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# Nuclear Shape Transition Between Spherical U(5) and $\gamma$-Unstable O(6) Limits of the Interacting Boson Model

A.M.Khalaf[(1)],T.M.Awwad[(2)] and N.Gaballah[(3)]
[(1)] Physics Department, Faculty of Science, Al-Azhar University, Egypt,
E-Mail: Ali_Khalaf43@hotmail.com
[(2)]Physics Department, Faculty of Girls,Ain Shams University, Egypt,
E-Mail:tawwad13@yahoo.com
[(3)]Physics Department, Faculty of Science(Girls Branch), Al-Azhar University, Egypt,
E-mail:nermgaballah@yahoo.com

## ABSTRACT

The interacting boson model (IBM) with intrinsic coherent state (characterized by $\beta$ and $\gamma$) is used to describe the nuclear second order shape phase transition (denoted E(5)) between the spherical oscillator U(5) and the $\gamma$-soft rotor O(6) structural limits. The potential energy surfaces (PES's) have been derived and the critical points of the phase transition have been determined . The model is examined for the spectra of even-even neutron rich xenon isotopic chain. The best adopted parameters in the IBM Hamiltonian for each nucleus have been adjusted to reproduce as closely as possible the experimental selected numbers of excitation energies of the yrast band, by using computer simulated search program.Using the best fitted parameters , the $E(I_i^\pi)/E(2_1^+)$ energy ratios for the $I_i^\pi = 4_1^+, 6_1^+, and\ 8_1^+$levels are calculated and compared to those of the O(6) and U(5) dynamical symmetry limits. $^{122}$Xe and $^{132}$Xe are considered as examples for the two O(6) and U(5) dynamical symmetry limits respectively.

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## 1.Introduction

The interacting boson model (IBM)[1] was widely used for describing the quadrupole collective states of the medium and heavy nuclei .In the original version of sd-IBM-1,the model includes s- and d-bosons and no distinction is made between proton and neutron bosons. The algebraic structure of this model is based upon U(6), and three dynamical symmetries arise, involving the subalgebras U(5), SU(3) and O(6) corresponding to spherical oscillator, axially symmetric rotor and gamma soft rotor. These three symmetry limits form a Casten triangle [2], that represents the nuclear phase diagram [3]. The intermediate between these three limits are of great interest. It was shown by Iachello [4-6], that new dynamical symmetries, called E(5),X(5) and Y(5) hold , respectively at the critical point between spherical shape and $\gamma$-unstable deformation, between spherical and axially symmetric shape and between deformed axial and triaxial shape. In all these cases, critical points are defined in the context of the collective geometric BohrHamiltonian [7]. Soon, thereafter the introduction of this concept of critical point symmetries different nuclear shapes and phase transitions between them are studied [8-15] .

The correspondence between the E(5) solution of Bohr Hamiltonian and the U(5)-O(6)transition in the IBM was studied in details [16-20] and the existence of an additional prolate-oblate transition was recognized [21,22].

Even mass Xenon nuclei$_{54}$Xe were received much attentions. The Xe nuclei with the mass number A $\sim$ 120 – 130 was studied experimentally [23,24] and interpreted theoretically

by the general Bohr Hamiltonian(GBH) [25] and by acquiring special solutions in the E(5) and

X(5) critical limits by using Davison potential Bohr Hamiltonian [26,27]. Also Montica et al [28] carried out the IBM2 calculations for the Xe isotopic chain.

The purpose of this paper is to analyze the potential energy surfaces (PES's) to investigate the evolution of nuclear shape transition in Xenon nulei from deformed $\gamma$-soft O(6) to spherical vibrator U(5) in framework of the sd-IBM-1 with intrinsic coherent state formalism [29,30].

The outline of the paper is as follows:

In section 2 wedescribe the formalism of the IBM 1 Hamiltonian under study, its intrinsic coherent state and the PES's. Comment on equilibrium deformation and critical points are considered in section 3. The U(5)-O(6) shape transition with more than Hamiltonian form is produced in section 4.Section 5 presents numerical calculations and discussion for Xe isotopic chain. Finally a conclusion and some remarks on our study are given in section 6.

