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FOUR BOSONS ELECTROMAGNETISM

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ABSTRACT

Based on light invariance and electric conservation a four bosons electromagnetism is proposed. It enlarges the electric charge conservation beyond displacement current and Dirac charge to a new physical situation where the electromagnetic phenomena is mediated by the usual photon plus a massive photon and two additional charged vector bosons.

Considering the enlarged abelian gauge symmetry U $(1)\times$ SO (2) transforming under a same gauge parameter a non-linear electromagnetism involving four bosons is introduced. It deploys a Lagrangian containing massless, massive and charged fields with three and four vector bosons interactions. The corresponding Noether's relations and classical equations of motion are studied. They provide a whole dynamics involving granular, collective terms through antisymmetric and symmetric sectors. It develops a new photon equation which extends the Maxwell's one. Self interacting photons are obtained.

A four boson electromagnetic flux is derived. It expresses an electromagnetism transfering $\Delta Q = 0$ and $|\Delta Q| = 1$, not more limited to just a massless photon. There is a new electromagnetic flowing to be understood, where aside of electric charge conservation, it appears a neutral electromagnetism. There are six neutral electromagnetic charges beyond electric charge as consequences from non-linearity. Two are derived from the second Noether identity and four from variational continuity equations.

An electromagnetic flux being conducted by a whole physics is generated. Based on fields set, it develops a determinism under the meaning of directive and circumstance. Interpreting that, light invariance concises the photon as directive, the photon becomes a whole maker. It assumes the symmetry command which will control the conservations laws and opportunities. Consequently, one combines the symmetry equation derived from Noether theorem with the four equations derived from variational principle, and an effective photon equation is obtained. A kind of Navier-Stokes electromagnetic flow is derived. It yields a four bosons electromagnetism preserving electric charge conservation plus introducting the meaning of chance through symmetry management.

Indexing terms/Keywords

Light invariance; Abelian gauge symmetry; Extended electromagnetism

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1 Introduction

Electromagnetism is understood as the phenomena given by light invariance and charge conservation. Under this focus, Maxwell equations are not a concluded paradigm [1]. The 900 pages of 'Treatise on Electricity and Magnetism' (1873) should be understood as a first part of a more complete description of electromagnetic phenomena. So, in a similar fashion to Maxwell's displacement current to preserve the electric charge conservation, one introduces here a context involving four vector bosons intermediations.

We consider four physical principles in order to explore this extended electromagnetism: light invariance, wholeness, gauge symmetry, charge conservation. They yield a dynamics not more based on an isolated A_μ-field but rather through a family with four interlinked fields: $\{A_\mu, U_\mu, V_\mu^\pm\}$, corresponding respectively to the photon, massive photon and charged photons. The origin for grouping this family is the Lorentz group irreducible representation $(\frac{1}{2},\frac{1}{2})$. They will share light invariance and charge conservation and will diversify through gauge symmetry.

In previous works, we have shown on a non-linear abelian gauge model [2] with charged fields [3]. The purpose here is to construct an electromagnetism transmited by four fields $\{A_\mu, U_\mu, V_\mu^\pm\}$ associated to a common gauge symmetry. The corresponding $U(1)\times SO(2)$ gauge transformations are defined as:

$$A_\mu' = A_\mu + k_1 \partial_\mu \alpha,$$

$$U_\mu' = U_\mu + k_2 \partial_\mu \alpha,$$

$$V_\mu^{+}' = e^{iq\alpha} V_\mu^{+} k_{+} \partial_\mu \alpha,$$

$$V_\mu^{-}' = e^{-iq\alpha} V_\mu^{-} + k_{-} \partial_\mu \alpha. \tag{1}$$

Thus, from Eq.(1) depending on a common gauge parameter, it will preserve the electric charge. As consequence, one constitutes the minimum set allowing an electromagnetism exchange containing the cases $\Delta Q = 0$ and $\Delta Q = \pm 1$. It proposes an extension to the electromagnetism phenomena where from charge conservation one gets a four fields behavior as a whole.

2 Lagrangian

Eq. (1) leads us to the following fields strength non-linear abelian Lagrangian based on light invariance and electric charge conservation

$$L = L_K + L_{GF} + L_I \tag{2}$$

where the kinetic term, $L_K = L_K^A + L_K^S$, is given by

$$L_K^A = a_1 F_{\mu\nu} F^{\mu\nu} + a_2 U_{\mu\nu} U^{\mu\nu} + 2a_3 V_{\mu\nu}^{+} V^{\mu\nu -}, \tag{3}$$

$$L_K^S = b_{(11)} S_{\mu\nu}^{1} S^{\mu\nu 1} + b_{(11)} S_{\mu\nu}^{2} S^{\mu\nu 2} + 2b_{(12)} S_{\mu\nu}^{1} S^{\mu\nu 2}$$

$$+ 2b_{(33)} S_{\mu\nu}^{+} S^{\mu\nu -} + c_{(11)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 1} + c_{(11)} S_{\mu}^{\mu 2} S_{\nu}^{\nu 2}$$

$$+ 2c_{(12)} S_{\mu}^{\mu 1} S_{\nu}^{\nu 2} + 2c_{(33)} S_{\mu}^{\mu +} S_{\nu}^{\nu -}, \tag{4}$$

with the granular fields strength definitions

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad U_{\mu\nu} \equiv \partial_\mu U_\nu - \partial_\nu U_\mu, \qquad V_{\mu\nu}^{\pm} \equiv \partial_\mu V_\nu^{\pm} - \partial_\nu V_\mu^{\pm},$$

$$S_{\mu\nu}^{1} \equiv \partial_\mu A_\nu + \partial_\nu A_\mu, \qquad S_{\mu\nu}^{2} \equiv \partial_\mu U_\nu + \partial_\nu U_\mu, \qquad S_{\mu\nu}^{\pm} \equiv \partial_\mu V_\nu^{\pm} + \partial_\nu V_\mu^{\pm}. \tag{5}$$

Notice that Eq.(4) is gauge invariant as a whole [4]. Appendix A relates on collective fields. The gauge-fixing term associated to these fields set abelian transformations is

$$L_{GF} = \frac{1}{4}\xi_{(11)}S^{\mu1}_{\mu}S^{\nu1}_{\nu} + \frac{1}{4}\xi_{(22)}S^{\mu2}_{\mu}S^{\nu2}_{\nu} + \frac{1}{2}\xi_{(12)}S^{\mu1}_{\mu}S^{\nu2}_{\nu}$$
$$+\frac{1}{4}(\xi_{(33)}+\xi_{(44)})S^{\mu+}_{\mu}S^{\nu-}_{\nu} + \frac{1}{4}(\xi_{(33)}-\xi_{(44)})\mathrm{Re}\{S^{\mu+}_{\mu}S^{\nu+}_{\nu}\}$$
$$+\frac{\sqrt{2}}{2}S^{\mu1}_{\mu}\mathrm{Re}\{[\xi_{(13)}+i\xi_{(14)}]S^{\nu+}_{\nu}\}$$
$$+\frac{\sqrt{2}}{2}S^{\mu2}_{\mu}\mathrm{Re}\{[\xi_{(23)}+i\xi_{(24)}]S^{\nu+}_{\nu}\} - \frac{1}{2}\xi_{(34)}\mathrm{Im}\{S^{\mu+}_{\mu}S^{\nu+}_{\nu}\}. \quad (6)$$

As usual the gauge fixing term takes just one degree of freedom of the system, however, at Eq.(6) it does not necessarily specify which quantum is being suppressed.

The interaction Lagrangian is decomposed in trilinear term and quadrilinear parts: $L_I = L_3 + L_4$ involving granular and collective fields. In addition, for physicity, one splits them in antisymmetric and symmetric sectors: $L_3 = L_3^A + L_3^S$ and $L_4 = L_4^A + L_4^S$. It yields the following separately gauge invariants terms:

$$L_3^A = 4(b_1F^1_{\mu\nu}+b_2U^2_{\mu\nu})\overset{[12]}{z}{}^{\mu\nu} + 8b_3\mathrm{Re}\{(\overset{[-1]}{z}{}^{\mu\nu}\overset{[-2]}{z}{}^{\mu\nu})W^+_{\mu\nu}\}$$
$$+4\overset{[+-]}{z}{}^{[\mu\nu]}(b_1F^1_{\mu\nu}+b_2U^2_{\mu\nu}) + 4\overset{(+-)}{z}{}^{[\mu\nu]}(\beta_1F^1_{\mu\nu}+\beta_2U^2_{\mu\nu}) \quad (7)$$

$$L_3^S = 2(\beta_1S^1_{\mu\nu}+\beta_2S^2_{\mu\nu})(\overset{(11)}{z}{}^{\mu\nu}+2\overset{(12)}{z}{}^{\mu\nu}+\overset{(22)}{z}{}^{\mu\nu})$$
$$+2(\rho_1S^{\mu1}_{\mu}+\rho_2S^{\mu2}_{\mu})(\overset{(11)}{z}{}^{\nu}_{\nu}+2\overset{(12)}{z}{}^{\nu}_{\nu}+\overset{(22)}{z}{}^{\nu}_{\nu})$$
$$+2[(\beta_1+4\rho_1)S^{\mu1}_{\mu}+(\beta_2+4\rho_2)S^{\mu2}_{\mu}](\overset{(11)}{\omega}{}^{\nu}_{\nu}+2\overset{(12)}{\omega}{}^{\nu}_{\nu}+\overset{(22)}{\omega}{}^{\nu}_{\nu})$$
$$+8\beta_3\mathrm{Re}\{(\overset{(-1)}{z}{}^{\mu\nu}+\overset{(-2)}{z}{}^{\mu\nu})S^+_{\mu\nu}\}+4\overset{+-3}{z}{}^{\mu\nu}(\beta_1S^1_{\mu\nu}+\beta_2S^2_{\mu\nu})$$
$$+8\rho_3\mathrm{Re}\{(\overset{(-1)}{z}{}^{\nu}_{\nu}+\overset{(-2)}{z}{}^{\nu}_{\nu})S^{\mu+}_{\mu}\}$$
$$+8(\beta_3+4\rho_3)\mathrm{Re}\{(\overset{(-1)}{\omega}{}^{\nu}_{\nu}+\overset{(-2)}{\omega}{}^{\nu}_{\nu})S^{\mu+}_{\mu}\}$$
$$+4\overset{+-3}{z}{}^{\nu}_{\nu}(\rho_1S^{\mu1}_{\mu}+\rho_2S^{\mu2}_{\mu})+4(\beta_1+4\rho_1)\overset{+-3}{\omega}{}^{\nu}_{\nu}S^{\mu1}_{\mu}$$
$$+4(\beta_2+4\rho_2)\overset{+-3}{\omega}{}^{\nu}_{\nu}S^{\mu2}_{\mu} \quad (8)$$

$$L_4^A = 2\overset{[12]}{z}_{\mu\nu}\overset{[12]}{z}{}^{\mu\nu}+2\overset{[12]}{z}_{\mu\nu}\overset{[21]}{z}{}^{\mu\nu}-4\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[13-]}{z}{}^{\nu}_{\nu}-4\overset{[23+]}{z}{}^{\mu}_{\mu}\overset{[23-]}{z}{}^{\nu}_{\nu}$$
$$+4\overset{[13+]}{z}_{\mu\nu}\overset{[13-]}{z}{}^{\mu\nu}+4\overset{[23+]}{z}_{\mu\nu}\overset{[23-]}{z}{}^{\mu\nu}+8\overset{[13+]}{z}_{\mu\nu}\overset{[23-]}{z}{}^{\mu\nu}$$
$$-8\mathrm{Re}\{\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[23-]}{z}{}^{\nu}_{\nu}\}+8\overset{[12]}{z}_{\mu\nu}\mathrm{Re}\{\overset{[+-]}{z}{}^{\mu\nu}\}+8\mathrm{Im}\{\overset{[13+]}{z}{}^{\mu}_{\mu}\overset{[24-]}{z}{}^{\nu}_{\nu}\}$$
$$+2\overset{[+-]}{z}_{\mu\nu}\overset{[+-]}{z}{}^{\mu\nu}-2\overset{[+-]}{z}_{\mu\nu}\overset{[-+]}{z}{}^{\mu\nu}-4\overset{[+-]}{z}{}^{\mu}_{\mu}\overset{[+-]}{z}{}^{\nu}_{\nu} \quad (9)$$

$$
\begin{aligned}
L_4^S = {} & \overset{(11)}{z}_{\mu\nu}\overset{(11)}{z}^{\mu\nu}+\overset{(22)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu}+2\overset{(11)}{z}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+2\overset{(22)}{z}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu} \\
& +4\overset{(11)}{\omega}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+4\overset{(22)}{\omega}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu}+4\overset{(11)}{z}_{\mu\nu}\overset{(12)}{z}^{\mu\nu}+4\overset{(12)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu} \\
& +8\overset{(12)}{z}_{\mu\nu}\overset{(11)}{\omega}^{\mu\nu}+8\overset{(12)}{z}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu}+16\overset{(11)}{\omega}_{\mu\nu}\overset{(12)}{\omega}^{\mu\nu}+16\overset{(12)}{\omega}_{\mu\nu}\overset{(22)}{\omega}^{\mu\nu} \\
& +2\overset{(11)}{z}_{\mu\nu}\overset{(22)}{z}^{\mu\nu}+2\overset{(12)}{z}_{\mu\nu}\overset{(12)}{z}^{\mu\nu}+8\overset{(12)}{z}_{\mu\nu}\overset{(12)}{\omega}^{\nu\mu}+16\overset{(12)}{\omega}_{\mu\nu}\overset{(12)}{\omega}^{\nu\mu} \\
& +4\overset{(11)}{z}{}^{\mu}_{\mu}\overset{(22)}{\omega}{}^{\nu}_{\nu}+8\overset{(11)}{\omega}{}^{\mu}_{\mu}\overset{(22)}{\omega}{}^{\nu}_{\nu}+2\overset{(12)}{z}_{\mu\nu}\overset{(12)}{z}^{\nu\mu}+4(\overset{(11)}{z}_{\mu\nu}+\overset{(22)}{z}_{\mu\nu})\overset{+-3}{z}^{\mu\nu} \\
& +4\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{z}{}^{\nu}_{\nu}+4\overset{(23+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{z}{}^{\nu}_{\nu}+16\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}+16\overset{(23+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu} \\
& +32\overset{(13+)}{\omega}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}+32\overset{(23+)}{\omega}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu}+4\overset{(13+)}{z}_{\mu\nu}\overset{(13-)}{z}^{\mu\nu}+4\overset{(23+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\mu\nu} \\
& +8\overset{(13+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\mu\nu}+8\{\overset{(11)}{z}{}^{\mu}_{\mu}+\overset{(22)}{z}{}^{\mu}_{\mu}+2\overset{(11)}{\omega}{}^{\mu}_{\mu}+2\overset{(22)}{\omega}{}^{\mu}_{\mu}+2\overset{(12)}{z}{}^{\mu}_{\mu}+4\overset{(12)}{\omega}{}^{\mu}_{\mu}\}\overset{+-3}{\omega}{}^{\nu}_{\nu} \\
& +8\overset{(12)}{z}_{\mu\nu}\mathrm{Re}\{\overset{+-3}{z}^{\mu\nu}\}+8\mathrm{Re}\{\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(23-)}{z}{}^{\nu}_{\nu}\}+32\mathrm{Re}\{\overset{(13+)}{z}{}^{\mu}_{\mu}\overset{(13-)}{\omega}{}^{\nu}_{\nu}\} \\
& +64\mathrm{Re}\{\overset{(13+)}{\omega}{}^{\mu}_{\mu}\overset{(23-)}{\omega}{}^{\nu}_{\nu}\}-8\mathrm{Im}\{\overset{(13+)}{z}_{\mu\nu}\overset{(24-)}{\omega}^{\nu\mu}\}-8\mathrm{Im}\{\overset{(23+)}{z}_{\mu\nu}\overset{(14-)}{\omega}^{\nu\mu}\} \\
& -32\mathrm{Im}\{\overset{(13+)}{\omega}_{\mu\nu}\overset{(24-)}{\omega}^{\nu\mu}\}+32\mathrm{Im}\{\overset{(14+)}{\omega}_{\mu\nu}\overset{(23-)}{\omega}^{\nu\mu}\}+4\mathrm{Im}\{\overset{(13+)}{z}_{\mu\nu}\overset{(24-)}{z}^{\nu\mu}\} \\
& -4\mathrm{Im}\{\overset{(14+)}{z}_{\mu\nu}\overset{(23-)}{z}^{\nu\mu}\}+8\mathrm{Im}\{\overset{(14+)}{z}_{\mu\nu}\overset{(23-)}{\omega}^{\nu\mu}\}+8\mathrm{Im}\{\overset{(24+)}{z}_{\mu\nu}\overset{(13-)}{\omega}^{\nu\mu}\} \\
& +2\{\overset{+-3}{z}_{\mu\nu}\overset{+-4}{z}^{\mu\nu}+\overset{+-3}{z}_{\mu\nu}\overset{-+4}{z}^{\mu\nu}\}-8\{\overset{(+-)}{z}_{\mu\nu}\overset{(+-)}{\omega}^{\mu\nu}-\overset{(+-)}{z}_{\mu\nu}\overset{(-+)}{\omega}^{\mu\nu}\} \\
& -16\{\overset{(+-)}{\omega}_{\mu\nu}\overset{(+-)}{\omega}^{\mu\nu}-\overset{(+-)}{\omega}_{\mu\nu}\overset{(-+)}{\omega}^{\mu\nu}\}-4\overset{(+-)}{z}{}^{\mu}_{\mu}\overset{(+-)}{z}{}^{\nu}_{\nu}+8\overset{+-3}{z}{}^{\mu}_{\mu}\overset{+-4}{\omega}{}^{\nu}_{\nu} \\
& +16\overset{+-3}{\omega}{}^{\mu}_{\mu}\overset{+-4}{\omega}{}^{\nu}_{\nu} \qquad (10
\end{aligned}
$$

Eqs. (2)-(10) are broadening the electromagnetic phenomena. They are introducing a step forward to observe a non-linear electromagnetism. Differently from Euler-Heisenberg [5], Born Infeld [6] and other effective models [7], they are providing a non-linear electromagnetic model with four bosons fields $\{A_\mu, U_\mu, V_\mu^{\pm}\}$ at fundamental level. Eqs. (7)-(10) are bringing three new features. First, an abelian non-linearity where differently from Yang-Mills each term is isolately gauge invariant; second, a dynamics with granular and collective fields showing that nature is a quanta and collective construction; third, the meaning of chance through free coefficients as $a_1, b_{(11)}$ and so on, which can take any value without violating gauge invariance. At this way, eq(1) is producing a self-interacting whole electromagnetism carrying individual and collective electromagnetic fields under a determinism directive and circumstance.

Given that this work intention is to generalize the electromagnetic phenomena, we should make a brief comparision with previous efforts in the sixties in order to extend QED with massive vector bosons. In 1962, Lee and Yang have introduced the longitudinal vector bosons propagation, and the term $F_{\mu\nu}W^{\mu}W^{\nu*}$ for obtaining the correct expression of the magnetic moment [8], in 1963 Salam introduced the term $\lambda(W_\mu W^{\mu*})^2$ for establishing renormalizability [9]. Comparing with the Lagrangian being proposed here one notices that these three terms appear at eqs.(4),(7),(9), respectively. Nevertheless is still necessary a further study on renormalitability [10], unitarity [11] and on corresponding gyromagnetic terms.

