$

for all $x \in A$ and $j=0,1,2,\cdots$. Therefore, we have

$$\left\|\frac{f(\lambda^k x)}{\lambda^k}-\frac{f(\lambda^m x)}{\lambda^m}\right\| \leq \frac{2\theta\|x\|^p}{\lambda^2}\sum_{j=k}^{m-1}(\lambda^{p-1})^j \qquad (2.7)$$

for all $x \in A$ and $m,k \in \mathbb{Z}^+$ with $m>k\geq 0$. Then the sequence $\left\{\frac{f(\lambda^m x)}{\lambda^m}\right\}$ is a Cauchy sequence for all $x \in A$, and so it converges. We can define a mapping $H: A \to B$ by $H(x)=\lim_{m\to\infty}\frac{f(\lambda^m x)}{\lambda^m}$ for all $x \in A$. Letting $m \to \infty$ in (2.7) with $k=0$, we obtain the desired inequality (2.4). It follows from (2.2) that

$$\begin{aligned}\|D_\mu H(x_1,\cdots,x_n)\| &\leq \lim_{m\to\infty}\frac{1}{\lambda^m}\|D_\mu f(\lambda^m x_1,\cdots,\lambda^m x_n)\| \\ &\leq \theta\lim_{m\to\infty}(\lambda^{p-1})^m(\|x_1\|^p+\|x_2\|^p+\cdots+\|x_n\|^p)=0,\end{aligned}$$

which gives $D_\mu H(x_1,\cdots,x_n)=0$ for all $x_1,\cdots,x_n \in A$ and $\mu \in T^1$. If we put $\mu=1$, in$D_\mu H(x_1,\cdots,x_n)=0$, then,by Lemma 2.1, the mapping H is additive. We let $x_1=x$and $x_2=\cdots=x_n=0$, then $H(\mu x)=\mu H(x)$. By the same reasoning as that the proof of Theorem 2.1 of [17], the mapping H is $\mathbb{C}$-linear. Also, it follows from (2.3) that

$$\begin{aligned}\|H([x,y,z])-[H(x),H(y),H(z)]\| &\leq \lim_{m\to\infty}\frac{1}{\lambda^{3m}}\|f([\lambda^m x,\lambda^m y,\lambda^m z])-[f(\lambda^m x),f(\lambda^m y),f(\lambda^m z)]\| \\ &\leq \lim_{m\to\infty}\theta(\lambda^{s-3})^m(\|x\|^s+\|y\|^s+\|z\|^s)=0\end{aligned}$$

for all $x,y,z \in A$. Thus we have

$$H([x,y,z])=[H(x),H(y),H(z)]$$

for all $x,y,z \in A$.This means that the mapping H is a C*-ternary homomorphism on A.

To prove the uniqueness of the C*-ternary homomorphism, let $G: A \to B$ be another C*-ternary homomorphism on A satisfying (2.4). Then we have

$$\|H(x)-G(x)\| = \frac{1}{\lambda^m}\|H(\lambda^m x)-G(\lambda^n x)\|$$
$$\leq \lim_{m\to\infty}\frac{1}{\lambda^m}\left(\|H(\lambda^m x)-f(\lambda^m x)\|+\|f(\lambda^m x)-G(\lambda^m x)\|\right)$$
$$\leq \frac{4\theta}{\lambda^2-\lambda^{p+1}}\lim_{m\to\infty}(\lambda^{p-1})^m\|x\|^p = 0$$

for all $x \in A$.Thus we obtain that $H(x) = G(x)$ for all $x \in A$. This completes the proof. ■

Theorem 2.3. Let $p > 1$, $s > 3$ and θ be positive real numbers. If $f: A \to B$ satisfies (2.2) and (2.3), then there exists a unique C*-ternary homomorphism $H: A \to B$ such that

$$\|f(x)-H(x)\| \leq \frac{2\theta\,\|x\|^p}{\lambda^{p+1}-\lambda^2} \qquad (2.8)$$

for all $x \in A$ and where $\lambda = n-1$ with $n \geq 3$.

