A non-abelian model $SU(N)\times SU(N)$

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Abstract

A composite non-abelian model $SU(N)\times SU(N)$ is proposed as possible extension of the Yang-Mills symmetry. We obtain the corresponding gauge symmetry of the model and the most general lagrangian invariant by $SU(N)\times SU(N)$. The corresponding Feynman rules of the model are studied. Propagators and vertices are derived in the momentum space. As physical application, instead of considering the color symmetry $SU_c(3)$ for QCD, we substitute it by the combination $SU_c(3)\times SU_c(3)$. It yields a possibility to go beyond QCD symmetry in the sense that quarks are preserved with three colors. This extension provides composite quarks in triplets and sextets multiplets accomplished with the usual massless gluons plus massive gluons. We present a power counting analysis that satisfies the renormalization conditions as well as one studies the structure of radiative corrections to one loop approximation. Unitary condition is verified at tree level. Tachyons are avoided. For end, one extracts a BRST symmetry from lagrangian and Slavnov-Taylor identities. .

Keywords

Initials in Capitals; Separate with Semicolons.

I. Introduction

A non-Abelian model for composite fields is presented for investigating an extension to the Yang-Mills case [1]. It yields the possibility to explore an extended symmetry having contributions that go beyond to the Yang-Mills symmetry [2]. For example, the possible insertion of mass terms into the lagrangian no need breaking gauge symmetry for non-abelian gauge fields. The description of the interactions by means of composite fields has already been considered by J. Schwinger [3]. Also, others approaches by means of group composition were discussed for a description of the Lepton-Hadron interaction beyond Weinberg-Salam-Glashow electroweak model, for details *see* [4]. An interesting search on description of massive non-abelian gauge fields is given by use the Stueckelberg formalism as alternative to description of the Standard Model, by including Higgs mechanism in breaking spontaneous symmetry [5].

The possibility to go beyond Yang-Mills symmetry is presented in this work by considering the group $SU(N)$ as the combination $SU(N) \times SU(N)$. Based on direct product we define this operation between two independent non-abelian groups [6]. Notice that it is an approach diverse from usual direct product in grand unification. Instead of taking the Cartesian product of two groups $SU(N) \times SU(N)$, it considers a common gauge group $SU(3)$ being rotated by two fundamental representation, which means that we are just tensoring two fundamental representations of the same $SU(N)$. Physically interpreting, we shall propose a composite Quantum Chromodynamics $SU_c(3) \times SU_c(3)$ model.

This means that instead of only $SU_c(3)$ as proposed by M. Gell-Mann [7] and others, it realizes an extension to QCD in the sense that preserves the experimental result that quarks contain three colors. At this way introduces the possibility of having triplets and sextets quarks and it yields the presence of massive gluons together with the usual massless gluons case. The study of possible exotics Quantum Chromodynamics was discussed by [8, 9] in which it can reveal the existence of exotic barions in hadron spectroscopy.

The outline of the paper is organized as follows. The second section introduces the symmetry gauge of $SU(N) \times SU(N)$ with non-abelian gauge fields and quarks sectors, and shows the complete lagrangian invariant by those symmetry transformations. In section 3 we begin a program for renormalization of this model by analyzing the power counting and radiative corrections to one-loop approximation. In section 4 one extracts a BRST symmetry from effective quantum lagrangian and such symmetry leads to an Slavnov-Taylor for $SU(N) \times SU(N)$. These quantities are necessaries to full renormalization of the model. Finally, section 5 is left for the concluding remarks on the prospected model LHC possibilities.

II. A non-abelian model for symmetry $SU(N) \times SU(N)$

A. Gauge fields sector of $SU(N) \times SU(N)$

Consider a fermionic matter field χ composite by the direct product [2]

$$\chi = \psi \otimes \phi , \tag{1}$$

in which ψ_i and ϕ_i $(i = 1,2,...,N)$ are independents spinors and scalars fields both in the fundamental representation of each $SU(N)$ in question, respectively. The fields (ψ, ϕ) have independent local transformation

$$\psi^{'} = U_1(x)\psi \quad and \quad \phi^{'} = U_2(x)\phi \quad , \quad with \quad U_1(x) = e^{it_1^a \omega_1^a(x)} \quad and \quad U_2(x) = e^{it_2^a \omega_2^a(x)} , \tag{2}$$

where (t_1^a, t_2^a) are two independents generators of two non-abelian groups $SU(N)$, satisfying the commutation relations

$$[t_1^a, t_1^b] = if^{abc}t_1^c \quad and \quad [t_2^a, t_2^b] = if^{abc}t_2^c , \quad with \quad a = 1,2,...,N^2 - 1 , \tag{3}$$

and ω_1 and ω_2 are real functions. Notice that we have considered the same structure constant of group f^{abc} for the two independents groups. Using properties of the direct products and the transformations above, we obtain the local transformation $SU(N) \times SU(N)$

$$\chi^{'} = U(x)\chi \quad , \quad U(x) = U_1 \otimes U_2 . \tag{4}$$

Clearly, the spinor ψ only belongs to $SU(N)$-left and scalar ϕ belongs to $SU(N)$-right of the product $SU(N) \times SU(N)$, and χ is a fermion that belongs to full symmetry $SU(N) \times SU(N)$ whose components are

$\chi_i \ (i = 1, 2, ..., N^2)$.

Nextly one shall be interested in establish a dynamic for these fermions χ . For introducing the non-abelian gauge fields, a composite covariant derivative based on representation product is proposed

$$D_\mu(A, B) = D_\mu(A) \otimes \mathbf{1} + \mathbf{1} \otimes D_\mu(B) , \tag{5}$$

where each covariant derivative $D_\mu(A)$ and $D_\mu(B)$ act on ψ and ϕ, respectively

$$D_\mu(A)\psi = (\partial_\mu + ig_1 A_\mu) \psi \quad and \quad D_\mu(B)\phi = (\partial_\mu + ig_2 B_\mu) \phi . \tag{6}$$

It is remarkable to notice that the covariant derivative of (5) fulfills the requirement of satisfying the Jacobi identity. This is why we can undertake that $D_\mu(A, B)$ is actually a covariant derivative. Notice that we adopt here a different and alternative procedure instead of grouping the different gauge potentials inside a single covariant derivative. We propose a combined covariant derivative built up from a different covariant derivative for each group factor.

