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GRAVITY FIELD: THEORETICAL AND EXPERIMENTAL CONTRIBUTION TO STUDY WEIGHING PENDULUM PERIOD

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ABSTRACT

We present in this paper a theoretical and experimental study of the period of a pendulum consisting of a heavy rigid rod on which we have set a mass m which can slide. The study was conducted at the National Advanced School of Engineering, University of Yaounde I. The instrument was manufactured in a room of metal carpenter for a relatively modest cost. The study was performed according to the position of the mass versus the axis of rotation. We show that the model developed in the framework of the theory of solid mechanics adjusts successfully experience while the theory developed in the framework of the approximation of the mechanical point has a limited validity margin. The theoretical and experimental study has allowed us to observe the asymptotic behavior of the mass relatively to the axis of rotation.

Key words

Weight pendulum, experience, solid mechanics, the period of the pendulum

INTRODUCTION

Called weighing pendulum any mobile solid around an axis (usually horizontal) not passing through its center of gravity and placed in a gravity field. Moved from its stable equilibrium position wherein the center of gravity is to the vertical axis, the solid started to fluctuate on both sides of this called equilibrium position. Some examples of heavy pendulum we know are: a clock pendulum, swing, etc. The historical weighing pendulum significance was studied by Galileo in scientific detail.

Our main objective in this study is to compare the theoretical and experimental results for a solid mechanics laboratory experience. The pendulum used in this study is shown in Figure 1. It was designed and built entirely in a metal carpenter in Yaounde to an estimated cost of 50.000 CFA francs. We used for this purpose a copper tube, heavy weight steel, a support and a ball bearing. Everything was assembled in the laboratory of physics of the National Advanced School of Engineering, University of Yaounde I. To make work more attractive we used a stopwatch and a webcam to record the pendulum motion, we analyzed motion with the synchrony software 2003.

Figure 1: Experimental setup of the weight pendulum

DESCRIPTION OF THE EXPERIMENTAL SETUP

We have in this work determined the period of the pendulum by two methods:

- A webcam was used for the analysis of a video sequence

- A Quartz timer was used for time recording

During the study of the weight pendulum period, we evaluated the role of the inertia of the pendulum which has allowed us to highlight the range of validity of the mechanical point compared to that of solid mechanics. Behavioral analysis of the period of the pendulum when one compares the mass of the axis of rotation of the pendulum is detailed.

A pendulum is constituted by a heavy mass m attached to heavy hollow rod whose mass m_T is much lower than that of the heavy mass (Figure 2). We asked whether this system could be described in the approximation of point mechanics. This question we could answer intuitively the negative is not as trivial as it sounds. Indeed, the period of oscillation of a pendulum is written as part of the mechanical point; with the distance L between the axis of rotation and g the acceleration of gravity. This expression is apparently valid as soon as it is mass away from the axis of rotation. It is therefore interesting to understand the range of validity of this relationship. It realizes intuitively that there is a limit to this description. Indeed, in the approximation of the mechanic point, the higher the center of gravity is close to the rotation axis, the longer the period becomes short. When $L = 0\,m$, we finally obtain $T_0 = 0s$. In practice, it is far to observe this behavior. When the axis of rotation passes through the center of gravity of the pendulum, $L = 0\,m$, the pendulum is stationary, which means that the period is infinite. So, there is a contradiction between the approximation of mechanics point and experimental reality.

In this study, we show in the first part, how to calculate the period of the pendulum in the context of solid mechanics. Then we define to what extent the model of approximation of the mechanical point intersects with that of solid mechanics. In the second part, we measure the period based on the distance between the axis of rotation and the ground and compare the measurements with the theory.

THEORETICAL STUDY OF THE WEIGHT PENDULUM PERIOD

The system studied is a weight pendulum, mass M. It is composed of a cylindrical rod denoted by (T) and a concentric cylinder noted (C) which can slide along the (T). The position of the center of gravity of the pendulum relative to the axis of rotation O is denoted L. The oscillation amplitude is set by the angle θ between the vertical and the rod of the pendulum. The experiment consists of measuring the period of the pendulum in relation to the position of the cylinder (C) which is progressively closer to the axis of rotation O of the pendulum.

