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Spin-1/2 Charged Particle in 1+4 dimensions with N=1-Supersymmetry

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ABSTRACT

We study the dynamics of a charged spin-(1/2) particle in an external 5-dimensional electromagnetic field. We then consider that we are at the TeV scale, so that we can access the fifth dimension and carry out our physical considerations in a 5-dimensional brane. In this brane, we focus our attention to the quantum-mechanical dynamics of the charged particle minimally coupled to the 5-dimensional electromagnetic field. We propose a way to identify the Abraham-Lorentz back reaction force as an effect of the extra (fifth) dimension. In another regime of fields allowed by the model, a massive charged particle (CHAMP) behavior can be, in which the bulk electric field play a crucial role.

Indexing terms/Keywords

Brane world, extra-dimensions, supersymmetry.

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1 Introduction

Supersymmetry (SUSY) has emerged as a proper response in order to obtain a unified view of all interactions in Nature. The contribution of SUSY is a possible answer to the gauge hierarchy problem, i.e., it seeks to explain why the energy of the symmetry breaking from the electroweak interaction is so low as compared to grand unification theory (GUT) scale. SUSY controls very well the divergences which emerge from the Standard Model (SM) and leads the symmetry breaking scale to the region of GUT. Another aspect in which the SUSY seems to be promising is related to the intensities of the three fundamental forces: the SM intensities are led to similar values at high temperatures, but never converge to the same magnitude of the interactions. However, by including the supersymmetry to the model, the intensities of the three forces converge to values very close when they are upon a regime of energies around 10^{16}GeV. This extension of the SM is known as minimally supersymmetric standard model (MSSM).

Mainly due to the discoveries done by Wilkinson Microwave Anisotropy Probe (WMAP), we should explain the presence of a hypothetical form of matter which we refer to as cold dark matter (CDM), that interacts very weakly with the electromagnetic radiation and most of its particles move slowly compared to the speed of light (cold) [1-4].

Supersymmetry provides a more natural framework for dark matter if we consider a mechanism of symmetry breaking [5], [6], [7]. If supersymmetry is indeed a symmetry of Nature, it can be better justified by its theoretical context: the emergence of a mass scale and hence the emergence of a split in the energy band known as mass gap. For that reason, a portion of matter in the Universe is not observed with the currently accessible energy scales. In fact, what is to be observed - from the cosmological point of view - should be the SUSY low-energy remnant [8]. A mechanism of symmetry breaking comes to justify the absence of concrete manifestation of a portion of supersymmetric particles in Nature.

On the other hand, the hypothesis of extra dimensions to model the fundamental forces of nature dates back to the early days of general relativity. The original theory of Kaluza-Klein sought to explain the gravitation from electromagnetism in a five space-time dimensions [9]. At present, the multidimensional approach of space-time has appeared attached to models of strings [10] or branes [11,12] and those models are being treated as possible alternatives to overcome the hierarchy problem. Those ways of handling the problem are based on the notion that matter can be restricted to a 3-brane embedded into a manifold with more than three spatial dimensions.

We plan in this paper to explore eventual remnants of the minimal supersymmetric standard model (MSSM) whose phase transition occurs at approximately 10^9 to 10^{11}GeV, if gravity mediates the breaking to the TeV scale, or at around 10^6GeV if it is a gauge mediation [8]. Within the context of this study, the scale of 10 TeV is actually a range of low energies; although when compared with the current capabilities of the particle accelerators is perceived as a high energy level.

In order to probe eventual remaining effects of the MSSM, we intend to establish a very particular physical scenario. We shall adopt Quantum Mechanics associated with a Supersymmetry, i.e, Supersymmetric Quantum Mechanics.

As our physical scenario, let us consider that a charged spin-1/2 particle massive enough to be treated as non-relativistic. This particle will have a high kinetic energy, but not enough to break the threshold for pair formation.

In addition, the particle will be subjected to the inherent magnetic field present in the particle accelerator (~10T) and being under this level of energy (~1TeV), we might also expect some manifestation [13] arising from the hypothesis of extra dimension.

The magnitude of the magnetic field is such that quantum-mechanical treatment is not necessary: the cyclotron radius is larger than the Compoton wavelength of our test particle. Actually, for a particle with rest mass of the order of 1 TeV, the critical magnetic field for quantum effects to become important would be of the order of 10^{24}G [18], which is never produced, even at astrophysical environments. Therefore, in our considerations here, we need not adopt a quantum-mechanical description. Therefore, we proceed to a discussion of the classical equations of motion.