3 Equations of motion

Taking the variational principle, the kinectic identity $\partial_\nu S^{\nu\mu} = \partial_\nu F^{\nu\mu} + \eta^{\mu\nu}\partial_\nu S_\alpha^\alpha$, and the Bianchi identity $\partial_\mu z_{(\nu}^{\nu)} + 2\partial_\nu z_{(\mu}^{\nu)} = \gamma_{(IJ)} A_\mu^I S_\nu^\nu + 2\gamma_{(IJ)} A_\nu^I S_\mu^\nu$, one expresses the following four compact equations of motion:

For field A_μ (photon),

$$\partial_\nu\{a_1 F^{\nu\mu} + a_2(\overset{[12]}{z}{}^{[\nu\mu]} + \overset{[+-]}{z}{}^{[\nu\mu]}) + a_3 \overset{(+-)}{z}{}^{[\nu\mu]} +$$

$$+ g^{\nu\mu}(a_4 S_\alpha^{\alpha 1} + a_5 S_\alpha^{\alpha 2} + a_6 \mathrm{Re} S_\alpha^{\alpha +} + a_7(\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2\overset{(12)}{z}{}_\alpha^\alpha + 2\overset{(+-3)}{z}{}_\alpha^\alpha) +$$

$$+ a_8(\overset{(11)}{\omega}{}_\alpha^\alpha + \overset{(22)}{\omega}{}_\alpha^\alpha + 2\overset{(12)}{\omega}{}_\alpha^\alpha + 2\overset{(+-3)}{\omega}{}_\alpha^\alpha))\} - \mu_1^2 U^\mu = j_A^\mu$$

where

$$j_A^\mu = A_\nu\{f_1^A F^{\nu\mu} + f_2^A U^{\nu\mu} + f_3^A(\overset{[12]}{z}{}^{[\nu\mu]} + \overset{[+-]}{z}{}^{[\nu\mu]}) + f_4^A \overset{(12)}{\omega}{}^{[\nu\mu]} + f_5^A S^{\nu\mu 1} + f_6^A S^{\nu\mu 2} +$$

$$+ f_7^A(\overset{(11)}{z}{}^{(\nu\mu)} + 2\overset{(12)}{z}{}^{(\nu\mu)} + 2\overset{(+-3)}{z}{}^{(\nu\mu)}) + f_8^A \overset{(22)}{z}{}^{(\nu\mu)} + f_9^A(\overset{(11)}{\omega}{}^{(\nu\mu)} + 2\overset{(12)}{\omega}{}^{(\nu\mu)}) + f_{10}^A \overset{(22)}{\omega}{}^{(\nu\mu)} +$$

$$+ g^{\nu\mu}(f_{11}^A S_\alpha^{\alpha 1} + f_{12}^A S_\alpha^{\alpha 2} + f_{13}^A \overset{(22)}{\omega}{}_\alpha^\alpha + 2 f_9^A \overset{(+-3)}{\omega}{}_\alpha^\alpha)\} +$$

$$+ U_\nu\{f_1^U F^{\nu\mu} + f_2^U U^{\nu\mu} + f_3^U(\overset{[12]}{z}{}^{[\nu\mu]} + \overset{[+-]}{z}{}^{[\nu\mu]}) + f_4^U \overset{(12)}{\omega}{}^{[\nu\mu]} + f_5^U S^{\nu\mu 1} + f_6^U S^{\nu\mu 2} + f_7^U \overset{(11)}{z}{}^{(\nu\mu)} +$$

$$+ f_8^U(\overset{(22)}{z}{}^{(\nu\mu)} + 2\overset{(12)}{z}{}^{(\nu\mu)} + 2\overset{(+-3)}{z}{}^{(\nu\mu)}) + f_9^U \overset{(11)}{\omega}{}^{(\nu\mu)} + f_{10}^U(\overset{(22)}{\omega}{}^{(\nu\mu)} + 2\overset{(12)}{\omega}{}^{(\nu\mu)}) +$$

$$+ g^{\nu\mu}(f_{11}^U S_\alpha^{\alpha 1} + f_{12}^U S_\alpha^{\alpha 2} + f_{13}^U \overset{(11)}{\omega}{}_\alpha^\alpha + 2 f_{10}^U \overset{(+-3)}{\omega}{}_\alpha^\alpha)\} +$$

$$+ V_\nu^+\{f_1^+ V^{\nu\mu -} + f_2^+ \overset{[13-]}{z}{}^{[\nu\mu]} + f_3^+ \overset{[14-]}{z}{}^{[\nu\mu]} + f_4^+ \overset{[23-]}{z}{}^{[\nu\mu]} + f_5^+ \overset{[24-]}{z}{}^{[\nu\mu]} + f_6^+ \overset{(13-)}{z}{}^{[\nu\mu]} +$$

$$+ f_7^+ \overset{(14-)}{z}{}^{[\nu\mu]} + f_8^+ \overset{(23-)}{z}{}^{[\nu\mu]} + f_9^+ \overset{(24-)}{z}{}^{[\nu\mu]} + f_{10}^+ \overset{(13-)}{\omega}{}^{[\nu\mu]} + f_{11}^+ \overset{(14-)}{\omega}{}^{[\nu\mu]} + f_{12}^+ \overset{(23-)}{z}{}^{[\nu\mu]} +$$

$$+ f_{13}^+ \overset{(24-)}{\omega}{}^{[\nu\mu]} + f_{14}^+ S^{\nu\mu -} + f_2^+ \overset{[13-]}{z}{}^{(\nu\mu)} + f_3^+ \overset{[14-]}{z}{}^{(\nu\mu)} + f_{15}^+ \overset{[23-]}{z}{}^{(\nu\mu)} + f_5^+ \overset{[24-]}{z}{}^{(\nu\mu)} +$$

$$+ f_{16}^+ \overset{(13-)}{z}{}^{(\nu\mu)} + f_7^+ \overset{(14-)}{z}{}^{(\nu\mu)} + f_{17}^+ \overset{(23-)}{z}{}^{(\nu\mu)} + f_9^+ \overset{(24-)}{z}{}^{(\nu\mu)} + f_{10}^+ \overset{(13-)}{\omega}{}^{(\nu\mu)} +$$

$$+ f_{11}^+ \overset{(14-)}{\omega}{}^{(\nu\mu)} + f_{12}^+ \overset{(23-)}{\omega}{}^{(\nu\mu)} + f_{13}^+ \overset{(24-)}{\omega}{}^{(\nu\mu)} +$$

$$+ g^{\nu\mu}(f_{18}^+ S_\alpha^{\alpha -} + f_{19}^+(\overset{[13-]}{z}{}_\alpha^\alpha + \overset{[23-]}{z}{}_\alpha^\alpha) + f_{20}^+(\overset{(13-)}{z}{}_\alpha^\alpha + \overset{(23-)}{z}{}_\alpha^\alpha) + f_{21}^+(\overset{(13-)}{\omega}{}_\alpha^\alpha + \overset{(23-)}{\omega}{}_\alpha^\alpha))\} +$$

$$+ V_\nu^-\{f_1^{*+} V^{\nu\mu +} + f_2^- \overset{[13+]}{z}{}^{[\nu\mu]} + f_3^{*+} \overset{[14+]}{z}{}^{[\nu\mu]} + f_4^- \overset{[23+]}{z}{}^{[\nu\mu]} + f_5^{*+} \overset{[24+]}{z}{}^{[\nu\mu]} + f_6^- \overset{(13+)}{z}{}^{[\nu\mu]} +$$

$$+ f_7^{*+} \overset{(14+)}{z}{}^{[\nu\mu]} + f_8^- \overset{(23+)}{z}{}^{[\nu\mu]} + f_9^{*+} \overset{(24+)}{z}{}^{[\nu\mu]} + f_{10}^{*+} \overset{(13+)}{\omega}{}^{[\nu\mu]} + f_{11}^{*+} \overset{(14+)}{\omega}{}^{[\nu\mu]} + f_{12}^{*+} \overset{(23+)}{z}{}^{[\nu\mu]} +$$

$$+ f_{13}^{*+}\overset{(24+)[\nu\mu]}{\omega} + f_{14}^{*+}S^{\nu\mu+} - f_{2}^{*-}\overset{[13+](\nu\mu)}{z} + f_{3}^{*+}\overset{[14+](\nu\mu)}{z} + f_{4}^{*+}\overset{[23+](\nu\mu)}{z} + f_{5}^{*+}\overset{[24+](\nu\mu)}{z} +$$

$$+ f_{6}^{*-}\overset{(13+)(\nu\mu)}{z} + f_{7}^{*+}\overset{(14+)(\nu\mu)}{z} + f_{8}^{-}\overset{(23+)(\nu\mu)}{z} + f_{9}^{*+}\overset{(24+)(\nu\mu)}{z} + f_{10}^{*+}\overset{(13+)(\nu\mu)}{\omega} + f_{11}^{*+}\overset{(14+)(\nu\mu)}{\omega} +$$

$$+ f_{12}^{*+}\overset{(23+)(\nu\mu)}{\omega} + f_{13}^{*+}\overset{(24+)(\nu\mu)}{\omega} +$$

$$+ g^{\nu\mu}(f_{18}^{*+}S_{\alpha}^{\alpha+} + f_{19}^{*+}(\overset{[13+]}{z}{}_{\alpha}^{\alpha} + \overset{[23+]}{z}{}_{\alpha}^{\alpha}) + f_{20}^{*+}(\overset{(13+)}{z}{}_{\alpha}^{\alpha} + \overset{(23+)}{z}{}_{\alpha}^{\alpha}) + f_{21}^{*+}(\overset{(13+)}{\omega}{}_{\alpha}^{\alpha} + \overset{(23+)}{\omega}{}_{\alpha}^{\alpha}))\}. \qquad (11)$$

Notice that Eq. (11) enlarges the Maxwell photon. It relates on a pure photonic with self-interacting photons. For field U_{μ} (massive photon),

$$\partial_{\nu}\{b_1 U^{\nu\mu} + b_2(\overset{[12][\nu\mu]}{z} + \overset{[+-][\nu\mu]}{z}) + b_3 \overset{(+-)[\nu\mu]}{z} +$$

$$+ g^{\nu\mu}(b_4 S_{\alpha}^{\alpha 1} + b_5 S_{\alpha}^{\alpha 2} + b_6 \mathrm{Re} S_{\alpha}^{\alpha+} + b_7(\overset{(11)}{z}{}_{\alpha}^{\alpha} + \overset{(22)}{z}{}_{\alpha}^{\alpha} + 2\overset{(12)}{z}{}_{\alpha}^{\alpha} + 2\overset{(+-3)}{z}{}_{\alpha}^{\alpha}) +$$

$$+ b_8(\overset{(11)}{\omega}{}_{\alpha}^{\alpha} + \overset{(22)}{\omega}{}_{\alpha}^{\alpha} + 2\overset{(12)}{\omega}{}_{\alpha}^{\alpha} + 2\overset{(+-3)}{\omega}{}_{\alpha}^{\alpha}))\} - \mu_2^2 U^{\mu} = j_U^{\mu}$$

where

$$j_U^{\mu} = A_{\nu}\{g_1^A F^{\nu\mu} + g_2^A U^{\nu\mu} + g_3^A(\overset{[12][\nu\mu]}{z} + \overset{[+-][\nu\mu]}{z}) + g_4^A \overset{(12)[\nu\mu]}{\omega} + g_5^A S^{\nu\mu 1} + g_6^A S^{\nu\mu 2} +$$

$$+ g_7^A(\overset{(11)(\nu\mu)}{z} + 2\overset{(12)(\nu\mu)}{z} + 2\overset{(+-3)(\nu\mu)}{z}) + g_8^A \overset{(22)(\nu\mu)}{z} + g_9^A(\overset{(11)(\nu\mu)}{\omega} + 2\overset{(12)(\nu\mu)}{\omega}) + g_{10}^A \overset{(22)(\nu\mu)}{\omega} +$$

$$+ g^{\nu\mu}(g_{11}^A S_{\alpha}^{\alpha 1} + g_{12}^A S_{\alpha}^{\alpha 2} + g_{13}^A \overset{(22)}{\omega}{}_{\alpha}^{\alpha} + 2g_9^A \overset{(+-3)}{\omega}{}_{\alpha}^{\alpha})\} +$$

$$+ U_{\nu}\{g_1^U F^{\nu\mu} + g_2^U U^{\nu\mu} + g_3^U(\overset{[12][\nu\mu]}{z} + \overset{[+-][\nu\mu]}{z}) + g_4^U \overset{(12)[\nu\mu]}{\omega} + g_5^U S^{\nu\mu 1} + g_6^U S^{\nu\mu 2} + g_7^U \overset{(11)(\nu\mu)}{z} +$$

$$+ g_8^U(\overset{(22)(\nu\mu)}{z} + 2\overset{(12)(\nu\mu)}{z} + 2\overset{(+-3)(\nu\mu)}{z}) + g_9^U \overset{(11)(\nu\mu)}{\omega} + g_{10}^U(\overset{(22)(\nu\mu)}{\omega} + 2\overset{(12)(\nu\mu)}{\omega}) +$$

$$+ g^{\nu\mu}(g_{11}^U S_{\alpha}^{\alpha 1} + g_{12}^U S_{\alpha}^{\alpha 2} + g_{13}^U \overset{(11)}{\omega}{}_{\alpha}^{\alpha} + 2g_{10}^U \overset{(+-3)}{\omega}{}_{\alpha}^{\alpha})\} +$$

$$+ V_{\nu}^{+}\{g_1^{+} V^{\nu\mu-} + g_2^{+}\overset{[13-][\nu\mu]}{z} + g_3^{+}\overset{[14-][\nu\mu]}{z} + g_4^{+}\overset{[23-][\nu\mu]}{z} + g_5^{+}\overset{[24-][\nu\mu]}{z} + g_6^{+}\overset{(13-)[\nu\mu]}{z} +$$

$$+ g_7^{+}\overset{(14-)[\nu\mu]}{z} + g_8^{+}\overset{(23-)[\nu\mu]}{z} + g_9^{+}\overset{(24-)[\nu\mu]}{z} + g_{10}^{+}\overset{(13-)[\nu\mu]}{\omega} + g_{11}^{+}\overset{(14-)[\nu\mu]}{\omega} + g_{12}^{+}\overset{(23-)[\nu\mu]}{z} +$$

$$+ g_{13}^{+}\overset{(24-)[\nu\mu]}{\omega} + g_{14}^{+}S^{\nu\mu-} - g_2^{*+}\overset{[13-](\nu\mu)}{z} + g_3^{+}\overset{[14-](\nu\mu)}{z} - g_4^{*+}\overset{[23-](\nu\mu)}{z} + g_5^{+}\overset{[24-](\nu\mu)}{z} +$$

$$- g_{16}^{+}\overset{(13-)(\nu\mu)}{z} + g_7^{+}\overset{(14-)(\nu\mu)}{z} - g_8^{+}\overset{(23-)(\nu\mu)}{z} + g_9^{+}\overset{(24-)(\nu\mu)}{z} + g_{10}^{+}\overset{(13-)(\nu\mu)}{\omega} + g_{11}^{+}\overset{(14-)(\nu\mu)}{\omega} +$$

$$+ g_{12}^{+}\overset{(23-)(\nu\mu)}{\omega} + g_{13}^{+}\overset{(24-)(\nu\mu)}{\omega} +$$

$$+ g^{\nu\mu}(g_{15}^{+}S_{\alpha}^{\alpha-} + g_{16}^{+}(\overset{[13-]}{z}{}_{\alpha}^{\alpha} + \overset{[23-]}{z}{}_{\alpha}^{\alpha}) + g_{17}^{+}(\overset{(13-)}{z}{}_{\alpha}^{\alpha} + \overset{(23-)}{z}{}_{\alpha}^{\alpha}) + g_{18}^{+}(\overset{(13-)}{\omega}{}_{\alpha}^{\alpha} + \overset{(23-)}{\omega}{}_{\alpha}^{\alpha}))\} +$$

$$+V_{\nu}^{-}\{g_1^{*+}V^{\nu\mu+}+g_2^{-}\overset{[13+][\nu\mu]}{z}+g_3^{*+}\overset{[14+][\nu\mu]}{z}+g_4^{-}\overset{[23+][\nu\mu]}{z}+g_5^{*+}\overset{[24+][\nu\mu]}{z}+g_6^{-}\overset{(13+)[\nu\mu]}{z}+$$

$$+g_7^{+}\overset{(14+)[\nu\mu]}{z}+g_8^{-}\overset{(23+)[\nu\mu]}{z}+g_9^{*+}\overset{(24+)[\nu\mu]}{z}+g_{10}^{*+}\overset{(13+)[\nu\mu]}{\omega}+g_{11}^{*+}\overset{(14+)[\nu\mu]}{\omega}+g_{12}^{*+}\overset{(23+)[\nu\mu]}{z}+$$

$$+g_{13}^{*+}\overset{(24+)[\nu\mu]}{\omega}+g_{14}^{*+}S^{\nu\mu+}-g_2^{*-}\overset{[13+](\nu\mu)}{z}+g_3^{*+}\overset{[14+](\nu\mu)}{z}-g_4^{*+}\overset{[23+](\nu\mu)}{z}+g_5^{*+}\overset{[24+](\nu\mu)}{z}+$$

$$-g_6^{*-}\overset{(13+)(\nu\mu)}{z}+g_7^{*+}\overset{(14+)(\nu\mu)}{z}-g_8^{-}\overset{(23+)(\nu\mu)}{z}+g_9^{*+}\overset{(24+)(\nu\mu)}{z}+g_{10}^{*+}\overset{(13+)(\nu\mu)}{\omega}+g_{11}^{*+}\overset{(14+)(\nu\mu)}{\omega}+$$

$$+g_{12}^{*+}\overset{(23+)(\nu\mu)}{\omega}+g_{13}^{*+}\overset{(24+)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(g_{15}^{*+}S_{\alpha}^{\alpha+}+g_{10}^{+}(\overset{[13+]}{z}{}_{\alpha}^{\alpha}+\overset{[23+]}{z}{}_{\alpha}^{\alpha})+g_{17}^{+}(\overset{(13+)}{z}{}_{\alpha}^{\alpha}+\overset{(23+)}{z}{}_{\alpha}^{\alpha})+g_{18}^{*+}(\overset{(13+)}{\omega}{}_{\alpha}^{\alpha}+\overset{(23+)}{\omega}{}_{\alpha}^{\alpha}))\}. \quad (12)$$

For field V_{μ}^{+} (massive positive charged photon),

$$\partial_{\nu}\{c_1V^{\nu\mu-}+c_2(\overset{[-1][\nu\mu]}{z}+\overset{[-2][\nu\mu]}{z})+$$

$$+g^{\nu\mu}(c_3S_{\alpha}^{\alpha1}+c_4S_{\alpha}^{\alpha2}+c_5S_{\alpha}^{\alpha+}+c_6S_{\alpha}^{\alpha-}+c_7(\overset{(-1)}{z}{}_{\alpha}^{\alpha}+\overset{(-2)}{z}{}_{\alpha}^{\alpha})+$$

$$+c_8(\overset{(-1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(-2)}{\omega}{}_{\alpha}^{\alpha}))-\mu_{+}^{2}V^{\mu-}=j_{V^{+}}^{\mu}$$

where

$$j_{V^{+}}^{\mu}=A_{\nu}\{h_1^{A}V^{\nu\mu-}+h_2^{A}\overset{[13-][\nu\mu]}{z}+h_3^{A}\overset{[23-][\nu\mu]}{z}+h_4^{A}\overset{[24-][\nu\mu]}{z}+h_5^{A}\overset{(13-)[\nu\mu]}{\omega}+h_6^{A}\overset{(23-)[\nu\mu]}{z}+$$

$$+h_7^{A}\overset{(24-)[\nu\mu]}{z}+h_8^{A}\overset{(23-)[\nu\mu]}{\omega}+h_9^{A}\overset{(24-)[\nu\mu]}{\omega}+h_{10}^{A}S^{\nu\mu-}+h_2^{A}\overset{[13-](\nu\mu)}{z}+h_3^{*A}\overset{[23-](\nu\mu)}{z}+$$

$$+h_4^{*A}\overset{[24-](\nu\mu)}{z}+h_5^{A}\overset{(13-)(\nu\mu)}{z}+h_6^{*A}\overset{(23-)(\nu\mu)}{z}+h_7^{*A}\overset{(24-)(\nu\mu)}{z}+h_8^{*A}\overset{(23-)(\nu\mu)}{\omega}+h_9^{*A}\overset{(24-)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(h_{11}^{A}S_{\alpha}^{\alpha-}-h_2^{A}(\overset{[13-]}{z}{}_{\alpha}^{\alpha}+\overset{[23-]}{z}{}_{\alpha}^{\alpha})+h_5^{A}(\overset{(13-)}{z}{}_{\alpha}^{\alpha}+\overset{(23-)}{z}{}_{\alpha}^{\alpha})+h_{12}^{A}(\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}+\overset{(23-)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+U_{\nu}\{h_1^{U}V^{\nu\mu-}+h_2^{U}\overset{[13-][\nu\mu]}{z}+h_3^{U}\overset{[14-][\nu\mu]}{z}+h_4^{U}\overset{[23-][\nu\mu]}{z}+h_5^{U}\overset{(13-)[\nu\mu]}{\omega}+h_6^{U}\overset{(14-)[\nu\mu]}{z}+h_7^{U}\overset{(23-)[\nu\mu]}{z}+$$

$$+h_8^{U}\overset{(13-)[\nu\mu]}{\omega}+h_9^{U}\overset{(14-)[\nu\mu]}{\omega}+h_{10}^{U}S^{\nu\mu-}+h_2^{*U}\overset{[13-](\nu\mu)}{z}+h_3^{*U}\overset{[14-](\nu\mu)}{z}+h_4^{*U}\overset{[23-](\nu\mu)}{z}+$$

$$+h_5^{*U}\overset{(13-)(\nu\mu)}{z}+h_6^{*U}\overset{(14-)(\nu\mu)}{z}+h_7^{*U}\overset{(23-)(\nu\mu)}{z}+h_8^{*U}\overset{(13-)(\nu\mu)}{\omega}+h_9^{*U}\overset{(14-)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(h_{11}^{U}S_{\alpha}^{\alpha1}-h_4^{U}(\overset{[13-]}{z}{}_{\alpha}^{\alpha}+\overset{[23-]}{z}{}_{\alpha}^{\alpha})+h_{12}^{U}\overset{(13-)}{z}{}_{\alpha}^{\alpha}+h_7^{U}\overset{(23-)}{z}{}_{\alpha}^{\alpha}+h_{13}^{U}\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}+h_{14}^{U}\overset{(23-)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+V_{\nu}^{-}\{h_1^{-}F^{\nu\mu}+h_2^{-}U^{\nu\mu}+h_3^{-}\overset{[12][\nu\mu]}{z}+h_4^{-}S^{\nu\mu1}+h_5^{-}S^{\nu\mu2}+$$

$$+h_6^{-}(\overset{(11)(\nu\mu)}{z}+\overset{(22)(\nu\mu)}{z}+2\overset{(12)(\nu\mu)}{z}+2\overset{(+-4)(\nu\mu)}{z})-h_3^{-}\overset{[+-](\nu\mu)}{z}+h_7^{-}\overset{(+-)(\nu\mu)}{\omega}+$$