## 2. Formalism

We start by considering the most general Hamiltonian of the sd-IBM in the multipole form as[1]

$$H = \epsilon_d \hat{n}_d + a_0 \hat{P}^\dagger . \hat{P} + a_1 \hat{L} . \hat{L} + a_2 \hat{Q} . \hat{Q} + a_3 \hat{T}_3 . \hat{T}_3 \qquad (1)$$

wherethe multipole operator $\hat{n}_d$ , $\hat{P}$ , $\hat{L}$ , $\hat{Q}$ , $and$ $\hat{T}_3$ are given by:

$$\hat{n}_d = \sum_m d_m^\dagger d_m (2)$$

$$\hat{P}^\dagger = \frac{1}{2}\left(d^\dagger . d^\dagger - s^\dagger s^\dagger\right) \qquad (3)$$

$$\hat{L} = \sqrt{10}\left[d^\dagger \times \tilde{d}\right]^{(1)} \qquad (4)$$

$$\hat{Q} = \left[s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}\right]^{(2)} + \chi\left[d^\dagger \times \tilde{d}\right]^{(2)} \qquad (5)$$

$$\hat{T}_3 = \left[d^\dagger \times \tilde{d}\right]^{(3)} \qquad (6)$$

with $\widetilde{d_\mu} = (-1)^\mu d_\mu$, $t^{(\lambda)} . U^{(\lambda)} = (-1)^\lambda \sqrt{2\lambda+1}[t^{(\lambda)} \times U^{(\lambda)}]^{(0)}$ and $[t^{(\lambda)} \times U^{(\lambda)}]_\mu^{(\lambda)} = \sum_{\mu_1\mu_2} \langle \lambda_1\mu_1\lambda_2\mu_2 | \lambda\mu\rangle\, t_{\mu_1}^{(\lambda_1)} t_{\mu_2}^{(\lambda_2)}$ where $\langle \lambda_1\mu_1\lambda_2\mu_2 | \lambda\mu\rangle$is the Clebsch – Gordan coefficients.The intrinsic coherent normalized state for the sd IBM for a nucleus with N valence bosons outside a doubly closed shell state $|\,0\rangle$ is given by [29,30]

$$|\,N, \beta, \gamma\rangle = \frac{1}{\sqrt{N!}}\left(\Gamma^\dagger(\beta,\gamma)\right)^N |\,0\rangle \qquad (7)$$

where$\Gamma^\dagger$ is the boson creation operator acting in the intrinsic system is given by

$$\Gamma^\dagger(\beta,\gamma) = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\, \left(d_2^\dagger + d_{-2}^\dagger\right)\right] \qquad (8)$$

where the intrinsic deformation parameters $\beta$ and $\gamma$ represent the shape parameters.

In terms of the parameters $\beta$ and $\gamma$, the expectation value of the Hamiltonian H is easily obtained from the evolution of the expectation values of each single term given by :

$$\langle N, \beta, \gamma | \hat{n}_d | N, \beta, \gamma\rangle = \frac{N}{1+\beta^2}\beta^2 \qquad (9)$$

$\langle N,\beta,\gamma|\hat{P}^{\dagger}.\ \hat{P}|N,\beta,\gamma\rangle=\frac{N(N-1)}{4(1+\beta^2)}(1-\beta^2)^2$ (10)

$\langle N,\beta,\gamma|\hat{L}\,.\hat{L}|N,\beta,\gamma\rangle=\frac{6N}{1+\beta^2}\beta^2$ (11)

$$\langle N,\beta,\gamma|\hat{Q}\,.\hat{Q}|N,\beta,\gamma\rangle=\frac{N}{1+\beta^2}[5+(1+\chi^2)\beta^2]+\frac{N(N-1)}{(1+\beta^2)^2}[4\beta^2+\frac{2}{7}\chi^2\beta^4-\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma] \quad (12)$$

$\langle N,\beta,\gamma|\hat{T}\,.\hat{T}|N,\beta,\gamma\rangle=\frac{N}{1+\beta^2}\frac{7}{5}\beta^2$ (13)

with these values the expectation value of the Hamiltonian for $\chi=0$ can be written in the form:

$E(N,\beta,\gamma)=\frac{A_2\beta^2+A_4\beta^4}{(1+\beta^2)^2}+A_0$ (14)

where

$$A_2=[\epsilon-(N-1)a_0+(4N-8)a_2+\lambda]N \quad (15)$$

$$A_4=[\epsilon-4a_2+\lambda]N \quad (16)$$

$$A_0=\left[\frac{1}{4}a_0(N-1)+5a_2\right]N \quad (17)$$

with

$$\lambda=6a_1+\frac{7}{5}a_3 \quad (18)$$

## 3.Equilibrium Deformation and Critical Points

Minimization of the energy with respect to $\beta$ for given values of the parameters gives the equilibrium value $\beta_0$ defining the phase of the system,$\beta_0=0$corresponds to the symmetric phase, and $\beta_0\neq 0$ to the broken symmetry phase. To determine the critical values of the order parameters of any nucleus, one needs to determine the locus of points for which conditions $\frac{\partial E}{\partial \beta}=0$and$\frac{\partial^2 E}{\partial \beta^2}=0$aresatisfied. The minima of E as a function of $\beta$ can be estimated by equating the first order derivative to zero, and the location of the critical point is obtaind when E becomes flat at $\beta=0$ or equating the second order derivative at $\beta=0$ to zero. This yield to $.\frac{\partial E}{\partial \beta}=0$: $A_2+(2A_4-A_2)\beta^2=0$ (19)