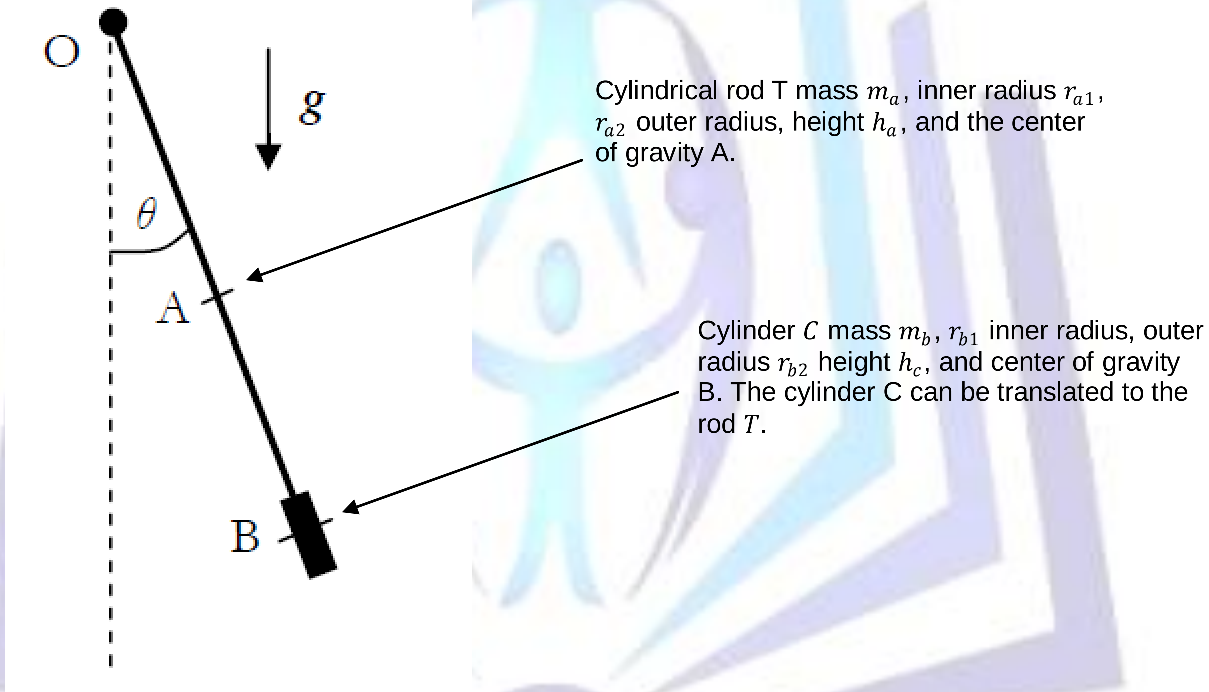

Figure 2: Diagram of the weight pendulum

In general, in the small angle approximation, the period of a weight pendulum depends on the moment of inertia J of the gravitational constant g, the distance L between the axis of rotation of the center of gravity of clock, and the mass M of the pendulum. Applying the theorem of moments of the pendulum system, we obtain the differential equation of motion :

$$\ddot{\theta} + \frac{(m_a OA + m_b OB)g}{J}\theta = 0 \quad \to T = 2\pi\sqrt{\frac{J}{(m_a OA + m_b OB)g}}$$

Note the following constraint on the position of the mass B: $0 < OB < 2OA$. We obtain the moment of inertia J of the pendulum by applying the Huygens theorem on the various elements making up the clock, ie, the rod (T) and the cylindrical mass (C) J_A and J_B represent the moment of inertia T and the cylindrical rod of the cylinder (C) with respect to an axis passing through the center of gravity and perpendicular to the plane of Figure 2. The total moment of inertia expressed in O (center of rotation) can be written as:

$$J = m_a OA^2 + J_A + m_b OB^2 + J_B \quad with \quad \begin{cases} J_A = m_a\left(\frac{r_{a1}^2 + r_{a2}^2}{4} + \frac{h_a^2}{12}\right) \\ J_B = m_b\left(\frac{r_{b1}^2 + r_{b2}^2}{4} + \frac{h_b^2}{12}\right) \end{cases}$$