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$$+g^{\nu\mu}(h_8^- S_\alpha^{\alpha 1}+h_9^- S_\alpha^{\alpha 2}+h_{10}^-(\overset{(11)}{z}{}_\alpha^\alpha+\overset{(22)}{z}{}_\alpha^\alpha+2\overset{(12)}{z}{}_\alpha^\alpha+2\overset{(11)}{\omega}{}_\alpha^\alpha+2\overset{(22)}{\omega}{}_\alpha^\alpha+4\overset{(12)}{\omega}{}_\alpha^\alpha)+$$

$$+h_3^- \overset{[+-]}{z}{}_\alpha^\alpha+h_{11}^- \overset{(+-)}{z}{}_\alpha^\alpha+h_{12}^- \overset{+-4}{z}{}_\alpha^\alpha)\}. \qquad (13)$$

For field V_μ^- (massive negative charged photon),

$$\partial_\nu\{d_1 V^{\nu\mu +}+d_2(\overset{[+1][\nu\mu]}{z}+\overset{[+2][\nu\mu]}{z})+$$

$$+g^{\nu\mu}(d_3 S_\alpha^{\alpha 1}+d_4 S_\alpha^{\alpha 2}+d_5 S_\alpha^{\alpha +}+d_6 S_\alpha^{\alpha -}+d_7(\overset{(+1)}{z}{}_\alpha^\alpha+\overset{(+2)}{z}{}_\alpha^\alpha)+$$

$$+d_8(\overset{(+1)}{\omega}{}_\alpha^\alpha+\overset{(+2)}{\omega}{}_\alpha^\alpha))-\mu_-^2 V^{\mu +}=j_{V^-}^\mu$$

where

$$j_{V^-}^\mu=A_\nu\{i_1^A V^{\nu\mu +}+i_2^A \overset{[13+][\nu\mu]}{z}+i_3^A \overset{[23+][\nu\mu]}{z}+i_4^A \overset{[24+][\nu\mu]}{z}+i_5^A \overset{(13+)[\nu\mu]}{\omega}+i_6^A \overset{(23+)[\nu\mu]}{z}+$$

$$+i_7^A \overset{(24+)[\nu\mu]}{z}+i_8^A \overset{(23+)[\nu\mu]}{\omega}+i_9^A \overset{(24+)[\nu\mu]}{\omega}+i_{10}^A S^{\nu\mu +}+i_2^A \overset{[13+](\nu\mu)}{z}+i_3^{*A} \overset{[23+](\nu\mu)}{z}+$$

$$+i_4^{*A} \overset{[24+](\nu\mu)}{z}+i_5^A \overset{(13+)(\nu\mu)}{z}+i_6^{*A} \overset{(23+)(\nu\mu)}{z}+i_7^{*A} \overset{(24+)(\nu\mu)}{z}+i_8^{*A} \overset{(23+)(\nu\mu)}{\omega}+i_9^{*A} \overset{(24+)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(i_{11}^A S_\alpha^{\alpha +}-i_2^A(\overset{[13+]}{z}{}_\alpha^\alpha+\overset{[23+]}{z}{}_\alpha^\alpha)+i_5^A(\overset{(13+)}{z}{}_\alpha^\alpha+\overset{(23+)}{z}{}_\alpha^\alpha)+i_{12}^A(\overset{(13+)}{\omega}{}_\alpha^\alpha+\overset{(23+)}{\omega}{}_\alpha^\alpha)\}+$$

$$+U_\nu\{i_1^U V^{\nu\mu +}+i_2^U \overset{[13+][\nu\mu]}{z}+i_3^U \overset{[14+][\nu\mu]}{z}+i_4^U \overset{[23+][\nu\mu]}{z}+i_5^U \overset{(13+)[\nu\mu]}{\omega}+i_6^U \overset{(14+)[\nu\mu]}{z}+i_7^U \overset{(23+)[\nu\mu]}{z}+$$

$$+i_8^U \overset{(13+)[\nu\mu]}{\omega}+i_9^U \overset{(14+)[\nu\mu]}{\omega}+i_{10}^U S^{\nu\mu +}+i_2^{*U} \overset{[13+](\nu\mu)}{z}+i_3^{*U} \overset{[14+](\nu\mu)}{z}+i_4^U \overset{[23+](\nu\mu)}{z}+i_5^{*U} \overset{(13+)(\nu\mu)}{z}+$$

$$+i_6^{*U} \overset{(14+)(\nu\mu)}{z}+i_7^U \overset{(23-)(\nu\mu)}{z}+i_8^{*U} \overset{(13-)(\nu\mu)}{\omega}+i_9^{*U} \overset{(14-)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(i_{11}^U S_\alpha^{\alpha 1}-i_4^U(\overset{[13+]}{z}{}_\alpha^\alpha+\overset{[23+]}{z}{}_\alpha^\alpha)+i_{12}^U \overset{(13+)}{z}{}_\alpha^\alpha+i_7^U \overset{(23+)}{z}{}_\alpha^\alpha+i_{13}^U \overset{(13+)}{\omega}{}_\alpha^\alpha+i_{14}^U \overset{(23+)}{\omega}{}_\alpha^\alpha)\}+$$

$$+V_\nu^+\{i_1^+ F^{\nu\mu}+i_2^+ U^{\nu\mu}+i_3^+ \overset{[12][\nu\mu]}{z}+i_4^+ S^{\nu\mu 1}+i_5^+ S^{\nu\mu 2}+$$

$$+i_6^+(\overset{(11)(\nu\mu)}{z}+\overset{(22)(\nu\mu)}{z}+2\overset{(12)(\nu\mu)}{z}+2\overset{(+-4)(\nu\mu)}{z})-i_3^+ \overset{[+-](\nu\mu)}{z}+i_7^+ \overset{(+-)(\nu\mu)}{\omega}+$$

$$+g^{\nu\mu}(i_8^+ S_\alpha^{\alpha 1}+i_9^+ S_\alpha^{\alpha 2}+i_{10}^+(\overset{(11)}{z}{}_\alpha^\alpha+\overset{(22)}{z}{}_\alpha^\alpha+2\overset{(12)}{z}{}_\alpha^\alpha+2\overset{(11)}{\omega}{}_\alpha^\alpha+2\overset{(22)}{\omega}{}_\alpha^\alpha+4\overset{(12)}{\omega}{}_\alpha^\alpha)+$$

$$+i_3^+ \overset{[+-]}{z}{}_\alpha^\alpha+i_{11}^+ \overset{(+-)}{z}{}_\alpha^\alpha+i_{12}^+ \overset{+-4}{z}{}_\alpha^\alpha)\}. \qquad (14)$$

As expected $U(1)\times SO(2)$ symmetry separates $\{A_\mu, U_\mu, V_\mu^\pm\}$ in two groups at equations motion. Appendix B presents the relationships between the above parameters and the original Lagrangian free coefficients. It shows how some terms can be avoided without breaking gauge symmetry. This means on-shell propositions be derived without violating the original out-shell symmetry proposition.

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4 Noether's identities

The model proposes the symmetry $U(1)\times SO(2)$ invariance under a common gauge parameter which should be studied under the gauge invariance of first and second species. It yields the following three Noether identities,

$$\partial_\mu J_N^\mu = 0 \quad \text{(charge conservation)}, \tag{15}$$

$$\partial_\mu K^{\mu\nu} + J_N^\nu = 0 \quad \text{(symmetry equation)}, \tag{16}$$

$$K^{\mu\nu}\partial_\mu\partial_\nu\alpha = 0 \quad \text{(symmetry constraint)}. \tag{17}$$

The first one means the total electric charge conservation is a consequence from the first specie gauge invariance. Any Lagrangian which respects electric charge conservation will automatically be invariant be invariant under the first kind transformation. Thus considering that the photon and massive photon carry no charge, they will be invariant under the transformations $A_\mu \to A_\mu$ and $U_\mu \to U_\mu$, while the charged photons will transform as $V_\mu^\pm \to e^{\pm iq\alpha}V_\mu^\pm$. It gives,

$$J_N^\mu \equiv iq\left(V_\nu^+ \frac{\partial L}{\partial(\partial_\mu V_\nu^+)} - V_\nu^- \frac{\partial L}{\partial(\partial_\mu V_\nu^-)}\right) \tag{18}$$

with

$$J_N^\mu = (J_K^A)^\mu + (J_K^S)^\mu + (J_3^A)^\mu + (J_3^S)^\mu, \tag{19}$$

where

$$(J_K^A)^\mu = -2qa_3\,\mathrm{Im}\{V_\nu^+ V^{\mu\nu-}\}$$

$$(J_K^S)^\mu = -8q\,\mathrm{Im}\{V_\nu^+(b_{(33)}S^{\mu\nu-} + c_{(33)}g^{\mu\nu}S_\rho^{\rho-})\} \tag{20}$$

$$(J_3^A)^\mu = -16qb_3\,\mathrm{Im}\{V_\nu^+(\overset{[-1]}{z}{}^{[\mu\nu]} + \overset{[-2]}{z}{}^{[\mu\nu]})\} \tag{21}$$

$$(J_3^S)^\mu = -16q\beta_3\,\mathrm{Im}\{V_\nu^+(\overset{(-1)}{z}{}^{(\mu\nu)} + \overset{(-2)}{z}{}^{(\mu\nu)})\}$$

$$-16q\rho_3\,\mathrm{Im}\{V_\nu^+ g^{\mu\nu}(\overset{(-1)}{z}{}_\rho^\rho + \overset{(-2)}{z}{}_\rho^\rho)\}$$

$$-16q(\beta_3 + 4\rho_3)\,\mathrm{Im}\{V_\nu^+ g^{\mu\nu}(\overset{(-1)}{\omega}{}_\rho^\rho + \overset{(-2)}{\omega}{}_\rho^\rho)\}. \tag{22}$$

Observe that every term defining J_N^μ conserves electric charge individually.

The second Noether identity is the symmetry equation. Being dynamics it is a consequence from the gauge invariance of the second kind. It gives

$$\partial_\mu K^{[\mu\nu]} + \partial_\mu \tilde{K}^{(\mu\nu)} + \partial^\nu K_\alpha^\alpha + J_N^\nu = 0 \tag{23}$$

where

$$K^{\mu\nu} \equiv k_1 \frac{\partial L}{\partial(\partial_\mu A_\nu)} + k_2 \frac{\partial L}{\partial(\partial_\mu U_\nu)} + k_+ \frac{\partial L}{\partial(\partial_\mu V_\nu^+)} + k_- \frac{\partial L}{\partial(\partial_\mu V_\nu^-)} \tag{24}$$

and $\tilde{K}^{(\mu\nu)}$ is a traceless symmetric tensor. Working on it,

$$K^{[\mu\nu]} = (K_K)^{[\mu\nu]} + (K_3^A)^{[\mu\nu]} \tag{25}$$

with

$$(K_K)^{[\mu\nu]} = 4k_1a_1F^{\mu\nu} + 4k_2a_2U^{\mu\nu} + 8a_3\,\mathrm{Re}\{k_+V^{\mu\nu-}\} \tag{26}$$

$$(K_3)^{[\mu\nu]} = 8(k_1b_1 + k_2b_2)(\overset{[12]}{z}{}^{[\mu\nu]} + \overset{[+-]}{z}{}^{[\mu\nu]}) + 8(k_1\beta_1 + k_2\beta_2)\overset{(+-)}{z}{}^{[\mu\nu]}$$

$$+16b_3\mathrm{Re}\{k_+(\overset{[-1]}{z}{}^{[\mu\nu]} + \overset{[-2]}{z}{}^{[\mu\nu]})\} \qquad (27)$$

and

$$K^{(\mu\nu)} = (K_K)^{(\mu\nu)} + (K_3)^{(\mu\nu)} \qquad (28)$$

with

$$(K_K)^{(\mu\nu)} = 4(k_1b_{(11)} + k_2b_{(12)})S^{\mu\nu1} + 4(k_2b_{(22)} + k_1b_{(12)})S^{\mu\nu2}$$

$$+4g^{\mu\nu}\{(k_1c_{(11)} + k_2c_{(12)})S_\rho^{\rho1} + (k_2c_{(22)} + k_1c_{(12)}))S_\rho^{\rho2}\}$$

$$+8b_{(33)}\mathrm{Re}\{k_+S^{\mu\nu-}\} + 8c_{(33)}g^{\mu\nu}\mathrm{Re}\{k_+S_\rho^{\rho-}\} \qquad (29)$$

$$(K_3)^{(\mu\nu)} = 4(k_1\beta_1 + k_2\beta_2)(\overset{(11)}{z}{}^{\mu\nu} + \overset{(22)}{z}{}^{\mu\nu} + 2\overset{(12)}{z}{}^{\mu\nu} + 2\overset{+-3}{z}{}^{(\mu\nu)})$$

$$+4(k_1\rho_1 + k_2\rho_2)g^{\mu\nu}(\overset{(11)}{z}{}_\rho^\rho + \overset{(22)}{z}{}_\rho^\rho + 2\overset{(12)}{z}{}_\rho^\rho + 2\overset{+-3}{z}{}_\rho^\rho)$$

$$+4(k_1[\beta_1 + 4\rho_1] + k_2[\beta_2 + 4\rho_2])g^{\mu\nu}(\overset{(11)}{\omega}{}_\rho^\rho + \overset{(22)}{\omega}{}_\rho^\rho + 2\overset{(12)}{\omega}{}_\rho^\rho + 2\overset{+-3}{\omega}{}_\rho^\rho)$$

$$+16\beta_3\mathrm{Re}\{k_+(\overset{(-1)}{z}{}^{(\mu\nu)} + \overset{(-2)}{z}{}^{(\mu\nu)})\} + 16\rho_3g^{\mu\nu}\mathrm{Re}\{k_+(\overset{(-1)}{z}{}_\rho^\rho + \overset{(-2)}{z}{}_\rho^\rho)\}$$

$$+16(\beta_3 + 4\rho_3)\mathrm{Re}\{k_+(\overset{(-1)}{\omega}{}_\rho^\rho + \overset{(-2)}{\omega}{}_\rho^\rho)\}. \qquad (30)$$

Another equivalent equation to the second Noether identity can be written in terms of L_m, L_3 and L_4. It is

$$K^\mu + J_N^\mu = 0 \qquad (31)$$

where

$$K^\mu \equiv k_1\frac{\partial L}{\partial A_\mu} + k_2\frac{\partial L}{\partial U_\mu} + k_+\frac{\partial L}{\partial V_\mu^+} + k_-\frac{\partial L}{\partial V_\mu^-}. \qquad (32)$$

Splitting the K^μ vector as

$$K^\mu = (K_m)^\mu + (K_3^A)^\mu + (K_3^S)^\mu + (K_4^A)^\mu + (K_4^S)^\mu,$$

one gets,

$$(K_m)^\mu = 2k_2m_2^2U^\mu + 4m_3^2\mathrm{Re}\{k^+V^{\mu-}\}, \qquad (33)$$

$$(K_3^A)^\mu = 4k_1\gamma_{[12]}U_\nu(b_1F^{\mu\nu} + b_2U^{\mu\nu}) + 8k_1b_3\mathrm{Re}\{(\gamma_{[13]} - i\gamma_{[14]})V_\nu^-V^{\mu\nu+}\}$$

$$-4k_2\gamma_{[12]}A_\nu(b_1F^{\mu\nu} + b_2U^{\mu\nu}) + 8k_2b_3\mathrm{Re}\{(\gamma_{[23]} - i\gamma_{[24]})V_\nu^-V^{\mu\nu+}\}$$

$$+8b_3\mathrm{Re}\{k_+(\gamma_{[13]} + i\gamma_{[14]})A_\nu V^{\nu\mu-}\} + 8b_3\mathrm{Re}\{k_+(\gamma_{[23]} + i\gamma_{[24]})U_\nu V^{\nu\mu-}\}$$

$$+8\gamma_{[34]}\mathrm{Im}\{k_+V_\nu^-(b_1F^{\mu\nu} + b_2U^{\mu\nu})\} + 8\gamma_{(34)}\mathrm{Im}\{k_+V_\nu^-(\beta_1F^{\mu\nu} + \beta_2U^{\mu\nu})\}, \qquad (34)$$

$$(K_3^S)^\mu = 4k_1\{(\gamma_{(11)}A_\nu + \gamma_{(12)}U_\nu)(\beta_1 S^{\mu\nu 1} + \beta_2 S^{\mu\nu 2})$$

$$+(\gamma_{(11)}A^\mu + \gamma_{(12)}U^\mu)(\rho_1 S_\nu^{\nu 1} + \rho_2 S_\nu^{\nu 2})$$

$$+2(\tau_{(11)}A^\mu + \tau_{(12)}U^\mu)([\beta_1 + 4\rho_1]S_\nu^{\nu 1} + [\beta_2 + 4\rho_2]S_\nu^{\nu 2})$$

$$+2\beta_3 \mathrm{Re}\{(\gamma_{(13)} - i\gamma_{(14)})V_\nu^- S^{\mu\nu +}\} + 2\rho_3 \mathrm{Re}\{(\gamma_{(13)} - i\gamma_{(14)})V^{\mu -} S_\nu^{\nu +}\}$$

$$+2(\beta_3 + 4\rho_3)\mathrm{Re}\{(\tau_{(13)} - i\tau_{(14)})V^{\mu -} S_\nu^{\nu +}\}\}$$

$$+4k_2\{(\gamma_{(22)}U_\nu + \gamma_{(12)}A_\nu)(\beta_1 S^{\mu\nu 1} + \beta_2 S^{\mu\nu 2})$$

$$+(\gamma_{(22)}U^\mu + \gamma_{(12)}A^\mu)(\rho_1 S_\nu^{\nu 1} + \rho_2 S_\nu^{\nu 2})$$

$$+2(\tau_{(22)}U^\mu + \tau_{(12)}A^\mu)([\beta_1 + 4\rho_1]S_\nu^{\nu 1} + [\beta_2 + 4\rho_2]S_\nu^{\nu 2})$$

$$+2\beta_3 \mathrm{Re}\{(\gamma_{(23)} - i\gamma_{(24)})V_\nu^- S^{\mu\nu +}\} + 2\rho_3 \mathrm{Re}\{(\gamma_{(23)} - i\gamma_{(24)})V^{\mu -} S_\nu^{\nu +}\}$$

$$+2(\beta_3 + 4\rho_3)\mathrm{Re}\{(\tau_{(23)} - i\tau_{(24)})V^{\mu -} S_\nu^{\nu +}\}\}$$

$$+8\mathrm{Re}\{k_+\beta_3([\gamma_{(13)} + i\gamma_{(14)}]A_\nu + [\gamma_{(23)} + i\gamma_{(24)}]U_\nu)S^{\mu\nu -}$$

$$+k_+\gamma_{(33)}V_\nu^-(\beta_1 S^{\mu\nu 1} + \beta_2 S^{\mu\nu 2}) + k_+\rho_3(\gamma_{(13)} + i\gamma_{(14)})A^\mu S_\nu^{\nu -}$$

$$+k_+(\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)})A^\mu S_\nu^{\nu -} + k_+\rho_3(\gamma_{(23)} + i\gamma_{(24)})U^\mu S_\nu^{\nu -}$$

$$+k_+(\beta_3 + 4\rho_3)(\tau_{(23)} + i\tau_{(24)})U^\mu S_\nu^{\nu -} + k_+\gamma_{(33)}V^{\mu -}(\rho_1 S_\nu^{\nu 1} + \rho_2 S_\nu^{\nu 2})$$