The gauge fields (A_μ, B_μ) transform in accord with

$$A_\mu' = U_1 A_\mu U_1^{-1} + \frac{i}{g_1} (\partial_\mu U_1) U_1^{-1} \quad and \quad B_\mu' = U_2 B_\mu U_2^{-1} + \frac{i}{g_2} (\partial_\mu U_2) U_2^{-1} , \tag{7}$$

in which (t_1^a, t_2^a) are basis of (A_μ, B_μ)

$$A_\mu = \sum_{a=1}^{N^2-1} A_\mu^a t_1^a \quad and \quad B_\mu = \sum_{a=1}^{N^2-1} B_\mu^a t_2^a , \tag{8}$$

respectively. The constants coupling g_1 and g_2 are associated to gauge fields (A_μ, B_μ), respectively. By using the definition (5) and some properties of direct product one gets the following transformation

$$D_\mu(A, B)' = U D_\mu(A, B) U^{-1} . \tag{9}$$

Therefore a symmetry $SU(N) \times SU(N)$ based on construction of direct product is established for gauge fields (A_μ, B_μ) and fermions χ . If one carries on the fields (A_μ, B_μ), it will be possible to introduce mass terms by already known mechanisms, like Higgs mechanism and Stueckelberg fields. However we intend to follow a different procedure.

As a next step we make the variable change

$$g_1 A_\mu^a = g_1 G_\mu^a + g_2 C_\mu^a \quad and \quad g_2 B_\mu^a = g_1 G_\mu^a - g_2 C_\mu^a , \tag{10}$$

in which (G_μ, C_μ) will be the physical fields that we are interested. Substituting (10) in (5), it yields

$$D_\mu(A, B)\chi = D_\mu(G, C)\chi = (\partial_\mu + ig_1 G_\mu + ig_2 C_\mu)\chi . \tag{11}$$

where the new fields (G_μ, C_μ) are Lie algebra valued on a new basis of generators, $\{T^a\}$ and $\{t^a\}$, as

$$G_\mu = \sum_{a=1}^{N^2-1} G_\mu^a T^a \quad and \quad C_\mu = \sum_{a=1}^{N^2-1} C_\mu^a t^a , \tag{12}$$

where

$$T^a = t_1^a \otimes \mathbf{1} + \mathbf{1} \otimes t_2^a \quad and \quad t^a = t_1^a \otimes \mathbf{1} - \mathbf{1} \otimes t_2^a . \tag{13}$$

Notice these new generators satisfy the commutation relation

$$T^a, T^b] = if^{abc} T^c \quad , \quad [t^a, t^b] = if^{abc} T^c \quad and \quad [T^a, t^b] = if^{abc} t^c , \tag{14}$$

which is just the same Lie algebra, but rewritten in another basis of generators. Consequently, $\{T^a\}$ and $\{t^a\}$ are the

independents generators basis for symmetry $SU(N) \times SU(N)$.

Now one needs to obtain the symmetry transformations for the physical fields (G_μ, C_μ), then we consider the transformation

$$D_\mu(G,C)^{'} = UD_\mu(G,C)U^{-1} ,$$ (15)

and using the equations above, one case is interested for us in which one chooses $\omega_1^a = \omega_2^a = \omega^a$, it seems to write the above transformation as solution in terms of transformations for (G_μ, C_μ)

$$G_\mu^{'} = UG_\mu U^{-1} + \frac{i}{g_1}(\partial_\mu U)U^{-1} \quad and \quad C_\mu^{'} = UC_\mu U^{-1} .$$ (16)

The infinitesimal transformations of the components (G_μ^a, C_μ^a) are

$$G_\mu^{'a} = G_\mu^a + f^{abc}G_\mu^b\omega^c - \frac{1}{g_1}\partial_\mu\omega^a \quad and \quad C_\mu^{'a} = C_\mu^a + f^{abc}C_\mu^b\omega^c .$$ (17)

The first transformation of (16) is just the usual case of a non-abelian gauge field. The second for C_μ is an unitary and homogeneous transformation that has interpretation of rotation of the field C_μ in the isospin space of the composite group $SU(N) \times SU(N)$. We will analyze the consequences of those transformations dictated by symmetry $SU(N) \times SU(N)$.

The case above mentioned $\omega_1^a = \omega_2^a = \omega^a$ it is important in order to introduce a mass term for the vector field C_μ into the lagrangian by respecting the invariance principle dictated by transformations (16), while the field G_μ remains massless like in the usual Yang-Mills symmetry. Therefore these model shows the introduction of a mass term with no necessity to establish a sector of Higgs scalar field.

Perhaps at this moment it should be more precise to change nomenclature. Instead of non-abelian $SU(N) \times SU(N)$ to consider a double $SU(N)$ under a common gauge parameter ω^a. By double $SU(N)$ one means to consider fields ψ_i and ϕ_i given by (1,2) as two fundamental representation rotating under the same group. One step ahead, we should also say that, by identifying ω_1^a and ω_2^a, the two quantum numbers of the different $SU(N)$ factors collapse into a unique quantum number. This then means that ψ_i and ϕ_i carry the same $SU(N)$-quantum number. Prior to identification, ψ_i and ϕ_i carrying quantum numbers of different natures. With $\omega_1^a = \omega_2^a$, the composite field χ and its charge is given by a combination of charges of same nature. In other words : ψ_i, ϕ_i and χ_i have all the same type of color, though their colours have different values.

For constructing the most general lagrangian invariant by those transformations (16) one catalogues all tensors that transform as

$$T \mapsto T^{'} = UTU^{-1} ,$$ (18)

like the strength field tensor $F_{\mu\nu}$ in usual Yang-Mills symmetry. At this way one derives the following fields strength tensors

$$ig_1F_{\mu\nu}(G) = [D_\mu(G), D_\nu(G)] , \quad f_{\mu\nu}(G,C) = [D_\mu(G), C_\nu] \quad and \quad C_{\mu\nu}(C) = g_3C_\mu C_\nu ,$$ (19)

where we have defined the usual covariant derivative

$$D_\mu(G) = \partial_\mu + ig_1G_\mu .$$ (20)

By rewriting those tensors into the basis generators components notation we obtain

$$F_{\mu\nu} = F_{\mu\nu}^a T^a \quad where \quad F_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_1 f^{abc}G_\mu^b G_\nu^c .$$ (21)

The second tensor has the mix between G_μ and C_μ

$$f_{\mu\nu} = f_{\mu\nu}^a t^a , \quad \text{with} \quad f_{\mu\nu}^a = \partial_\mu C_\nu^a - g_1 f^{abc} G_\mu^b C_\nu^c , \tag{22}$$

in which it is split into the antisymmetric and symmetric parts

$$f_{[\mu\nu]}^a = \partial_\mu C_\nu^a - \partial_\nu C_\mu^a - g_1 f^{abc} G_\mu^b C_\nu^c - g_1 f^{abc} C_\mu^b G_\nu^c , \tag{23}$$

and

$$f_{(\mu\nu)}^a = \partial_\mu C_\nu^a + \partial_\nu C_\mu^a - g_1 f^{abc} G_\mu^b C_\nu^c + g_1 f^{abc} C_\mu^b G_\nu^c , \tag{24}$$

respectively. The symmetrical part of $f_{\mu\nu}$ reveal a longitudinal propagation for the field C_μ beyond a transversal as a consequence of transformations (16). Indeed, the vector field C_μ can be interpreted as a Proca field with a longitudinal propagation beyond a transversal one. The third tensor is defined only in terms of C_μ for antisymmetric part

$$C_{[\mu\nu]} = C_{[\mu\nu]}^a T^a , \quad \text{with} \quad C_{[\mu\nu]}^a = g_3 f^{abc} C_\mu^b C_\nu^c , \tag{25}$$

and the symmetric part

$$C_{(\mu\nu)} = g_3 \{ C_\mu , C_\nu \} = g_3 C_\mu^a C_\nu^b \{ t^a , t^b \} = g_3 C_\mu^a C_\nu^b (4\delta^{ab} - 2t_1^a \otimes t_2^b - 2t_1^b \otimes t_2^a + d^{abc} T^c) , \tag{26}$$

in which g_3 is the constant coupling associated to self-interactions of massive non-abelian gauge field.