Proof. It follows from (2.5) that

$$\left\|\lambda^k f\left(\frac{x}{\lambda^k}\right)-\lambda^m f\left(\frac{x}{\lambda^m}\right)\right\| \leq \frac{2\theta}{\lambda^{p+1}}\sum_{j=k}^{m-1}(\lambda^{1-p})^j\|x\|^p \qquad (2.9)$$

for all $x \in A$ and $m, k \in \mathbb{Z}^+$ with $m > k \geq 0$. Then the sequence $\left\{\lambda^m f\left(\frac{x}{\lambda^m}\right)\right\}$ is a Cauchy sequence for all $x \in A$ and it converges. So we can define a mapping $H: A \to B$ by $H(x) = \lim_{m\to\infty}\lambda^m f\left(\frac{x}{\lambda^m}\right)$ for all x ∈ A. Letting $m \to \infty$ in (2.9) with $k = 0$, we obtain (2.8). The rest of proof is similar method to proof of Theorem 2.2. This completes the proof. ■

Theorem 2.4. Let $p < 1$ with $p = \sum_{j=1}^{n}|p_j| \neq 1$, $s < 3$ and θ be positive real numbers. If a mapping $f: A \to B$ satisfies

$$\|D_\mu f(x_1,\cdots,x_n)\| \leq \theta\prod_{j=1}^{n}\|x_j\|^{p_j}, \qquad (2.10)$$

$$\|f([x,y,z])-[f(x),f(y),f(z)]\| \leq \theta\,(\|x\|^s\|y\|^s\|z\|^s) \qquad (2.11)$$

forall $x_1,\cdots,x_n, x, y, z \in A$ and $\mu \in T^1$, then there exists a unique C*-ternary homomorphism $H: A \to B$ such that

$$\|f(x)-H(x)\| \leq \frac{2\theta\,\|x\|^p}{n(\lambda^2-\lambda^{p+1})} \qquad (2.12)$$

for all $x \in A$ and where $\lambda = n-1$ with $n \geq 3$.

Proof. Let us assume $x_1 = \cdots = x_n = x$ and $\mu = 1$ in (2.10). Then we have

$$\left\|\binom{n}{2}f((n-1)x)-\frac{n(n-1)^2}{2}f(x)\right\| \leq \theta\,\|x\|^p, \qquad (2.13)$$

which gives

$$\left\|\frac{f(\lambda^k x)}{\lambda^k}-\frac{f(\lambda^m x)}{\lambda^m}\right\| \leq \frac{2\theta\|x\|^p}{n\lambda^2}\sum_{j=k}^{m-1}(\lambda^{p-1})^j \qquad (2.14)$$

for all $x \in A$ and $m, k \in \mathbb{Z}^+$ with $m > k \geq 0$. Then $\left\{f\left(\frac{\lambda^m x}{\lambda^m}\right)\right\}$ is a Cauchy sequence for all $x \in A$ and it is convergence. So we can define a mapping $H: A \to B$ by $H(x) = \lim_{m\to\infty} f\left(\frac{\lambda^m x}{\lambda^m}\right)$ for all x ∈ A. Letting $m \to \infty$ in (2.14) with $k = 0$, we obtain (2.12). The rest of proof is similar method to proof of Theorem 2.2. This completes the proof. ■

Corollary 2.5. Let $p > 1$ with $p = \sum_{j=1}^{n}|p_j| \neq 1$, $s > 3$ and θ be positive real numbers. If $f: A \to B$ satisfies (2.10) and (2.11), then there exists a unique C*-ternary homomorphism $H: A \to B$ such that

$$\|f(x)-H(x)\| \leq \frac{2\theta\,\|x\|^p}{n(\lambda^{p+1}-\lambda^2)} \qquad (2.15)$$

for all $x \in A$ and where $\lambda = n-1$ with $n \geq 3$.

Proof. It follows from (2.10) that

$$\left\|\lambda^k f\left(\frac{x}{\lambda^k}\right)-\lambda^m f\left(\frac{x}{\lambda^m}\right)\right\| \leq \frac{2\theta\|x\|^p}{n\lambda^{p+1}}\sum_{j=k}^{m-1}(\lambda^{1-p})^j \qquad (2.16)$$

for all $x \in A$ and $m, k \in \mathbb{Z}^+$ with $m > k \geq 0$. Then $\left\{\lambda^m f\left(\frac{x}{\lambda^m}\right)\right\}$ is a Cauchy sequence for all $x \in A$ and it converges. So we can define a mapping $H: A \to B$ by $H(x) = \lim_{m\to\infty} \lambda^m f\left(\frac{x}{\lambda^m}\right)$ for all x ∈ A. Letting $m \to \infty$ in (2.16) with $k = 0$, we obtain (2.15). The rest of proof is similar method to proof of Theorem 2.3. This completes the proof. ■

Now let A and B be a unital C*-ternary algebras with unit $e$and e', respectively. Then we show C*-ternary isomorphism between C*-ternary algebras associated with the functional equation $D_\mu f(x_1, \cdots, x_n) = 0$.