$$+k_+(\beta_1 + 4\rho_1)\tau_{(33)}V^{\mu -}S_\nu^{\nu 1} + k_+(\beta_2 + 4\rho_2)\tau_{(33)}V^{\mu -}S_\nu^{\nu 2}\} \quad (35)$$

$$(K_4^A)^\mu = 8k_1\{\gamma_{[12]}U_\nu \overset{[12]}{z}{}^{[\mu\nu]} - \gamma_{[13]}\mathrm{Re}\{V^{\mu +}(\overset{[13-]}{z}{}_\nu^\nu + \overset{23-]}{z}{}_\nu^\nu)\}$$

$$+\gamma_{[13]}V_\nu^+(\overset{[13-]}{z}{}^{\mu\nu} + \overset{[23-]}{z}{}^{\mu\nu}) + \gamma_{[12]}U_\nu \mathrm{Re}\{\overset{[+-]}{z}{}^{\mu\nu}\}$$

$$-\frac{1}{2}\gamma_{[13]}\mathrm{Im}\{V_\nu^+ \overset{[24-]}{z}{}^{\nu\mu}\} + \frac{1}{2}\gamma_{[14]}\mathrm{Im}\{V_\nu^+ \overset{[23-]}{z}{}^{\nu\mu}\}\}$$

$$+8k_2\{\gamma_{[12]}A_\nu \overset{[12]}{z}{}^{[\nu\mu]} - \gamma_{[23]}\mathrm{Re}\{V^{\mu +}(\overset{[13-]}{z}{}_\nu^\nu + \overset{23-]}{z}{}_\nu^\nu)\}$$

$$+\gamma_{[23]}V_\nu^+(\overset{[13-]}{z}{}^{\mu\nu} + \overset{[23-]}{z}{}^{\mu\nu}) - \gamma_{[12]}A_\nu \mathrm{Re}\{\overset{[+-]}{z}{}^{\mu\nu}\}$$

$$-\frac{1}{2}\gamma_{[23]}\mathrm{Im}\{V_\nu^+ \overset{[14-]}{z}{}^{\nu\mu}\} + \frac{1}{2}\gamma_{[24]}\mathrm{Im}\{V_\nu^+ \overset{[13-]}{z}{}^{\nu\mu}\}\}$$

$$+8\mathrm{Re}\{k_+[\gamma_{[13]}A_\nu \overset{[13-]}{z}{}^{\nu\mu} + \gamma_{[23]}U_\nu \overset{[23-]}{z}{}^{\nu\mu}$$

$$-(\gamma_{[13]}A^\mu + \gamma_{[23]}U^\mu)(\overset{[13-]}{z}{}_\nu^\nu + \overset{[23-]}{z}{}_\nu^\nu) + 2\gamma_{[13]}A_\nu \overset{[23-]}{z}{}^{\nu\mu}]\}$$

$$+4\mathrm{Im}\{k_+[4\gamma_{[34]}V_\nu^+(\overset{[12]}{z}{}^{[\mu\nu]} + \overset{[+-]}{z}{}^{(\mu\nu)}) - 4\gamma_{[34]}V^{\mu -}\overset{[+-]}{z}{}_\nu^\nu$$

$$+\gamma_{[14]}A_\nu \overset{[23-]}{z}{}^{\mu\nu} - \gamma_{[13]}A_\nu \overset{[24-]}{z}{}^{\mu\nu}$$

$$+\gamma_{[24]}U_\nu \overset{[13-]}{z}{}^{\mu\nu} - \gamma_{[23]}U_\nu \overset{[14-]}{z}{}^{\mu\nu}]\}, \qquad (36)$$

$$(K_4^S)^\mu = 4k_1\{(\gamma_{(11)}A_\nu+\gamma_{(12)}U_\nu)(\overset{(11)}{z}{}^{\mu\nu}+\overset{(22)}{z}{}^{\mu\nu})$$

$$+2(\gamma_{(11)}+2\tau_{(11)})(A_\nu \overset{(12)}{z}{}^{(\mu\nu)}+A_\nu\overset{(11)}{\omega}{}^{\mu\nu}+A^\mu\overset{(22)}{\omega}{}^\nu_\nu+2A^\mu\overset{+-3}{\omega}{}^\nu_\nu)$$

$$+2(\gamma_{(12)}+2\tau_{(12)})(U_\nu\overset{(11)}{\omega}{}^{\mu\nu}+U_\nu\overset{(22)}{\omega}{}^{\mu\nu}+2U_\nu\overset{(12)}{\omega}{}^{\mu\nu}+2U^\mu\overset{+-3}{\omega}{}^\nu_\nu)$$

$$+8\tau_{(11)}A_\nu\overset{(12)}{\omega}{}^{(\mu\nu)}+2\gamma_{(12)}U_\nu\overset{(12)}{z}{}^{(\mu\nu)}+2(\gamma_{(11)}A_\nu+\gamma_{(12)}U_\nu)\overset{+-3}{z}{}^{(\mu\nu)}$$

$$+2\gamma_{(13)}\mathrm{Re}\{V^{\mu+}(\overset{(13-)}{z}{}^\nu_\nu+\overset{(23-)}{z}{}^\nu_\nu)\}+2\gamma_{(13)}V^+_\nu(\overset{(13-)}{z}{}^{\mu\nu}+\overset{(23-)}{z}{}^{\mu\nu})$$

$$+8(\gamma_{(13)}+2\tau_{(13)})\mathrm{Re}\{V^{\mu+}(\overset{(13-)}{\omega}{}^\nu_\nu+\overset{(23-)}{\omega}{}^\nu_\nu)\}$$

$$-(\gamma_{(14)}-2\tau_{(14)})\mathrm{Im}\{V^+_\nu\overset{(23-)}{z}{}^{\nu\mu}\}+(\gamma_{(13)}-2\tau_{(13)})\mathrm{Im}\{V^+_\nu\overset{(24-)}{z}{}^{\nu\mu}\}$$

$$+2(\gamma_{(14)}+4\tau_{(14)})\mathrm{Im}\{V^+_\nu\overset{(23-)}{\omega}{}^{\nu\mu}\}-2(\gamma_{(13)}+4\tau_{(13)})\mathrm{Im}\{V^+_\nu\overset{(24-)}{\omega}{}^{\nu\mu}\}\}$$

$$+4k_2\{(\gamma_{(22)}U_\nu+\gamma_{(12)}A_\nu)(\overset{(11)}{z}{}^{\mu\nu}+\overset{(22)}{z}{}^{\mu\nu})$$

$$+2(\gamma_{(22)}+2\tau_{(22)})(U_\nu\overset{(12)}{z}{}^{(\mu\nu)}+U_\nu\overset{(22)}{\omega}{}^{\mu\nu}+U^\mu\overset{(11)}{\omega}{}^\nu_\nu+2U^\mu\overset{+-3}{\omega}{}^\nu_\nu)$$

$$+2(\gamma_{(12)}+2\tau_{(12)})(A_\nu\overset{(11)}{\omega}{}^{\mu\nu}+A_\nu\overset{(22)}{\omega}{}^{\mu\nu}+2A_\nu\overset{(12)}{\omega}{}^{\mu\nu}+2A^\mu\overset{+-3}{\omega}{}^\nu_\nu)$$

$$+8\tau_{(22)}U_\nu\overset{(12)}{\omega}{}^{(\mu\nu)}+2\gamma_{(12)}A_\nu\overset{(12)}{z}{}^{(\mu\nu)}+2(\gamma_{(22)}U_\nu+\gamma_{(12)}A_\nu)\overset{+-3}{z}{}^{(\mu\nu)}$$

$$+2\gamma_{(23)}\mathrm{Re}\{V^{\mu+}(\overset{(13-)}{z}{}^\nu_\nu+\overset{(23-)}{z}{}^\nu_\nu)\}+2\gamma_{(23)}V^+_\nu(\overset{(13-)}{z}{}^{\mu\nu}+\overset{(23-)}{z}{}^{\mu\nu})$$

$$+8(\gamma_{(23)}+2\tau_{(23)})\mathrm{Re}\{V^{\mu+}(\overset{(13-)}{\omega}{}^\nu_\nu+\overset{(23-)}{\omega}{}^\nu_\nu)\}$$

$$-(\gamma_{(24)}-2\tau_{(24)})\mathrm{Im}\{V^+_\nu\overset{(13-)}{z}{}^{\nu\mu}\}+(\gamma_{(23)}-2\tau_{(23)})\mathrm{Im}\{V^+_\nu\overset{(14-)}{z}{}^{\nu\mu}\}$$

$$+2(\gamma_{(24)}+4\tau_{(24)})\mathrm{Im}\{V^+_\nu\overset{(13-)}{\omega}{}^{\nu\mu}\}-2(\gamma_{(23)}+4\tau_{(23)})\mathrm{Im}\{V^+_\nu\overset{(14-)}{\omega}{}^{\nu\mu}\}\}$$

$$+8\mathrm{Re}\{k_+[\gamma_{(33)}V^-_\nu(\overset{(11)}{z}{}^{\mu\nu}+\overset{(22)}{z}{}^{\mu\nu}+2\overset{(12)}{z}{}^{\mu\nu})$$

$$+2\tau_{(33)}V^{\mu-}(\overset{(11)}{z}{}^\nu_\nu+\overset{(22)}{z}{}^\nu_\nu+\overset{(12)}{z}{}^\nu_\nu+2\overset{(11)}{\omega}{}^\nu_\nu+2\overset{(22)}{\omega}{}^\nu_\nu+4\overset{(12)}{\omega}{}^\nu_\nu)$$

$$+4\{(\gamma_{(13)}+2\tau_{(13)})A^\mu+(\gamma_{(23)}+2\tau_{(23)})U^\mu\}(\overset{(13-)}{\omega}{}^\nu_\nu+\overset{(23-)}{\omega}{}^\nu_\nu)$$

$$+\gamma_{(13)}A_{\nu}\overset{(13-)}{z}{}^{\nu\mu}+\gamma_{(23)}U_{\nu}\overset{(23-)}{z}{}^{\nu\mu}+2\gamma_{(13)}A_{\nu}\overset{(23-)}{z}{}^{\nu\mu}$$

$$+2\gamma_{(33)}V_{\nu}^{-}\overset{+-4}{z}{}^{(\mu\nu)}+4(\gamma_{(33)}+2\tau_{(33)})V^{\mu-}\overset{+-4}{\omega}{}^{\nu}_{\nu}]\}$$

$$+4\mathrm{Im}\{k_{+}[2(\gamma_{(14)}+4\tau_{(14)})A_{\nu}\overset{(23-)}{\omega}{}^{\mu\nu}-2(\gamma_{(13)}+4\tau_{(13)})A_{\nu}\overset{(24-)}{\omega}{}^{\mu\nu}$$

$$-(\gamma_{(14)}-2\tau_{(14)})A_{\nu}\overset{(23-)}{z}{}^{\mu\nu}+(\gamma_{(13)}-2\tau_{(13)})A_{\nu}\overset{(24-)}{z}{}^{\mu\nu}$$

$$2(\gamma_{(24)}+4\tau_{(24)})U_{\nu}\overset{(13-)}{\omega}{}^{\mu\nu}-2(\gamma_{(23)}+4\tau_{(23)})U_{\nu}\overset{(14-)}{\omega}{}^{\mu\nu}$$

$$-(\gamma_{(24)}-2\tau_{(24)})U_{\nu}\overset{(13-)}{z}{}^{\mu\nu}+(\gamma_{(23)}-2\tau_{(23)})U_{\nu}\overset{(14-)}{z}{}^{\mu\nu}$$

$$-16(\gamma_{(34)}+2\tau_{(34)})V_{\nu}^{-}\overset{(+-)}{\omega}{}^{(\mu\nu)}-4\gamma_{(34)}V^{\mu-}\overset{(+-)}{z}{}^{\nu}_{\nu}]\}. \quad (37)$$

Eqs. (23) and (31) are relating two ways to figure out the symmetry equation. They relating propagating-sources and massive-sources, respectively. They contain two alternatives for complementing the basic Eqs.(11)-(14).

The third Noether relationship, Eq. (17), leads to the following symmetry constraint on the $K^{\mu\nu}$ tensor:

$$K^{\mu\nu}=-K^{\nu\mu}, \quad \text{or, equivalently}, \quad K^{(\mu\nu)}=0. \quad (38)$$

Notice that eq(41) does not cancel L_S. This condition can be weaken through the relationship

$$\partial_{\mu}\partial_{\nu}K^{\mu\nu}=0. \quad (39)$$

Furthermore, making the decomposition: $K^{\mu\nu}=K^{[\mu\nu]}+\tilde{K}^{(\mu\nu)}-\frac{1}{2}\eta^{\mu\nu}K^{\alpha}_{\alpha}$, where $\mathrm{tr}\,\tilde{K}^{(\mu\nu)}=0$, we get the following equation:

$$\partial_{\mu}\partial_{\nu}\tilde{K}^{(\mu\nu)}-\frac{1}{2}\mathrm{W}K^{\alpha}_{\alpha}=0, \quad (40)$$

which can be useful for further considerations on a symmetric charge conservation.

5 Directive Photon Equation

The minimum action principle lead us to the expression

$$\delta S=\int d^{4}x[\frac{\partial L}{\partial\Phi_{I}}-\partial_{\mu}\frac{\partial L}{\partial(\partial_{\mu}\Phi_{I})}]\delta\Phi_{I}+\int d\sigma_{\mu}[\frac{\partial L}{\delta(\partial_{\mu}\Phi_{I})}\delta\partial^{\mu}\Phi_{I}+L\delta x^{\mu}]=0 \quad (41)$$

where $\Phi_{I}\equiv\{A_{\mu},U_{\mu},V_{\mu}^{\pm}\}$. Consequently the electromagnetic model proposed in this work we will have four equations of motion plus three Noether identities. For simplicity, we will take $\delta x^{\mu}=0$.

Given the previous equations one has to build up the fields system. A whole dynamics must be constructed originated on five equations of motion. Our viewpoint is to select the photon. Being a special particle, that one associated to light invariance property, we consider as compulsory to develop an effective photon equation. An expression where at left hand side one writes the kinematics obtained from the photon variational equation plus the symmetry equation, while at the right hand side the others fields equations are included acting as source. It yields an expression propagating just the photon quanta

$$\partial_{\nu}\{a_{1}F^{\nu\mu}+a_{2}(\overset{[12]}{z}{}^{[\nu\mu]}+\overset{[+-]}{z}{}^{[\nu\mu]})+a_{3}\overset{(+-)}{z}{}^{[\nu\mu]}+$$

$$+g^{\nu\mu}[a_4 S_\alpha^{\alpha 1}+a_5 S_\alpha^{\alpha 2}+a_6 S_\alpha^{\alpha +}+a_7 S_\alpha^{\alpha +}+a_8(\overset{(11)}{z}{}_\alpha^\alpha+\overset{(22)}{z}{}_\alpha^\alpha+2\overset{(12)}{z}{}_\alpha^\alpha+2\overset{(+-3)}{z}{}_\alpha^\alpha)+$$

$$+a_9(\overset{(11)}{\omega}{}_\alpha^\alpha+\overset{(22)}{\omega}{}_\alpha^\alpha+2\overset{(12)}{\omega}{}_\alpha^\alpha+2\overset{(+-3)}{\omega}{}_\alpha^\alpha)]\}=J^\mu_{\text{syst}} \qquad (42)$$

Eq.(5.2) will be called as the photon directive equation. It is an interpretation from eq.(5.1) where it contains a whole electromagnetism combining minimal action principle, light invariance, Noether theorem and electric charge conservation. It provides a photon effective equation with a dynamics beyond Maxwell and the original photon variational equation. It describes a systemic electromagnetism mixing fundamental laws plus chance. It yields a kind of Navier-Stokes electromagnetism.

The right handed systemic current is written as

$$J^\mu_{\text{syst}}=J^\mu_A-J^\mu_N \qquad (43)$$

where

$$J^\mu_A=A_\nu\{r_1^A F^{\nu\mu}+r_2^A U^{\nu\mu}+r_3^A V^{\nu\mu+}+r_4^A V^{\nu\mu-}+r_5^A S^{\nu\mu 1}+r_6^A S^{\nu\mu 2}+$$

$$+r_7^A S^{\nu\mu+}+r_8^A S^{\nu\mu-}+r_9^A \overset{[12]}{z}{}^{[\nu\mu]}+r_{10}^A\overset{(12)}{\omega}{}^{[\nu\mu]}+r_{11}^A\overset{(11)}{z}{}^{(\nu\mu)}+r_{12}^A\overset{(12)}{z}{}^{(\nu\mu)}+$$

$$+r_{13}^A\overset{(22)}{z}{}^{(\nu\mu)}+r_{12}^A\overset{+-3}{z}{}^{(\nu\mu)}+r_{14}^A\overset{[+-]}{z}{}^{(\nu\mu)}+r_{15}^A\overset{(11)}{\omega}{}^{(\nu\mu)}+r_{16}^A\overset{(12)}{\omega}{}^{(\nu\mu)}+r_{17}^A\overset{(22)}{\omega}{}^{(\nu\mu)}+$$

$$+r_{18}^A\overset{[13+]}{z}{}^{\nu\mu}+r_{19}^A\overset{[23+]}{z}{}^{[\nu\mu]}+r_{20}^A\overset{(23+)}{z}{}^{(\nu\mu)}+r_{21}^A\overset{[24+]}{z}{}^{\mu\nu}+r_{22}^A\overset{(13+)}{z}{}^{\nu\mu}+r_{23}^A\overset{(23+)}{z}{}^{\nu\mu}+r_{24}^A\overset{[23+]}{z}{}^{(\nu\mu)}+$$

$$+r_{25}^A\overset{[24+]}{z}{}^{\mu\nu}+r_{26}^A\overset{(23+)}{\omega}{}^{\mu\nu}+r_{27}^A\overset{(24+)}{z}{}^{\mu\nu}+r_{28}^A\overset{[13-]}{z}{}^{\nu\mu}+r_{29}^A\overset{[23-]}{z}{}^{[\mu\nu]}+r_{30}^A\overset{(23-)}{z}{}^{([nu\mu]}+$$

$$+r_{31}^A\overset{(24-)}{z}{}^{\mu\nu}+r_{32}^A\overset{[23-]}{z}{}^{(\mu\nu)}+r_{33}^A\overset{(13-)}{z}{}^{(\mu\nu)}+r_{34}^A\overset{(23-)}{z}{}^{(\mu\nu)}+r_{35}^A\overset{(24-)}{z}{}^{\mu\nu}+r_{36}^A\overset{(23-)}{\omega}{}^{\mu\nu}$$

$$+r_{37}^A\overset{(24-)}{\omega}{}^{\mu\nu}+g^{\nu\mu}(r_{38}^A S_\alpha^{\alpha 1}+r_{39}^A S_\alpha^{\alpha 2}+r_{40}^A S_\alpha^{\alpha+}+r_{41}^A S_\alpha^{\alpha-}+r_{42}^A\overset{(22)}{\omega}{}_\alpha^\alpha+r_{43}^A\overset{(+-3)}{\omega}{}_\alpha^\alpha+$$

$$+r_{44}^A\overset{[13+]}{z}{}_\alpha^\alpha+r_{44}^A\overset{[23+]}{z}{}_\alpha^\alpha+r_{45}^A\overset{(13+)}{z}{}_\alpha^\alpha+r_{46}^A\overset{(23+)}{z}{}_\alpha^\alpha+r_{47}^A\overset{(13+)}{\omega}{}_\alpha^\alpha+r_{48}^A\overset{(23+)}{\omega}{}_\alpha^\alpha+r_{49}^A\overset{[13-]}{z}{}_\alpha^\alpha+$$

$$+r_{49}^A\overset{[23-]}{z}{}_\alpha^\alpha+r_{50}^A\overset{(13-)}{z}{}_\alpha^\alpha+r_{50}^A\overset{(23-)}{z}{}_\alpha^\alpha+r_{51}^A\overset{(13-)}{\omega}{}_\alpha^\alpha+r_{51}^A\overset{(23-)}{\omega}{}_\alpha^\alpha)+$$

$$+U_\nu\{r_1^U F^{\nu\mu}+r_2^U U^{\nu\mu}+r_3^U V^{\nu\mu+}+r_4^U V^{\nu\mu-}+r_5^U S^{\nu\mu 1}+r_6^U S^{\nu\mu 2}+$$

$$+r_7^U S^{\nu\mu+}+r_8^U S^{\nu\mu-}+r_9^U\overset{[12]}{z}{}^{[\nu\mu]}+r_{10}^U\overset{(12)}{\omega}{}^{[\nu\mu]}+r_{11}^U\overset{(11)}{z}{}^{(\nu\mu)}+r_{12}^U\overset{(22)}{z}{}^{(\nu\mu)}+$$