Now we can define a general tensor $Z_{\mu\nu}$ as linear combination of the tensors defined in (19), and we split in their antisymmetric and symmetric parts. Therefore we split $Z_{\mu\nu}$ in antisymmetric and symmetric parts

$$Z_{[\mu\nu]} = F_{\mu\nu} + af_{[\mu\nu]} + bC_{[\mu\nu]} \quad \text{and} \quad Z_{(\mu\nu)} = cf_{(\mu\nu)} + dz_{(\mu\nu)} + g_{\mu\nu}\left(ef_{(\alpha}^{\;\;\alpha)} + fz_{(\alpha}^{\;\;\alpha)} \right), \tag{27}$$

in which (a,b,c,d,e,f) are real parameters, thus

$$Z_{[\mu\nu]} = Z_{[\mu\nu]}^{(T)a} T^a + Z_{[\mu\nu]}^{(t)a} t^a , \tag{28}$$

where

$$Z_{[\mu\nu]}^{(T)a} = F_{\mu\nu}^a + bC_{[\mu\nu]}^a \quad \text{and} \quad Z_{[\mu\nu]}^{(t)a} = af_{[\mu\nu]}^a . \tag{29}$$

Similarly

$$Z_{(\mu\nu)} = Z_{(\mu\nu)}^{(T)a} T^a + Z_{(\mu\nu)}^{(t)a} t^a + Z_{(\mu\nu)}^{(\Lambda)ab} \Lambda^{ab} , \tag{30}$$

where

$$Z_{(\mu\nu)}^{(T)a} = dg_3 d^{abc} C_\mu^b C_\nu^c + fg_3 d^{abc} g_{\mu\nu} C_\alpha^b C^{\alpha c} , \quad Z_{(\mu\nu)}^{(t)a} = cf_{(\mu\nu)}^a + eg_{\mu\nu} f_{(\alpha}^{\;\;\alpha)a} ,$$

$$Z_{(\mu\nu)}^{(\Lambda)ab} = dg_3 C_\mu^a C_\nu^b + fg_3 g_{\mu\nu} C_\alpha^a C^{\alpha b} \quad \text{and} \quad \Lambda^{ab} = 4\delta^{ab} - 2t_1^a \otimes t_2^b - 2t_1^b \otimes t_2^a . \tag{31}$$

The real parameters (a,b,c,d,e,f) have introduced for a better control of all terms that contribute into the lagrangian, it will be important when one analyze the aspect of unitarity directly from the vector propagators. Here it was been convenient to define another base $\{\Lambda^{ab}\}$, beyond $\{T^a\}$ and $\{t^a\}$, in which it satisfy to commutation relations

$$\left[T^a , \Lambda^{bc} \right] = -2i\left[f^{abd}(t_1^d \otimes t_2^c + t_1^c \otimes t_2^d) + f^{acd}(t_1^b \otimes t_2^d + t_1^d \otimes t_2^b) \right],$$

$$\left[t^a , \Lambda^{bc} \right] = -2i\left[f^{abd}(t_1^d \otimes t_2^c - t_1^c \otimes t_2^d) + f^{acd}(t_1^b \otimes t_2^d - t_1^d \otimes t_2^b) \right]. \tag{32}$$

Now we are enable to write the most complete lagrangian invariant by symmetry transformations (16) as

$$L = -\frac{1}{4}tr(Z_{\mu\nu}Z^{\mu\nu}) + \frac{1}{2}m^2 tr(C_\mu C^\mu) - \frac{1}{2\xi}tr(\partial_\mu G_\mu)^2 - \frac{1}{4}\eta tr(\tilde{Z}_{\mu\nu}Z^{\mu\nu}) , \tag{33}$$

where we have taken into account the semi-topological term $\tilde{Z}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}Z^{\alpha\beta}$, that give us a non-trivial contribution and invariant by (16), and the lasts terms are contributions that is not presents in the earlier definitions of the tensors (19)-(??), with $\lambda_1 \in R$. Clearly, the semi-topological term brings out a CP violation in the model, but on general aspects we look for complete symmetry with the presence of all terms. The parameter η of (33) sets a parity violating regime. The mass term for gauge field C_μ has been introduced due to transformations (16), and one has chosen the gauge fixing term

$$L_{gf} = -\frac{1}{2\xi}(\partial_\mu G^{\mu a})^2 , \tag{34}$$

convenient to quantize the model hereafter, where $\xi \in$. Using the traces relations

$$tr(T^a T^b) = tr(t^a t^b) = N\delta^{ab} \quad , \quad tr(T^a t^b) = tr(T^a \Lambda^{bc}) = tr(t^a \Lambda^{bc}) = 0$$

$$and \quad tr(\Lambda^{ab}\Lambda^{cd}) = 4\delta^{ab}\delta^{cd} + 2\delta^{ac}\delta^{bd} + 2\delta^{ad}\delta^{bc} , \tag{35}$$

we can obtain all the free and interaction terms from the lagrangian (33). The lagrangian free part of the vector fields (G_μ, C_μ) is given by

$$L_{0G} = -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)^2 - \frac{1}{2\xi}(\partial_\mu G^{\mu a})^2 \quad and$$

$$L_{0C} = -\frac{a^2}{4}(\partial_\mu C_\nu^a - \partial_\nu C_\mu^a)^2 - \frac{c^2}{4}(\partial_\mu C_\nu^a + \partial_\nu C_\mu^a)^2 - 2e(c+2e)(\partial_\mu C^{\mu a})^2 + \frac{1}{2}m^2 C_\mu^a C^{\mu a} , \tag{36}$$

respectively. The Faddeev-Popov ghost lagrangian can be added by the usual methods using the infinitesimal transformations (17) and the gauge fixing term (34), then one gets

$$L_{FP} = \bar{\eta}^a(\delta^{ab}W)\eta^b + g_1 f^{abc}\partial_\mu \bar{\eta}^a G^{\mu c}\eta^b , \tag{37}$$

where $(\bar{\eta}, \eta)$ are Faddeev-Popov fields. From the lagrangian free part we calculate the (G_μ^a, C_μ^a) vector fields propagators by writing it into the form

$$L_0 = \frac{1}{2}G^{\mu a}(W\theta_{\mu\nu} + \xi^{-1}W\omega_{\mu\nu})G^{\nu a} + \frac{1}{2}C^{\mu a}[((a^2+c^2)W + m^2)\theta_{\mu\nu} + ((2c^2 + 2e(c+2e))W + m^2)\omega_{\mu\nu}]C^{\nu a} ,$$

$$\tag{38}$$

in terms of projection operators

$$\theta_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{W} \quad and \quad \omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{W} . \tag{39}$$