To investigate the possibility that QED in (1+3)-D, embedded in a submanifold of a (1+4)-D brane, in this paper, we study Maxwell´s electrodynamics in the 5-D scenario. The analysis of this dynamics in (1+4)-D dimensions gives us the possibility to describe a hidden sector of Electromagnetism which we can conjecture as dark energy, and a particle coupled to this field is a candidate to dark matter.

One question we address to in our study of the motion of a charged particle in a 5-dimensional supersymmetric scenario consists in checking how the electromagnetic interactions can control its motion in the 3-dimensional spatial sector corresponding to our world or may take it to the bulk [12]. If it is driven to the bulk , ($x^1=x^2=x^3=0, x^4\neq 0$), we can understand under what circumstances the external electromagnetic fields in 5-D determine that the particle behaves as dark matter. On the other hand, this discussion may guide us to relate the back radiation force in (1+3)-D with the mechanical power of the magnetic force in 5-D.

In this letter, we are going to pursue an investigation of a Maxwell electrodynamics in (1+4) dimensions and, in this frame, we shall be discussing particular aspects of the dynamics of charged massive particles under the action of the 5-dimensional electric and magnetic fields. We pick up a particle of mass m and spin-1/2 in an external field [14], [15], [16], [17]. We build up a superspace action for this model, read off the supersymmetry.transformations of the component coordinates. At the end, we study the dynamics of a particle in a (1+4)-dimensional brane.

The scenario we draw to pursue our investigation is as follows: we adopt the viewpoint of extra dimensions as large as $(TeV)^{-1}$, according to the frame established by Dvali et al. [3]. We then consider that we are at the TeV scale, so that we can access the fifth dimension and carry out our physical considerations in a 5-dimensional brane. In this brane, we focus our attention to the quantum-mechanical dynamics of a charged particle minimally coupled to the 5-dimensional electromagnetic field. Clearly, this particle must be massive enough to an extent that it still makes sense to consider its dynamics described by Quantum Mechanics. Then, we consider that the mass, m, of our (charged) test particle is of the TeV-order, but still larger than the energy scale we are considering. This means that there is not enough energy to penetrate the Compton wavelength of the particle, so that a quantum-mechanical approach is justifiable. We are, therefore, considering a situation in which a charged massive particle is investigated at an energy scale E (E≤2m) for which the fifth dimension shows up and, then, the dynamics of the particle is governed by a quantum-mechanical treatment. The particle feels the action of the 5-dimensional Maxwell field, which encompasses the electric ($\vec{E}$) and magnetic ($\vec{B}$) fields of the ordinary 4-dimensional Maxwell theory, and includes two extra electric- and magnetic-like fields genuinely connected to the fifth dimension, but which possible observable effects in our 4-dimensional world.

Supersymmetry (actually Supersymmetric Quantum Mechanics) is present in our approach for we know that the treatment of spin-(1/2) particles may be associated to a supersymmetric approach in which the spin variables are identified with the Grassmannian partner of the variables describing the particle position. The Supersymmetry we take about here is not a high-energy SUSY; it is simply the sort of dynamical symmetry which is underneath the description of the quantum-mechanics aspects of s=(1/2)-particles. We quote relevant references for such a discussion in [19].

This work is outlined as follows: in Section 1, we start off from the Dirac equation in 5-D, work out the non-relativistic limit, and obtain the Hamiltonian of our system. In Section 2, we highlight aspects of 5-D Electromagnetism relevant for our purpose; then in Section 3, we set the supersymmetric Hamiltonian and discuss the dynamics of our test-particle in different situations. Finally in Section 4. Finally, in Section 4 we present our Concluding Comments.

2 Setting up the Hamiltonian

Once our physical framework has been settled down, we start our considerations on the dynamics of our test particle under the action of the 5-dimensional electromagnetic field.

We intend to investigate the 5-D electrodynamics. Maxwell electromagnetism in 4-dimensional space-time could be the derivative of a more fundamental theory in 5-D dimensions. This electrodynamics lives in a 5-D hypersurface of a brane. Vectors and tensors shall be decomposed in terms of SO(3) indices; we split the 4-th component, which behaves as a scalar under SO(3) rotations.

