ISSN: 2347-3487

Interactive Two-Stage Stochastic fuzzy Rough Programming for Water Resources Management

E. E. Ammar[1], and H. A. Khalifa[2]

1- Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt.
2- Department of Operations Research, Institute of Statistical Studies and Research, Cairo University, Cairo, Egypt.

Abstract

This paper deals with a fuzzy programming approach for treating an interactive two-stage stochastic rough-interval water resource management. The approach has been developed by incorporating an interactive fuzzy resolution method within a rough two-stage stochastic programming framework. The approach can not only tackle dual rough intervals presented as an inexact boundary intervals that exist in the objective function and the left- and right-hand sides of the constraints that are associated with different levels of economic penalties when the promised policy targets are violated. The results indicate that a set of solutions under different feasibility degrees has been generated for planning the water resources allocation. They can help the decision makers to conduct in depth analysis of tradeoffs between economic efficiency and constraint-violation risk, as well as enable them to identify, in an interactive way, a desired compromise between satisfaction degree of the goal and feasibility of the constraints. A management example in terms of rough-intervals water resources allocation has been treated for the sake of applicability of the proposed approach.

Keywords: Rough sets and intervals; Fuzzy programming; Interactive approach; Inexact water resources management.

Council for Innovative Research

Peer Review Research Publishing System

Journal: Journal of Advances in Physics

Vol 4, No.3

japeditor@gmail.com

www.cirjap.com

ISSN: 2347-3487

1. Introduction

Recently, the conflict-laden issues of water resources allocation among competing municipal, industrial and agricultural interests have been of increasing concerns. The competition among water users has been intensified due to growing population shifts, shrinking water availabilities, varying natural conditions and deteriorating quality of water resources. The increased water demands and the inadequate water supplies have exacerbated the storage of water resources; this has been considered as a major obstacle to sustainable water resources management. The problem arises when the essential demand cannot be satisfied due to insufficient resources, losses can hardly avoided resulting in variety of adverse impacts on socio-economic development. Therefore, a sound planning water resources allocation is desired to help reduce such losses. However, the water resources management system are complicated with a variety of uncertainties and their interactions when may intensify the conflict-laden issues of water allocation.

To study the above concern problems, a number of optimization techniques ial distributions. In TSP, an initial decision must be made before the realization of random variables (first-stage decision), and then a corrective action can be taken place (second-stage decision). This implies that a second-stage decision can be used to minimize "penalties" that may appear due to any infeasibility.

In general, TS deals with uncertainties expressed as probabilistic distributions and account for economic penalties with recourse against any infeasibility. In many real-world problems' however, the quality and quantity of uncertain/information are often not satisfactory enough to be presented as probabilistic distributions. Even if such distributions for uncertain parameters are available, reflecting them in large-scale optimization models can be extremely challenging.

In comparison, rough-interval mathematical programming (RIMP) is effective in talking of unknown quantities expressed as intervals with known lower and upper bounds. RIMP proves to be an effective approach to deal with the uncertainties in the parameters of the considered problems. Rough-interval two-stage stochastic programming (RITSP) approach for water resources management could not only tockle uncertainties expressed as probabilistic distribution and rough intervals, but also analyze a variety of policy scenarios that are associated with different levels of economic penalties when the promised policy targets are violated.

An interactive fuzzy resolution (IFR) method is proposed for solving a two-stage stochastic rough-intervals water resources allocation management problems with fuzzy parameters. IFR permits interactive participations of DMS in all steps of decision process expressing their preference in linguistic terms. A set of solutions could obtained from IFR under different feasibility degrees of the constraints which are meaningful to support in-depth analysis of tradeoffs between economic efficiency and constraint violation risk. IFR enables DMS to identify a compromised solution between two factors in conflict: feasibility of the constraint and satisfaction degree of the goal.

In earlier work in the conflict-laden issues of water resources allocation among competition interests have been of in creasing concerns many researches as Huang and Chng (2003); Wang et al. (2003), Li and Huang (2008), Lu et al., (2006) and (200). The above concerns a number of optimization techniques were developed/by Slousinski (986); Wu et al., (1997); Huang (1998); Tairaj and Vedula (2000); Seifi and Hipel (2001); Luo et al., (2003); Maqsod et al., (2005); Li et al., (2007) and Wang et al., (2007).

Recently, Jimenez et a;/. (2007) proposed an interactive fuzzy resolution (IFR) method for solving linear programming problems with fuzzy parameters. Also, Wang and Huang (2011) have been proposed an interactive two-stage stochastic fuzzy programming (ITSFP) approach for water resource management problem. They develop its approach by incorporating IFR method within an inexact two-stage stochastic programming framework. ITSFP cannot only tackle dual uncertainties presented as a fuzzy boundary intervals that exist in the objective function and the left-and right-hand of the constraint, but also permit in-depth analysis of various policy scenarios that are associated with different levels of economic penalties when the promised policy target are violated.

This paper deal with a fuzzy programming approach for treating an interactive two-stage stochastic fuzzy (ITSFP) for rough-interval water resources management. The approach has been developed by incorporating an interactive fuzzy resolution method within a rough-interval two-stage stochastic programming (RITSP) framework. ITSFP can not only tackle dual rough-intervals presented as a fuzzy boundary intervals that exist in the objective function and the left-and right-hand/side of the constraints, but also permit in-depth analysis of various policy scenarios that are associated with different levels of economic penalties when the promised policy targets are violated. A management problem in terms of rough-intervals water resources allocation has been studied for the sake of applicability of the proposed approach.

2. Preliminaries

In this section, we recall some definitions needed through the paper.

Moore (1979) introduced the concept of closed interval numbers. Let $I(R)=\{[a^L, a^U]: a^L, a^U \in R=(-\infty,\infty),\ a^L \le a^U\}$ denote the set of all closed interval numbers on *R*.

Definition 1. Suppose that $[a^L, a^U],\ [b^L, b^U] \in I(R)$. We define:

1- $[a^L, a^U] + [b^L, b^U] = [a^L + b^L, a^U + b^U]$,

2- $[a^L, a^U] - [b^L, b^U] = [a^L - b^L, a^U - b^U]$,

3- The order relation $"\leq"$ in $I(R)$ is defined by:

$[a^L, a^U] \leq [b^L, b^U]$ if and only if $a^L \leq b^L,\ a^U \leq b^U$.

Definition 2. (Zadeh (1965)): A fuzzy set $\tilde{A}$ in $X \subset R$ is characterized by a membership function $\mu_{\tilde{A}}(\cdot)$, where *X* represents a space of points, with an element of *X* denoted by *x*; $\mu_{\tilde{A}}(x)$ represents the membership grade of *x* in $\tilde{A}$; $\mu_{\tilde{A}}(x) : X \to [0,1]$.

Definition 3. A convex fuzzy set $\tilde{A}$ is a set defined on the real line *R* with a continuous membership function $\mu_{\tilde{A}}$ that ca be defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if} \quad x \leq a_1; \\ f_{\tilde{A}}(x), & \text{if} \quad a_1 < x \leq a_2; \\ 1, & \text{if} \quad a_2 \leq x \leq a_3; \\ g_{\tilde{A}}(x), & \text{if} \quad a_3 \leq x < a_4; \\ 0, & \text{if} \quad x \geq a_4, \end{cases} \qquad (1)$$

where $f_{\tilde{A}}$ and $g_{\tilde{A}}$ represent a continuous and monotonically increasing function on

the left-hand side of $\tilde{A}$ and a continuous and monotonically decreasing function on the right-hand side of $\tilde{A}$, respectively.