$$+r_{13}^U\overset{(12)}{z}{}^{(\nu\mu)}+r_{14}^U\overset{+-3}{z}{}^{(\nu\mu)}+r_{15}^U\overset{[+-]}{z}{}^{(\nu\mu)}+r_{16}^U\overset{(11)}{\omega}{}^{(\nu\mu)}+r_{17}^U\overset{(12)}{\omega}{}^{(\nu\mu)}+r_{18}^U\overset{(22)}{\omega}{}^{(\nu\mu)}+r_{19}^U\overset{[13+]}{z}{}^{\nu\mu}+$$

$$+r_{20}^U\overset{[14+]}{z}{}^{[\nu\mu]}+r_{21}^U\overset{[23+]}{z}{}^{(\nu\mu)}+r_{22}^U\overset{(13+)}{z}{}^{[\nu\mu]}+r_{23}^U\overset{(13+)}{z}{}^{(\nu\mu)}+r_{24}^U\overset{(23+)}{z}{}^{\nu\mu}+r_{25}^U\overset{(14+)}{z}{}^{\mu\nu}+r_{26}^U\overset{(13+)}{\omega}{}^{\mu\nu}+$$

$$+r_{27}^U\overset{(14+)}{\omega}{}^{\mu\nu}+r_{28}^U\overset{[13-]}{z}{}^{\mu\nu}+r_{29}^U\overset{[14-]}{z}{}^{\mu\nu}+r_{30}^U\overset{[23-]}{z}{}^{\nu\mu}+r_{31}^U\overset{(13-)}{z}{}^{\mu\nu}+r_{32}^U\overset{(14-)}{z}{}^{\mu\mu}+$$

$$+r_{33}^U\overset{(14-)}{z}{}^{\mu\nu}+r_{34}^U\overset{(13-)}{\omega}{}^{\mu\nu}+r_{35}^U\overset{(14-)}{\omega}{}^{\mu\nu}+g^{\nu\mu}(-km_2^2+r_{36}^U S_\alpha^{\alpha 1}+r_{37}^U S_\alpha^{\alpha 2}+r_{38}^U S_\alpha^{\alpha+}+r_{39}^U S_\alpha^{\alpha-}+$$

$$+r_{40}^U\overset{(11)}{\omega}{}_\alpha^\alpha+r_{41}^U\overset{(+-3)}{\omega}{}_\alpha^\alpha+r_{42}^U\overset{[13+]}{z}{}_\alpha^\alpha+r_{42}^U\overset{[23+]}{z}{}_\alpha^\alpha+r_{43}^U\overset{(13+)}{z}{}_\alpha^\alpha+r_{43}^U\overset{(23+)}{z}{}_\alpha^\alpha+r_{44}^U\overset{(13+)}{\omega}{}_\alpha^\alpha$$

$$+r_{44}^{U}\overset{(23+)}{\omega}{}_{\alpha}^{\alpha}+r_{45}^{U}\overset{[13-]}{z}{}_{\alpha}^{\alpha}+r_{45}^{U}\overset{[23-]}{z}{}_{\alpha}^{\alpha}+r_{46}^{U}\overset{(13-)}{z}{}_{\alpha}^{\alpha}+r_{46}^{U}\overset{(23-)}{z}{}_{\alpha}^{\alpha}+r_{47}^{U}\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}+r_{47}^{U}\overset{(23-)}{\omega}{}_{\alpha}^{\alpha})+$$

$$+V_{\nu}^{+}\{r_1^+F^{\nu\mu}+r_2^+U^{\nu\mu}+r_3^+V^{\nu\mu-}+r_4^+S^{\nu\mu1}+r_5^+S^{\nu\mu2}+r_6^+S^{\nu\mu-}+r_7^+\overset{[12]}{z}{}^{[\nu\mu]}+$$

$$+r_8^+(\overset{(11)}{z}{}^{(\nu\mu)}+\overset{(22)}{z}{}^{(\nu\mu)})+r_9^+(\overset{(12)}{z}{}^{(\nu\mu)}+\overset{+-4}{z}{}^{(\nu\mu)})+r_{10}^+\overset{[+-]}{z}{}^{(\nu\mu)}+r_{11}^+\overset{[13-]}{z}{}^{\mu\nu}+r_{12}^+\overset{[14-]}{z}{}^{\nu\mu}+$$

$$+r_{13}^+\overset{[23-]}{z}{}^{[\nu\mu]}+r_{14}^+\overset{[23-]}{z}{}^{(\nu\mu)}+r_{15}^+\overset{[24-]}{z}{}^{\nu\mu}+r_{16}^+\overset{(13-)}{z}{}^{[\nu\mu]}+r_{17}^+\overset{(13-)}{z}{}^{(\mu\nu)}+r_{18}^+\overset{(14-)}{z}{}^{(\nu\mu)}+$$

$$+r_{19}^+\overset{(23-)}{z}{}^{[\nu\mu]}+r_{20}^+\overset{(23-)}{z}{}^{(\nu\mu)}+r_{21}^+\overset{(24-)}{z}{}^{(\nu\mu)}+r_{22}^+\overset{(13-)}{\omega}{}^{\nu\mu}+r_{23}^+\overset{(14-)}{\omega}{}^{\nu\mu}+$$

$$+r_{24}^+\overset{(23-)}{\omega}{}^{\nu\mu}+r_{25}^+\overset{(24-)}{\omega}{}^{\nu\mu}+r_{26}^+\overset{(+-)}{\omega}{}^{(\mu\nu)}+g^{\nu\mu}(-k_-m_+^2+r_{27}^+S_{\alpha}^{\alpha1}+r_{28}^+S_{\alpha}^{\alpha2}+r_{29}^+S_{\alpha}^{\alpha-}+$$

$$+r_{30}^+(\overset{(11)}{z}{}_{\alpha}^{\alpha}+\overset{(22)}{z}{}_{\alpha}^{\alpha}+2\overset{(12)}{z}{}_{\alpha}^{\alpha})+r_{31}^+(\overset{(11)}{\omega}{}_{\alpha}^{\alpha}+2\overset{(12)}{\omega}{}_{\alpha}^{\alpha}+\overset{(22)}{\omega}{}_{\alpha}^{\alpha})+r_{32}^+\overset{[+-]}{z}{}_{\alpha}^{\alpha}+r_{33}^+\overset{(+-)}{z}{}_{\alpha}^{\alpha}+$$

$$+r_{33}^+\overset{[13-]}{z}{}_{\alpha}^{\alpha}+r_{34}^+\overset{[23-]}{z}{}_{\alpha}^{\alpha}+r_{35}^+\overset{(13-)}{z}{}_{\alpha}^{\alpha}+r_{36}^+\overset{(23-)}{z}{}_{\alpha}^{\alpha}+r_{37}^+\overset{(+-4)}{\omega}{}_{\alpha}^{\alpha}+r_{38}^+\overset{(13-)}{\omega}{}_{\alpha}^{\alpha}+$$

$$+r_{37}^+\overset{(23-)}{\omega}{}_{\alpha}^{\alpha})\}+$$

$$+V_\nu^-\{r_1^-F^{\nu\mu}+r_2^-U^{\nu\mu}+r_3^-V^{\nu\mu-}+r_4^-S^{\nu\mu1}+r_5^-S^{\nu\mu2}+r_6^-S^{\nu\mu+}+r_7^-\overset{[12]}{z}{}^{[\nu\mu]}+$$

$$+r_8^-(\overset{(11)}{z}{}^{(\nu\mu)}+\overset{(22)}{z}{}^{(\nu\mu)})+r_9^-(\overset{(12)}{z}{}^{(\nu\mu)}+\overset{+-4}{z}{}^{(\nu\mu)})+r_{10}^-\overset{[+-]}{z}{}^{(\nu\mu)}+r_{11}^-\overset{[13+]}{z}{}^{[\mu\nu]}+$$

$$+r_{12}^-\overset{[13+]}{z}{}^{(\mu\nu)}+r_{13}^-\overset{[14+]}{z}{}^{\nu\mu}+r_{14}^-\overset{[23+]}{z}{}^{[\nu\mu]}+r_{15}^-\overset{[23+]}{z}{}^{(\nu\mu)}+r_{16}^-\overset{[24+]}{z}{}^{\nu\mu}+r_{17}^-\overset{(13+)}{z}{}^{[\nu\mu]}+$$

$$+r_{18}^-\overset{(13+)}{z}{}^{(\mu\nu)}+r_{19}^-\overset{(14+)}{z}{}^{(\nu\mu)}+r_{20}^-\overset{(23+)}{z}{}^{[\nu\mu]}+r_{21}^-\overset{(23+)}{z}{}^{(\nu\mu)}+$$

$$+r_{22}^-\overset{(24+)}{z}{}^{\nu\mu}+r_{23}^-\overset{(13+)}{\omega}{}^{\nu\mu}+r_{24}^-\overset{(14+)}{\omega}{}^{\nu\mu}+r_{25}^-\overset{(23+)}{\omega}{}^{\nu\mu}+r_{26}^-\overset{(24+)}{\omega}{}^{\nu\mu}+r_{27}^-\overset{(+-)}{\omega}{}^{(\mu\nu)}+$$

$$+g^{\nu\mu}(-k_+m_-^2+r_{28}^-S_\alpha^{\alpha1}+r_{29}^-S_\alpha^{\alpha2}+r_{30}^-S_\alpha^{\alpha-}+r_{31}^-(\overset{(11)}{z}{}_\alpha^\alpha+\overset{(22)}{z}{}_\alpha^\alpha+2\overset{(12)}{z}{}_\alpha^\alpha)+$$

$$+r_{32}^-(\overset{(11)}{\omega}{}_\alpha^\alpha+2\overset{(12)}{\omega}{}_\alpha^\alpha+\overset{(22)}{\omega}{}_\alpha^\alpha)+r_{33}^-\overset{[+-]}{z}{}_\alpha^\alpha+r_{34}^-\overset{(+-)}{z}{}_\alpha^\alpha+r_{35}^-\overset{[13+]}{z}{}_\alpha^\alpha+r_{35}^-\overset{[23+]}{z}{}_\alpha^\alpha+$$

$$+r_{36}^-\overset{(13+)}{z}{}_\alpha^\alpha+r_{36}^-\overset{(23+)}{z}{}_\alpha^\alpha+r_{37}^-\overset{(+-4)}{\omega}{}_\alpha^\alpha+r_{38}^-\overset{(13+)}{\omega}{}_\alpha^\alpha+r_{38}^-\overset{(23+)}{\omega}{}_\alpha^\alpha)\}$$

Thus, based on photon as directive, a systemic model is defined through eq.(5.2-4). At appendix C the coefficients corresponding to the source terms are determined. It enlarges the electromagnetic description with photon plus three messengers, where the photon assumes the symmetry directive while the others fields the symmetry circumstances. It propitiates a complexity performance by introducing the meaning of chance [12].

6 Charges

Eq.(1.1) proposes an electromagnetism beyond Maxwell. So one should understand on the charges provided by the model. It yields seven conserved charges. The first one is the electric charge conservation. Eq(4.1) means the electromagnetic current. It antecedes the Lagrangian performance. It leads us to $\frac{d}{dt}Q_N=0$, where Q_N is the charge

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operator, $Q_N = \int d^3 x J_N^0$. The time-independence of Q_N implies that Q_N commutes with Hamiltonian, and hence also with the S-matrix, $[Q_N, S] = 0$, which means an algebraic condition on the S-matrix.

The second and third ones are derived from symmetry equation. It is that we learn by adding gauge invariance of the second kind to gauge invariance of first kind. Eq (4.2) which can be splitted as

$$\partial_\mu j_A^\mu = 0 \tag{44}$$

$$\partial_\mu j_S^\mu = \partial_\nu \partial_\mu \tilde{K}^{(\mu\nu)} - \frac{1}{2} \mathrm{W} K_\alpha^\alpha = 0 \tag{45}$$

where

$$j_A^\mu = \partial_\nu K^{[\mu\nu]}, \quad j_S^\mu = \partial_\nu K^{(\mu\nu)} \tag{46}$$

are introducting an electromagnetic region beyond electric charge. They are showing on the existence of charges Q_A and Q_S independent from Q_N. They are consequences from fields non-linearity and contain some terms depending on charged fields.

There is also at eq.(4.18) an alternative description. It relates masses, trilinear and quadrilinear vertices. It gives

$$\partial_\mu K^\mu = 0 \tag{47}$$

There are more four charges conservations corresponding to each equation of motion. Four continuity equations expressing sources and current depending these four electromagnetic fields.

For the field A_μ:

$$J_A^\mu = \partial^\mu (S_\alpha^{\alpha A} + z_\alpha^{A\alpha}) - j_A^\mu$$

where

$$S_\alpha^{\alpha A} = (a_7 + a_4) S_\alpha^{\alpha 1} + (a_8 + a_5) S_\alpha^{\alpha 2} + a_9 S_\alpha^{\alpha +} + a_{10} S_\alpha^{\alpha -},$$

$$z_\alpha^{A\alpha} = (a_{11} + \frac{a_6}{2})(\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2\overset{(12)}{z}{}_\alpha^\alpha + 2\overset{(+-3)}{z}{}_\alpha^\alpha) + a_{12}(\overset{(11)}{\omega}{}_\alpha^\alpha + \overset{(22)}{\omega}{}_\alpha^\alpha + 2\overset{(12)}{\omega}{}_\alpha^\alpha + 2\overset{(+-3)}{\omega}{}_\alpha^\alpha),$$

$$j_A^\mu = \frac{\partial L}{\partial A_\mu} - \frac{a_6}{2}\{A_\nu (2(\gamma_{(11)} S^{\nu\mu 1} + \gamma_{(12)} S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(11)} S_\alpha^{\alpha 1} + \gamma_{(12)} S_\alpha^{\alpha 2})) +$$

$$+ U_\nu (2(\gamma_{(12)} S^{\nu\mu 1} + \gamma_{(22)} S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(12)} S_\alpha^{\alpha 1} + \gamma_{(22)} S_\alpha^{\alpha 2})) +$$

$$+ V_\nu^+ (\gamma_{(33)} S^{\nu\mu -} + g^{\nu\mu} \frac{\gamma_{(33)}}{2} S_\alpha^{\alpha -}) + V_\nu^- (\gamma_{(33)} S^{\nu\mu +} + g^{\nu\mu} \frac{\gamma_{(33)}}{2} S_\alpha^{\alpha +})\} \tag{48}$$

For the field U_μ:

$$J_U^\mu = \partial^\mu (S_\alpha^{\alpha U} + z_\alpha^{U\alpha} - m_2^2 U^\mu) - j_U^\mu$$

where

$$S_\alpha^{\alpha U} = (b_7 + b_4) S_\alpha^{\alpha 1} + (b_8 + b_5) S_\alpha^{\alpha 2} + b_9 S_\alpha^{\alpha +} + b_{10} S_\alpha^{\alpha -},$$

$$z_\alpha^{U\alpha} = (b_{11} + \frac{b_6}{2})(\overset{(11)}{z}{}_\alpha^\alpha + \overset{(22)}{z}{}_\alpha^\alpha + 2\overset{(12)}{z}{}_\alpha^\alpha + 2\overset{(+-3)}{z}{}_\alpha^\alpha) + b_{12}(\overset{(11)}{\omega}{}_\alpha^\alpha + \overset{(22)}{\omega}{}_\alpha^\alpha + 2\overset{(12)}{\omega}{}_\alpha^\alpha + 2\overset{(+-3)}{\omega}{}_\alpha^\alpha),$$

$$j_U^\mu = \frac{\partial L}{\partial U_\mu} - \frac{b_6}{2}\{A_\nu (2(\gamma_{(11)} S^{\nu\mu 1} + \gamma_{(12)} S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(11)} S_\alpha^{\alpha 1} + \gamma_{(12)} S_\alpha^{\alpha 2})) +$$

$$+U_{\nu}(2(\gamma_{(12)}S^{\nu\mu 1}+\gamma_{(22)}S^{\nu\mu 2})+g^{\nu\mu}(\gamma_{(12)}S_{\alpha}^{\alpha 1}+\gamma_{(22)}S_{\alpha}^{\alpha 2}))+$$

$$+V_{\nu}^{+}(\gamma_{(33)}S^{\nu\mu -}+g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_{\alpha}^{\alpha -})+V_{\nu}^{-}(\gamma_{(33)}S^{\nu\mu +}+g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_{\alpha}^{\alpha +})\} \quad (49)$$

For the field V_{μ}^{+}:

$$J_{V^{+}}^{\mu}=\partial^{\mu}(S_{\alpha}^{\alpha +}+\overset{+}{z}{}_{\alpha}^{\alpha}-m_{-}^{2}V^{\mu -})-j_{V^{+}}^{\mu}$$

where

$$S_{\alpha}^{\alpha +}=c_{5}S_{\alpha}^{\alpha 1}+c_{6}S_{\alpha}^{\alpha 2}+c_{7}S_{\alpha}^{\alpha +}+(c_{3}+c_{8})S_{\alpha}^{\alpha -},$$

$$\overset{+}{z}{}_{\alpha}^{\alpha}=(c_{9}+\frac{c_{4}}{2})(\overset{(-1)}{z}{}_{\alpha}^{\alpha}+\overset{(-2)}{z}{}_{\alpha}^{\alpha})+c_{10}(\overset{(-1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(-2)}{\omega}{}_{\alpha}^{\alpha}),$$

$$j_{V^{+}}^{\mu}=\frac{\partial L}{\partial V_{\mu}^{+}}-\frac{c_{4}}{4}[A_{\nu}(\gamma_{(13)}-i\gamma_{(14)})(2S^{\nu\mu -}+g^{\nu\mu}S_{\alpha}^{\alpha -})+U_{\nu}(\gamma_{(23)}-i\gamma_{(24)})(2S^{\nu\mu -}+g^{\nu\mu}S_{\alpha}^{\alpha -})+$$

$$+V_{\nu}^{-}((\gamma_{(13)}-i\gamma_{(14)})(2S^{\nu\mu 1}+g^{\nu\mu}S_{\alpha}^{\alpha 1})+(\gamma_{(23)}-i\gamma_{(24)})(2S^{\nu\mu 2}+g^{\nu\mu}S_{\alpha}^{\alpha 2}))] \quad (50)$$

For the field V_{μ}^{-}:

$$J_{V^{-}}^{\mu}=\partial^{\mu}(S_{\alpha}^{\alpha -}+\overset{-}{z}{}_{\alpha}^{\alpha}-m_{-}^{2}V^{\mu +})-j_{V^{-}}^{\mu}$$

where

$$S_{\alpha}^{\alpha -}=d_{5}S_{\alpha}^{\alpha 1}+d_{6}S_{\alpha}^{\alpha 2}+d_{8}S_{\alpha}^{\alpha +}+(d_{3}+d_{7})S_{\alpha}^{\alpha -},$$

$$\overset{-}{z}{}_{\alpha}^{\alpha}=(d_{9}+\frac{d_{4}}{2})(\overset{(+1)}{z}{}_{\alpha}^{\alpha}+\overset{(+2)}{z}{}_{\alpha}^{\alpha})+d_{10}(\overset{(+1)}{\omega}{}_{\alpha}^{\alpha}+\overset{(+2)}{\omega}{}_{\alpha}^{\alpha})$$

$$j_{V^{-}}^{\mu}=\frac{\partial L}{\partial V_{\mu}^{-}}-\frac{d_{4}}{4}[A_{\nu}(\gamma_{(13)}+i\gamma_{(14)})(2S^{\nu\mu +}+g^{\nu\mu}S_{\alpha}^{\alpha +})+$$

$$+U_{\nu}(\gamma_{(23)}+i\gamma_{(24)})(2S^{\nu\mu +}+g^{\nu\mu}S_{\alpha}^{\alpha +})+$$

$$+V_{\nu}^{+}((\gamma_{(13)}+i\gamma_{(14)})(2S^{\nu\mu 1}+g^{\nu\mu}S_{\alpha}^{\alpha 1})+(\gamma_{(23)}+i\gamma_{(24)})(2S^{\nu\mu 2}+g^{\nu\mu}S_{\alpha}^{\alpha 2}))]$$

Eq(5.2) also yields the systemic charge conservation:

$$J_{sys}^{\mu}=\partial_{\nu}(S_{sys\alpha}^{\alpha}+z_{sys\alpha}^{\alpha})-J_{S}^{\mu}+J_{N}^{\mu}$$

where

$$S_{sys\alpha}^{\alpha}=a_{4}S_{\alpha}^{\alpha 1}+a_{5}S_{\alpha}^{\alpha 2}+a_{6}S_{\alpha}^{\alpha +}+a_{7}S_{\alpha}^{\alpha +}$$

$$z_{sys\alpha}^{\alpha}=a_{8}(\overset{(11)}{z}{}_{\alpha}^{\alpha}+\overset{(22)}{z}{}_{\alpha}^{\alpha}+2\overset{(12)}{z}{}_{\alpha}^{\alpha}+2\overset{(+-3)}{z}{}_{\alpha}^{\alpha})+$$

$$+a_{9}(\overset{(11)}{\omega}{}_{\alpha}^{\alpha}+\overset{(22)}{\omega}{}_{\alpha}^{\alpha}+2\overset{(12)}{\omega}{}_{\alpha}^{\alpha}+2\overset{(+-3)}{\omega}{}_{\alpha}^{\alpha})$$

$$J_{S}^{\mu}=-\frac{\partial L}{\partial A_{\mu}}-(k_{2}+\frac{\xi a_{5}}{b_{1}+b_{5}})\frac{\partial L}{\partial U_{\mu}}-k_{+}\frac{\partial L}{\partial V_{\mu}^{+}}-k_{-}\frac{\partial L}{\partial V_{\mu}^{-}}+$$

$$-A_\nu[\gamma_{11}\Delta S^{\nu\mu 1}+\gamma_{12}\Delta S^{\nu\mu 2}+g^{\nu\mu}(\frac{\gamma_{11}}{2}\Delta S_\alpha^{\alpha 1}+\frac{\gamma_{12}}{2}\Delta S_\alpha^{\alpha 2})]$$

$$-U_\nu[\gamma_{12}\Delta S^{\nu\mu 1}+\gamma_{22}\Delta S^{\nu\mu 2}]+g^{\nu\mu}(\frac{\gamma_{12}}{2}\Delta S_\alpha^{\alpha 1}+\frac{\gamma_{22}}{2}\Delta S_\alpha^{\alpha 2})]$$

$$-V_\nu^+[\frac{\gamma_{33}}{2}\Delta S^{\nu\mu -}+g^{\nu\mu}\frac{\gamma_{33}}{4}\Delta S_\alpha^{\alpha -}]-V_\nu^-[\frac{\gamma_{33}}{2}\Delta S^{\nu\mu +}+g^{\nu\mu}\frac{\gamma_{33}}{4}\Delta S_\alpha^{\alpha +}]$$

Eq.(6.9) shows that the systemic charge is just a linear combination of all the four charges. We can imagine the systemic charge as a river receiving its volume from the charge of every field charge.