We invert the operators from (38) between fields to obtain the propagators in the momentum space. In the case of massless gauge fields is exactly like the propagator in Yang-Mills. For massive vector fields one gets

$$\langle C_\mu^a C_\nu^b \rangle = -i\delta^{ab}\left[\frac{1}{(a^2+c^2)k^2 - m^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \frac{1}{(2c^2 + 2e(c+2e))k^2 - m^2}\frac{k_\mu k_\nu}{k^2} \right], \tag{40}$$

in which one has observed two masses

$$\mu_1^2 = \frac{m^2}{a^2 + c^2} \quad and \quad \mu_2^2 = \frac{m^2}{2c^2 + 2e(c+2e)} , \tag{41}$$

for transversal part of spin-1, and another mass for longitudinal part of spin-0. The expressions of all propagators are showed in the figure (1).

$$\langle G_\mu^a G_\nu^b \rangle = -\frac{i\delta^{ab}}{k^2}\left[g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2} \right]$$

$$\langle C_\mu^a C_\nu^b \rangle = -\frac{i\delta^{ab}}{(a^2+c^2)k^2 - m^2}\left[g_{\mu\nu} + (a^2 - c^2 - 2e(c+2e))\frac{k_\mu k_\nu}{(2c^2+2e(c+2e))k^2 - m^2} \right]$$

$$\langle \bar{\eta}^a \eta^b \rangle = -i\frac{\delta^{ab}}{p^2}$$

Fig. 1: Feynman propagators of the vector fields and Ghosts.

In order to show an initial consistency of the model one should to observe the behavior of those propagators when $k \to \infty$, it goes to zero in ultraviolet regime. Differently from usual Proca's case, the massive field C_μ has a health behavior. Considering the unitarity one guarantees the positivity of the propagators residue if one imposes the following inequalities : $a^2 + c^2 > 1$ and $2c^2 + 2ce + 4e^2 < 1$, on those conditions the model is unitary at the tree level. Since (a,c) are real parameters, by imposing the above conditions there is no any possibility for emergence of tachyons propagation here.

It is also interesting to see the independency of the massive field propagator in relation to any parameters if we had introduced the most general gauge fixing

$$\mathsf{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu G^{\mu a} + \sigma\partial_\mu C^{\mu a})^2 , \tag{42}$$

with $\sigma \in \mathbb{R}$.

The vector fields and Faddeev-Popov interaction terms are given through the lagrangian terms

$$\mathsf{L}_I^{(3)} = g_1\partial_\mu G_\nu^a\left[G^\mu, G^\nu\right]^a + a^2 g_1\partial_\mu C_\nu^a\left(\left[G^\mu, C^\nu\right]^a + \left[C^\mu, G^\nu\right]^a\right)$$

$$+ bg_3\partial_\mu G_\nu^a\left[C^\mu, C^\nu\right]^a + c^2 g_1\partial_\mu C_\nu^a\left(\left[G^\mu, C^\nu\right]^a - \left[C^\mu, G^\nu\right]^a\right)$$

$$+ 4e(c+e)g_1\partial_\mu C^{\mu a}\left[G_\nu, C^\nu\right]^a , \tag{43}$$

and

$$\mathsf{L}_I^{(4)} = -\frac{1}{4}g_1^2[G_\mu, G_\nu]^a[G^\mu, G^\nu]^a - \frac{a^2}{2}g_1^2[G_\mu, C_\nu]^a\left([G^\mu, C^\nu]^a + [C^\mu, G^\nu]^a\right)$$

$$- \frac{b}{2}g_1 g_3[G_\mu, G_\nu]^a[C^\mu, C^\nu]^a - \frac{b^2}{4}g_3^2[C_\mu, C_\nu]^a[C^\mu, C^\nu]^a$$

$$- \frac{c^2}{2}g_1^2[G_\mu, C_\nu]^a\left([G^\mu, C^\nu]^a - [C^\mu, G^\nu]^a\right) - 4e^2 g_1^2[G_\mu, C^\mu]^a[G_\nu, C^\nu]^a$$

$$-\frac{1}{N}\left[\frac{3d^2}{2}+2f(d+2f)\right]g_3^2 C_\mu^a C_\nu^a C^{\mu b} C^{\nu b} -\frac{1}{N}\left[\frac{d^2}{2}+2f(d+2f)\right]g_3^2 C_\mu^a C_\nu^b C^{\mu a} C^{\nu b}$$

$$-\frac{d^2}{4}g_3^2 d^{abc}d^{ade}C_\mu^b C_\nu^c C^{\mu d} C^{\nu e} -\left(\frac{d^2}{4}+f^2\right)g_3^2 d^{abc}d^{ade}C_\mu^b C^{\mu c}C_\nu^d C^{\nu e}. \tag{44}$$

for the three-line and four-line vertex, respectively. The Feynman rules for vertex are obtained in the momentum space

$$iV_{(G)\mu\nu\rho}^{(3)abc}(k_1,k_2,k_3)=g_1 f^{abc}\left[g_{\mu\nu}(k_1-k_2)_\rho + g_{\nu\rho}(k_2-k_3)_\mu + g_{\mu\rho}(k_3-k_1)_\nu\right]$$

$$iV_{(G)\mu\nu\rho\sigma}^{(4)abcd}=-ig_1^2\left[f^{eab}f^{ecd}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})\right.$$
$$\left.+f^{eac}f^{ebd}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\rho\nu})+f^{ead}f^{ebc}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma})\right]$$

Fig. 2: Three and four lines vertices for massless gauge fields. It is just usual Yang-Mills vertices.