Finally, in explaining the dependence of the period based on OB, we obtain:

$$T(OB) = 2\pi\sqrt{\frac{m_a OA^2 + J_A + m_b OB^2 + J_B}{g(m_a OA + m_b OB)}}$$

To better observe the dependence of T with OB, OB call distance equal x. It follows that: this function passes through a minimum when :

$$x = x_c = -C_2 + \sqrt{C_2^2 + C_1}$$

It can be seen by referring to Table 1 that the function tends to infinity at: $x = -\frac{OAm_a}{m_b}$, passes through a minimum at x_c and tends to infinity when new OB tends to infinity. (The - sign here means that the mass m_b is located above the axis of rotation with which we have the pendulum is not physically possible).

x	**$-OAm_a/m_b$**	**x_c**	**+∞**
dT^2/dx	**-**	**0**	**+**
T^2	**-∞** ↘	**T^2_{min}**	↗ **+∞**

Table 1: Table of variation of the representative curve of the function T (OB)

Within or OB tends to infinity, T_2 tends asymptotically to the right slope $\frac{Q^2P^2}{g}$ according to the law: $T^2(x \to \infty) = \frac{4\pi^2 x}{g}$ which is the law governing the period of a simple pendulum that we subsequently call T_0. This is linear in $x = OB$. The difference between this line and the experimental observation can be quantified by the relation:

$$T^2 - T_0^2 = \left(\frac{4\pi^2}{g}\right)\left(\frac{OB^2 + C_1}{OB + C_2}\right) - \left(\frac{4\pi^2 OB}{g}\right) = \left(\frac{4\pi^2}{g}\right)\left(\frac{C_1 - C_2 OB}{OB + C_2}\right)$$

leading when OB is great (OB >> C2)

$T^2 - T_0^2 \approx \left(\frac{4\pi^2}{g}C_2\right) = -\frac{4\pi^2 m_a OA}{g m_b}$

This difference is infact constant. It can be seen that for the weighing pendulum whose period is given by:

$T(OB) = 2\pi\sqrt{\frac{m_a OA^2 + J_A + m_b OB^2 + J_B}{g(m_a OA + m_b OB)}}$

is comparable to a simple pendulum mut:

$\begin{cases} m_b OB \gg m_a OA \\ m_b OB^2 \gg m_a OA^2 + J_A + J_B \end{cases}$

The first relationship combined with $OB < OA^2$and J_B negligible compared to the other values led to $m_a \gg \frac{m_b}{2}$ and the second in $m_a \gg \frac{m_b}{3}$, so the first is enough. When this approximation is not valid then, the square of the period of the pendulum deviates from a linear function of the position of the mass. A singularity model of solid mechanics is that the period of the weight pendulum passes through a minimum for a given position of the mass m_b. The model proposed in the context of solid mechanics well aware of the fact that the static equilibrium period of the pendulum tends to infinity and if $\frac{2m_b}{m_a} \gg 1$ it coincides with that obtained in the approximation of the mechanical point.

In the following section, we study experimentally the pendulum that we confront the model presented in this section.

EXPERIMENTAL STUDY OF THE PERIOD VERSUS OB

The experience carried out was to measure the period of the pendulum for different values of OB. To measure the period, we recorded the oscillations of the pendulum on amplitude for 10 periods starting around 10° with a webcam. We adjusted the variation of elongation as a function of time with a sinusoidal pattern by the least squares method.