Definition 4. The α-level set of a fuzzy set $\tilde{A}$ can be defined as follows:

$$\tilde{A}_\alpha = \{x \in R : \mu_{\tilde{A}}(x) \geq \alpha,\ \alpha \in [0,1]\} \qquad (2)$$

since $\mu_{\tilde{A}}$ is upper semi-continuous, the α-level set of $\tilde{A}$ defined in (2) forms a closed bounded interval (Wang and Huang (2011)):

$$\tilde{A}_\alpha = [f_A^{-1}(\alpha),\ g_A^{-1}(\alpha)],$$

where,

$$f_A^{-1}(x) = \inf\{x : \mu_{\tilde{A}}(x) \geq \alpha\}, \qquad (3)$$

and

$$g_A^{-1}(x) = \sup\{x : \mu_{\tilde{A}}(x) \geq \alpha\}. \qquad (4)$$

Definition 5. (Wang and Huang (2011)): The expected interval of a fuzzy set $\tilde{A}$, denoted by EI$(\tilde{A})$, can be defined as follows:

EI $(\tilde{A})=[E_1^{\tilde{A}}, E_2^{\tilde{A}}]$

$$=\left[\int_0^1 f_{\tilde{A}}(\alpha)d\alpha \int_0^1 g_{\tilde{A}}(\alpha)d\alpha\right]. \qquad (5)$$

Definition 6. (Wang and Huang (2011)): the expected value of a fuzzy set $\tilde{A}$, denoted by $E\vee(\tilde{A})$, is the half of its expected interval:

$$E\vee(\tilde{A})=\frac{E_1^{\tilde{A}}+E_2^{\tilde{A}}}{2}. \qquad (6)$$

Definition 7. Let *R* be the set of real numbers, a fuzzy number $\tilde{a}$ is a mapping: $\mu_{\tilde{a}}:R\rightarrow[0,1]$, with the following properties:

1- $\mu_{\tilde{a}}(x)$ is upper semi continuous membership function,

2- $\tilde{a}$ is convex fuzzy set, i.e., $\mu_{\tilde{a}}(wx^1+(1-w)x^2)\geq\min(\mu_{\tilde{a}}(x^1),\mu_{\tilde{a}}(x^2))$, for all $x^1, x^2\in R,\ w\in[0,1]$,

3- $\tilde{a}$ is normal, i.e., there exists $x_0\in R$, for which $\mu_{\tilde{a}}(x_0)=1$,

4- $\text{supp}(\tilde{a})=\{x\in R:\mu_{\tilde{a}}(x)>0\}$ is a support of $\tilde{a}$.

Definition 8. (Kaufmann and Gupta (1988)): A trapezoidal fuzzy number (Tr. F. N) can be represented completely by a quadruplet: $\tilde{A}=(a_1,a_2,a_3,a_4)$ whose membership function is characterized as:

$$\mu_{\tilde{A}}=\begin{cases}0, & \text{if} \quad x<a_1;\\ \dfrac{x-a_1}{a_2-a_1}, & \text{if} \quad a_1\leq x<a_2;\\ 1, & \text{if} \quad a_2\leq x<a_3;\\ \dfrac{a_4-x}{a_4-a_3}, & \text{if} \quad a_3\leq x<a_4;\\ 0, & \text{if} \quad x>a_4.\end{cases} \qquad (7)$$

Remark 1. A triangular fuzzy number (T. F. N) is a special case of a trapezoidal fuzzy number (Tr. F. N.) with $a_2=a_3$.

3. Problem definition

Consider a problem in which a water manager is response for allocating water to multiple users. The objective of the water resources allocation problem is to maximize the system benefit. Based on local water management polices, a prescribed quantity of water is promised to each user. If the promised water is delivered, it will result in net benefits to the local economy, otherwise, users will have to either obtain water from more expensive sources or curtail their development plans resulting in economic penalties. In such problem, the water flow levels are uncertain (expressed as random variables), while a decision of water allocation target (first-stage decision) must be mate before the realization of random variables, and then a recourse action can be taken after disclosure of a random variables (second-stage decision). Therefore, this problem under consideration can be formulated as a two-stage stochastic programming (TSP) model as follows (Wang and Huang (2011)):

$$\max f = \sum_{i=1}^{m} NB_i T_i - E\left[\sum_{i=1}^{m} C_i S_{iQ}\right] \tag{8a}$$

subject to

$$\sum_{i=1}^{m} (T_i - S_{iQ})(1+\delta) \le Q, \tag{8b}$$

(Water availability constraints)

$$S_{iQ} \le T_i \le T_{i\max}, \qquad i = 1, \ldots, m; \tag{8c}$$

(Water-allocation target constraints)

$$S_{iQ} \ge 0, \qquad i = 1, \ldots, m; \tag{8d}$$

(Non-negativity and technical constraints),

where *f* is the system benefit ($); NB_i is the net benefit to user (*i*) per m^3 of water allocation $(\$/m^3)$ (first-stage revenue parameters); T_i is the allocation target for water that is promised to user $(i)(m^3)$ (first-stage decision variables); $E(\cdot)$ is the expected value of a random variable; C_i is the loss to user (i) per m^3 of water not delivered, $C_i > NB_i\ (\$/m^3)$ (second-stage parameters); S_{iQ} is the storage of water to user (i) when the seasonal flow is $Q\,(m^3)$ (second-stage decision variables); *Q* is the total amount of seasonal flows (m^3) (random variable); δ is the rate of water loss during transportation; $T_{i\max}$ s the maximum allowable allocation amount for user $(i)(m^3)$; *m* is the total number of water user, *I* is the water user, e.g., $i = 1$ for the municipality, $i = 2$ for the industrial user, $i = 3$ for the agricultural sector.

The distribution of *Q* must be approximated by a set of discrete values. Letting *Q* take values q_j with probability $p_j\,(j = 1, \ldots, n)$, we have

$$E\left(\sum_{i=1}^{m} C_i S_{iQ}\right) = \sum_{=1}^{m} C_i \left(\sum_{j=1}^{n} p_j q_{ij}\right) \tag{9}$$

where s_{ij} denotes the amount by which the water-allocation target (T_i) is not met when the seasonal flow is q_j with probability p_j . Thus, model (1) van be reformulated as follows:

$$\max f = \sum_{i=1}^{m} NB_i T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j c_i s_{ij} \tag{10a}$$

subject to

$$\sum_{i=1}^{m}(T_i - s_{ij})(1+\delta) \le a_j, \qquad j = 1, \dots, n; \qquad (10b)$$

$$s_{ij} \le T_i \le T_{i\max}, \qquad i = 1, \dots, m \text{ ; and} \qquad (10c)$$

$$s_{ij} \ge 0, \qquad i = 1, \dots, m; \; j = 1, \dots, n. \qquad (10d)$$

For a difficults may be face the planner to promise a deterministic water-allocation target (T_i) to a water user, the available flows are uncertain. Thus, we need to define uncertain parameters (rough and interval) of model (3). The notion of rough set (Nanda and Majumdar (1992)) given through the following definition:

Definition. Let *U* be a nonempty set and Ω b a complete sub algebra of the Boolean algebra $P(U)$ of subsets of *U*. the pair (U, Ω) is called a rough universe. Let *R* be a relation defined as follows:

$$A = (A^L, A^U) \in R \text{ if and only if } A^L, A^U \in \Omega, \; A^L \subseteq A \; .$$

The elements of *R* are called rough sets and the elements of Ω are called exact sets.