$$iV^{(3)abc}_{(GC)\mu\nu\rho}(k_1,k_2,k_3) = ibg_3 f^{abc}(g_{\nu\rho}k_{2\mu} - g_{\mu\nu}k_{2\rho})$$

$$+(a^2+c^2)g_1 f^{abc}g_{\mu\rho}(k_1-k_3)_\nu + (a^2-c^2)g_1 f^{abc}(g_{\mu\rho}k_{1\nu} - g_{\nu\rho}k_{2\mu})$$

$$+4e(c+e)g_1 f^{abc}(g_{\nu\rho}k_{1\mu} - g_{\mu\nu}k_{3\rho})$$

$$iV^{(4)abcd}_{(GC)\mu\nu\rho\sigma} = ibg_3 g_1 f^{eab} f^{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$

$$-(a^2+c^2)ig_1^2 \, g_{\mu\nu}g_{\rho\sigma}(f^{eac}f^{ebd} + f^{ead}f^{ebc})$$

$$+(a^2-c^2)ig_1^2 \, (f^{eac}f^{ebd}g_{\mu\sigma}g_{\nu\rho} + f^{ead}f^{ebc}g_{\mu\rho}g_{\nu\sigma})$$

$$-i8e^2 g_1^2 \, (f^{eab}f^{ecd}g_{\mu\nu}g_{\rho\sigma} - f^{ead}f^{ebc}g_{\mu\sigma}g_{\nu\rho})$$

$$iV^{(4)abcd}_{(C)\mu\nu\rho\sigma} = -ib^2 g_3^2 \, [f^{eab}f^{ecd}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$

$$+f^{eac}f^{ebd}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\rho\nu})$$

$$+f^{ead}f^{ebc}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] - \frac{6d^2+8f(d+f)}{N}ig_3^2 \, [\, \delta^{ab}\delta^{cd}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) + \delta^{ac}\delta^{bd}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

$$+\delta^{ad}\delta^{bc}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma})] - \frac{4d^2+16f(d+f)}{N}ig_3^2 \, (\delta^{ab}\delta^{cd}g^{\mu\nu}g^{\rho\sigma} + \delta^{ac}\delta^{bd}g^{\mu\rho}g^{\nu\sigma} + \delta^{ad}\delta^{bc}g^{\mu\sigma}g^{\nu\rho})$$

$$-d^2 ig_3^2[d^{eab}d^{ecd}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) + d^{eac}d^{ebd}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\rho\nu}) + d^{ead}d^{ebc}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma})]$$

$$-(d^2+4f^2)ig_3^2[d^{eab}d^{ecd}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) + d^{ead}d^{ebc}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\rho\nu}) + d^{eac}d^{ebd}(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma})]$$

Fig. 3: New vertices of the $SU(N)\times SU(N)$ symmetry mixing massless and massive vector fields.

$$i\widetilde{V}^{(3)abc}_{(GC)\mu\nu\rho}(k_1,k_3) = a^2 g_1 f^{abc}\varepsilon_{\alpha\mu\nu\rho}(k_1+k_3)^\alpha$$

$$i\widetilde{V}^{(4)abcd}_{(GC)\mu\nu\rho\sigma} = -a^2 ig_1^2 \varepsilon_{\mu\nu\rho\sigma}(f^{ead}f^{ebc} - f^{eac}f^{ebd})$$

Fig. 4: Contributions of the semitopological terms.
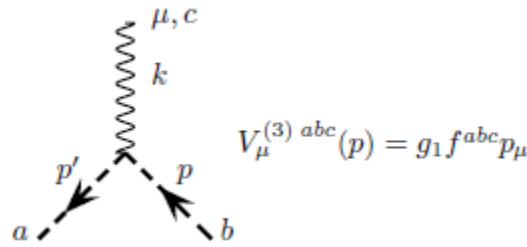
$$V_\mu^{(3)\,abc}(p) = g_1 f^{abc} p_\mu$$

Fig. 5: The Faddeev-Popov vertex.

B. Quarks sector for symmetry $SU_c(3) \times SU_c(3)$

In the last subsection we have seen the construction of a composite gauge symmetry $SU(N) \times SU(N)$ and one has concentred just in the sector of vector fields (G_μ, C_μ). Here one shall present the fermions sector to complete the lagrangian (33) by adding the fermionic fields. Our construction initial was based on a fermionic field χ, that is a composition of a fermion ψ and a scalar ϕ, but we will be interested in establishing a dynamic for field χ. Therefore the lagrangian for fermions sector of $SU(N) \times SU(N)$ is coupling those fermions represented by χ to covariant derivative (11)

$$\mathsf{L}_{fermions} = \overline{\chi}\left[i\gamma^\mu D_\mu(G,C) - m_\chi \mathbf{1}\right]\chi , \tag{45}$$

where m_χ is the fermion mass, and clearly χ is a fermion field of N^2 components. In accord with the symmetry $SU(N) \times SU(N)$, the field χ can be split into the components

$$q_i \quad with \quad i = 1,2,...,\frac{N(N-1)}{2} \quad and \quad Q_i \quad with \quad i = 1,2,...,\frac{N(N+1)}{2} , \tag{46}$$

where $N \times N = \dfrac{N(N-1)}{2} \oplus \dfrac{N(N+1)}{2}$. The components of q and Q have the following transformations

$$q_i \mapsto q_i^{'} = (e^{i\omega^a t^a})_{ij} q_j \quad and \quad Q_i \mapsto Q_i^{'} = (e^{i\omega^a \Lambda^a})_{ij} Q_j , \tag{47}$$

in which t^a and Λ^a are square matrices $\dfrac{N(N-1)}{2} \times \dfrac{N(N-1)}{2}$ and $\dfrac{N(N+1)}{2} \times \dfrac{N(N+1)}{2}$, both in the fundamental representation of $SU(N)$. Now one wishes introducing different masses for the two sets of fermions q and Q, but the massive term of (45) is incompatible with this requirement because the field χ is a mixing of q and Q. Consequently, the set of transformations (47) and (2) form our fermionic sector of $SU(N) \times SU(N)$, so we propose the lagrangian invariant by those transformations

$$\mathsf{L}_{fermions} = \overline{\chi} i\gamma^\mu D_\mu(G,C)\chi - m_q \overline{q}q - m_Q \overline{Q}Q , \tag{48}$$

where m_q and m_Q are masses of q and Q, respectively.

$$V_G^{\mu a} = -ig_1 \gamma^\mu T^a$$

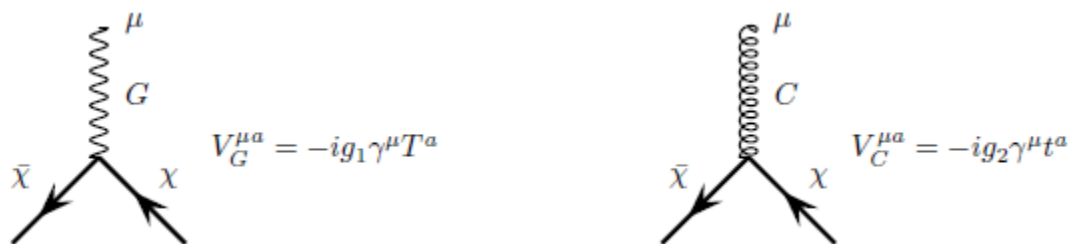$$V_C^{\mu a} = -ig_2 \gamma^\mu t^a$$

Fig. 6: Fermions vertices interacting with massless and massive vector fields.