We consider the following action for the fermion field in (1+4) dimensions,

$$L = \Psi(i\Gamma^{\mu}D_{\mu} - m)\Psi \qquad (1)$$

where we define:

$$\mu \in \{0,1,2,3,4\},\ \eta = (+,-,-,-,-); \qquad (2)$$

$$x^{\{\mu\}} = (t, x = x^1, y = x^2, z = x^3, x^4),\ F_{\{\mu\nu\}} = \partial_{\{\mu\}}A_{\{\nu\}} - \partial_{\{\nu\}}A_{\{\mu\}}; \qquad (3)$$

$$D_{\{\mu\}} = \partial_{\{\mu\}} + ieA_{\{\mu\}},\ A_{\{\mu\}} = \left(A^0, A_{\{i\}}, A^4\right),\ \left\{\Gamma^{\{\mu\}}, \Gamma^{\{\nu\}}\right\} = 2\eta^{\{\mu\nu\}}, \qquad (4)$$

$$\partial^{\{i\}} \Leftrightarrow -\nabla,\ \partial^0 \Leftrightarrow \left(\frac{\partial}{\partial t}\right),\ E^{\{i\}} \Leftrightarrow \vec{E},\ B^{\{i\}} \Leftrightarrow \vec{B}, \qquad (5)$$

$$F^{04} = -E,\ F_{\{i4\}} = Z_{\{i\}},\ F_{ij} = -\varepsilon_{ijk}B_{k},\ F_{\{0i\}} = E_{\{i\}}; \qquad (6)$$

i,j,k∈{1,2,3}. Notice that the scalar, ε, and the vector,$\vec{Z}$ accompany the vectors $\vec{E}$and $\vec{B}$, later on to be identified with the electric and magnetic fields, respectively. Our explicit representation for the gamma-matrices is given by:

$$\Gamma^5 = \begin{bmatrix} 0 & 1^2 \\ 1^2 & 0 \end{bmatrix};\ \Gamma^{\{i\}} = \begin{bmatrix} 0 & \sigma^{\{i\}} \\ -\sigma^{\{i\}} & 0 \end{bmatrix} \Gamma^5 = \begin{bmatrix} 1^2 & 0 \\ 0 & -1^2 \end{bmatrix}; \Gamma^5 = \begin{bmatrix} 0 & i^2 \\ i^2 & 0 \end{bmatrix}; \qquad (7)$$

We quote above the Γ-matrices representing the Clifford algebra of (1+4)D. We set the Dirac spinor, Ψ and $\Psi=\Psi^{\dagger}\Gamma^{0}$.

We now take the equation of motion for Ψ from the Euler-Lagrange equations (1) . We suppose a stationary solution,

$$\Psi(x,t) = exp(-iEt)\chi(x);; \qquad (8)$$

here, we are considering that the external field does not depend on t, and A_o=0.

To study the properties of the matter at a low-energy scale, we work out the non-relativistic limit. In terms of the component fields defined above, the 5D Dirac equation takes over the form :

$$(\varepsilon - m)\chi^1 + \left(i\sigma^{\{i\}}\left(\partial_{\{i\}} + ieA_{\{i\}}\right) + (\partial^4 + ieA^4)\right)\chi^2 = 0; \qquad (9)$$

$$(i\sigma^{\{i\}}(\partial_{\{i\}} + ieA_{\{i\}}) - (\partial_4 + ieA_4))\chi_1 + (E + m))\chi_2 = 0. \qquad (10)$$

Here, we do not go through the Foldy-Wolthuizen transformations to get to the non-relativistic limit. We follow the usual procedure to get to the Pauli equation and we arrive at:

$$H = -(1/(2m))[(\nabla - ieA)^2 + (\partial_4 - ieA^4)^2 + e\sigma(\nabla \times A + 2\vec{Z}) + e\varphi] \quad (11)$$

With $p\_\{i\} = -i\nabla$, we rewrite (7) as follows:

$$H = (1/(2m))[(\vec{p} - e\vec{A})^2 + (p^4 - eA^4)^2] - e(\vec{B} + 2\vec{Z}) \cdot \vec{S} + e\varphi;; \quad (12)$$

$$\vec{S} = \left(\frac{e}{2m}\right)\vec{\sigma}, \quad (13)$$

and we obtain the Pauli Hamiltonian revealing with the contribution coming from the extra-dimension.