Definition 10. The degree of rough of the set (A^L, A^U) is

$$\lambda = \frac{\text{card}\,A^L}{\text{card}\,A^U} = \frac{A^{L+} - A^{L-}}{A^{U+} - A^{U-}} = \frac{\|A^L\|}{\|A^U\|} \qquad (11)$$

where $\|A\|$ denotes the length of the interval.

In response to the above concerns, rough interval and interval parameters are introduced into model (3) to communicale uncertainties. This leads to an interactive rough interval two-stage stochastic programming (IRITSP)model as follows:

$$\max\ (f^{L\mp}, f^{U\mp}) = \sum_{i=1}^{m}(N\,B_i^{L\mp}, N\,B_i^{U\mp})(T^{\mp})$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\,(C_i^{L\mp}, C_i^{U\mp})(S^{\mp}) \qquad (12a)$$

subject to

$$\sum_{i=1}^{m}(T_i^{\mp} - S_{ij}^{\mp})(1+\delta^{\mp}) \le q_j^{\mp}, \quad j = 1, \dots, n; \qquad (12b)$$

$$S_{ij}^{\mp} \le T_i^{\mp} \le T_{i\max}^{\mp}, \qquad i = 1, \dots, m; \qquad (12c)$$

and

$$S_{ij}^{\mp} \ge 0, \qquad i = 1, \dots, m; \; j = 1, \dots, n; \qquad (12d)$$

where $(A^{L\pm}, A^{U\pm})$ is rough interval and $B^{\mp}$ is interval.

ISSN: 2347-3487

4. Interactive fuzzy resolution approach

In many real world problems, when the subjective judgment of decision maker (DMS) is influential in the decision making process, the fuzzy set theory is recognized as an effective tool to express their opinions. For example, in RITSP can hardly reflect the ambiguous or vague information from the subjective estimations. Say, DMS estimate that the most possible value for the net benefit to a water user is $90 per m^3 of water allocated, and there is no possibility for it to be lower than $80 or more than $100 per m^3 of water; such a subjective estimation can be expressed as a fuzzy set. Therefore, an interactive fuzzy resolution (IFR) method is introduced to tackle uncertainties presented as fuzzy sets. Firstly, consider the following enteractive fuzzy rough interval two-stage stochastic programming (IFRITSP) model:

$$\max\ (f^{L\mp}, f^{U\mp}) = \sum_{i=1}^{m} (\tilde{N}\ B_i^{L\mp}, \tilde{N}\ B_i^{U\mp})(T_i^{\mp})$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\,(C_i^{L\mp}, C_i^{U\mp})(S_{ij}^{\mp}) \qquad (13a)$$

subject to

$$\sum_{i=1}^{m} (T_i^{\mp} - S_{ij}^{\mp})(1+\tilde{\delta}^{\mp}) \le \tilde{q}_j^{\mp}, \quad j = 1, ..., n; \qquad (13b)$$

$$S_{ij}^{\mp} \le T_i^{\mp} \le T_{i\max}^{\mp}, \qquad i = 1, ..., m; \qquad (13c)$$

and

$$S_{ij}^{\mp} \ge 0, \qquad i = 1, ..., m; \ j = 1, ..., n; \qquad (13d)$$

where $T_i^{\mp}, S_{ij}^{\mp}$ and $T_{i\max}^{\mp}$ are interval variables; $(\tilde{N}\ B_i^{L\mp}, \tilde{N}\ B_i^{U\mp})$, $(\tilde{C}_i^{L\mp}, \tilde{C}_i^{U\mp})$, are fuzzy rough interval parameters; $\tilde{\delta}^{\mp}$ and $\tilde{q}_j^{\mp}$ are fuzzy parameters.

Here, we introduce some concepts and definitions of fuzzy set theory. A fuzzy set $(\tilde{A})$ in *X* is characterized by a membership function $(\mu_{\tilde{A}}(\cdot))$, where *X* re[resents a space of points, with an element of *X* denoted by *x* (Zadeh (1965)); represents the membership grade of *x* in $\tilde{A}$ taking value *x*. Zadeh (1978) define the possibility associated with $\tilde{A}$ as $\mu_{\tilde{A}}$.

According to formulae (5) and (6), if the fuzzy set $(\tilde{A})$ is trapezoidal or triangular, its expected interval and expected value can be calculated as follows:

$$EI(\tilde{A}) = \left[\frac{1}{2}(a_1+a_2), \frac{1}{2}(a_3+a_4)\right], EV(\tilde{A}) = \frac{1}{4}(a_1+a_2+a_3+a_4) \qquad (14)$$

The uncertain and / or imprecise nature of many parameters leads to concerns of system feasibility and optimality. It is therefore necessary to answer the following two questions:

(1) How to define the feasibility of a decision vector, when the constraints involve fuzzy sets?

(2) How to define the optimality for an objective function with fuzzy coefficients?

Several focuses have been developed to answer these questions (Lai and Huang (1994); Rommelfanger and Slovinski (1998); Wang and Huang (2011)). A varity of methods for comparing or ranking fuzzy sets have been reported (Wang and Kerre (1996); Timenez et al., (2007), Wang and Huang (2011)) constructed a fuzzy relationship analysis to compare fuzzy sets: such a method was computationally efficient.

For any pair of fuzzy sets $(\tilde{A}$ and $\tilde{B})$, the degree in which $\tilde{A}$ is smaller than $\tilde{B}$ can be defined as follows:

$$\mu_M(\tilde{A},\tilde{B}) = \begin{cases} 0, & \text{if } E_2^{\tilde{B}} - E_1^{\tilde{A}} < 0; \\ \dfrac{E_2^{\tilde{B}} - E_1^{\tilde{A}}}{(E_2^{\tilde{B}} - E_1^{\tilde{A}}) - (E_2^{\tilde{B}} - E_2^{\tilde{A}})}, & \text{if } 0 \in [E_1^{\tilde{B}} - E_2^{\tilde{A}},\, E_2^{\tilde{B}} - E_1^{\tilde{A}}]; \\ 1, & \text{if } E_1^{\tilde{B}} - E_2^{\tilde{A}} > x < a_3; \end{cases} \quad (15)$$

where $[E_1^{\tilde{A}}, E_2^{\tilde{A}}]$ and $[E_1^{\tilde{B}}, E_2^{\tilde{B}}]$ are the expected intervals of $\tilde{A}$ and $\tilde{B}$. When $\mu_M(\tilde{A},\tilde{B}) \geq \alpha,\ \alpha \in [0,1]$, it implies that $\tilde{A}$ is smaller than or equal to $\tilde{B}$ at least in degree (α) and can be represented by $\tilde{A} \leq_\alpha \tilde{B}$.