We get through the period of adjustment. Measurements are in good agreement with those measured on the clock. The variation of the period squared based OB is shown in Figure 3. We can observe that the model of the previous paragraph perfectly aware of the evolution of the OB period with the position of the mass m_b. Particularly the presence of the minimum is clearly observed and the discrepancy of the period of both sides of the minimum. However, it is difficult to measure the evolution of the period to very low values of B. The calculations were performed with the following parameters $m_b = 1.460\text{kg}$; $h_b = 0.034\text{m}$; $r_{b1} = 0.006\text{m}$; $r_{b2} = 0.04\text{m}$; $r_{a1} = 5\text{mm}$; $r_{a2} = 6\text{mm}$; $m_a = 0.276\text{kg}$.

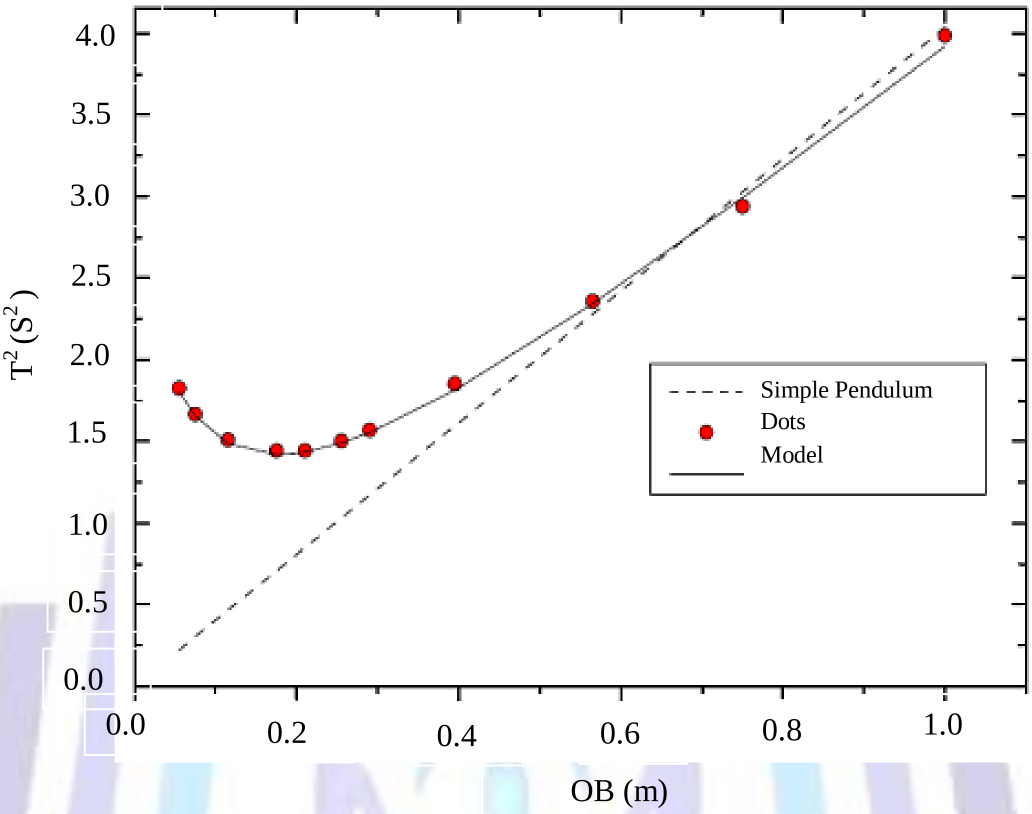

Figure 3: Square representations of the period of weight pendulum versus the distance OB. the solid lines represent the full model, the dotted lines are the result of the simple pendulum and the experimental points in red.

It can also be seen in Figure 3 that, there is a deviation between the asymptotic behavior of the simple pendulum and the weight pendulum when OB becomes large. This shows that, the condition $m_a \gg \frac{m_b}{2}$ is not fully realized. In this case we actually $\frac{2m_b}{m_a} = 10.58$ which is not much greater than 10, which explains the observed difference. To better understand the conditions of validity of the mechanical point we have completed this experience with three other experiments with masses m_b variables b_I, b_{II} and b_{III}. Times the square of the pendulum are shown in Figure 4 and the characteristics of the clocks are listed in Table 2.