Therefore the equilibriumvalue is $\beta_0=\pm\sqrt{\frac{A_2}{A_2-2A_4}}$

$$\frac{\partial^2 E}{\partial \beta^2}\Big|_{\beta=0}=0: \qquad A_2=0 \quad (20)$$

Therefore the relation betweenthe parameters to give the critical point is $E=(N-1)a_0-\lambda-(4N-8)$.Thus the most general PES of the critical point in the U(5)-O(6)phase transition term is:

$$E_{critical}=\frac{A_4\beta^4}{(1+\beta^2)^2}+A_0 \quad (21)$$

The analysis of the two dynamical symmetry limits of the IBM provides a good test to the PES's presented in the above formalism.

(i) For the U(5) limit ($a_0=0, a_2=0$), the equilibrium shape of the nucleus is always spherical, this yields $E(N,\beta)=(\epsilon+\lambda)N\frac{\beta^2+\beta^4}{(1+\beta^2)^2}$ (22)

That is

$$\frac{E(N,\beta)}{(\epsilon+\lambda)N}=\frac{\beta^2}{1+\beta^2} \quad (23)$$

The energy functional is γ independent and has a minimum at $\beta=0$.Figure (1a) illustrate the scaled energy functional as a function of $\beta$.

(ii)The analysis of the equilibrium shape in the O(6) limit $(\epsilon=0, a_2=0)$show that a minimum occurs at $\beta=0$and at $\beta=\pm\sqrt{\frac{a_0(N-1)-\lambda}{a_0(N-1)+\lambda}}$

with

$$A_2 = [\lambda - a_0(N-1)]N$$

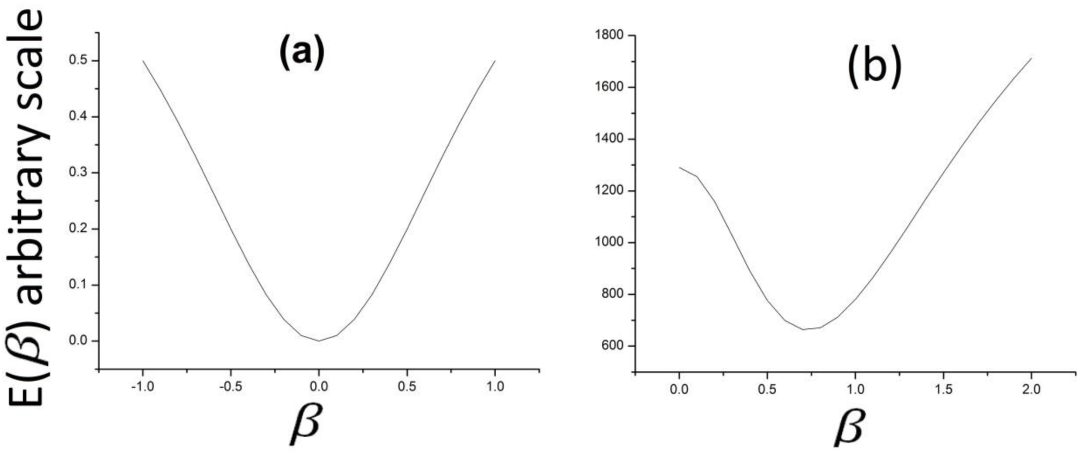
$A_4 = \lambda N$ (24)

$$A_0 = \frac{1}{4}a_0(N-1)N$$

Under the condition $a_0(N-1) > \lambda$ and $N \geq 5$, the critical point is found at $a_0(N-1) = \lambda$ Therefore

$$E_{critical} = \frac{\lambda N\beta^4}{(1+\beta^2)^2} + \frac{1}{4}a_0(N-1)N \quad (25)$$

Figure (1b)illustrates the PES in the O(6)limit with the parameters $A_2 = -3600\ KeV, A_4 = 1560\ KeV, and\ A_0 = 1290KeV$,we notice that the equilibrium shape is deformed at $\beta = 0.73$




**Figure(1) The energy functional $E(\beta)$as a function of deformation parameter $\beta$: (a)For U(5) limit (b) for O(6) limit (the minimum of $E(\beta)$is at $\beta \neq 0$ $(0.73)$ ).**

## 4. The U(5)-O(6)Shape transition

The transition between the spherical and $\gamma$-unstable shapes can be studied by considering five cases.