For the first question, according to Wang and Huang (2011) a decision interval $T_i^{\pm} \subseteq R,\ S_{ij} \subseteq R,\ i = 1, \ldots, m$ and $j = 1, \ldots, n$ will be feasible of degree α (or α -feasible) if

$$\min_j \left\{ \mu_M \left(\sum_{i=1}^{m} (T_i^{\mp} - S_{ij}^{\mp}), q_j^{\mp} \right) \right\} = \alpha \quad (16)$$

where α is the degree of the feasibility of a decision vector, and $1-\alpha$ provides a measure of the risk of infeasibility. According to formula (12), this is equivalent to:

$$\sum_{i=1}^{m} (T_i^{\mp} - S_{ij}^{\mp})(1 + (1-\alpha)E_1^{\tilde{\delta}\mp} + \alpha E_2^{\tilde{\delta}\mp}) \leq \alpha E_1^{\tilde{q}_j^{\mp}} + (1-\alpha) E_2^{\tilde{q}_j^{\mp}},$$

$$j = 1, \ldots, n \quad (17)$$

The set of decision vectors that are α -feasible denoted by $k(\alpha)$, it is evident that:

$$\alpha_1, \alpha_2 \Rightarrow k(\alpha_1) \supset k(\alpha_2) \quad (18)$$

To address the second question, vectors $T^0 \in R^m$ and $5^0 \in R^{mn}$ is an acceptable solution if it can be verified that (Wang and Huang (20110):

$$\mu_M \left[\sum_{i=1}^{m} (\tilde{N} B_i^{L\mp}, \tilde{N} B_i^{U\mp} (T_i^{\mp})_1^{\tilde{q}_j^{\mp}} - \sum_{i=1}^{m} \sum_{j=1}^{n} p_j (\tilde{C}_i^{L\mp}, \tilde{C}_i^{U\mp})(S_{ij}^{\mp}), \right.$$

$$\left. \sum_{i=1}^{m} (\tilde{N} B_i^{L\mp} (\tilde{N} B_i^{L\mp}, N \tilde{B}_i^{\mp})(T_i^{\mp}) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_j (\tilde{C}_i^{L\mp}, \tilde{C}_i^{U\mp})(S_{ij}^{\mp}) \right] \geq \frac{1}{2}$$

Thus,

$$\mu_M\,[(\tilde{f}^{L\mp}, \tilde{f}^{U\mp}),\ (\tilde{f}^{0L\mp}, \tilde{f}^{0U\mp})] \geq \frac{1}{2} \qquad (19)$$

where (T^0, S^0) is a better choice (with maximized objective) at least in degree $\frac{1}{2}$. According to formula (12), it is equivalent to:

$$\frac{1}{2}(E_1^{\tilde{f}^{0k\mp}}, E_2^{\tilde{f}^{0k\mp}}) \geq \frac{1}{2}(E_1^{k\mp}, E_2^{k\mp}),\ k = L, U \qquad (20)$$

Based in formulae (14) and (17), vectors $T^0 \in R^m$ and $S^0 \in R^{mn}$ is an α-acceptable solution of the initial FRISP model (6) if it an optimal solution to the following α-parametric RITSP model:

$$\max\ EV\,(f^{L\mp}, f^{U\mp}) = \sum_{i=1}^{m} EV\,(\tilde{N}\,B_i^{L\mp}, \tilde{N}\,B_i^{U\mp})(T_i^{\mp})$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\, EV\,(C_i^{L\mp}, C_i^{U\mp})(S_{ij}^{\mp}) \qquad (21a)$$

subject to

$$\sum_{i=1}^{m}(T_i^{\mp} - S_{ij}^{\mp})(1 + (1-\alpha)E_1^{\tilde{\delta}^{\mp}} + \alpha E_2^{\tilde{\delta}^{\mp}}) \leq \alpha E_1^{\tilde{q}_j^{\mp}} + (1-\alpha)E_2^{\tilde{q}_j^{\mp}},$$

$$j = 1, \ldots, n; \qquad (21b)$$

$$S_{ij}^{\mp} \leq T_i^{\mp} \leq T_{i\,\max}^{\mp}, \qquad i = 1, \ldots, m; \qquad (21c)$$

and

$$S_{ij}^{\mp} \geq 0, \qquad i = 1, \ldots, m;\ j = 1, \ldots, n; \qquad (21d)$$

where $EV\,(f^{L\mp}, f^{U\mp}) = (EV\,(f^{L\mp}), EV\,(f^{U\mp}));\ EV\,(C_i^{L\mp}, C_i^{U\mp}) = (EV\,(C_i^{L\mp}),\ EV\,(C_i^{U\mp}))$ and $EV\,(\tilde{N}\,B_i^{L\mp}, \tilde{N}\,B_i^{U\mp}) = (EV\,(\tilde{N}\,B_i^{L\mp}), EV\,(\tilde{N}\,B_i^{U\mp}))$ are rough intervals of boundary parameters; α is the feasibility degree of a decision vectors and $(1-\alpha)$ provides a measure about the risk of infeasibility for a decision vectors.

According to Wang and Huang (2011), if $T_i^{\mp}$ are consider as uncertain inputs, let the interval variable $T_i^{\mp} = T_i^{-} + \Delta T_i\, y_i$, where $\Delta T_i = T_i^{+} - T_i^{-}$ and $y_i \in [0, 1]$. y_i, $i = 1, \ldots, m$ are decision variables that are used for identifying an optimized set of target values $(T_i^{\mp})$ in order to support the related policy analysis. Thus, by introducing decision variables (g_i) model (21) can be reformulated as:

$$\max\ EV\,(\tilde{f}^{L\mp}, \tilde{f}^{U\mp}) = \sum_{i=1}^{m} EV\,(\tilde{N}\,B_i^{L\mp}, \tilde{N}\,B_i^{U\mp})(T_i^{\mp} + \Delta T_i\, y_i)$$

ISSN: 2347-3487

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\, EV\,(\tilde{C}_i^{L\mp}, \tilde{C}_i^{U\mp})(S_{ij}^{\mp}) \qquad \text{(22a)}$$

subject to

$$\sum_{i=1}^{m}(T_i^{-}+\Delta T_i\, y_i - S_{ij}^{\mp})(1+(1-\alpha)E_1^{\tilde{\delta}^{\mp}}+\alpha E_2^{\tilde{\delta}^{\mp}}) \le \alpha E_1^{\tilde{q}_j^{\mp}}+(1-\alpha)E_2^{\tilde{q}_j^{\mp}},$$

$$j=1,\ldots,n; \qquad \text{(22b)}$$

$$S_{ij}^{\mp} \le T_i^{-}+\Delta T_i\, y_i \le T_{i\,\max}^{\mp}, \quad i=1,\ldots,m; \qquad \text{(22c)}$$

and

$$S_{ij}^{\mp} \ge 0, \qquad i=1,\ldots,m;\; j=1,\ldots,n; \qquad \text{(22d)}$$

when $T_i^{\mp}$ are known model (22) can be transformed into four deterministic such model, which are correspond to the lower and upper of the rough objective-function value and correspond to the lower and upper bounds of the objective function interval value. This transformation process is based on an interactive algorithm. For the lower of the rough objective function value, consider the following two submodels. The first model is:

$$\max\; EV\,(\tilde{f}^{L+}) = \sum_{i=1}^{m} EV\,(\tilde{N}\,B_i^{L+})(T_i^{-}+\Delta T_i\, y_i)$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\, EV\,(\tilde{C}_i^{L-})(S_{ij}^{-}) \qquad \text{(23a)}$$

subject to

$$\sum_{i=1}^{m}(T_i^{-}+\Delta T_i\, y_i - S_{ij}^{-})(1+(1-\alpha)E_1^{\tilde{\delta}^{-}}+\alpha E_2^{\tilde{\delta}^{-}}) \le \alpha E_1^{\tilde{q}_j^{+}}+(1-\alpha)E_2^{\tilde{q}_j^{+}},$$

$$j=1,\ldots,n; \qquad \text{(23b)}$$

$$S_{ij}^{-} \le T_i^{-}+\Delta T_i\, y_i \le T_{i\,\max}^{+}, \quad i=1,\ldots,m; \qquad \text{(23c)}$$

$$S_{ij}^{\mp} \ge 0, \qquad i=1,\ldots,m;\; j=1,\ldots,n; \qquad \text{(23d)}$$

where S_{ij}^{-} and y_i are decision a variables.

The second model correspond to the lower bound of the objective-function interval is:

$$\max\; EV\,(\tilde{f}^{L-}) = \sum_{i=1}^{m} EV\,(\tilde{N}\,B_i^{L-})(T_i^{-}+\Delta T_i\, y_i)$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n} p_j\, EV\,(\tilde{C}_i^{L+})(S_{ij}^{+}) \qquad \text{(24a)}$$

subject to

$$\sum_{i=1}^{m}(T_i^- + \Delta T_i y_i - S_{ij}^+)(1+(1-\alpha)E_1^{\tilde{\delta}+} + \alpha E_2^{\tilde{\delta}+}) \le \alpha E_1^{\tilde{q}_j^-} + (1-\alpha)E_2^{\tilde{q}_j^-},$$

$$j = 1, \ldots, n; \qquad (24b)$$

$$S_{ij}^+ \le T_i^- + \Delta T_i y_i \le T_{i\,\max}^-, \quad i = 1, \ldots, m; \qquad (24c)$$

$$S_{ij}^+ \ge S_{ij}^-, \quad i = 1, \ldots, m; \; j = 1, \ldots, n; \qquad (24d)$$

where S_{ij}^+ and y_i are decision a variables.

Also, for the upper of the rough objective-function value, consider the following two submodels, the first model correspond to the upper bound of the objective-function interval

$$\max \; EV(\tilde{f}^{U+}) = \sum_{i=1}^{m} EV(\tilde{N}B_i^{U+})(T_i^- + \Delta T_i y_i)$$

$$- \sum_{i=1}^{m}\sum_{j=1}^{n} p_j EV(\tilde{C}_i^{U-})(S_{ij}^-) \qquad (25a)$$

subject to

$$\sum_{i=1}^{m}(T_i^- + \Delta T_i y_i - S_{ij}^-)(1+(1-\alpha)E_1^{\tilde{\delta}^-} + \alpha E_2^{\tilde{\delta}^-}) \le \alpha E_1^{\tilde{q}_j^+} + (1-\alpha)E_2^{\tilde{q}_j^+},$$

$$j = 1, \ldots, n; \qquad (25b)$$

$$S_{ij}^- \le T_i^- + \Delta T_i y_i \le T_{i\,\max}^+, \quad i = 1, \ldots, m; \qquad (25c)$$

$$S_{ij}^{\mp} \ge 0, \quad i = 1, \ldots, m; \; j = 1, \ldots, n; \qquad (25d)$$

where S_{ij}^- and y_i are decision a variables.

The second model correspond to the lower bound of the objective-function interval is:

$$\max \; EV(\tilde{f}^{U-}) = \sum_{i=1}^{m} EV(\tilde{N}B_i^{U-})(T_i^- + \Delta T_i y_i)$$

$$- \sum_{i=1}^{m}\sum_{j=1}^{n} p_j EV(\tilde{C}_i^{U+})(S_{ij}^+) \qquad (26a)$$

subject to

$$\sum_{i=1}^{m}(T_i^- + \Delta T_i y_i - S_{ij}^+)(1+(1-\alpha)E_1^{\tilde{\delta}+} + \alpha E_2^{\tilde{\delta}+}) \le \alpha E_1^{\tilde{q}_j^-} + (1-\alpha)E_2^{\tilde{q}_j^-},$$

$$j = 1, \ldots, n \,; \tag{26b}$$

$$S_{ij}^{+} \le T_i^{-} + \Delta T_i \, y_i \le T_{i\,\max}^{-}, \quad i = 1, \ldots, m \,; \tag{26c}$$

$$S_{ij}^{+} \ge S_{ij}^{-}, \quad i = 1, \ldots, m; \; j = 1, \ldots, n \,; \tag{26d}$$

where S_{ij}^{+} and y_i are decision a variables. Submodels (23) – (26) are deterministic linear programming problems. The solutions of model 919) under the optimized water-allocation targets are given by the following (27) and (28):

$$[S_{ij\,\mathrm{opt}}^{L\mp}] = [S_{ij\,\mathrm{opt}}^{L-}, S_{ij\,\mathrm{opt}}^{L+}], \; [\tilde{f}_{\mathrm{opt}}^{L\mp}] = [\tilde{f}_{\mathrm{opt}}^{L-}, \tilde{f}_{\mathrm{opt}}^{L+}] \tag{27}$$

where $S_{ij\,\mathrm{opt}}^{L-}$ and $\tilde{f}_{\mathrm{opt}}^{L+}$ are the solution of (20), and $S_{ij\,\mathrm{opt}}^{L+}$ and $\tilde{f}_{\mathrm{opt}}^{L-}$ are those of sub-model (21). Also,

$$[S_{ij\,\mathrm{opt}}^{U\mp}] = [S_{ij\,\mathrm{opt}}^{U-}, S_{ij\,\mathrm{opt}}^{U+}], \; [\tilde{f}_{\mathrm{opt}}^{U\mp}] = [\tilde{f}_{\mathrm{opt}}^{U-}, \tilde{f}_{\mathrm{opt}}^{U+}] \tag{28}$$

where $S_{ij\,\mathrm{opt}}^{U-}$ and $\tilde{f}_{\mathrm{opt}}^{U+}$ are the solution of (25), and $S_{ij\,\mathrm{opt}}^{U+}$ and $\tilde{f}_{\mathrm{opt}}^{U-}$ are those of sub-model (26). Thus, the optimized rough-water allocation scheme is:

$$A_{ij\,\mathrm{opt}}^{L\mp} = T_{i\,\mathrm{opt}}^{L\mp} - S_{ij\,\mathrm{opt}}^{L\mp}, \; A_{ij\,\mathrm{opt}}^{U\mp} = T_{i\,\mathrm{opt}}^{U\mp} - S_{ij\,\mathrm{opt}}^{U\mp} \tag{29}$$

The degree of rough of the optimized rough-water allocation, denoted by λ , is:

$$\lambda = \frac{|A_{ij\,\mathrm{opt}}^{L\mp}|}{A_{ij\,\mathrm{opt}}^{U\mp}|} \tag{30}$$

5. The Algorithm

An interactive algorithm is proposed to solve the α-parametric RITSP problems by using interactive fuzzy resolution (IFR). The IFR (Wang and Huang (2011)) method allows DMS to consider two factors in an interactive way, feasibility of the constraints and satisfaction degree of the goals. A strong desire to obtain a better objective-function value will lead to a lower degree of constraint feasibility; contrarly, a willingness to accept a lower satisfaction degree of the goal will guarantee a higher feasibility of the constraints. Therefore, the DMS have identify a balanced solution between the two objectives in conflict; to improve the objective-function value or to improve the constraint feasibility.