Theorem 2.6. Let $p < 1$, $s < 3$ and θ be positive real numbers and $f: A \to B$ be a bijective mapping satisfying (2.2) and

$$f([x, y, z]) = [f(x), f(y), f(z)] \qquad (2.17)$$

for all $x, y, z \in A$. If $\lim_{m\to\infty} f\left(\frac{\lambda^m e}{\lambda^m}\right) = e'$, then the mapping $f: A \to B$ is a unique C*-ternary isomorphism.

Proof. By the same method as in the proof of Theorem 2.2, we obtain a unique $\mathbb{C}$-linear mapping $H: A \to B$ by $H(x) = \lim_{m\to\infty} f\left(\frac{\lambda^m x}{\lambda^m}\right)$ for all $x \in A$ which satisfies (2.4). By (2.17),

$$H([x, y, z]) = \lim_{m\to\infty} \frac{f(\lambda^{3m}[x, y, z])}{\lambda^{3m}} = \lim_{m\to\infty} \left[\frac{f(\lambda^m x)}{\lambda^m}, \frac{f(\lambda^m x)}{\lambda^m}, \frac{f(\lambda^m x)}{\lambda^m}\right] = [H(x), H(y), H(z)]$$

for all $x, y, z \in A$. Then the mapping $H: A \to B$ is a C*-ternary homomorphism. It follows from $e \in A$ is unit and (2.17) that we have

$$H(x) = H([e, e, x]) = \lim_{m\to\infty} \frac{1}{\lambda^{2m}} f(\lambda^{2m}[e, e, x]) = \lim_{m\to\infty} \left[\frac{f(\lambda^m e)}{\lambda^m}, \frac{f(\lambda^m e)}{\lambda^m}, f(x)\right] = [e', e', f(x)] = f(x)$$

for all $x \in A$. Thus the bijective mapping $f: A \to B$ is a C*-ternary isomorphism. This completes the proof. ■

Corollary 2.7. Let $p > 1$, $s > 3$ and θ be positive real numbers, and $f: A \to B$ be a bijective mapping satisfying (2.2) and (2.17). If $\lim_{m\to\infty} \lambda^m f\left(\frac{e}{\lambda^m}\right) = e'$, then the mapping $f: A \to B$ is a unique C*-ternary isomorphism.

Proof. The proof is similar to the proofs Theorem 2.3 and 2.6. This completes the proof. ■

Theorem 2.8. Let $p = \sum_{j=1}^{n} |p_j| < 1$, $s < 3$ and θ be positive real numbers. Assume that $f: A \to B$ satisfies (2.10) and (2.17). If $\lim_{m\to\infty} f\left(\frac{\lambda^m e}{\lambda^m}\right) = e'$, then the mapping $f: A \to B$ is a unique C*-ternary isomorphism.

Proof. The proof is similar to the proofs Theorem 2.4 and 2.6. This completes the proof. ■

Corollary 2.9. Let $p = \sum_{j=1}^{n} |p_j| > 1$, $s > 3$ and θ be positive real numbers. Assume that $f: A \to B$ satisfies (2.10) and (2.17). If $\lim_{m\to\infty} \lambda^m f\left(\frac{e}{\lambda^m}\right) = e'$, then the mapping $f: A \to B$ is a unique C*-ternary isomorphism.

Proof. The proof is similar to the proofs Theorem 2.5 and 2.7. This completes the proof. ■

Stability of C*-ternary derivations

In this subsection, we investigate the generalized Hyers-Ulam stability of C*-ternary derivations on C*-ternary algebra A for the functional equation $D_\mu f(x_1, \cdots, x_n) = 0$.