Now, it could be suitable to work out some specific aspects of Classical Electrodynamics in (1+4)-D.

We shall present the whole set of fields as written in terms of SO(3)-tensor representations and put in a manifest form the Maxwell equations in five dimensions, but written in terms of the SO(3) fields. Next, we written down the components of the energy-momentum tensor, we present its associated continuity equations and the meaning of the $\Theta_{\{\mu\nu\}}$ −components. This task may be clarifying for the sake of our final discussion on the study of the classical dynamics of the 5-D charged particle under the action of external 5-D electromagnetic fields.

3 The 5-D Electromagnetic Fields.

We start by considering the equations of motion in vaccum:$\varepsilon^{\{\mu\rho\sigma\beta\gamma\}}\partial_{\{\sigma\}}F_{\{\beta\gamma\}} = 0$and $\partial_{\{\sigma\}}F^{\{\sigma\gamma\}} = 0$, where$F_{\{\beta\gamma\}}$ is the field strength defined in (2)and:$\varepsilon^{\{\mu\rho\sigma\beta\gamma\}}$is the Levi-Civita tensor in 5 dimensions. Then, we have:

$$\vec{\nabla} \cdot \vec{E} - ((\partial\varepsilon)/(\partial x^4)) = 0, \quad (14)$$

$$\vec{\nabla} \times \vec{B} - ((\partial\vec{E})/(\partial t)) - ((\partial\vec{Z})/(\partial x^4)) = 0, \quad (15)$$

$$((\partial\varepsilon)/(\partial t)) + \vec{\nabla} \cdot \vec{Z} = 0, \quad (16)$$

$$\vec{\nabla} \times \vec{E} = -((\partial\vec{B})/(\partial t)), \quad (17)$$

$$\left(\frac{\partial\vec{Z}}{\partial t}\right) + \vec{\nabla}\varepsilon + ((\partial\vec{E})/(\partial x^4)) = 0, \quad (18)$$

$$\nabla \times \vec{Z} - ((\partial\vec{B})/(\partial x^4)) = 0, \quad (19)$$

$$\vec{\nabla} \cdot \vec{B} = 0, \quad (20)$$

where the component fields are defined as follows:

$$E \Leftrightarrow F\_\{0i\} \equiv \partial_0 A\_\{i\} - \partial\_\{i\}A_0 = 0; \quad (21)$$

$$F_{04} \equiv \partial_0 A_4 - \partial_4 A_0 = -\varepsilon; \quad (22)$$

$$F\_\{ij\} = \partial\_\{i\}A\_\{j\} - \partial\_\{j\}A\_\{i\} \equiv -\varepsilon\_\{ijk\}B\_\{k\}; \quad (23)$$

$$F\_\{i4\} = \partial\_\{i\}A_4 - \partial_4 A\_\{i\} \equiv Z\_\{i\}. \quad (24)$$

With these equations we can understand more clearly the influence of the fifth dimension on the particle motion. We could consider a 5-dimensional version of the Chern-Simons terms for the electromagnetic field [21]. It is however cubic in the potential and it yields non-linear contributions to the Maxwell equations. This is why we are not contemplating such a possibility. However, we point out a very remarkable result by Qi, Witten and Zhang [22], who discuss topological superconductivity in (1+3) dimensions based on a more fundamental 5D electromagnetic model with Chern-Simons term wich yields an axion-type $F\check{F}$ contribution in (1+3)D.

From the energy-momentum tensor, $\Theta^{\{\mu\}}_{\{\kappa\}} = F^{\{\mu\alpha\}}F_{\{\alpha\kappa\}} + \left(\frac{1}{4}\right)\delta^{\{\mu\}}_{\{\kappa\}}F^{\{\alpha\beta\}}F_{\{\alpha\beta\}}$, evaluating each set of separate components, we have:

$$\Theta^0{}_0 = (1/2)[(E^2) + B^2 + (\varepsilon^2) + Z^2], \quad (25)$$

$$\Theta^0\_\{i\} = -\left[\left(\vec{E} \times \vec{B}\right) + \varepsilon\vec{Z}\right]_{\{i\}}, \quad (26)$$