The natural language for establishing a semantic correspondence for different degrees of feasibility is the best way to reflect the DMS preferences. Therefore, Wang and Huang (2011) and others establish 11 scales, which allowed a sufficient distinction between different levels without being excessive. The 11 scales are as follows:

0.0- Unacceptable solution, 0.1- Practically unacceptable solution,

0.2- Almost unacceptable solution, 0.3- Very unacceptable solution,

0.4- Quite unacceptable solution, 0.5- Neither acceptable solution,

0.6- Quite acceptable solution, 0.7- Very acceptable solution,

0.8- Almost acceptable solution, 0.9 Practically acceptable solution,

1.0- Completely acceptable solution.

Obviously, other scales can be considered depending on the wishes of the DMS. The 11 scales correspond to different risk level of violating the modeling constraints. The higher the scale value, the lower the risk level. In real world problems, the DMS may not willing to admit a high risk of constraints violation. If α_0 is the minimum constraint feasibility that the DMS are willing to accept, the feasibility interval of α will be reduce to $\alpha_0 \leq \alpha \leq 1$. According to the semantic scale, the discrete values of α can be achieved as follows:

$$M = \left\{ \alpha_k = \alpha_0 + 0.1k \,/\, k = 0, 1, ..., \frac{1-\alpha_0}{0.1} \right\} \subset [0, 1] \tag{31}$$

In the first step of the IFR method, the space $O = \{T_{\text{opt}}, S_{\text{opt}} / \alpha_k \in M\}$ of the α_k -acceptable solutions and the corresponding possibilistic distributions of the objective-function values $f_{\text{opt}}(\alpha_k)$ can be obtained through solving (13) under each α_k . In order to obtain a decision vector that complies with the expectation of the DMS, two conflicting factors should be evaluated: feasibility of the constraints and acceptability of the objective-function value. After obtaining the results under different $f_{\text{opt}}(\alpha_k)$, the DMS are asked to specify a goal $(\bar{G})$ and its tolerance threshold $(\underline{G})$. The foal is expressed by means of a fuzzy goal set $(\tilde{G})$ whose membership function is:

$$\mu_{\tilde{G}}(z) = \begin{bmatrix} 1, & \text{if} & z \geq \bar{G}; \\ \lambda \in (0, 1), & \text{if} & \underline{G} < z < \bar{G}; \\ 0, & \text{if} & z \leq \underline{G}, \end{bmatrix} \tag{32}$$

In the second step of the IFR method, the satisfaction degree of the fuzzy goal $(\tilde{G})$ for each α -acceptable solution, i.e., the membership grade of $f_{\text{opt}}(\alpha_k)$ to $(\tilde{G})$ can be analyzed through an index:

$$\mu_{\tilde{G}}(f_{\text{opt}}(\alpha_k)) = \frac{\int_S \mu_{\tilde{f}_{\text{opt}}}(z) \cdot \mu_{\tilde{G}}(z)\, dz}{\int_S \mu_{\tilde{f}_{\text{opt}}}(z)\, dz} \tag{33}$$

where $\int$ denotes the classical integral; *S* is the support of the fuzzy set $(\tilde{f}_{\text{opt}})$. This an extension of the widely accepted center of gravity defuzzification method, using the goal function $(\mu_{\tilde{G}})$ as a weighting factor (Jimenez et al., (2007)).

In the third step of the IFR method, we attempt to identify the balance between feasibility of the constraints and the satisfication degree of the foal in the space of α_k -acceptable solutions (O).

Now, we consider to build two fuzzy sets: *F* and *S* with the following membership functions $\mu_{\tilde{F}}(T_{\text{opt}}(\alpha_k), S_{\text{opt}}(\alpha_k)) = \alpha_k$ and $\mu_{\tilde{S}}(T_{\text{opt}}(\alpha_k), S_{\text{opt}}(\alpha_k)) = K_{\tilde{G}}(\tilde{f}_{\text{opt}}(\alpha_k))$, respectively. For obtaining a final recommendation, a fuzzy decision $(\tilde{D})$ is defined by aggregating the two a forenamed fuzzy sets (i.e., $\tilde{D} = \tilde{F} \cap \tilde{S}$) (Bellman and Zaeh (1970); Jimênez et al., (2007)):

ISSN: 2347-3487

$$\mu_{\tilde{D}}(T_{\text{opt}}(\alpha_k), S_{\text{opt}}(\alpha_k)) = \min(\alpha_k, k_{\tilde{G}}(f_{\text{opt}}(\alpha_k)) \quad (34)$$

The solution with the highest membership grade will be the final decision for the IFRITSP problem (i.e., model (6))

$$\mu_{\tilde{D}}(T^*_{\text{opt}}, S^*_{\text{opt}}) = \max_{\alpha_k \in M}\{\min(\alpha_k, k_{\tilde{G}}(f_{\text{opt}}(\alpha_k)) \quad (35)$$

The detailed solution process can be summarized as follows:

Step 1: Formulate the IFRITSP model (6).

Step 2: Reformulate the IFRIRSP model by introducing $T_i^{\mp} = T_i^- + \Delta T_i\, y_i$, where $\Delta T_i = T_i^+ - T_i^-$ and $y_i \in [0,1]$.

Step 3: Transform the IFRITSP model into four submodels ((20)-(23)), where the first two submodels corresponding to $\tilde{f}^{L\mp}$ and the second two submodels corresponding to $\tilde{f}^{U\mp}$.

Step 4: Obtain the α_k -acceptable solutions through solving the $\tilde{f}^{L+}$ submodel under each α_k .

Step 5: Compute the satisfication degree of the fuzzy foal for each α -acceptable solution.

Step 6: Identify a balance (i.e., membership grade of the fuzzy decision) between feasibility of the constraints and satification degree of the foal for each α -acceptable solution.

Step 7: Formulate and solve the $\tilde{f}^{L-}$ submodel by the following the same interactive procedure as that in $\tilde{f}^{L+}$ submodel.

Step 8: Reach a desired compromise between feasibility of the constraints and the satisfication degree of the goal by considering the solutions from the two submodels.

Step 9: Obtain the α_k -feasibility optimal solution:

$$\tilde{f}^{L\mp}_{\text{opt}} = [\tilde{f}^{L-}_{\text{opt}}, \tilde{f}^{L+}_{\text{opt}}], \qquad S^{L\mp}_{ij\,\text{opt}} = [S^{L-}_{ij\,\text{opt}}, S^{L+}_{ij\,\text{opt}}]\ \forall\ i, j\ .$$

Step 10: Obtain the optimal water-allocation scheme:

$$A^{L\mp}_{ij\,\text{opt}} = T^{L\mp}_{i\,\text{opt}} - S^{L\mp}_{ij\,\text{opt}}\quad \forall\ i, j\ .$$

Step 11: Repeat steps (4)-(10) for obtaining the optimal water allocation scheme:

$$A^{U\mp}_{ij\,\text{opt}} = T^{U\mp}_{i\,\text{opt}} - S^{U\mp}_{ij\,\text{opt}}\quad \forall\ i, j$$

To the two submodels corresponding to $\tilde{f}^{U\mp}$.