Theorem 2.10. Let $p < 1$, $s < 3$ and θ be positive real numbers. Suppose that $f: A \to B$ is a mapping such that (2.2) and

$$\|f([x, y, z]) - [f(x), y, z] - [x, f(y), z] - [x, y, f(z)]\| \leq \theta\,(\|x\|^s + \|y\|^s + \|z\|^s) \qquad (2.18)$$

for all $x, y, z \in A$ and $\mu \in T^1$. Then there exists a unique derivation$D: A \to A$ such that

$$\|f(x) - D(x)\| \leq \frac{2\theta\,\|x\|^p}{\lambda^2 - \lambda^{p+1}} \qquad (2.19)$$

for all $x \in A$ and where $\lambda = n - 1$ with $n \geq 3$.

Proof. By the same method as in the proof of Theorem 2.2, there exists a unique $\mathbb{C}$-linear mapping $D: A \to A$ satisfying (2.19). The mapping is given by $D(x) = \lim_{m\to\infty} \frac{f(\lambda^m x)}{\lambda^m}$ for all $x \in A$. It follows from (2.18) that

$$\begin{aligned}
&\|D([x, y, z]) - [D(x), y, z] - [x, D(y), z] - [x, y, D(z)]\| \\
&= \lim_{m\to\infty} \left\| \frac{f(\lambda^{3m}[x, y, z])}{\lambda^{3m}} - \frac{1}{\lambda^{2m}}\left[\frac{f(\lambda^m x)}{\lambda^m}, \lambda^m y, \lambda^m z\right] - \frac{1}{\lambda^{2m}}\left[\lambda^m x, \frac{f(\lambda^m y)}{\lambda^m}, \lambda^m z\right] - \frac{1}{\lambda^{2m}}\left[\lambda^m x, \lambda^m y, \frac{f(\lambda^m z)}{\lambda^m}\right] \right\| \\
&\leq \lim_{m\to\infty} \frac{\theta\lambda^{mp}}{\lambda^{3m}} (\|x\|^s + \|y\|^s + \|z\|^s) = 0
\end{aligned}$$

for all $x, y, z \in A$. Then we have

$$D([x, y, z]) = [D(x), y, z] + [x, D(y), z] + [x, y, D(z)]$$

for all $x, y, z \in A$. Thus the mapping $D: A \to A$ is a unique C*-ternary derivation satisfying (2.19). ■

Corollary 2.11. Let $p > 1$, $s > 3$ and θ be positive real numbers, and $f: A \to A$ be a bijective mapping satisfying (2.2) and (2.18). Then there exists a unique C*-ternary derivation$D: A \to A$ such that

$$\|f(x) - D(x)\| \leq \frac{2\theta \|x\|^p}{\lambda^{p+1} - \lambda^2} \quad (2.20)$$

for all $x \in A$ and where $\lambda = n - 1$ with $n \geq 3$.

Proof. By the same method as in the proof of Theorem 2.3, there exists a unique $\mathbb{C}$-linear mapping $D: A \to A$ satisfying (2.20). The mapping is given by $D(x) = \lim_{m\to\infty} \lambda^m f\left(\frac{x}{\lambda^m}\right)$ for all $x \in A$. The rest of proof is similar method to proof of Theorem 2.10. This completes the proof. ■

Theorem 2.12. Let $p = \sum_{j=1}^{n} |p_j| \neq 1$, $s \neq 3$ and θ be positive real numbers. If $f: A \to A$ satisfies (2.10) and

$$\|f([x,y,z]) - [f(x), y, z] - [x, f(y), z] - [x, y, f(z)]\| \leq \theta (\|x\|^s \|y\|^s \|z\|^s) \quad (2.21)$$

for all $x, y, z \in A$ and $\mu \in T^1$, then there exists a unique C*-ternary derivation$D: A \to A$ such that

$$\|f(x) - D(x)\| \leq \begin{cases} \frac{2\theta \|x\|^p}{n(\lambda^2 - \lambda^{p+1})} & p < 1, s < 3 \\ \frac{2\theta \|x\|^p}{n(\lambda^{p+1} - \lambda^2)} & p > 1, s > 3 \end{cases} \quad (2.22)$$

for all $x \in A$ and where $\lambda = n - 1$ with $n \geq 3$.

Proof. The proof is similar to the proofs Theorem 2.4 and 2.10 if $p < 1, s < 3$, and Corollary 2.5 and 2.11 if $p > 1, s > 3$. This completes the proof. ■

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