Case 1: $(putting\ a_0 = a_1 = a_3 = 0)$

The critical point appear when $A_2 = 0$, yielding $E_c = -a_2(4N-8)$

Introducing the control parameter $\eta_1$,such that

$$\frac{1-\eta_1}{\eta_1} = -\frac{Na_2}{\epsilon}$$

Then the critical point is located at $\eta_c = \frac{4N-8}{5N-8}$

For large N limit $\eta_c = \frac{4}{5}$ (26)

If we eliminate the contribution of the one body terms, the coefficients $A_2$and $A_4$ becomes

$A_2 = [\epsilon + 4(N-1)\ a_2]N$ (27)

$A_4 = \epsilon N$ (28)

yielding the critical point at $E_c = -4(N-1)a_2$ and $\eta_c = \frac{4}{5}$

The corresponding PES's for the values of this case is given in Figure (2) for three values of $\eta$ .

Case 2: $(putting\ a_1 = a_2 = a_3 = 0)$

The critical point appearswhen $A_2 = 0$, yielding $\epsilon_c = a_0(N-1)$

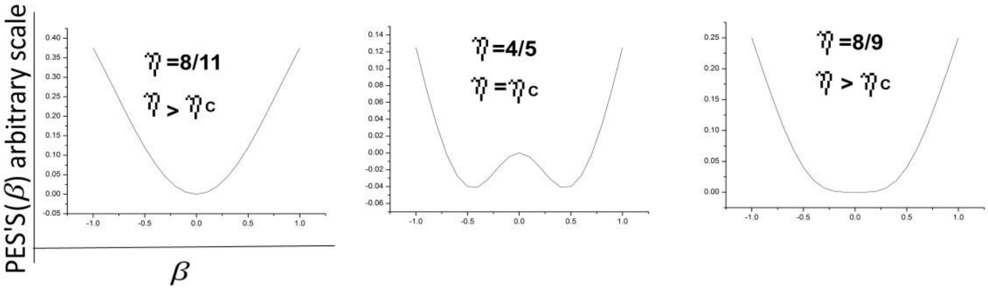
Introducing the control parameter $\eta'$such that

$$\frac{1-\eta'}{\eta'} = \frac{(N-1)a_0}{\epsilon}$$

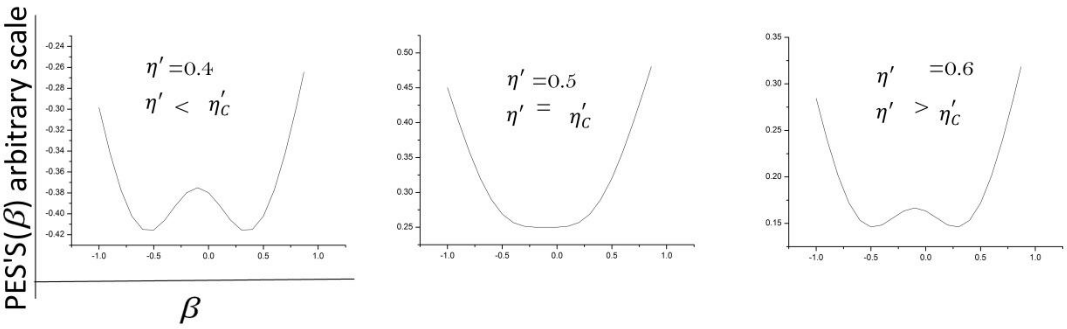
Then the critical point is located at

$$\eta_c = \frac{1}{2} \qquad (29)$$

In this case the PES has a flat behavior ($\sim\beta^4$)for small $\beta$,an inflection point is at $\beta = 1$ and approaches a constant for large $\beta$. The global minimum at $\beta = 0$ is not well localized and the PES exhibits considerable instability in $\beta$ resembling a squarewell potential for $0 \le \beta \le 1$.




**Figure(2) Calculated PES's as a function deformation parameter $\beta$ U(5)-O(6) shape transition for case one at three values of control parameter $\eta = 8/9$ , $4/5$, $8/11$ the critical point is at $\eta_c = 4/5$.**




**Figure(3)the same as in Figure(2) but for the case 2 at the control parameter $\eta' = 0.4, 0.5$ and $0.6$ . The critical point is at $\eta' = 0.5$ .**

The PES is illustrated in Figure(3) for three values of $\eta'$

The critical points connecting U(5) and O(6)in the above two cases are however different and can be viewed as two different lines in nuclear shape phase diagram, with the respective critical point lying in two different points in Casten triangle. In both cases the PES's display a spherical minimum in $\beta = 0$ for$\eta$ larger than the critical value $\eta_c$, while having a deformed minimum for values of $\eta$ smaller than the critical value. At the critical point, the PES in both cases occurs in leading order a $\beta^4$behavior, but differs for the higher order terms.