$$\Theta^{\{i\}}_{\{j\}} = E_{\{i\}}E_{\{j\}} + B_{\{i\}}B_{\{j\}} - e_{\{i\}}e_{\{j\}} - \left(\frac{1}{2}\right)\delta^{\{i\}}_{\{j\}}(E^2 + B^2 + \varepsilon^2 - Z^2), \quad (27)$$

$$\Theta^0{}_4 = \vec{E} \cdot \vec{Z}, \quad (28)$$

$$\Theta^{\{i\}}{}_4 = -[\varepsilon\vec{E} + \vec{Z} \times \vec{B}]\_\{i\}, \quad (29)$$

$$\Theta^4{}_4 = -(1/2)[(E^2) - B^2 + Z^2 - (\varepsilon^2)] \quad (30)$$

which is interpreted as a new "density" of pressure towards the extra dimension x^4=s. In our study of the 5-D Electromagnetism the Poynting's theorem is summarized by the following expression

$$(\partial/(\partial t))u + \vec{\nabla} \cdot \vec{S} + (\partial/(\partial s))\xi = -\vec{j} \cdot \vec{E} + j_{\{s\}}\varepsilon, \quad (31)$$

where$\vec{S}$ is the "Poynting vector" representing the momentum flow, $\vec{j}$ is the current density, $\vec{E}$ is the electric field, and u is the density of electromagnetic energy,$\xi = \Theta^{04}$ and $j_{\{s\}} \equiv j^4$.

$u = (1/2)[(E^2) + B^2 + (\varepsilon^2) + Z^2]$,

$\vec{S} = (\vec{E} \times \vec{B}) + \varepsilon\vec{Z}$,

$$\xi = -\vec{E} \cdot \vec{B}.$$

The 5-D expression for the Lorentz force follows in connection with the following continuity equation (in presence of external sources:$\rho, \vec{j}, j_{\{s\}}$):

$$\left(\frac{1}{c}\right)\left(\frac{\partial \vec{S}}{\partial t}\right) - \vec{\nabla} \cdot \vec{\sigma} + \left(\frac{\partial \chi}{\partial s}\right) = -\rho\vec{E} - \vec{j} \times \vec{B} + j_{\{s\}}B, \tag{32}$$

and,

$\vec{S} = (\vec{E} \times \vec{B}) + \varepsilon\vec{Z}$,

$$\vec{\sigma} = \Theta^{\{i\}}_{\{j\}},$$

$$\chi = (\varepsilon\vec{E} + \vec{Z} \times \vec{B}). \tag{33}$$

In our scenario, 5-D, there is a third term which embraces the conservation of scalar moments, .

$$-((\partial\xi)/(\partial t)) - \nabla \cdot \chi + (\partial/(\partial s))\Omega = -\rho E + j \cdot B \tag{34}$$

and,

$$\xi = -\vec{E} \cdot \vec{Z},$$

$$\Omega = -(1/2)[(E^2 - B^2 - (\varepsilon^2) + Z^2]. \tag{35}$$

Now, we are back to the situation of a charged particle under the action of an external 5-D eletromagnetic field. We shall take this particular system in order to better understand how the 5-D fields may act upon charged particles dynamics may take place partly in (1+3)-D. This will open up for us some interesting discussions on the back reaction of force and may even lead us to an interpretation of a possible sort of dark-matter-like charged particles.

4 The Supersymmetric Mechanical Description.

To render more systematic to our discussion, we think it is advisable to set up a superfield approach. We can define the N=1-supersymmetric model in analogy with the model presented above, eq.(<ref>Lpauli54dn1spin</ref>). It is not a trivial task, and to solve this question we start by defining the superfields as below:.

$$\Phi_{\{i\}}(t,\theta) = x_{\{i\}}(t) + i\theta\psi_{\{i\}}(t), \Phi_4(t,\theta) = x_4(t) + i\theta\psi_4(t), \tag{36}$$

The supercharge operators and the covariant derivatives are given by:

$Q = \partial_{\{\theta\}} + i\theta\partial_{\{t\}};\ D = \partial_{\{\theta\}} - i\theta\partial_{\{t\}},\ and\ H = i\partial_{\{t\}}$(37)