Cylindre a	**m_a**	**r_{a1}**	**r_{a2}**	**h_a**	**J_A**	**$2m_b/m_a$**
	45g	2.2mm	2.5mm	60cm	1.4 10^{-3} kg.m^2	
Cylindre b_I	m_b	r_{b1}	r_{b1}	h_b	J_B	3.11
	70g	3.0	9.0	45mm	1.3 10^{-5} kg.m^2	
Cylindre b_{II}	m_b	r_{b1}	r_{b2}	h_b	J_B	7.42
	167g	3.0mm	12.5mm	45mm	9.4 10^{-5} kg.m^2	
Cylindre b_{III}	m_b	r_{b1}	r_{b2}	h_b	J_B	38.26
	861g	3.0mm	25.0mm	45mm	28.1 10^{-5} kg.m^2	

Table 2 : The characteristics of cylindrical component of the pendulum. Note the value of the ratio in the last column 2mb/ma of this table.

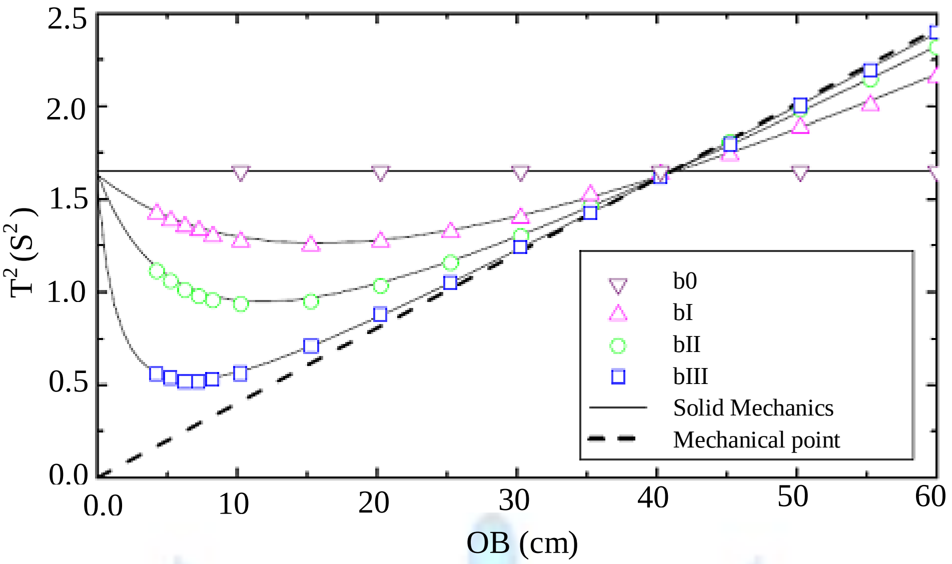

Figure 4: Squared periods of oscillation against OB. Measurements are represented by squares, circles or triangles according to the mass while dashed curve represents the model of point mechanics and those in continuous model of solid mechanics. Horizontal curve captioned b_0 corresponds to the period of the rod without the mass.

Whatever the mass, there is a perfect match between the model of solid mechanics and experience. In particular, the position of the minimum is perfectly reproduced. There is, however, that when OB becomes large, single cylinder b_{III} leads to a result which approximates the model deduced from the approximation of the mechanical point.

In fact, this is the only case for which the report $\frac{2m_b}{m_a} = 38.26$ is greater than 10 which is the criterion to be met for the simple pendulum is comparable to the pendulum when weighing the mass m_b is located at the end of rod. We can finally observe that all curves intersect at a particular point in the vicinity of OB = 40cm, corresponding to a period which is that of the rod alone. A somewhat tedious calculation shows that all curves intersect at a particular point does not depend on the mass m_b but essentially the characteristics of the stem and to a lesser extent the size of the mass m_b.

CONCLUSION

We have shown using a weight pendulum the experiment and the theoretical model of solid mechanics are in good agreement. The criterion used to assimilate the weight pendulum to the simple pendulum has been set.

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