Case 3:Modified O(6) to produce transition

Putting $\epsilon = a_2 = 0$ in the original Hamiltonian and adding the term $\alpha N(N+4)$, then the parameters of the PES's become

$$A_2 = [-4(N-1)a_0 + \lambda]N$$

$$A_4 = \lambda N \qquad (30)$$

$$A_0 = [a_0(N-1) + \alpha(N+4)]N$$

In Figure (4),we show the PES's corresponding to modified O(6) limit, with chosen parameters to produce a shape transition at N=7. The parameters are $\lambda = 580\ KeV$ , $a_0 = 12\ KeV$ $\quad \alpha = 65\ KeV$ and $N = 4,7,13$.

Case 4: modified U(5) to produce transition

Putting $a_0 = a_2 = 0$ in the original Hamiltonian and adding the term $(a_4 T_4 . T_4 - \alpha N\, \hat{n}_d\, )$where $\widehat{T_4}$ is the hexadecapole oprator

$$\widehat{T_4} = [d^\dagger \times \tilde{d}]^{(4)}$$

$$\langle N,\beta,\gamma|\widehat{T_4}.\widehat{T_4}|N,\beta,\gamma\rangle = \frac{9}{5}\frac{N\beta^2}{1+\beta^2} + \frac{18}{35}\frac{N(N-1)\beta^4}{(1+\beta^2)^2} \qquad (31)$$

,then the parameters of the PES's become

$$A_2 = \left[\epsilon + \lambda + \frac{9}{5}a_4 - \alpha N\right]N$$

$$A_4 = \left[\epsilon + \lambda + \frac{9}{5}a_4 + \left(\frac{18}{35} - \alpha\right)N\right]N \qquad (32)$$

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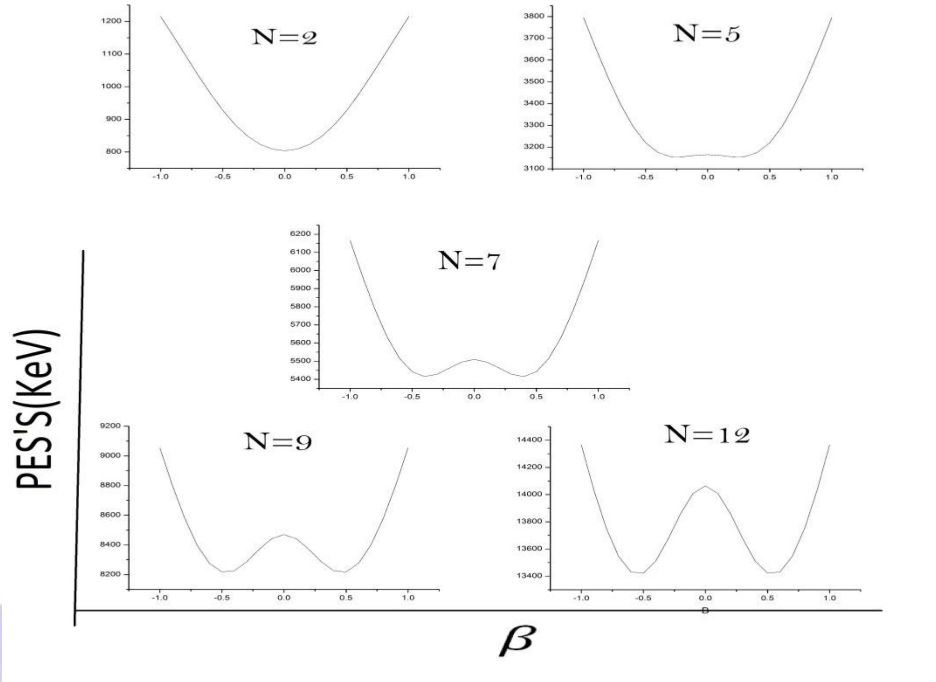
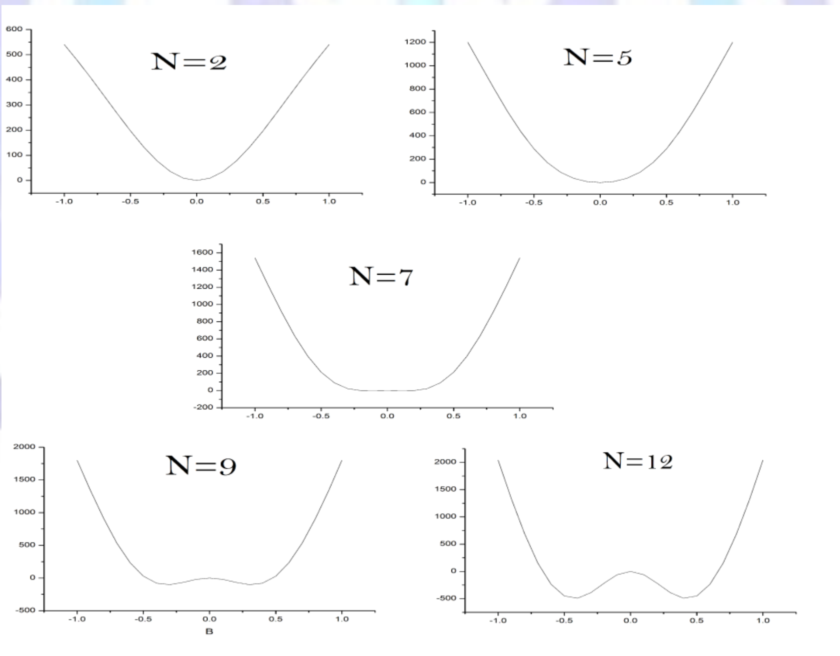
In Figure(5), we show the PES's corresponding to modified U(5)limit, with chosen parameters to produce a shape transition at N=7. The parameters are $A_2 = (660 - 100N)N$and $A_4 = (500 + 60N)N$