We have to set up a Lagrangian in terms of the $\Phi_{\{i\}}$'s and Φ_4 so as to recover the Hamiltonian of eq.(<ref>Hpauli54dn1</ref>). The N=1-supersymmetric Lagrangian that generates the appropriate bosonic sector is given by:

$$. L_1 = (i/2)m(\Phi_{\{i\}}D\Phi_{\{i\}} + \Phi_4 D\Phi_4) + ie[(D\Phi_{\{i\}})A_{\{i\}}(\Phi_{\{j\}}, \Phi_4) + (D\Phi_4)A_4(\Phi_{\{j\}}, \Phi_4)] \tag{38}$$

Integrating over the Grassman variable, we obtain:

$$L_1 = (1/2)m((x_{\{i\}})^2 + (x_4)^2) - (i/2)(\psi_{\{i\}}\psi_{\{i\}} + \psi_4\psi_4) + e(A_{\{i\}}x_{\{i\}} + A_4x_4) + e\varphi - \left(\frac{ie}{2}\right)\left(B_{\{i\}} + 2Z_{\{i\}}\right)\varepsilon_{\{ijk\}}\psi_{\{j\}}\psi_{\{k\}}, \tag{39}$$

where the dot stands for a derivative with respect to time. We can also write,

$$L_1 = (1/2)m((x_{\{i\}})^2 + (x_4)^2) - (i/2)(\psi_{\{i\}}\psi_{\{i\}} + \psi_4\psi_4) + e(A_{\{i\}}x_{\{i\}} + A_4x_4) + e\varphi + eB_{\{i\}}S_{\{i\}} + e2B_{\{i\}}S_{\{i\}}, \tag{40}$$

where we define the spin by the product below: ,

$$S_{\{i\}} = -\left(\frac{i}{2}\right)\varepsilon_{\{ijk\}}\psi_{\{j\}}\psi_{\{k\}}. \tag{41}$$

$$H^1 = \left(\frac{1}{2}\right)m\left(\left(\dot{x}_{\{i\}}\right)^2 + (\dot{x}^4)^2\right) + i\left(\psi_{\{i\}}\dot{\psi}_{\{i\}} + \psi^{4\dot{\psi}4}\right) + e\varphi + \left(\frac{ie}{2}\right)\left(B_{\{i\}} + 2Z_{\{i\}}\right)\varepsilon_{\{ijk\}}\psi_{\{j\}}\psi_{\{k\}}, \tag{42}$$

$$H_1 = (\frac{1}{2m})[(\vec{p} - e\vec{A})^2 + (p^4 - eA^4)^2] + \left(\frac{ie}{2}\right)\left(B_{\{i\}} + 2Z_{\{i\}}\right)\varepsilon_{\{ijk\}}\psi_{\{j\}}\psi_{\{k\}} + e\varphi + i(\psi_{\{i\}}\dot{\psi}_{\{i\}} + \psi_4\dot{\psi}_4), \tag{43}$$

where we observe a new Pauli coupling with the field B_{i} and the contributions of the fermionic coordinates ψ_{j} and ψ_4.

The eqs.of motion in the fermionic sector are:

$$\dot{\psi}_4 = 0, \tag{44}$$

$$\dot{\psi}\_\{i\} = e\left(\vec{B}+2\vec{Z}\right)_{\{j\}}\varepsilon_{\{ijk\}}\psi_{\{\kappa\}}, \tag{45}$$

where it is manifest the Pauli-type coupling in the fermionic sector. For the coordinates x and x^4, the equations of motion assume the form:

$$m((\partial^2\vec{x})/(\partial t^2)) = e\vec{v}\times\vec{B} - e\vec{Z}x^4 + e\vec{E}, \tag{46}$$

$$m((\partial^2 x^4)/(\partial t^2)) = -e\vec{Z}.\vec{v} - e\varepsilon, \tag{47}$$

where the extended Lorentz force gets contribution from B. Notice that actually RHS of both eqs. (46) and (47) express the Lorentz force which can also be read off from the continuity eqs. (32) and (34). To focus the possible novelties that this model may reveal, we have to pay attention to the extra-dimension contribution. To understand its exclusive effect, we set E=0, B=0, and E=0; then, we get:

$$m((\partial^2\vec{x})/(\partial t^2)) = -e\vec{Z}\dot{x}_4, \tag{48}$$

$$m((\partial^2 x^4)/(\partial t^2)) = -e\vec{Z}.\vec{v}. \tag{49}$$

Manipulating the equations above, we obtain:

$$((\partial^3 x^4)/(\partial t^3)) = \alpha^2.\dot{x}_4, \tag{50}$$

$$\alpha = \left(\frac{e}{m}\right)\left|B\vec{Z}\right|. \tag{51}$$

The solutions are:

$(i)\ \ \dot{x}^4 \sim exp(+\alpha t),$

$(ii)\ \dot{x}^4 \sim exp(-\alpha t).$

Taking into account the run-away solution (i), and replacing in (<ref>t1</ref>), we obtain:

cc

$$m((\partial^2\vec{x})/(\partial t^2)) \sim -\vec{Z}exp(+\alpha t) \tag{52}$$

$$\vec{x} \sim -\vec{Z}exp(+\alpha t). \tag{53}$$

In more explicit way, we have:

$$F_+ = m((\partial^2\vec{x}_+)/(\partial t^2)) = -e\vec{Z}|\alpha|exp(+e((|\vec{Z}|)/m)t),$$

In this case, the particle moves in the opposite direction of B and has an exponential growth force F_+

$$F_- = m((\partial^2\vec{x}_-)/(\partial t^2)) = e\vec{Z}|\alpha|exp(-e((|\vec{Z}|)/m)t),$$

In this case, the particle moves in the same direction of B and has a decreasing exponential strength F_-.

By (48), we can suppose that the extra term in the Lorentz force ($\vec{Z}\dot{x}^4$) can be associated with,the back reaction or the Abraham-Lorentz force. Then, we could interpret the back reaction force as an effect of a fifth dimension of space-time where the $\vec{Z}$-field is the 3-dimensional projection of the true magnetic field in (1+4)-D.

On the other hand taking into account (ii), and replacing in (47), we obtain:

$$m((\partial^2\vec{x})/(\partial t^2)) \sim \vec{Z}exp(-\alpha t), \tag{54}$$

$$\vec{x} \sim \vec{Z}exp(-\alpha t). \tag{55}$$

But, if we study the case in which $\vec{Z}$=0, and $\varepsilon\neq 0$, from (47), we get:

$$((\partial^2 x^4)/(\partial t^2)) = -((e\varepsilon)/m),$$

which shows that the particle indefinitely escapes to the extra dimension x^4. Here, we can conclude that the particle may describe some component of dark matter, and the fieldε, the piece remnant of the 5-D electric field drives the particle away from the Minkowskibrane. Charged dark matter has been considered by several authors in the literature. We shall come back to this point at the end of our Concluding Remarks.

5 Concluding Remarks.

So, to conclude, we would like to stress and comment on a few issues. We have studied an N=1, D=5-supersymmetric particle, in an external electromagnetic field by considering the non-relativistic regime, which, as already motivated in the introductory comments, is reasonable for we consider heavy particles with low kinetic energy. We admit that this model could be interesting to describe cold dark matter. We suppose that this particle is immersed in a (1+4)-dimensional brane. In this extended electromagnetic field, the (Z_{i} and ε)-fields appear in the Maxwell equations. The effect of the fifth coordinate (x^4) is felt by means of an extension of the Lorentz force in 4 dimensions. And, by focusing on the evolution of

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the x^4-coordinate, we have identified two possible relevant situations in connection with the time evolution of the x^4-coordinate. In the situation, x^4 is of the run-away type and we associate its effect as the Abraham-Lorentz back reaction force in our 4-dimensional world. By adopting this point of view, we propose that the effect on an extra dimension may show up under the guise of the back reaction force in the dynamics of a charged particle subjected to an electromagnetic field.

On the other hand, the presence of an electric field may drive the charged particle to the bulk and, by virtue of this mechanism, we propose that the charged particle, in this regime, has a similar behaviour to the dark matter particles. Of course, the particle is not electrically neutral; so, in this sense, it cannot be a genuine dark matter constituent. However, the extra electric field, also confined to the bulk, drives the test particle from the Minkowskibrane and, once in the bulk, it interacts with the bulk field $\vec{Z}$. Though charged, it escapes to the bulk and this is the reason we say it similar to the dark matter particles. In connection with this mechanism, we point out a series of interesting works where the possibility of electrically charged dark matter is fully discussed and a considerable number of constraints is settled down in case dark matter particles are not neutral. In the Ref. [20] the authors discuss the stringent constraints on the existence of charged dark matters.

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