Now we will apply it to the particular case $SU_c(3) \times SU_c(3)$ which can be interesting due to preserve the QCD color symmetry. The vector fields sector is like those showed earlier, but now we have eight massless gauge field plus eight massive vector fields (G_μ^a, C_μ^a) with $a = 1,2,...,8$. Both can be interpreted as eight massless gluons plus eight massive gluons. The sixteen matrices $\{T^a\}$ and $\{t^a\}$ $(a = 1,2,...8)$ build up the basis for gluons massless and massive gluons, respectively, being the combinations (13) of the Gell-Mann matrices [11] of group $SU(3)$ in the fundamental representation

$$T^a = \frac{\lambda^a}{2} \otimes \mathbf{1} + \mathbf{1} \otimes \frac{\lambda^a}{2} \quad and \quad t^a = \frac{\lambda^a}{2} \otimes \mathbf{1} - \mathbf{1} \otimes \frac{\lambda^a}{2}, \tag{49}$$

satisfying the commutation relation (14), and the structure constant of group f^{abc} listed by table in [11].

The sector of the composite quarks is defined by the direct product [2]

$$\chi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{50}$$

in which the quark is a composition of fermions $\psi = (\psi_1, \psi_2, \psi_3)$ and scalars $\phi = (\phi_1, \phi_2, \phi_3)$, of three colors each, both in the fundamental representation. Clearly, these product is a column matrix of nine components. It can be split in quarks triplets

$$q_i = \frac{1}{\sqrt{2}} \varepsilon_{ijk} \psi_j \phi_k, \quad with \quad i,j,k = 1,2,3, \tag{51}$$

where ε_{ijk} is Levi-Civita symbol, and sextets quarks are defined by

$$\Xi_{(ij)} = \frac{1}{\sqrt{2}} (\psi_i \phi_j + \psi_j \phi_i), \quad with \quad i,j = 1,2,3, \tag{52}$$

in accord with the components

$$\sqrt{2}Q_1 := \Xi_{(11)} = \sqrt{2}\psi_1\phi_1 \quad , \quad \sqrt{2}Q_4 := \Xi_{(22)} = \sqrt{2}\psi_2\phi_2 \quad , \quad \sqrt{2}Q_6 := \Xi_{(33)} = \sqrt{2}\psi_3\phi_3$$

$$Q_2 := \Xi_{(12)} = \frac{1}{\sqrt{2}} (\psi_1\phi_2 + \psi_2\phi_1) \quad , \quad Q_3 := \Xi_{(13)} = \frac{1}{\sqrt{2}} (\psi_1\phi_3 + \psi_3\phi_1)$$

$$Q_5 := \Xi_{(23)} = \frac{1}{\sqrt{2}} (\psi_2\phi_3 + \psi_3\phi_2), \tag{53}$$

then, one obtains

$$\chi = \begin{pmatrix} Q_1 \\ \dfrac{1}{\sqrt{2}}(q_3 + Q_2) \\ \dfrac{1}{\sqrt{2}}(Q_3 - q_2) \\ \dfrac{1}{\sqrt{2}}(Q_2 - q_3) \\ Q_4 \\ \dfrac{1}{\sqrt{2}}(q_1 + Q_5) \\ \dfrac{1}{\sqrt{2}}(Q_3 + q_2) \\ \dfrac{1}{\sqrt{2}}(Q_5 - q_1) \\ Q_6 \end{pmatrix}. \tag{54}$$

Thus, introducing the definition where q and Q are respectively the column matrices of triplets and sextets quarks

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad and \quad Q = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{pmatrix}, \tag{55}$$

one gets,

$$\mathsf{L}_{(q,Q)} = \mathsf{L}_{0(q,Q)} + \mathsf{L}_{Int}, \tag{56}$$

where

$$\mathsf{L}_{0(q,Q)} = \bar{q}(i\gamma^\mu \partial_\mu - m_q)q + \overline{Q}(i\gamma^\mu \partial_\mu - m_Q)Q, \tag{57}$$

in which m_q and m_Q set the masses of triplets and sextets, respectively. The triplets and sextets have symmetry transformations given by (47), in which $t^a = \lambda^a/2 \ (a = 1,2,...,8)$ are Gell-Mann matrices, and $\Lambda^a \ (a = 1,2,...,8)$ are square matrices 6×6 listed in [2]. Intuitively, one expects massive sextets quarks will appear at higher energies than usual QCD.

$$\langle \bar{q}_i q_j \rangle = \frac{i\delta_{ij}}{\not{p} - m_q + i\varepsilon} \qquad \langle \overline{Q}_i Q_j \rangle = \frac{i\delta_{ij}}{\not{p} - m_Q + i\varepsilon} \qquad \overset{i}{\underset{p}{\longrightarrow}} \overset{j}{}$$

Fig. 7: Quarks propagators.

The interactions terms involving the triplets, sextets quarks and gluons are

$$\mathsf{L}^I_{qQG} = -g_1 \bar{\chi}_i G^a_\mu (T^a)_{ij} \chi_j = -\frac{1}{2} g_1 \bar{q} \gamma^\mu \begin{pmatrix} G^3_\mu + \dfrac{G^8_\mu}{\sqrt{3}} & G^1_\mu - iG^2_\mu & G^4_\mu - iG^5_\mu \\[2mm] G^1_\mu + iG^2_\mu & G^3_\mu - \dfrac{G^8_\mu}{\sqrt{3}} & G^6_\mu - iG^7_\mu \\[2mm] G^4_\mu + iG^5_\mu & G^6_\mu + iG^7_\mu & -\dfrac{2G^8_\mu}{\sqrt{3}} \end{pmatrix} q$$

$$-\frac{1}{2} g_1 \bar{Q} \gamma^\mu \begin{pmatrix} G^3_\mu + \dfrac{G^8_\mu}{\sqrt{3}} & G^1_\mu + iG^2_\mu & G^4_\mu + iG^5_\mu & G^4_\mu + iG^5_\mu & 0 & 0 \\[2mm] G^1_\mu - iG^2_\mu & \dfrac{2G^8_\mu}{\sqrt{3}} & G^6_\mu + iG^7_\mu & 0 & G^4_\mu + iG^5_\mu & 0 \\[2mm] G^4_\mu - iG^5_\mu & G^6_\mu - iG^7_\mu & G^3_\mu - \dfrac{G^8_\mu}{\sqrt{3}} & 0 & G^1_\mu + iG^2_\mu & G^4_\mu + iG^5_\mu \\[2mm] 0 & G^1_\mu - iG^2_\mu & 0 & G^3_\mu - \dfrac{G^8_\mu}{\sqrt{3}} & G^6_\mu + iG^7_\mu & 0 \\[2mm] 0 & G^4_\mu - iG^5_\mu & G^1_\mu - iG^2_\mu & G^6_\mu - iG^7_\mu & -G^3_\mu - \dfrac{G^8_\mu}{\sqrt{3}} & G^6_\mu + iG^7_\mu \\[2mm] 0 & 0 & G^4_\mu - iG^5_\mu & 0 & G^6_\mu - iG^7_\mu & -\dfrac{2G^8_\mu}{\sqrt{3}} \end{pmatrix} Q , \quad (58)$$