**Figure(4)Calculated PES's as a function of deformation parameter $\beta$ corresponding to modified O(6) to produce shape transition , case 3, with the parameters $\lambda = 580\ KeV$ ,$a_0 = 12\ KeV$and $\alpha = 65\ KeV$. The total number of bosons is N=2,5,7,9 and `12.**




**Figure(5)Calculated PES's as a function of deformation parameter $\beta$ corresponding to modified U(5) to produce shape transition , case 4, with the parameters $A_2 = (660 - 100N)N$ and$A_4 = (5.0 + 60N)N$.The total number of bosons is N=2,5,7,9 and `12.**

Case 5:

If $a_0 \neq 0$ and $a_4 = 0$ in case 4, then the parameters of the PES's become

$$A_2 = [\epsilon + \lambda - (N-1)a_0 - \alpha N]N$$

$$A_4 = [\epsilon + \lambda - \alpha N] \qquad (33)$$

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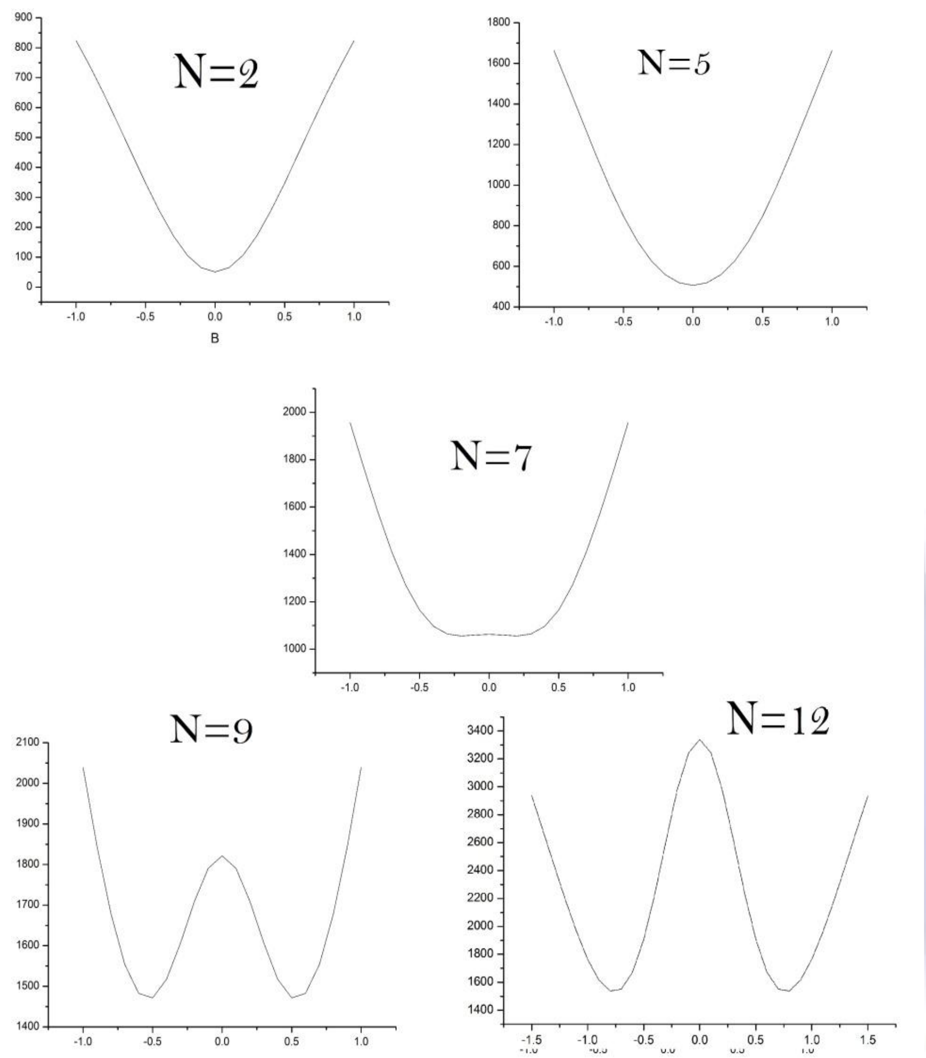
$$A_0 = \frac{1}{4}(N-1)a_0 N$$