Step 12: Find the rough degrees of the optimal water allocation scheme:

$$\lambda_{ij} = \frac{|A^{L\mp}_{ij}|}{|A^{U\mp}_{ij}|}, \qquad \lambda_{ij} \in [0,1]\ \forall\ i, j\ .$$

Step 13: Stop.

ISSN: 2347-3487

6. Case study

The following water resources management problem will be used demonstrate applicability of the developed IFRITSP approach. A water manager is responsible for allocating water from unregulated sector to three users: a municipality, an industry concern and an agricultural sector. These water users need to know how much water they can expect so as to make appropriate decisions on their various activities and investments. If the promised water is delivered a net benefit to the local economy will be generated for each unit water allocated; otherwise, either the water must be obtained from higher-priced alternatives of the demand must cartailed by reduced industrial and / or agricultural productions, resulting in a reduced system benefit (Wang and Huang (2011)). Table 1 shows the maximum allowable water allocation, the water-allocation target, the fuzzy-rough net benefit to user *i* per m^3 of water allocated and the loss to user *i* per m^3 of water not delivered. Moreover, Table 1 presents the fuzzy boundary intervals for the rate of water loss during transportation and the seasonal flows under different probability levels.

Table 1. Related economic data ($/ m^3) and seasonal flows (in $10^6\ m^3$) and different probability levels

Activity	User $(i = 1)$	User $(i = 2)$	User $(i = 3)$
Maximum allowable allocation $(T^{\mp}_{\max})$	7.0	7.0	7.0
Water allocation target	[1.5, 2.5]	[2.0, 4.0]	[3.5, 6.5]
Net benefit water demand is satisfied $(\tilde{N}B_i^{L\mp}, \tilde{N}B_i^{U\mp})$	([(85, 90, 95), (105, 110, 115)], [(80, 90, 100), (100, 110, 120)])	([940, 45, 50), (60, 65, 75)], [935, 45, 55), (55, 65, 75)]	([(26, 28, 30), (30, 32, 34)])
Reduction of net benefit water demand is not delivered $(\tilde{C}_i^{L\mp}, \tilde{C}_i^{U\mp})$	([(210, 220, 230), (270, 280, 290)], [(215, 220, 230), (270, 280, 290)])	[(50, 60, 70), (80, 90, 100)], [(55, 60, 65), (85, 90, 95)])	([(40, 50, 60), (60, 70, 80)])
Flow level	**Probability (%)**	**Seasonal flow** $(\tilde{q}_j^{\mp})$	
Low $(j = 1)$	0.2	[(3.3, 3.5, 3.7), (.3, 4.5, 4.7)]	
Medium $(j = 2)$	0.6	[(7.0, 8.0, 9.0), (11.0, 12.0, 13.0)]	
High $(j = 3)$	0.2	[(114.0, 15.0, 16.0), (18.0, 19.0, 20.0)]	
Rate of water loss $(\tilde{\delta}^{\mp})$		[(0.10, 0.15, 0.20), (0.30, 0.35, 0.40)]	

$$\max\ (\tilde{f}^{L\mp}, \tilde{f}^{U\mp}) = 1.9([(85, 90, 95), (105, 110, 115)], [(80, 90, 100), (100, 110, 120)])$$

$$+3.49[(40, 45, 50), (60, 65, 70)], [(35, 45, 55), (55, 65, 75)])$$

$$+4.7([(27, 28, 29), (31, 32, 33)], [(26, 28, 30), (30, 32, 34)])$$

$$-0.2([9215, 220, 225), (275, 280, 285)], [(210, 220, 232),$$

$$(270, 280, 290)]S_{11}^{\mp})$$

$$-0.2([55, 60, 65), (85, 90, 95)], [(50, 60, 70), (80, 90, 100)]S_{21}^{\mp}$$

$$-0.2([(45, 50, 55), (65, 70, 75)], [(40, 50, 60), (60, 70, 80)])S_{31}^{\mp}$$

$$-0.6([(215, 220, 225), (275, 280, 285)], [(210, 220, 230),$$

$$(270, 280, 290)])S_{12}^{\mp}$$

$$-0.6([55, 60, 65), (85, 90, 95)], [(50, 60, 70), (80, 90, 100)]S_{22}^{\mp}$$

$$-0.6([(45, 50, 55), (65, 70, 75)], [(40, 50, 60), (60, 70, 80)])S_{32}^{\mp}$$

$$-0.2([(215, 220, 225), (275, 280, 285)], [(210, 220, 230),$$

$$(270, 280, 290)])S_{13}^{\mp}$$

$$-0.2([55, 60, 65), (85, 90, 95)], [(50, 60, 70), (80, 90, 100)]S_{23}^{\mp}$$

$$-0.2([(45, 50, 55), (65, 70, 75)], [(40, 50, 60), (60, 70, 80)])S_{33}^{\mp}$$

subject to

$$(10 - S_{11}^{\mp} - S_{21}^{\mp} - S_{31}^{\mp})(1 + 0.6E_1^{\tilde{\delta}\mp} + 0.4E_2^{\tilde{\delta}\mp}) \le 0.4E_1^{\tilde{q}_1^{\mp}\mp} + 0.6E_2^{\tilde{q}_1^{\mp}},$$

$$(10 - S_{12}^{\mp} - S_{22}^{\mp} - S_{32}^{\mp})(1 + 0.6E_1^{\tilde{\delta}\mp} + 0.4E_2^{\tilde{\delta}\mp}) \le 0.4E_1^{\tilde{q}_2^{\mp}\mp} + 0.6E_2^{\tilde{q}_2^{\mp}},$$

$$(10 - S_{13}^{\mp} - S_{23}^{\mp} - S_{33}^{\mp})(1 + 0.6E_1^{\tilde{\delta}\mp} + 0.4E_2^{\tilde{\delta}\mp}) \le 0.4E_1^{\tilde{q}_3^{\mp}\mp} + 0.6E_2^{\tilde{q}_3^{\mp}},$$

$$\tilde{S}_{11}^{\mp} \le 1.9 \le 7,$$

$$\tilde{S}_{21}^{\mp} \le 3.4 \le 7,$$

$$\tilde{S}_{31}^{\mp} \le 4.7 \le 7,$$

$$\tilde{S}_{12}^{\mp} \le 1.9 \le 7,$$

$$\tilde{S}_{22}^{\mp} \le 3.4 \le 7,$$

$$\tilde{S}_{32}^{\mp} \le 4.7 \le 7,$$

$$\tilde{S}_{13}^{\mp} \le 1.9 \le 7,$$

$$\tilde{S}_{23}^{\mp} \le 3.4 \le 7,$$

$$\tilde{S}_{33}^{\mp} \le 4.7 \le 7\,,$$

$$\tilde{S}_{i\,j}^{\mp} \ge 0,\ \forall\ i\,,\ j = 1, 2, 3.$$

ISSN: 2347-3487

Table 2. The α -acceptable solutions from the IRISTP model (in $10^6 m^3$)