and

$$\mathsf{L}^I_{qQC} = -g_2 \bar{\chi}_i C^a_\mu (t^a)_{ij} \chi_j = -\frac{1}{2} g_2 \bar{Q} \gamma^\mu \begin{pmatrix} 0 & -C^4_\mu - iC^5_\mu & -C^1_\mu - iC^2_\mu \\[2mm] -C^4_\mu - iC^5_\mu & C^6_\mu + iC^7_\mu & 0 \\[2mm] C^1_\mu + iC^2_\mu & 0 & -C^6_\mu + iC^7_\mu \\[2mm] -C^6_\mu - iC^7_\mu & 0 & C^1_\mu - iC^2_\mu \\[2mm] 0 & 0 & C^4_\mu - iC^5_\mu \\[2mm] C^6_\mu - iC^7_\mu & -C^4_\mu + iC^5_\mu & 0 \end{pmatrix} q$$

$$-\frac{1}{2} g_2 \bar{q} \gamma^\mu \begin{pmatrix} 0 & -C^4_\mu + iC^5_\mu & 0 & -C^6_\mu + iC^7_\mu & 0 & C^6_\mu + iC^7_\mu \\[2mm] C^4_\mu - iC^5_\mu & C^6_\mu - iC^7_\mu & -C^3_\mu - \sqrt{3}C^8_\mu & 0 & -C^1_\mu - iC^2_\mu & -C^4_\mu - iC^5_\mu \\[2mm] -C^1_\mu + iC^2_\mu & 0 & 0 & C^1_\mu + iC^2_\mu & C^4_\mu + iC^5_\mu & 0 \end{pmatrix} Q . \quad (59)$$

$$V^{\mu a}_{G\, ij} = -ig_1 \gamma^\mu (T^a)_{ij} \qquad V^{\mu a}_{G\, ij} = -ig_1 \gamma^\mu (T^a)_{ij}$$
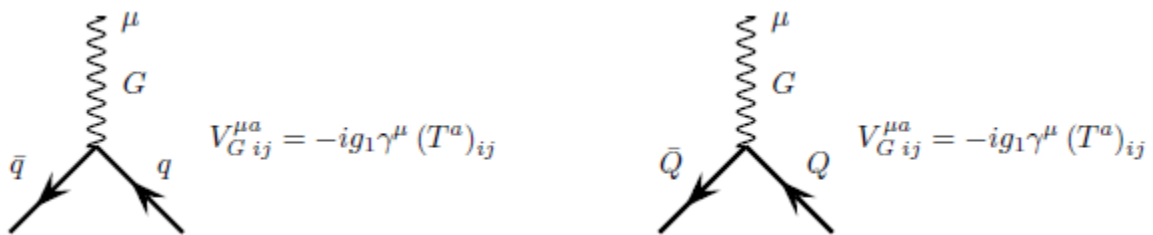
Fig. 8: Vertices of triplets and sextets Quarks interacting with massless gluons.
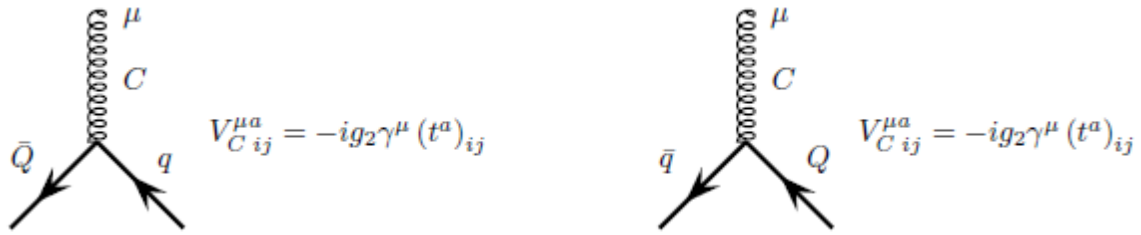
Fig. 9: Vertices of triplets and sextets Quarks interacting with massive gluons.

The constants coupling g_1 and g_2 sets the interaction between massless gluons and massive gluons with quarks, respectively. These terms show the interaction between triplets quarks intermediates by massless gluons like in usual QCD, furthermore the sextets quarks also interact by means of massless gluons. In the case of massive gluons, they appear just as mediators of the interactions between triplet and sextets of quarks.

For end of this section we resume what one has obtained here. Thus we have got a complete effective quantum lagrangian for symmetry $SU(N) \times SU(N)$ governed by transformations (16), where vector fields, Faddeev-Popov and quarks sectors are given by

$$\mathsf{L}_{eff} = \mathsf{L}_{vector\ fields} + \mathsf{L}_{FP} + \mathsf{L}_{gauge fixing} + \mathsf{L}_{quarks}. \tag{60}$$

These composition of fields is a way to introduce sextets of quarks beyond already known triplets of quarks in the particular case $SU_c(3) \times SU_c(3)$. For case $SU_c(3) \times SU_c(3)$, one gets eight massless gluons plus eight massive gluons by self-interacting and with triplets and sextets of quarks. Curiously, anti-triplets and sextets (or anti-sextets and triplets) of quarks interact by means of massive gluons only, while the massless gluons are just mediators of the interaction of anti-triplets with triplets, or anti-sextets with sextets.

III. Remarks on renormalization

In this section one begins an essential program to establish the full renormalization of the model $SU(N) \times SU(N)$. We have obtained the complete lagrangian invariant by transformations (16), and here, we will analyze the corresponding power counting. The structure of the radiative corrections has Feynman integrals divergent that behave like those in the Yang-Mills symmetry.