In the Figure (6),we show the PES's corresponding to chosen parameters to produce a shape transitionthe parameters are $A_2 = [1031.21 - 154.2N]N$ , $A_4 = [930.01 - 53N]$, and$A_0 = 25.3(N-1)N$.




**Figure(6) The same as in Figure(5) but when $a_4 = 0$ with the parameters $A_2 = (1031.21 - 154.2N)N$, $A_4 = (930.01 + 53N)N$, and $A_0 = 25.3(N-1)N$**

## 5. Application to Xenon Isotopic chain

The xenon isotopic chain along the mass region$A \sim 120 - 130$ represent excellent example for studying O(6)-U(5) shape phase transition which give a good test of the proposed nuclear IBM .We will study the second order shape phase transition between the U(5) and O(6) by analyzing the PES's of the Xenon isotopic chain $^{122-132}$Xe. The structure parameter of the quadrupole operator is taken to be zero .For each nucleus parameters of the PES's $A_2$ and $A_4$ which are linear combination of the original parameters of the Hamiltonian have been adjusted by fitting the experimental excitation energies of the yrast band to the calculated ones using a computer search program to minimize $\chi$ such that

$$\chi = \left(\frac{1}{N}\sum_{i=1}^{N}\left|\frac{E^{exp}\cdot(I_i) - E^{cal}(I_i)}{\Delta E^{exp}\cdot(I_i)}\right|^2\right)^{\frac{1}{2}} \qquad (34)$$

where N is the number of the experimental fitting points and $\Delta E^{exp}\cdot(I)$ are the experimental errors. The adopted best parameters are listed in Table [1]. These model parameters give a satisfactory description of the experimental data [31]and theoretical

calculations[32]In Figure(7), the corresponding PES's plotted for this isotopicchain of nuclei which evolve from $\gamma$-unstable nuclei to spherical vibrator when moving from the lighter $^{122}$Xe to heavier $^{132}$Xe isotopes.