Concluding, one notices that this four bosons electromagnetism produces seven conserved charges. The first one corresponds to the electric charge it means that one associated to the parameter q, as eq.(1.1) gauge transformation is showing and eq.(4.4) conservation law expliciting. The others six are neutral charges. Eqs.(6.3-6.8) are showing conserved current depending on free coefficients as parameters, although they are involving charged fields.

7 Chance

Eq.(1.1) develops the meaning of symmetry management by associating different fields under a common gauge parameter. As symmetry management one understands to take opportunities without violating symmetry. It manipulates with the free coefficients of theory and the gauge fixing terms. There are two ways of doing that. The first one means interfering on equations but respecting the original Lagrangian at eq.(2); the second to cut off some terms at original Lagrangian but preserving the gauge invariance.

The fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$ develops a determinism under directive and circumstance. By directive is understood those relationships derived directly from light invariance and gauge symmetry and, as circumstance that ones derived indirectly. Both features are fundamental for understanding the model. Thus under this whole physics light should be responsible for the directive on symmetry management. It yields from eq.(1.1) a massless photon and from eqs.(5.1-2) a directive photon equation. So, it is still missing an interpretation on the others fields participation. From [2] one gets that a linear whole gauge transformation develops a volume of circumstances equal to $1+(N-1)[N^3+2N-4]$, where N is the number of involved potential fields. For our case $N=4$, one gets a space for circumstances equal to 231, which means the number of free parameters that the model provides for opportunities to be taken. They are not occasional chances, but are being determined from fundamental laws.

The new feature is that Lagrangian, equations of motion, currents and charges can be shortened according to the chances that the model develops. Physically, it means that inside of the fundamental equations there are opportunities to be taken. Mathematically, expressions can be simplified. Consequently, there are more than one physical solution inside of the whole gauge model. A propitious variety for complexity be understood [12].

Let us take some symmetry management examples. The cases involving equations of motion and charges conservations. Given the generic equation of motion

$$\partial_\nu(T^{\mu\nu}+g^{\mu\nu}T_\alpha^\alpha)=g_I X_\nu^I(X^{[\mu\nu]}+X^{(\mu\nu)}+g^{\nu\mu}X_\alpha^\alpha) \tag{51}$$

where $X_\mu^I \equiv \{A_\mu, U_\mu, V_\mu^\pm\}$, we get as circumstance the minimal photon self interactive equation

$$\partial_\nu F^{\mu\nu}=gA_\mu F^{\mu\nu} \tag{52}$$

by playing with coefficients at Appendix C. Eq.(7.2) shows the meaning of chance inserted on theory. Based on light invariance and charge conservation it derives a pure photonic charge. It enlarges the possibility of the photon to interact with EM-fields with a coupling constant different from electric charge.

Similarly, the model offers opportunities to four conserved charges

$$\partial_\nu j_I^\mu = 0 \tag{53}$$

be obtained. From the variational principle, one gets the following photonic charge:

$$\partial_\mu j_A^\mu = 0$$

under the chance

$$a_7 = -a_4, a_8 = -a_5, a_{11} = -\frac{a_6}{2}, a_9 = a_{10} = a_{12} = 0 \quad (54)$$

where

$$j_A^\mu = \frac{\partial L}{\partial A_\mu} - \frac{a_6}{2}\{A_\nu(2(\gamma_{(11)}S^{\nu\mu 1} + \gamma_{(12)}S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(11)}S_\alpha^{\alpha 1} + \gamma_{(12)}S_\alpha^{\alpha 2})) +$$

$$+U_\nu(2(\gamma_{(12)}S^{\nu\mu 1} + \gamma_{(22)}S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(12)}S_\alpha^{\alpha 1} + \gamma_{(22)}S_\alpha^{\alpha 2})) +$$

$$+V_\nu^+(\gamma_{(33)}S^{\nu\mu -} + g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_\alpha^{\alpha -}) + V_\nu^-(\gamma_{(33)}S^{\nu\mu +} + g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_\alpha^{\alpha +})\} \quad (55)$$

Similary for the photon massive field U_μ:

$$\partial_\mu j_U^\mu = 0$$

under the chance

$$b_7 = -b_4, b_8 = -b_5, b_{11} = -\frac{b_6}{2}, b_9 = b_{10} = b_{12} = 0 \quad (56)$$

where

$$j_U^\mu = \frac{\partial L}{\partial U_\mu} - \frac{b_6}{2}\{A_\nu(2(\gamma_{(11)}S^{\nu\mu 1} + \gamma_{(12)}S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(11)}S_\alpha^{\alpha 1} + \gamma_{(12)}S_\alpha^{\alpha 2})) +$$

$$+U_\nu(2(\gamma_{(12)}S^{\nu\mu 1} + \gamma_{(22)}S^{\nu\mu 2}) + g^{\nu\mu}(\gamma_{(12)}S_\alpha^{\alpha 1} + \gamma_{(22)}S_\alpha^{\alpha 2})) +$$

$$+V_\nu^+(\gamma_{(33)}S^{\nu\mu -} + g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_\alpha^{\alpha -}) + V_\nu^-(\gamma_{(33)}S^{\nu\mu +} + g^{\nu\mu}\frac{\gamma_{(33)}}{2}S_\alpha^{\alpha +})\} \quad (57)$$

For the field V_μ^+:

$$\partial_\mu j_{V^+}^\mu = 0$$

with the chance

$$c_3 = -c_8, c_9 = -\frac{c_4}{2}, c_5 = c_6 = c_7 = c_{10} = 0 \quad (58)$$

where

$$j_{V^+}^\mu = \frac{\partial L}{\partial V_\mu^+} - \frac{c_4}{4}[A_\nu(\gamma_{(13)} - i\gamma_{(14)})(2S^{\nu\mu -} + g^{\nu\mu}S_\alpha^{\alpha -}) +$$

$$+U_\nu(\gamma_{(23)} - i\gamma_{(24)})(2S^{\nu\mu -} + g^{\nu\mu}S_\alpha^{\alpha -}) +$$

$$+V_\nu^-((\gamma_{(13)} - i\gamma_{(14)})(2S^{\nu\mu 1} + g^{\nu\mu}S_\alpha^{\alpha 1}) +$$

$$+(\gamma_{(23)} - i\gamma_{(24)})(2S^{\nu\mu 2} + g^{\nu\mu}S_\alpha^{\alpha 2}))] \quad (59)$$

For the field V_μ^-:

$$\partial_\mu j_{V^-}^\mu = 0$$

under the chance

$$d_3 = -d_7, d_9 = -\frac{d_4}{2}, d_5 = d_6 = d_8 = d_{10} = 0 \quad (60)$$

where

$$j^{\mu}_{V^-} = \frac{\partial L}{\partial V^-_{\mu}} - \frac{d_4}{4}[A_{\nu}(\gamma_{(13)} + i\gamma_{(14)})(2S^{\nu\mu+} + g^{\nu\mu}S_{\alpha}^{\alpha+}) +$$
$$+U_{\nu}(\gamma_{(23)} + i\gamma_{(24)})(2S^{\nu\mu+} + g^{\nu\mu}S_{\alpha}^{\alpha+}) +$$
$$+V^+_{\nu}((\gamma_{(13)} + i\gamma_{(14)})(2S^{\nu\mu 1} + g^{\nu\mu}S_{\alpha}^{\alpha 1}) +$$
$$+(\gamma_{(23)} + i\gamma_{(24)})(2S^{\nu\mu 2} + g^{\nu\mu}S_{\alpha}^{\alpha 2}))] \quad (61)$$

Symmetry management and chance are a new physical possibility being introduced by this whole gauge physics. It opens to the fact of nature be moved by chance.

8 Conclusion

Eq.(1) is based on the meaning of grouping. It gets together four electromagnetic fields, $\{A_{\mu}, U_{\mu}, V^{\pm}_{\mu}\}$ based on light invariance and electric charge conservation. It yields a systemic electromagnetism which classical aspect was partially studied in this work. It processes an electromagnetism transfering $\Delta Q = 0$ and $|\Delta Q| = 1$.

Eq.(2) shows a new possibility for understanding the electromagnetic phenomena. Originally electrodynamics started with charged spin-$\frac{1}{2}$ particles and later on it was extended for scalar particles and vector bosons. Based on four bosons electromagnetism this work enlarges this limit by grouping spin-1 and spin-0 fields as Lorentz group irreductible representation $(\frac{1}{2}, \frac{1}{2})$. New electromagnetic fields, charges and interactions are derived. Classical equations were studied and an electromagnetic flux with seven types of conserved charges is obtained.

Eq.(5.1) develops an electromagnetic flux. Six features appear. They are granular and collective dynamics, fields interdependence, symmetry management, chance, neutral electromagnetism, photonics. The first two ones are explicitly shown in the equations of motion where fields strengths $F_{\mu\nu}$, $U_{\mu\nu}$, $V^{\pm}_{\mu\nu}$ and $z_{\mu\nu}$ are moving on space-time through coupled equations of motion. The third one emerges from a set determinism based on directive and circumstance which interpretation challenge us. For this, being responsible for light invariance, the photon should be taken as a singular particle, one interprets it as responsible for symmetry management. Consequently, its variational equation is incorporated into the Noether symmetry equation, resulting an effective photon equation, eq.(5.2). As fourth, one gets the meaning of chance derived from fundamental laws. Things can also be determined at random, by accident, by chance, but connected to basic principles. For instance, eqs.(7.4-11) relationships are defined based on free coefficients which can be take any value without breaking gauge symmetry. Fifth, a neutral electromagnetism beyond electric charge is performed, according to new charges derived at section 6. For this, notice that the gauge symmetry U$(1)\times$ SO(2) estabilishes not only the J^N_{μ} electric charge conservation law, as the neutral currents $\partial_{\mu} j^{\mu}_A = \partial_{\mu} j^{\mu}_S = \partial_{\mu} K^{\mu} = 0$ and four continuity equations at eqs.(6.5-6.8). At last, a photonics with self-interacting photons not depending on electric charge is realized. They can be found out by propagating on space-time or by acting as sources.

Concluding, originated from light invariance and conserved charge, one obtains an extended electromagnetic flux $\{A_{\mu}, U_{\mu}, V^{\pm}_{\mu}\}$ carrying whole physical laws. There is a more vast electromagnetic aspect to be investigated than Maxwell equations. Now, Maxwell becomes just a sector of the electromagnetic phenomena introducting electric charge conservation. Adding to it, there is an enlargement on the electromagnetic phenomena associated to more electromagnetic fields, coupling constants, self-interactions and a new determinism. A further contribution is to couple the whole symmetry current $(\partial_{\nu} K^{\nu\mu} + J^{\mu}_N)$ to the photon field. The challenge will be to associate this four bosons electromagnetism to the electroweak interaction.

9 Collective fields

A new physical entity derived from the whole gauge approach is the gauge invariant collective field

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$z_{\mu\nu} = \gamma_{[IJ]} A^I_\mu A^J_\nu$. However notice that individually each collective field in the above matrices are not gauge invariant. Their physical meaning restricts to the space-time dynamics as eqs.(11)-(13) are showing.

There are the following z -fields for four bosons electromagnetism:

$$z_{[\nu\mu]} = \begin{pmatrix} 0 & z^{[1,2]} & z^{[1,+]} & z^{[1,-]} \\ z^{[2,1]} & 0 & z^{[2,+]} & z^{[2,-]} \\ z^{[+,1]} & z^{[+,2]} & 0 & z^{[+,-]} \\ z^{[-,1]} & z^{[-,2]} & z^{[-,+]} & 0 \end{pmatrix} z_{(\nu\mu)} = \begin{pmatrix} z^{(1,1)} & z^{(1,2)} & z^{(1,+)} & z^{(1,-)} \\ z^{(2,1)} & z^{(2,2)} & z^{(2,+)} & z^{(2,-)} \\ z^{(+,1)} & z^{(+,2)} & z^{(+,+)} & z^{(+,-)} \\ z^{(-,1)} & z^{(-,2)} & z^{(-,+)} & z^{(-,-)} \end{pmatrix}$$

where

$$\overset{(11)\mu\nu}{z} \equiv \gamma_{(11)} A^\mu A^\nu, \quad \overset{(22)\mu\nu}{z} \equiv \gamma_{(22)} U^\mu U^\nu, \quad \overset{(12)\mu\nu}{z} \equiv \gamma_{(12)} A^\mu U^\nu,$$

$$\overset{(21)\mu\nu}{z} \equiv \gamma_{(21)} U^\mu A^\nu \quad \overset{(11)}{z}{}^\mu_\mu = \gamma_{(11)} A_\mu A^\mu, \quad \overset{(22)}{z}{}^\mu_\mu = \gamma_{(22)} U_\mu U^\mu,$$

$$\overset{(12)}{z}{}^\mu_\mu = \gamma_{(12)} A_\mu U^\mu, \quad \overset{[12]\mu\nu}{z} \equiv \gamma_{[12]} A^\mu U^\nu, \quad \overset{[21]\mu\nu}{z} \equiv \gamma_{[21]} U^\mu A^\nu,$$

$$\overset{(13+)\mu\nu}{z} \equiv \gamma_{(13)} A^\mu V^{\nu+}, \quad \overset{(13-)\mu\nu}{z} \equiv \{\overset{(13+)\mu\nu}{z}\}^* = \gamma_{(13)} A^\mu V^{\nu-},$$

$$\overset{(14+)\mu\nu}{z} \equiv \gamma_{(14)} A^\mu V^{\nu+}, \quad \overset{(14-)\mu\nu}{z} \equiv \{\overset{(14+)\mu\nu}{z}\}^* = \gamma_{(14)} A^\mu V^{\nu-},$$

$$\overset{[13+]\mu\nu}{z} \equiv \gamma_{[13]} A^\mu V^{\nu+}, \quad \overset{[13-]\mu\nu}{z} \equiv \{\overset{[13+]\mu\nu}{z}\}^* = \gamma_{[13]} A^\mu V^{\nu-},$$

$$\overset{[14+]\mu\nu}{z} \equiv \gamma_{[14]} A^\mu V^{\nu+}, \quad \overset{[14-]\mu\nu}{z} \equiv \{\overset{[14+]\mu\nu}{z}\}^* = \gamma_{[14]} A^\mu V^{\nu-},$$

$$\overset{(23+)\mu\nu}{z} \equiv \gamma_{(23)} U^\mu V^{\nu+}, \quad \overset{(23-)\mu\nu}{z} \equiv \{\overset{(23+)\mu\nu}{z}\}^* = \gamma_{(23)} U^\mu V^{\nu-},$$

$$\overset{(24+)\mu\nu}{z} \equiv \gamma_{(24)} U^\mu V^{\nu+}, \quad \overset{(24-)\mu\nu}{z} \equiv \{\overset{(24+)\mu\nu}{z}\}^* = \gamma_{(24)} U^\mu V^{\nu-},$$

$$\overset{[23+]\mu\nu}{z} \equiv \gamma_{[23]} U^\mu V^{\nu+}, \quad \overset{[23-]\mu\nu}{z} \equiv \{\overset{[23+]\mu\nu}{z}\}^* = \gamma_{[23]} U^\mu V^{\nu-},$$

$$\overset{[24+]\mu\nu}{z} \equiv \gamma_{[24]} U^\mu V^{\nu+}, \quad \overset{[24-]\mu\nu}{z} \equiv \{\overset{[24+]\mu\nu}{z}\}^* = \gamma_{[24]} U^\mu V^{\nu-},$$

$$\overset{+-3\mu\nu}{z} \equiv \gamma_{(33)} V^{\mu+} V^{\nu-}, \quad \overset{-+3\mu\nu}{z} \equiv \{\overset{+-3\mu\nu}{z}\}^* = \gamma_{(33)} V^{\mu-} V^{\nu+},$$

$$\overset{+-4\mu\nu}{z} \equiv \gamma_{(44)} V^{\mu+} V^{\nu-}, \quad \overset{-+4\mu\nu}{z} \equiv \{\overset{+-4\mu\nu}{z}\}^* = \gamma_{(44)} V^{\mu-} V^{\nu+},$$

$$\overset{(+-)\mu\nu}{z} \equiv -i\gamma_{(34)} V^{\mu+} V^{\nu-}, \quad \overset{(-+)\mu\nu}{z} \equiv \{\overset{(+-)\mu\nu}{z}\}^* = i\gamma_{(34)} V^{\mu-} V^{\nu+},$$

$$\overset{[+-]\mu\nu}{z} \equiv -i\gamma_{[34]} V^{\mu+} V^{\nu-}, \quad \overset{[-+]\mu\nu}{z} \equiv \{\overset{[+-]\mu\nu}{z}\}^* = i\gamma_{[34]} V^{\mu-} V^{\nu+},$$

$$\overset{(+1)\mu\nu}{z} \equiv (\gamma_{(13)} + i\gamma_{(14)}) A^\mu V^{\nu+}, \quad \overset{(-1)\mu\nu}{z} \equiv \{\overset{(+1)\mu\nu}{z}\}^*,$$

$$\overset{(+2)\mu\nu}{z} \equiv (\gamma_{(23)} + i\gamma_{(24)})U^{\mu}V^{\nu+}, \quad \overset{(-2)\mu\nu}{z} \equiv \{\overset{(+2)\mu\nu}{z}\}^{*},$$

$$\overset{[+1]\mu\nu}{z} \equiv (\gamma_{[13]} + i\gamma_{[14]})A^{\mu}V^{\nu+}, \quad \overset{[-1]\mu\nu}{z} \equiv \{\overset{[+1]\mu\nu}{z}\}^{*},$$

$$\overset{[+2]\mu\nu}{z} \equiv (\gamma_{[23]} + i\gamma_{[24]})U^{\mu}V^{\nu+}, \quad \overset{[-2]\mu\nu}{z} \equiv \{\overset{[+2]\mu\nu}{z}\}^{*}.$$

Similarly, one defines ω-fields by replacing, in the above definitions, z by ω and γ by τ. For example:

$$\overset{(11)\mu\nu}{\omega} \equiv \tau_{(11)}A^{\mu}A^{\nu}, \quad \overset{(22)\mu\nu}{\omega} \equiv \tau_{(22)}U^{\mu}U^{\nu}, \quad \overset{(12)\mu\nu}{\omega} \equiv \tau_{(12)}A^{\mu}U^{\nu},$$

$$\overset{+-3\mu\nu}{\omega} \equiv \tau_{(33)}V^{\mu+}V^{\nu-}, \quad \overset{(+1)\mu\nu}{\omega} \equiv (\tau_{(13)} + i\tau_{(14)})A^{\mu}V^{\nu+},$$

and so on.