Feasibility degree	Shortage $(S_{ij}^{L\mp}, S_{ij}^{U\mp})$								
	$(S_{11}^{L\mp}, S_{11}^{U\mp})$	$(S_{21}^{L\mp}, S_{21}^{U\mp})$	$(S_{31}^{L\mp}, S_{31}^{U\mp})$	$(S_{12}^{L\mp}, S_{12}^{U\mp})$	$(S_{22}^{L\mp}, S_{22}^{U\mp})$	$(S_{32}^{L\mp}, S_{32}^{U\mp})$	$(S_{13}^{L\mp}, S_{13}^{U\mp})$	$(S_{23}^{L\mp}, S_{23}^{U\mp})$	$(S_{33}^{L\mp}, S_{33}^{U\mp})$
$\alpha = 0.4$	0	[1.3524, 2.6829]	4.7	0	0	[0, 1.0781]	0	0	0
$\alpha = 0.5$	0	[1.3870, 2.7074]	4.7	0	0	[0, 1.111]	0	0	0
$\alpha = 0.6$	0	[1.4212, 2.7317]	4.7	0	0	[0, 1.1439]	0	0	0
$\alpha = 0.7$	0	1.4552, 2.7559]	4.7	0	0	[0, 1.1765]	0	0	0
$\alpha = 0.8$	0	[1.4888, 2.7799]	4.7	0	0	[0, 1.2088]	0	0	0
$\alpha = 0.9$	0	1.5222, 2.8036]	4.7	0	0	[0, 1.2409]	0	0	0
$\alpha = 1.0$	0	1.5553, 2.823]	4.7	0	0	[0, 4.7]	0	0	0

Table 3. Possibilistic distribution of system benefit, $(\tilde{f}_{(\alpha)}^{L\mp}, \tilde{f}_{(\alpha)}^{U\mp})$ (L. E. 10^6)

$\alpha = 0.4$	([(291.6278, 234.096, 396.4966), (446.952, 490.2565, 533.54616), [(271.045, 344.0622, 417.0794), (473.1956, 490.2568, 558.7092)])
$\alpha = 0.5$	([(290.0968, 342.7782, 395.4596), (446.26, 489.634, 533.008)], [(269.6375, 342.7782, 415.9189), (421.147, 489.634, 524.121)])
$\alpha = 0.6$	([(288.5758, 341.5026, 394.4294), (445.576, 489.0184, 532.4608)], [(268.2392, 341.5026, 414.766), (420.4972, 489.0184, 557.5396)])
$\alpha = 0.7$	([(287.0634, 340.2342, 393.405), (444.896, 488.4064, 531.9168)], [(266.8488, 340.2342, 413.6196), (419.8512, 488.4064, 556.9616)])
$\alpha = 0.8$	([(285.5646, 338.9772, 392.3898), (444.224, 487.8016, 531.3792)], [(265.4709, 338.9772, 412.4835), (419.2128, 487.8016, 556.3904)])
$\alpha = 0.9$	([(159.278, 233.724, 308.17), (443.556, 487.2004, 530.8445)], [(264.1035, 337.7298, 411.3561), (418.5782, 487.2004, 555.82261)])
$\alpha = 1.0$	([(159.278, 233.724, 308.17), (442.894, 486.6046, 530.3152)], [(149.701, 233.724, 317.747), (417.9493, 486.6046, 533.709)])

ISSN: 2347-3487

References

[1] Huang, G. H., and Chang, N. B., (2003). The perspectives of environmental informatics and systems analysis, Journal of Environmental Informatics, (1): 1-6.

[2] Jairaj, P. G., and Vedula, S., (2000). Multi-reservoir system optimization using fuzzy mathematical programming, Water Resources Management, (14): 457-472.

[3] Jimanez, M., Arenas, M., Bilbao, and Rodriguez, M. V., (2007). Linear programming with fuzzy parameters: an interactive methods resolution, European Journal of Operational Research, (177): 1599-1609.

[4] Lai, Y. L., and Hwang, C. L., (1994). Fuzzy Multiple objective Decision Making: Methods and Applications, Springer-Verlage, Berline-Heidelberg, New-York.

[5] Li, Y. P., Huang, G. H., Nie, S. L., and Qin, X. S., (2007). ITCLP: an inexact-stage chance-constrain (2007). ITCLP: an inexact two-stage chance-constrain program for planning waste management systems, Resources Conservation and recycling, (49): 284-307.

[6] Li, Y. P., and Huang, G. H., (2008). Interval-parameter two-stage stochastic nonlinear programming for water resources management under uncertainty, Water Resources Management, (22): 681-698.

[7] Lu, H. W., Huang, G. H., and He, L., (2010). Development of an interval-valued fuzzy linear programming method based on infinite α -cuts for water resources management, Environmental Modeling and Software, (25): 345-361.

[8] Luo, B., Maqsood, I., Yin, Y. Y., Huang, G. H., and Chohen, S. J., (2003). Adaption to climate change through water trading under uncertainty- an inexact two-stage nonlinear programming approach, Journal of Environmental Informatics, (2): 58-68.

[9] Moqsood, I., Huang, G. H., and Yeomans, J. S., (2005). An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty, European Journal of Operational Research, (167): 208-225.

[10] Moore, R. E., (1979). Methods and Applications of Interval Analysis, SIAM, Philadeiphia, PA.

[11] Nande, S., and Majumbar. S., (1992). Fuzzy Rough Sets, Fuzzy sets and Systems, (45): 157-160.

[12] Seif, A., and Hipel, K. W., (2001). Interior-point method for reservoir operation with stochastic inflows, ASCE-Journal of Water Resources Planning Management, (127): 48-57.

[13] Slowinski, R., (1986). A multicriteria fuzzy linear programming method for water supply system development planning, Fuzzy Sets and Systems, 919): 217-237.

[14] Wang, X., and Kerre, E., (1996). On the classification and the dependencies of the ordering methods. In: Ruan, D. (Ed.), Fuzzy Logic Foundation and Inclustrial Applications, International Series in Intelligent Technologies, Kluwer, Dordrecht, 73-90.

[15] Wang, L. e., fang, L., and Hipel, K. W., (2003). Water resources allocation: a cooperative game theoretic approach, Journal of Environmental Informatics, (2): 11-22.

[16] Wang, S., Huang, G. H., Lu, H. W., and Li, Y. P., (2010). An interval-valued fuzzy linear programming with infinite α -cuts method for environmental management under uncertainty, Stochastic Environmental Research and Risk Assessment, (25): 211-222.

[17] Wang, S., and Huang, G. H., (2011). Interactive two-stage stochastic fuzzy programming for water resources management, Journal of Environmental Management, (92): 1986-1995.

[18] Wu, S. M., Huang, G. H., and Guo, H. C., (1997). An interactive inexact-fuzzy approach for multiobjective planning of water resources systems.

[19] Zadeh, L. A., (1965). Fuzzy sets, Information and control, (8): 338-353.

[20] Zadeh, L. A., (1978). Fuzzy sets as 0 basis for a theory of possibility, Fuzzy Sets and Systems, (1): 3-28.8.