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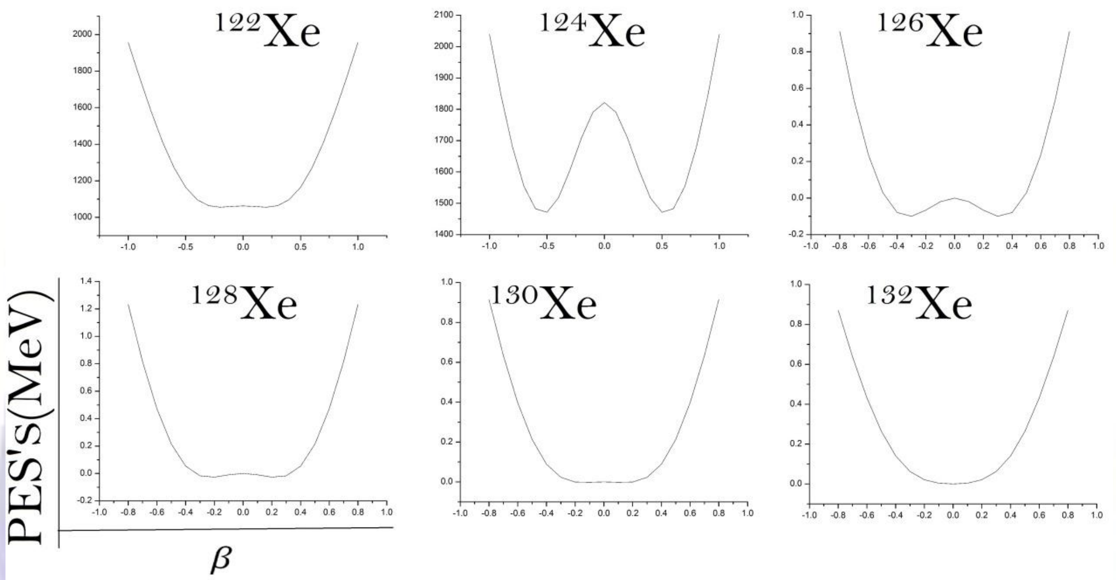
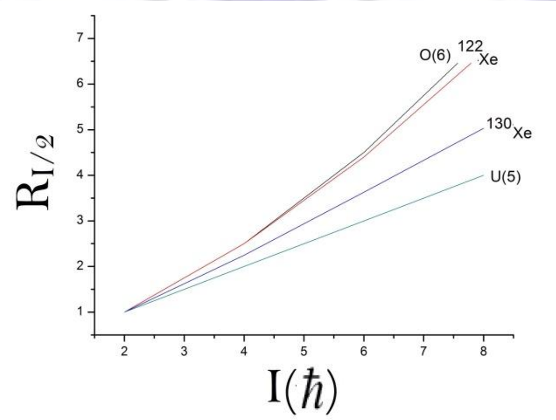
**Table (1) the PES's parameters $A_2$ and $A_4$ (in MeV) as derived in fitting procedure for Xe isotopic chain.**

| Isotope | $A_2$(MeV) | $A_4$(MeV) |
|---|---|---|
| $^{122}$Xe | -4.84 | 12.76 |
| $^{124}$Xe | -3.40 | 11.00 |
| $^{126}$Xe | -2.16 | 9.36 |
| $^{128}$Xe | -1.12 | 7.84 |
| $^{130}$Xe | -0.28 | 6.44 |
| $^{132}$Xe | 0.36 | 5.16 |

To get the characteristic of collectivity in our Xe isotopic chain, the behavior of the energyratios $R_{I/2} = E(I_i^{\pi})/E(2_1^+)$ have been calculated for $^{122}$Xe and $^{132}$Xe and compared to those of the predicted dynamical symmetries for the O(6) and U(5) limits of the IBM , Specially the ratio $R_{4/2}$ is a good criterion for the shape transition. The value $R_{4/2}$ has a limiting value 2 for quadrupole vibrator and 2.5 for a non-axial $\gamma$ soft rotor. As it is seen from the Figure (8) $R_{4/2}$decreasing gradually from 2.5 for $^{122}$Xe to 2.25 for $^{130}$Xe. This means that this structure varies from $\gamma$-soft rotor along harmonic vibrator. So the energy spectrum of the $^{122-132}$Xe nuclei can be studied between the rotational and vibrational limits.




**Figure(7)Calculated PES's (in MeV)as a function of deformation parameter $\beta$ in U(5)-O(6) to shape transition , for $^{122-132}$Xeisotopic chain( with $N_{\pi} = 2$ *proton and* $N_{\nu} = 7$ *to* 2 neutron bosons). The total number of bosons is $N = 9$ to 4.**




**Figure(8)The evolution of yrast energy ratios $R_{I/2} = E(I)/E(2)$ as a function of angular momentum I for the nuclei $^{122}$Xe and $^{130}$Xe and comparison with the prediction of U(5) and O(6) dynamical symmetry limits.**

## 6. Conclusion

The paper is focused on the U(5)-O(6)second order shape phase transition of the IBM. The considered Hamiltonian has been written in multipole form and studied in some different cases in order to produce shape phase transition. The PES's have been calculated as expectation values of the Hamiltonian operator within the intrinsic coherent states. The PES's in each case are investigated and analyzed and the equilibrium deformation and critical points are determined. The values of the PES's for doubly even Xe isotopic chain are studied systematically. The model parameters are adjusted by fitting the excitation yrast energies with the calculated ones by performing a computer search program in order to minimize the root-mean-square(rms) deviation between the experimental excitation energies and calculated ones. The phase diagram for Xe nuclei exhibits second order shape phase transition from spherical U(5) to $\gamma$-unstable O(6) when moving from heavier isotope $^{132}$Xe(with boson number =4)to lighter ones $^{122}$Xe (with boson number N=9)

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