10 Equations of motion free coefficients

The model develops the following parameters written in terms of the original free Lagrangian coefficients, eq(2):

10.1 Field A_{μ}

Defining:

$$\Delta = \frac{b_{(12)}}{a_2 + b_{22}} \tag{62}$$

10.1.1 Left Side

$$a_1 = a_1 + b_{(11)} - b_{(12)}\Delta, \quad a_2 = 8(b_1 - \Delta b_2), \quad a_3 = 8(\beta_1 - \Delta\beta_2), \quad \mu_1 = -2\Delta m_2^2$$

$$a_4 = 4(b_{(11)} + c_{(11)}) + \xi_{(11)} - \Delta(4b_{(12)} + 4c_{(12)} + \xi_{(12)})$$

$$a_5 = 4(b_{(12)} + c_{(12)}) + \xi_{(22)} - \Delta(4b_{(12)} + 4c_{(12)} + \xi_{(22)})$$

$$a_6 = \mathrm{Re}[\sqrt{2}((\xi_{(13)} + i\xi_{(14)}) - \Delta(\xi_{(23)} + i\xi_{(24)}))]$$

$$a_7 = 2(2\rho_1 - \beta_1 - \Delta(2\rho_2 - \beta_2)), \quad a_8 = 2(\beta_1 + 4\rho_1 - \Delta(\beta_2 + 4\rho_2)) \tag{63}$$

10.1.2 Field A_{μ} Right Side : Field A_{μ}

$$f_1^A = 4\Delta\gamma_{[12]}b_1, \quad f_2^A = 4\Delta\gamma_{[12]}b_2, \quad f_3^A = -8\Delta\gamma_{[12]}, \quad f_4^A = 16\Delta(\gamma_{(12)} + \tau_{(12)})$$

$$f_5^A = 4\Delta(\beta_2\gamma_{(11)} - \beta_1\gamma_{(12)}), \quad f_6^A = 4\Delta(\beta_2\gamma_{(11)} - \beta_1\gamma_{(12)}), \quad f_7^A = 4(\gamma_{(11)} - \Delta\gamma_{(12)})$$

$$f_8^A = 4(\gamma_{(11)} - \Delta(\gamma_{(12)} + 2\tau_{(12)})), \quad f_9^A = 8(\gamma_{(11)} + 2\tau_{(11)} - \Delta(\gamma_{(12)} + 2\tau_{(12)})), \quad f_{10}^A = -16\Delta\tau_{(12)}$$

$$f_{11}^A = -2(\beta_1 - \Delta\beta_2)\gamma_{(11)} + f_7^A\rho_1 + 8(\tau_{(11)} - \Delta\tau_{(12)})(\beta_1 + 4\rho_1)$$

$$f_{12}^A = -2(\beta_1 - \Delta\beta_2)\gamma_{(12)} + f_7^A\rho_2 + 8(\tau_{(11)} - \Delta\tau_{(12)})(\beta_2 + 4\rho_2), \quad f_{13}^A = 8(\gamma_{(11)} + 2\tau_{(11)})$$

10.1.3 Field A_{μ} Right Side : Field U_{μ}

$$f_1^U = 4\Delta\gamma_{[12]}b_1, \quad f_2^U = 4\Delta\gamma_{[12]}b_2, \quad f_3^U = 8\Delta\gamma_{[12]}, \quad f_4^U = 16\Delta(\gamma_{(12)} + 2\tau_{(12)})$$

$$f_5^U = 4\Delta(\beta_2\gamma_{(12)} - \beta_1\gamma_{(22)}), \quad f_6^U = 4\Delta(\beta_2\gamma_{(12)} - \beta_1\gamma_{(22)}) \quad , f_7^U = 4(\gamma_{(12)} + 2\gamma_{(12)} - \Delta\gamma_{(12)})$$

$$f_8^U = 4(\gamma_{(12)} - \Delta\gamma_{(22)}), \quad f_9^U = 16\tau_{(12)}, \quad f_{10}^U = 8(\gamma_{(12)} + 2\tau_{(12)} - \Delta(\gamma_{(22)} + 2\tau_{(22)}))$$

$$f_{11}^U = -2(\beta_1 - \Delta\beta_2)\gamma_{(12)} + f_6^U\rho_1 + 8(\tau_{(12)} - \Delta\tau_{(22)})(\beta_1 + 4\rho_1)$$

$$f_{12}^U = -2(\beta_1 - \Delta\beta_2)\gamma_{(22)} + f_6^U\rho_1 + 8(\tau_{(12)} - \Delta\tau_{(22)})(\beta_2 + 4\rho_2), \quad f_{13}^U = -8(\gamma_{(22)} + 2\tau_{(22)})$$

10.1.4 Field A_μ Right Side : Fields V_μ^+ and V_μ^-

$$f_1^+ = 4b_3[(\gamma_{[13]} + i\gamma_{[14]}) - \Delta(\gamma_{[23]} + i\gamma_{[24]})], \quad f_2^+ = -4\gamma_{[13]} + 2i\gamma_{[24]}\Delta, \quad f_3^+ = -2i\Delta\gamma_{[23]}$$

$$f_4^+ = -8\gamma_{[23]} + 4\Delta\gamma_{[23]} - 2i\gamma_{[14]}, \quad f_5^+ = 2i\gamma_{[13]}, \quad f_6^+ = -4\gamma_{(13)} + 2i\Delta(2\tau_{(24)} - \gamma_{(24)})$$

$$f_7^+ = 2i\Delta(\gamma_{(13)} - 2\tau_{(13)}), \quad f_8^+ = 8\gamma_{(13)} - 2i(2\tau_{(14)} - \gamma_{(14)}), \quad f_9^+ = -2i(\gamma_{(13)} - 2\tau_{(13)})$$

$$f_{10}^+ = 4i\Delta(4\tau_{(24)} + \gamma_{(24)}), \quad f_{11}^+ = -4i\Delta(4\tau_{(23)} + \gamma_{(23)}), \quad f_{12}^+ = -4i(4\tau_{(14)} + \gamma_{(14)}),$$

$$f_{13}^+ = 4i(4\tau_{(13)} + \gamma_{(13)}), \quad f_{14}^+ = (-4(\beta_1 - \Delta\beta_2)\gamma_{(33)} + 4(\gamma_{(13)} + i\gamma_{(14)}) - 4(\gamma_{(23)} + i\gamma_{(24)}))\beta_3$$

$$f_{15}^+ = 8\gamma_{[13]} + 4\Delta\gamma_{[23]} + 2i\gamma_{[14]}, f_{16}^+ = -4\gamma_{(13)} + 2i\Delta(2\tau_{(24)} - \gamma_{(24)}),$$

$$f_{17}^+ = 8\gamma_{(13)} + 2i\Delta(2\tau_{(14)} - \gamma_{(14)}), \quad f_{18}^+ = -2(\beta_1 - \Delta\beta_2)\gamma_{(33)} +$$

$$+4(\gamma_{(13)} + i\gamma_{(14)} - \Delta(\gamma_{(23)} + i\gamma_{(24)}))\rho_3 + 4(\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)} - \Delta(\tau_{(23)} + i\tau_{(24)}))$$

$$f_{19}^+ = -4\gamma_{[13]} + 4\Delta\gamma_{[23]}, \quad f_{20}^+ = 4(\gamma_{(13)} - \Delta\gamma_{(23)}), \quad f_{21}^+ = 16(\gamma_{(13)} + 2\tau_{(13)} - \Delta(\gamma_{(23)} + 2\tau_{(23)})$$

$$f_2^- = -4\gamma_{[13]} + 2i\Delta\gamma_{[24]} + 8\Delta\gamma_{[23]}, \quad f_4^- = -4\Delta\gamma_{[23]} + 2i\gamma_{[14]},$$

$$f_6^- = -4\gamma_{(13)} + (8\gamma_{(23)} - 2i(4\tau_{(23)} + \gamma_{(23)}))\Delta, \quad f_8^- = 2i(2\tau_{(14)} - \gamma_{(14)})$$

10.2 Field U_μ

Defining $\Delta_2 = \dfrac{b_{(12)}}{a_1 + b_{11}}$

10.2.1 Left Side

$$b_1 = a_2 + b_{(22)} - b_{(12)}\Delta_2, \quad b_2 = 8(b_2 - \Delta_2 b_1), \quad b_3 = 8(\beta_2 - \Delta_2\beta_2), \quad \mu_2 = 2m_2^2$$

$$b_4 = 4(b_{(12)} + c_{(12)}) + \xi_{(12)} - \Delta_2(4b_{(11)} + 4c_{(11)} + \xi_{(11)})$$

$$b_5 = 4(b_{(22)} + c_{(22)}) + \xi_{(22)} - \Delta_2(4b_{(12)} + 4c_{(12)} + \xi_{(12)})$$

$$b_6 = \mathrm{Re}[\sqrt{2}((\xi_{(23)} + i\xi_{(24)}) - \Delta_2(\xi_{(13)} + i\xi_{(14)}))]$$

$$b_7 = 2(2\rho_2 - \beta_2 - \Delta_2(2\rho_1 - \beta_1)), \quad b_8 = 2(\beta_2 + 4\rho_2 - \Delta_2(\beta_1 + 4\rho_1))$$

10.2.2 Field U_μ Right Side : Field A_μ

$$g_1^A = 4\gamma_{[12]}b_1, \quad g_2^A = 4\gamma_{[12]}b_2, \quad g_3^A = -8\gamma_{[12]}, \quad g_4^A = -16(\gamma_{(12)} + \tau_{(12)})$$

$$g_5^A = 4(\beta_1\gamma_{(12)} - \beta_2\gamma_{(11)}), \quad g_6^A = 4\Delta_2 g_5^A, \quad g_7^A = 4(\gamma_{(12)} - \Delta_2\gamma_{(11)})$$

$$g_8^A = 4(\gamma_{(12)} + 2\tau_{(12)} - \Delta_2\gamma_{(11)}), \quad g_9^A = 8(\gamma_{(12)} + 2\tau_{(12)} - \Delta_2(\gamma_{(11)} + 2\tau_{(11)}))$$

$$g_{10}^A = 16\tau_{(12)}, \quad g_{13}^A = -8\Delta_2(\gamma_{(11)} + 2\tau_{(11)})$$

$$g_{11}^A = -2(\beta_2 - \Delta_2\beta_1)\gamma_{(11)} + 4(\gamma_{(12)} - \Delta_2\gamma_{(11)})\rho_1 + 8(\tau_{(12)} - \Delta_2\tau_{(11)})(\beta_1 + 4\rho_1)$$

$$g_{12}^A = -2(\beta_2 - \Delta_2\beta_1)\gamma_{(12)} + 4(\gamma_{(12)} - \Delta_2\gamma_{(11)})\rho_2 + 8(\tau_{(12)} - \Delta_2\tau_{(11)})(\beta_2 + 4\rho_2)$$

10.2.3 Field U_μ Right Side : Field U_μ

$$g_1^U = 4\Delta_2\gamma_{[12]}b_1, \quad g_2^U = 4\Delta_2\gamma_{[12]}b_2, \quad g_3^U = -8\Delta_2\gamma_{[12]}, \quad g_4^U = -16\Delta_2(\gamma_{(12)} + 2\tau_{(12)})$$

$$g_5^U = 4\Delta_2(\beta_1\gamma_{(22)} - \beta_2\gamma_{(12)}), \quad g_6^U = 4\Delta_2 g_5^U, \quad g_7^U = 4(\gamma_{(22)} + \Delta_2(\gamma_{(12)} + 2\tau_{(12)}))$$

$$g_8^U = 4(\gamma_{(22)} - \Delta_2\gamma_{(12)}), \quad g_9^U = 16\Delta_2\tau_{(12)}, \quad g_{10}^U = 8(\gamma_{(22)} + 2\tau_{(22)} - \Delta_2(\gamma_{(12)} + 2\tau_{(12)}))$$

$$g_{11}^U = -2(\beta_2 - \Delta_2\beta_1)\gamma_{(12)} + 4(\gamma_{(22)} - \Delta_2\gamma_{(12)})\rho_1 + 8(\tau_{(22)} - \Delta_2\tau_{(12)})(\beta_1 + 4\rho_1)$$

$$g_{12}^U = -2(\beta_2 - \Delta_2\beta_1)\gamma_{(22)} + 4(\gamma_{(22)} - \Delta_2\gamma_{(12)})\rho_2 + 8(\tau_{(22)} - \Delta_2\tau_{(12)})(\beta_2 + 4\rho_2)$$

$$g_{13}^U = 8(\gamma_{(22)} + 2\tau_{(22)})$$

10.2.4 Field U_μ Right Side : Fields V_μ^+ and V_μ^-

$$g_1^+ = 4b_3[(\gamma_{[23]} + i\gamma_{[24]}) - \Delta_2(\gamma_{[13]} + i\gamma_{[14]})], \quad g_2^+ = -4\Delta_2\gamma_{[13]} + 2i\gamma_{[24]}\Delta_2, \quad g_3^+ = -2i\Delta_2\gamma_{[23]}$$

$$g_4^+ = -8\Delta_2\gamma_{[13]} - 4\gamma_{[23]} + 2i\Delta_2\gamma_{[14]}, \quad g_5^+ = 2i\Delta_2\gamma_{[13]}, \quad g_6^+ = -4\Delta_2\gamma_{(13)} + 2i(2\tau_{(24)} - \gamma_{(24)})$$

$$g_7^+ = -2i(\gamma_{(23)} - 2\tau_{(23)}), \quad g_8^+ = 8\Delta_2\gamma_{(13)} - 4\gamma_{(23)} - 2i(2\tau_{(14)} - \gamma_{(14)})$$

$$g_9^+ = -2i\Delta_2(\gamma_{(13)} - 2\tau_{(13)}), \quad g_{10}^+ = -4i\Delta_2(4\tau_{(24)} + \gamma_{(24)}), \quad g_{11}^+ = 4i\Delta_2(4\tau_{(23)} + \gamma_{(23)})$$

$$g_{12}^+ = 4i\Delta_2(4\tau_{(14)} + \gamma_{(14)}), \quad g_{13}^+ = -4i\Delta_2(4\tau_{(13)} + \gamma_{(13)})$$

$$g_{14}^+ = -4(\beta_2 - \Delta_2\beta_1)\gamma_{(33)} + 4(\gamma_{(23)} + i\gamma_{(24)})\beta_3 - 4(\gamma_{(23)} + i\gamma_{(24)})\beta_3$$

$$g_{15}^+ = -2(\beta_2 - \Delta_2\beta_1)\gamma_{(33)} + 4[(\gamma_{(23)} + i\gamma_{(24)}) - \Delta_2(\gamma_{(13)} + i\gamma_{(14)})]\rho_3 +$$

$$+ 4(\beta_3 + 4\rho_3)(\tau_{(23)} + i\,\tau_{(24)} - \Delta_2(\tau_{(13)} + i\,\tau_{(14)}))$$

$$g_{16}^+ = -4\gamma_{[23]} + 4\Delta_2(\gamma_{[13]}), \quad g_{17}^+ = 4(\gamma_{(23)} - \Delta_2\gamma_{(13)}), \quad g_{18}^+ = 16(\gamma_{(23)} + 2\tau_{(23)} + \Delta_2(\gamma_{(13)} + 2\tau_{(23)})$$

$$g_2^- = 4\Delta_2\gamma_{[13]} + 8\gamma_{[23]} + 2i\gamma_{[24]}, \quad g_4^- = -4\gamma_{[23]} + 2i\Delta_2\gamma_{[14]}$$

$$g_6^- = -4\Delta_2\gamma_{(13)} + 2i(2\tau_{(24)} - \gamma_{(24)}) - 8\gamma_{(23)}, \quad g_8^- = -4\gamma_{(23)} - 2i\Delta_2(2\tau_{(14)} - \gamma_{(14)})$$

10.3 Field V_μ^+

10.3.1 Left Side

$$c_1 = 4(a_3 + b_{(33)}), \quad c_2 = 8b_3, \quad c_3 = \frac{\sqrt{2}}{2}(\xi_{(13)} + i\xi_{(14)}), \quad c_4 = \frac{\sqrt{2}}{2}(\xi_{(23)} + i\xi_{(24)})$$

$$c_5 = \frac{1}{2}(\xi_{(33)} - \xi_{(44)}) + i\xi_{(34)}, \quad c_6 = 4(b_{(33)} + c_{(33)}) + \frac{1}{2}(\xi_{(33)} + \xi_{(44)})$$

$$c_7 = 4(2\rho_3 - \beta_3), \quad c_8 = 8(\beta_3 + 4\rho_3), \quad \mu_+ = 2m_3^2 \qquad (64)$$

10.3.2 Field V_μ^+ Right Side : Field A_μ

$$h_1^A = 4b_3(\gamma_{[13]} + i\gamma_{[14]}), \quad h_2^A = 4\gamma_{[13]}, \quad h_3^A = 8\gamma_{[13]} + 2i\gamma_{[14]}, \quad h_4^A = -2i\gamma_{[13]}$$

$$h_5^A = 4\gamma_{(13)}, \quad h_6^A = 8\gamma_{(13)} - 2i(\gamma_{(14)} - 2\tau_{(14)}), \quad h_7^A = 2i(\gamma_{(13)} - 2\tau_{(13)})$$

$$h_8^A = 4i(\gamma_{(14)} - 4\tau_{(14)}), \quad h_9^A = -4i(\gamma_{(13)} + 4\tau_{(13)}), \quad h_{10}^A = 8i\beta_3\gamma_{(14)}$$

$$h_{11}^A = 2((2\rho_3 - \beta_3)\gamma_{(13)} + i(2\rho_3 + \beta_3)\gamma_{(14)} + 2(\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)}))$$

$$h_{12}^A = 16(\gamma_{(13)} + 2\tau_{(13)})$$

10.3.3 Field V_μ^+ Right Side : Field U_μ

$$h_1^U = 4b_3(\gamma_{[23]} + i\gamma_{[24]}), \quad h_2^U = 2i\gamma_{[24]}, \quad h_3^U = -2i\gamma_{[23]}, \quad h_4^U = 4\gamma_{[23]}$$

$$h_5^U = -2i(\gamma_{(24)} - 2\tau_{(24)}), \quad h_6^U = 2i(\gamma_{(23)} - 2\tau_{(23)}), \quad h_7^U = 4\gamma_{(23)}, \quad h_8^U = 4i(\gamma_{(24)} + 4\tau_{(24)})$$

$$h_9^U = -4i(\gamma_{(23)} + 4\tau_{(23)}), \quad h_{10}^U = 8i\rho_3\gamma_{(24)}, \quad h_{12}^U = (\gamma_{(23)} + 4\tau_{(23)})$$

$$h_{11}^U = 2((2\rho_3 - \beta_3)\gamma_{(23)} + i(2\rho_3 + \beta_3)\gamma_{(24)} + 2(\beta_3 + 4\rho_3)(\tau_{(23)} + i\tau_{(24)}))$$

$$h_{13}^U = 32\tau_{(23)}, \quad h_{14}^U = 16(\gamma_{(23)} + 2\tau_{(23)}) \qquad (65)$$

10.3.4 Field V_μ^+ Right Side : Field V_μ^-

$$h_1^- = -4i(b_1\gamma_{[34]} + \beta_1\gamma_{(34)}), \quad h_2^- = -4i(b_2\gamma_{[34]} + \beta_2\gamma_{(34)}), \quad h_3^- = -2i\gamma_{[34]}$$

$$h_4^- = 4(\beta_1\gamma_{(33)} - \beta_3(\gamma_{(13)} - i\gamma_{(14)})), \quad h_5^- = 4(\beta_2\gamma_{(33)} - \beta_3(\gamma_{(23)} - i\gamma_{(24)})$$

$$h_6^- = 4\gamma_{(33)}, \quad h_7^- = 32i(\gamma_{(34)} + 2\tau - (34)), \quad h_8^- = 2\rho_1\gamma_{(33)} + 2(\beta_1 + 4\rho_1)\tau_{(33)} - \beta_3(\gamma_{(13)} - i\gamma_{(14)})$$

$$h_9^- = 2\rho_1\gamma_{(33)} + 2(\beta_2 + 4\rho_2)\tau_{(33)} - \beta_3(\gamma_{(23)} - i\gamma_{(24)}), \quad h_{10}^- = 8\tau_{(33)}, \quad h_{11}^- = 8i\gamma_{(34)}$$

$$h_{12}^- = 16(\gamma_{(33)} + 2\tau_{(33)})$$

10.4 Field V_μ^-

10.4.1 Left Side

$$d_1 = c_1, \quad d_2 = c_2, \quad d_3 = c_3^*, \quad d_4 = c_4^*, \quad d_5 = c_6^*, \quad d_6 = c_5^*$$

$$d_7 = c_7, \quad d_8 = c_8, \quad \mu_- = 2m_3^2 \qquad (66)$$

10.4.2 Field V_μ^- Right Side : Field A_μ

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$$i_1^A = h_1^{A*}, \quad i_2^A = h_2^A, \quad i_3^A = h_3^{A*}, \quad i_4^A = h_4^{A*}, \quad i_5^A = h_5^A, \quad i_6^A = h_6^{A*}$$

$$i_7^A = h_7^{A*}, \quad i_8^A = h_8^{A*}, \quad i_9^A = h_9^{A*}, \quad i_{10}^A = h_{10}^{A*}, \quad i_{11}^A = h_{11}^{A*}, \quad i_{12}^A = h_{12}^A$$