C. Power counting

The analysis of power counting is useful to indicate us on behavior of all possible Feynman diagrams that contribute to the model in higher order in the perturbation series. Based on structure of propagators and vertices have been showed earlier, we shall obtain the superficial divergence degree D of any Feynman diagrams that those symmetry permits. Hence one defines the following notation for external and internal lines, vertices, loops of massless, massive gluons, quarks and ghosts :

D=Superficial divergence degree for any Feynman diagram

2ω=Space−time dimension

L=Number of loops

I_G=Number of internal lines of massless gluons

I_C=Number of internal lines of massive gluons

I_g=Number of internal lines of ghosts

I_χ=Number of internal lines of quarks

V_{3G}=Three line vertex of massless gluons

V_{4G}=Four line vertex of massless gluons

V_{3GC}=Three line vertex that mix massless and massive gluons

V_{4GC}=*Four line vertex that mix massless and massive gluons*

V_{4C}=*Four line vertex of massive gluons*

V_g=*Vertex of ghost interaction*

$V_{\chi G}$=*Vertex of quarks and massless gluons interaction*

$V_{\chi C}$=*Vertex of quarks and massive gluons interaction*

E_G=*Number of external lines of massless gluons*

E_C=*Number of external lines of massive gluons*

$$E_\chi = Number\ of\ external\ lines\ of\ quarks\ . \tag{61}$$

With all those definitions, the divergence degree of any Feynman graphic has the expression

$$D = 2\omega L - 2I_G - 2I_C - 2I_g - I_\chi + V_{3G} + V_{3GC} + V_g\ , \tag{62}$$

the number of loops is

$$L = I_G + I_C + I_g + I_\chi - V_{3G} - V_{4G} - V_{3GC} - V_{4GC} - V_{4C} - V_g - V_{\chi G} - V_{\chi C} + 1\ , \tag{63}$$

and the topological relations

$$2I_G + E_G = 4V_{4G} + 3V_{3G} + 2V_{4GC} + V_{3GC} + V_g\ ,$$

$$2I_C + E_C = 2V_{3GC} + 2V_{4GC} + 4V_{4C}\ ,$$

$$2I_\chi + E_\chi = 2V_{\chi G} + 2V_{\chi C}\ , \tag{64}$$

By substituting (63) and (??) in (62), we find the divergence degree D in terms of external lines and vertices

$$D = 2\omega + (1-\omega)(E_G + E_C) + \left(\frac{1}{2} - \omega\right)E_\chi + (2\omega - 4)\left(V_{4G} + \frac{1}{2}V_{3G} + \frac{1}{2}V_g + \frac{1}{2}V_{3GC} + V_{4GC} + V_{4C}\right),$$
$$(65)$$

where we have used that there is no any external ghost, so $I_g = V_g$. These result give us a important interpretation in the case of the physical dimension $\omega = 2$. In this case the expression (65) depends on external lines of massless and massive gluons only

$$D = 4 - E_G - E_C - \frac{3}{2}E_\chi\ , \tag{66}$$

The equation (66) is a great indication that this model is renormalizable in four-dimension, like in the usual Yang-Mills symmetry. The dimensionality analysis for constants coupling (g_1, g_2, g_3) is totally analogous to the $SU(N)$ case. Consider the generator functional

$$Z : \int DG DC D\bar{\eta} D\eta \exp\left[i\int d^{2\omega}x\, L_{total}\right], \tag{67}$$

where we have substituted the physical dimension by the dimensional regularizator. Clearly if the action is dimensionless, the fields dimensions are given by

$$[G] = [C] = [\eta] = [\Lambda]^{\omega-1}\ , \tag{68}$$

in which $[\Lambda]$ sets a mass dimension parameter. By using those relations in the interactions terms of (??) and (??) one gets the dimension of coupling constants

$$[g_1] = [g_2] = [g_3] = [\Lambda]^{2-\omega}. \tag{69}$$

These relations shows that in the case of physical dimension $\omega = 2$ all coupling constants of this model are dimensionless, it is another statement that establish the renormalizability of the model. In the next subsection one shall present the perturbative character of the model by writing all contributions to one-loop for all propagators and vertex. These contributions are clearly divergent, but their divergent structure has a behavior controllable like in the usual Yang-Mills case.

Conclusions

We have studied a Yang-Mills extension based on composition of two independents non-abelian groups $SU(N)$. This symmetry is constructed in such a way that fermions are composite of a direct product between others fermions and scalars. It yields a gauge sector where fields follow (16) transformations. The first one is just gauge transformation of a non-abelian field G_μ Lie algebra valued in a given basis, while the second is just an unitary massive vector field C_μ Lie algebra valued in a second basis. Under this deduction, one intends to go beyond QCD through lagrangian (60). It introduces new possibilities beyond those already known from usual Yang-Mills symmetry.

The first effort of this work is to show the lagrangians (33) and (45) validity for perturbation theory. In a previous work one has proved on its hamiltonian positivity [2]. Given such stability for perturbative approach a next step should be to study on its unitarity and renormalizability. The unitarity at the tree level of model is satisfied by establishing conditions between the parameters (a, c) that set the transversal and longitudinal parts, in accord with positivity of residue into the propagator of C_μ. The equation (66) indicates renormalizability in terms of power counting. Thus one gets a health model in terms of perturbation theory. Its hamiltonian is not negative, the correspondent power counting analysis supports renormalizability, unitarity is satisfied at tree level and free of anomalies. Consequently, it is a model candidate for being studied under Callan-Symmanzik equation.

The full renormalizability of the model is something to be studied in the next paper. Here we have presented the BRST symmetry and Slavnov-Taylor identities as a beginning way, in which one shall enable to establish relationship between the one-particle irreducible Green functions. Thus as a next effort we will calculate the Feynman diagrams to one loop approximation and so to realize a study on renormalization group and the Callan-Symanzik beta function for $SU(N) \times SU(N)$. Considering the trilinear vertices abundance one expects that in this case it be more asymptotically free than those QCD usual case. The addition of composite scalars quarks on behavior of beta function must also analyzed, that is, if this model is asymptotically free in the presence of scalars quarks under a lagrangian $(D_\mu \Phi_i)^2$ where Φ_i scalar field is constituted by scalar colorful stones $\Phi_i = f_{ijk} \phi_i \phi_j$ [2]. The calculus of deep inelastic scattering of $SU_c(3) \times SU_c(3)$ is something to be investigated and the influence due to the presence of scalar quarks and massive gluons on results.

Thus a new color phenomenology is proposed for LHC. Based on $SU_c(3)$ symmetry there is new suggestions for the colorful world. The model $SU_c(3) \times SU_c(3)$ or double $SU_c(3)$ contains QCD and extends it for composite quarks (fermionic and bosonic) in triplets and sextets, massless and massive gluons (transverse and longitudinal) as another possibilities derived from the Twelve colorful stones table in [2]. It provides a dynamics for quarks and leptons with different spins. In terms of interactions, one expects a weaker running coupling constant than QCD due to a larger number of three and four gluons vertices. This means that there is a colorful weak interaction for being investigated theoretically. It is a model where one replaces colorful massive gluons instead of $(W^\pm, Z^0)$ as the intermediate bosons for flavors exchange.

Thus, given LHC new energy range, one expects new experimental possibilities for the colorful world. Instead of just following the pattern with quarks in triplets and eight gluons there is a new colorful diversity to be measured from an origin based on twelve colorful stones coupled to a double symmetry $SU_c(3)$. As a new phenomenological sector, one expects massive gluons, scalar quarks, quarks in sextets with different masses than the usual ones, massive glueballs and also a new variety of exotic mesons and barions as Θ^+ to be detected by LHC [9].

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