10.4.3 Field V_μ^- Right Side : Field U_μ

$$i_1^U = h_1^{U*}, \quad i_2^U = h_2^{U*}, \quad i_3^U = h_3^{U*}, \quad i_4^U = h_4^U, \quad i_5^U = h_5^{U*}, \quad i_6^U = h_6^{U*}$$

$$i_7^U = h_7^U, \quad i_8^U = h_8^{U*}, \quad i_9^U = h_9^{U*}, \quad i_{10}^U = h_{10}^{U*}, \quad i_{11}^U = h_{11}^{U*}$$

$$i_{12}^U = h_{12}^U, \quad i_{13}^U = h_{13}^U, \quad i_{14}^U = h_{14}^U$$

10.4.4 Field V_μ^- Right Side : Field V_μ^+

$$i_1^+ = h_1^-, \quad i_2^+ = h_2^-, \quad i_3^+ = h_3^-, \quad i_4 = h_4^{-*}, \quad i_5^+ = h_5^{-*}, \quad i_6^+ = h_6^{-*}$$

$$i_7^+ = h_7^-, \quad i_8^+ = h_8^{-*}, \quad i_9^+ = h_9^-, \quad i_{10}^+ = h_{10}^-, \quad i_{11}^+ = h_{11}^-, \quad i_{12}^+ = h_{12}^-$$

11 Directive Photon Equation Coefficients

11.1 Field A_μ

$$r_1^A = -4k\gamma_{[12]}b_1 r_2^A = -4k\gamma_{[12]}b_2$$

$$r_3^A = -4k\_b_3(\gamma_{[13]} - i\gamma_{[14]})r_4^A = -4k_+ b_3(\gamma_{[13]} + i\gamma_{[14]})$$

$$r_5^A = -4(\gamma_{(11)} - k\gamma_{(12)})\beta_1 - \gamma_{(11)}\Delta r_6^A = -4(\gamma_{(11)} - k\gamma_{(12)})\beta_2 - \gamma_{(12)}\Delta$$

$$r_7^A = -4k\_\beta_3(\gamma_{(13)} - i\gamma_{(14)})r_8^A = -4k_+\beta_3(\gamma_{(13)} + i\gamma_{(14)})$$

$$r_9^A = -8k\gamma_{(12)} r_{10}^A = -16k(\gamma_{(12)} + 2\tau_{(12)})$$

$$r_{11}^A = 4(\gamma_{(11)} - k\gamma_{(12)})r_{12}^A = 4(\gamma_{(11)} - 2k\gamma_{(12)})$$

$$r_{13}^A = 4(\gamma_{(11)} - k(\gamma_{(12)} + 2\tau_{(12)}))r_{14}^A = -8k\gamma_{[12]}$$

$$r_{15}^A = 8[(\gamma_{(11)} + 2\tau_{(11)}) - k(\gamma_{(12)} + 2\tau_{(12)})]r_{16}^A = 16[(\gamma_{(11)} + 2\tau_{(11)}) - k(\gamma_{(12)} + 2\tau_{(12)})]$$

$$r_{17}^A = -16k\tau_{(12)} r_{18}^A = -4k_-\gamma_{[13]}$$

$$r_{19}^A = -2k\_(4\gamma_{[13]} - i\gamma_{[14]})r_{20}^A = -2ik\_(\gamma_{(14)} + 2\tau_{(14)})$$

$$r_{21}^A = 2ik_-\gamma_{(13)} r_{22}^A = -4k_-\gamma_{(13)}$$

$$r_{23}^A = 2ik\_(\gamma_{(14)} - 2\tau_{(14)})r_{24}^A = -2k\_(4\gamma_{[13]} + i\gamma_{[14]})$$

$$r_{25}^A = -2ik\_(\gamma_{(13)} - 2\tau_{(13)})r_{26}^A = -4ik\_(\gamma_{(14)} + 4\tau_{(14)})$$

$$r_{27}^A = 4ik\_(\gamma_{(13)} + 4\tau_{(13)})r_{28}^A = -4k_+\gamma_{[13]}$$

$$r_{29}^A = -2k_+(\gamma_{[13]} + i\gamma_{[14]})r_{30}^A = -2k_+(4\gamma_{(13)} - i(\gamma_{(14)} - 2\tau_{(14)}))$$

$$r_{31}^{A} = -2ik_{+}\gamma_{[13]}$$

$$r_{32}^A = -2k_+(4\gamma_{[13]} - i\gamma_{[14]}) r_{33}^A = -4k_+\gamma_{(13)}$$

$$r_{34}^A = -2k_+(4\gamma_{(13)} + i(\gamma_{(14)} - 2\tau_{(14)})) r_{35}^A = -2ik_+(\gamma_{(13)} - 2\tau_{(13)})$$

$$r_{36}^A = 4ik_+(\gamma_{(14)} + 2\tau_{(14)}) r_{37}^A = -4ik_+(\gamma_{(13)} + 4\tau_{(13)})$$

$$r_{38}^A = 4[\gamma_{(11)}\rho_1 + 2\tau_{(11)}(\beta_1 + 4\rho_1) - k(\rho_1\gamma_{(12)} + 2\tau_{(12)}(\beta_1 + 4\rho_1))] + \frac{\Delta\gamma_{(11)}}{2}$$

$$r_{39}^A = 4[\gamma_{(11)}\rho_2 + 2\tau_{(11)}(\beta_2 + 4\rho_2) - k(\rho_2\gamma_{(12)} + 2\tau_{(12)}(\beta_2 + 4\rho_2))] + \frac{\Delta\gamma_{(12)}}{2}$$

$$r_{40}^A = -2k_-(4\gamma_{[13]} - i\gamma_{[14]}) r_{41}^A = -4k_+[\rho_3(\gamma_{(13)} + i\gamma_{(14)}) + (\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)})]$$

$$r_{42}^A = 8(\gamma_{(11)} + 2\tau_{(11)}) r_{43}^A = 16((\gamma_{(11)} + 2\tau_{(11)}) - k\tau_{(12)})$$

$$r_{44}^A = 4k_-\gamma_{[13]} r_{45}^A = 2ik_-\gamma_{[13]}$$

$$r_{46}^A = -4k_-\gamma_{(13)} r_{47}^A = -32k_-\tau_{(13)}$$

$$r_{48}^A = -16k_-(\gamma_{(13)} + 2\tau_{(13)}) r_{49}^A = 4k_+\gamma_{[13]}$$

$$r_{50}^A = -4k_+\gamma_{(13)} r_{51}^A = -16k_+(\gamma_{(13)} + 2\tau_{(13)}) equation 1$$

11.2 Field U_μ

$$r_1^U = -4b_1\gamma_{[12]}$$

$$r_2^U = -4b_2\gamma_{[12]} r_3^U = -4k_- b_3(\gamma_{[13]} - i\gamma_{[14]})$$

$$r_4^U = -4k_+ b_3(\gamma_{[13]} + i\gamma_{[14]} r_5^U = 4\beta_1(\gamma_{(12)} - k\gamma_{(22)}) - \Delta\gamma_{(12)}$$

$$r_6^U = 4\beta_2(\gamma_{(12)} - k\gamma_{(22)}) - \Delta\gamma_{(22)} r_7^U = -4\beta_3 k_-(\gamma_{(23)} - i\gamma_{(24)})$$

$$r_8^U = -4\beta_3 k_+(\gamma_{(23)} + i\gamma_{(24)}) r_9^U = -8\gamma_{[12]}$$

$$r_{10}^U = 16(\gamma_{(12)} + 2\tau_{(12)}) r_{11}^U = 4(\gamma_{(12)} + 2\tau_{(12)} - k\gamma_{(22)})$$

$$r_{12}^U = 4(\gamma_{(12)} - k\gamma_{(22)}) r_{13}^U = 8(\gamma_{(12)} - k\gamma_{(22)})$$

$$r_{14}^U = 8(2\tau_{(12)} - k\gamma_{(22)}) r_{15}^U = -8\gamma_{[12]}$$

$$r_{16}^U = 16\tau_{(12)} r_{17}^U = 16((\gamma_{(12)} + 2\tau_{(12)}) - k(\gamma_{(22)} + 2\tau_{(22)}))$$

$$r_{18}^U = -8k(\gamma_{(22)} + 2\tau_{(22)}) r_{19}^U = -2ik_-\gamma_{[24]}$$

$$r_{20}^U = 2ik_-\gamma_{[23]} r_{21}^U = -4k_-\gamma_{[23]}$$

$$r_{22}^U = -2k_-(4\gamma_{(23)} + i(\gamma_{(24)} - \tau_{(24)})) r_{23}^U = -2k_-(4\gamma_{(23)} - i(\gamma_{(24)} + \tau_{(24)}))$$

$$r_{24}^U = -4k_-\gamma_{(23)} r_{25}^U = -2ik_-(\gamma_{(23)} - 2\tau_{(23)})$$

$$r_{26}^U = -4ik_-(\gamma_{(24)} + 4\tau_{(24)})r_{27}^U = -4ik_-(\gamma_{(23)} + 4\tau_{(23)})$$

$$r_{28}^U = 2ik_+\gamma_{[24]}r_{29}^U = -2ik_+\gamma_{[23]}$$

$$r_{30}^U = -4k_+\gamma_{[23]}r_{31}^U = -2ik_+(\gamma_{(24)} - 2\tau_{(24)})$$

$$r_{32}^U = -2ik_+(\gamma_{(23)} + 2\tau_{(23)})r_{33}^U = -4k_+\gamma_{(23)}$$

$$r_{34}^U = 4ik_+(\gamma_{(24)} + 2\tau_{(24)})r_{35}^U = -4ik_+(\gamma_{(23)} + 4\tau_{(23)})$$

$$r_{36}^U = 8(\gamma_{(12)} + 2\tau_{(12)} - 2k(\gamma_{(22)} + 2\tau_{(22)})) - \Delta\frac{\gamma_{(12)}}{2}$$

$$r_{37}^U = 4(\rho_1\gamma_{(12)} + 2(\beta_1 + 4\rho_1)\tau_{(12)} - k(\rho_1\gamma_{(22)} + 2(\beta_1 + 4\rho_1)\tau_{(22)}) - \Delta\frac{\gamma_{(12)}}{2}$$

$$r_{38}^U = -4k_+[\rho_3(\gamma_{(13)} + i\gamma_{(14)}) + (\beta_3 + 4\rho_3)(\tau_{(13)} + i\tau_{(14)})]$$

$$r_{39}^U = -4k_-[\rho_3(\gamma_{(23)} - i\gamma_{(24)}) + (\beta_3 + 4\rho_3)(\tau_{(23)} - i\tau_{(24)})]$$

$$r_{40}^U = -8k(\gamma_{(22)} + 2\tau_{(22)})r_{41}^U = 4[\rho_2\gamma_{(12)} + 2\tau_{(12)}(\beta_2 + 4\rho_2) - 4k(\gamma_{(22)} + 2\tau_{(22)})]$$

$$r_{42}^U = 4k_-\gamma_{[23]}r_{43}^U = -4k_-(\gamma_{(23)} + 4\tau_{(23)})$$

$$r_{44}^U = -32k_-\tau_{(23)}r_{45}^U = 4k_+\gamma_{[23]}$$

$$r_{46}^U = -4k_+\gamma_{(23)}r_{47}^U = -16k_-(\gamma_{(23)} + 2\tau_{(23)})$$

$$equation1$$

11.3 Field V_μ^+

$$r_1^+ = 4ik_-(b_1\gamma_{[34]} + \beta_1\gamma_{(34)})r_2^+ = -4ik_-(b_2\gamma_{[34]} + \beta_2\gamma_{(34)})$$

$$r_3^+ = -8b_3((\gamma_{[13]} + i\gamma_{[14]}) + k(\gamma_{[23]} + i\gamma_{[24]}))r_4^+ = -4\beta_1\gamma_{(33)}k_-$$

$$r_5^+ = -4\beta_2\gamma_{(33)}k_-$$

$$r_6^+ = 8\beta_3[(\gamma_{(13)} + i\gamma_{(14)}) - k(\gamma_{(23)} + i\gamma_{(24)})] - \Delta\frac{\gamma_{(33)}}{2}$$

$$r_7^+ = -8i\gamma_{[34]}k_-r_8^+ = -8k_-\gamma_{(33)}$$

$$r_9^+ = -4k_-\gamma_{(33)}r_{10}^+ = 8k_-\gamma_{[34]}$$

$$r_{11}^+ = 4\gamma_{[14]} - 2ik\gamma_{[24]}r_{12}^+ = -2ik\gamma_{[23]}$$

$$r_{13}^+ = 2(-4\gamma_{[13]} + i\gamma_{[14]} - 2k\gamma_{[23]})r_{14}^+ = 2(4\gamma_{[13]} + i\gamma_{[14]} - 2k\gamma_{[23]})$$

$$r_{15}^+ = -2ik(\gamma_{(24)} + 2\tau_{(24)})r_{16}^+ = 4\gamma_{(13)}$$

$$r_{17}^+ = 4\gamma_{(13)} - 2ik(\gamma_{(23)} - 2\tau_{(23)})r_{18}^+ = 2ik(\gamma_{(23)} - 2\tau_{(23)})$$

$$r_{19}^{+}=2(-4\gamma_{(13)}+i(\gamma_{(14)}+2\tau_{(14)})-2k\gamma_{(23)})r_{20}^{+}=2(4\gamma_{(13)}+i(\gamma_{(14)}+2\tau_{(14)})-2k\gamma_{(23)})$$

$$r_{21}^{+}=2i(\gamma_{(13)}+2\tau_{(13)})r_{22}^{+}=4ik(\gamma_{(24)}+2\tau_{(24)})$$

$$r_{23}^{+}=-4ik(\gamma_{(23)}+4\tau_{(23)})r_{24}^{+}=4i(\gamma_{(14)}+4\tau_{(14)})$$

$$r_{25}^{+}=4i(\gamma_{(13)}+4\tau_{(13)})r_{26}^{+}=-16ik\_(\gamma_{(34)}+2\tau_{(34)})$$

$$r_{27}^{+}=-4k_{-}(\rho_1\gamma_{(33)}+\tau_{(33)}(\beta_1+4\rho_1))r_{28}^{+}=-4k_{-}(\rho_2\gamma_{(33)}+\tau_{(33)}(\beta_2+4\rho_2))$$

$$r_{29}^{+}=8[\rho_3(\gamma_{(13)}+i\gamma_{(14)})+(\beta_3+4\rho_3)(\tau_{(13)}+i\tau_{(14)})-$$

$$k(\rho_3(\gamma_{(23)}+i\gamma_{(24)})+(\beta_3+4\rho_3)(\tau_{(23)}+i\tau_{(24)}))]-\Delta\frac{\gamma_{(33)}}{4}$$

$$r_{30}^{+}=-8k_{-}\tau_{(33)}r_{31}^{+}=-16k_{-}\tau_{(33)}$$

$$r_{32}^{+}=8k_{-}\gamma_{[34]}r_{33}^{+}=-8ik_{-}\gamma(34)$$

$$r_{34}^{+}=-4\gamma_{[13]}+4k\gamma_{[23]}r_{35}^{+}=4\gamma_{(13)}+4k\gamma_{(23)}$$

$$r_{36}^{+}=16(\gamma_{(13)}+2\tau_{(13)})-4k\gamma_{(23)}r_{37}^{+}=16((\gamma_{(13)}+2\tau_{(13)})-k(\gamma_{(23)}+2\tau_{(23)}))$$

$equation1$

11.4 Field V_{μ}^{-}

$$r_1^{-}=4ik_{+}(b_1\gamma_{[34]}+\beta_1\gamma_{(13)})r_2^{-}=-4ik_{+}(b_2\gamma_{[34]}+\beta_2\gamma_{(34)})$$

$$r_3^{-}=-8b_3((\gamma_{[13]}-i\gamma_{[14]})+k(\gamma_{[23]}-i\gamma_{[24]}))r_4^{-}=-4\beta_1\gamma_{(33)}k_{+}$$

$$r_5^{-}=-4\beta_2\gamma_{(33)}k_{+}$$

$$r_6^{-}=8\beta_3[(\gamma_{(13)}-i\gamma_{(14)})+k(\gamma_{(23)}+\frac{1}{4}i(\gamma_{(24)}-2\tau_{(24)}))]-\Delta\frac{\gamma_{(33)}}{2}$$

$$r_7^{-}=-8i\gamma_{[34]}k_{+}r_8^{-}=-4k_{+}\gamma_{(33)}$$

$$r_9^{-}=-8k_{+}\gamma_{(33)}r_{10}^{-}=8ik_{+}\gamma_{[34]}$$

$$r_{11}^{-}=-4\gamma_{[13]}+8k(\gamma_{[23]}-i\gamma_{[24]})r_{12}^{-}=4\gamma_{[13]}+2ik\gamma_{[23]}$$

$$r_{13}^{-}=2ik\gamma_{[23]}r_{14}^{-}=2i\gamma_{[14]}+4k\gamma_{[23]}$$

$$r_{15}^{-}=2i\gamma_{[14]}-4k\gamma_{[23]}r_{16}^{-}=-2i\gamma_{[13]}$$

$$r_{17}^{-}=-4\gamma_{(13)}+k(8\gamma_{(23)}+2i(\gamma_{(24)}-2\tau_{(24)}))$$

$$r_{18}^{-}=4\gamma_{(13)}-k(8\gamma_{(23)}-2i(\gamma_{(24)}-2\tau_{(24)}))$$

$$r_{19}^{-}=2ik(\gamma_{(23)}+2\tau_{(23)})r_{20}^{-}=2i(\gamma_{(14)}+2\tau_{(24)})+4k\gamma_{(23)}$$

$$r_{21}^{-} = 2i(\gamma_{(14)} + 2\tau_{(24)}) - 4k\gamma_{(23)} r_{22}^{-} = 2i(\gamma_{(13)} + 2\tau_{(13)})$$

$$r_{23}^{-} = 4ik(\gamma_{(24)} + 4\tau_{(24)}) r_{24}^{-} = 4ik(\gamma_{(23)} + 4\tau_{(23)})$$

$$r_{25}^{-} = -4i(\gamma_{(14)} + 4\tau_{(14)}) r_{26}^{-} = -4ik_{(}\gamma_{(13)} + 2\tau_{(13)})$$

$$r_{27}^{-} = -4k_{+}(\rho_2\gamma_{(33)} + \tau_{(33)}(\beta_2 + 4\rho_2))$$

$$r_{28}^{-} = -4k_{+}(\rho_1\gamma_{(33)} + \tau_{(33)}(\beta_1 + 4\rho_1)) r_{29}^{-} = 4k_{+}(\rho_2\gamma_{(33)} + \tau_{(33)}(\beta_2 + 4\rho_2))$$

$$r_{30}^{-} = 8[\rho_3(\gamma_{(13)} - i\gamma_{(14)}) + (\beta_3 + 4\rho_3)(\tau_{(13)} - i\tau_{(14)}) - k((\beta_3 + 4\rho_3)(\tau_{(23)} + i\tau_{(24)}))] - \Delta\frac{\gamma_{(33)}}{4}$$

$$r_{31}^{-} = -8k_{+}\tau_{(33)} r_{32}^{-} = -16k_{+}\tau_{(33)}$$

$$r_{33}^{-} = -8ik_{-}\gamma_{[34]} r_{34}^{-} = -8ik_{+}\gamma_{(34)}$$

$$r_{35}^{-} = -4\gamma_{[13]} + 4k\gamma_{[23]} r_{36}^{-} = 4(\gamma_{[13]} + 4k\gamma_{[23]}$$

$$r_{37}^{-} = -16k_{+}(\gamma_{(33)} + 2\tau_{(33)})$$

$$r_{38}^{-} = 16[\gamma_{(13)} + 2\tau_{(13)} - k(\gamma_{(23)} + 2\tau_{(23)})] equation1$$

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