Analysis of flow of polar and non polar incompressible ferrofluids

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ABSTRACT

In this paper, flow between two parallel plates is analyzed for both polar and non polar ferrofluids. Velocity is obtained without pressure gradient for polar fluid and with pressure gradient for non polar fluid. The solution of the spin velocity is found in terms of applied magnetic field and magnetic flux density for polar fluid. Shear stress is calculated for both polar and non polar ferrofluid.

Indexing terms/Keywords

Polar; Non polar; Magnetic field; Magnetic flux density; Magnetic susceptibility; Pressure gradient; Symmetric stress tensor; Antisymmetric stress tensor; Body couple; Couple stress tensor; Shear stress.

Academic Discipline And Sub-Disciplines

Mathematics; Fluid Mechanics; Ferrohydro dynamics;

Mathematics Subject Classification:

76XX

TYPE (METHOD/APPROACH)

Analytical method, Mathematica.

1. INTRODUCTION

Ferrofluids are colloidal liquids made of nano scale ferromagnetic or ferrimagnetic particles suspended in a carrier fluid usually an organic solvent (benzene, carbon dioxide, chloroform) or water [5]. The carrier fluid can be classified as polar or non polar. The molecules in which the arrangement or geometry of the atoms is such that one end of the molecule has a positive electrical charge and the other side has a negative charge are called as polar molecules. A polar fluid is the one that is capable of transmitting stress couples which induces body torque. Examples of polar molecules are Water, Ammonia, Hydrochloric acid, Sulfur Dioxide, Hydrogen Sulfide, Carbon Monoxide etc. A non-polar molecule is that in which the electrons are distributed more symmetrically and thus does not have an excess /abundance of charges at the opposite sides. The charges all cancel out each other. In non polar fluids torques within it arises only as the moments of direct forces. Examples of non polar molecules are carbon dioxide, hydrogen, nitrogen, oxygen, chlorine etc [13].

Ferrofluids have a wide range of applications. Ferrofluids are used to form liquid seals around the spinning drive shafts in hard disks, vacuum feed throughs for semiconductor manufacturing, pressure seals for compressors and blowers, In medicine, ferrofluids are used as contrast agents for magnetic resonance imaging and can be used for cancer detection. In optics research is under way to create a shape-shifting magnetic mirror from ferrofluid for Earth-based astronomical telescopes [1,5].

He et al. [2] have studied effective magnetoviscosity of Planar-Couette magnetic fluid flow with applied uniform dc magnetic field transverse to the duct axis and described the conditions for multi valued effective magnetoviscosity and spin velocity.

Felderhof [3] have studied magnetoviscosity and relaxation in ferrofluids and analyzed both planar Couette flow and Poiseuille pipe flow in parallel and perpendicular magnetic field and the entropy production for these situations is calculated and related to the magnetoviscosity.

Rosensweig [4] analyzed plane Couette flow problem of sheared magnetized ferrofluid and determined the shear stress on the fixed wall. Korlie et al. [6] have analyzed the nature of steady solutions of a sheared ferrofluid between parallel boundaries and subject to an applied magnetic field perpendicular to the boundaries and found the velocity and spin fields of a ferrofluid shear flow in the limit of small but non-vanishing spin viscosity under zero antisymmetric stress boundary conditions.

The paper is organized as follows. We describe the governing equations for both polar and non polar ferrofluid in Section 2. We then outline the mathematical formulation and solution

in Section 3. The results and discussions are presented in Section 4. Section 5 contains the graphs.

2. GOVERNING EQUATIONS

In polar fluids body torque per unit mass is introduced in addition to the body force and a couple stress is introduced in addition to the normal stress. Whereas in non polar fluids only body force and Normal stress are introduced. Either the stress tensor is symmetric or the angular momentum is conserved in non polar fluids [4,5,6,7,8,9,10,11,12]. Governing equations for incompressible polar and non polar fluids [5] are given by

$$\nabla . \vec{v} = 0, \tag{1}$$

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} . \nabla \right) \vec{v} \right] = \vec{f} + \nabla . T + \rho g, \tag{2}$$

$$\nabla . \vec{B} = 0, \tag{3}$$

$$\nabla \times \vec{H} = 0, \tag{4}$$

$$\rho \vec{I} \left[\frac{\partial \vec{\omega}}{\partial t} + \vec{v} . \nabla \vec{\omega} \right] = \nabla . C + \vec{T} + \vec{L}, \tag{5}$$

$$\frac{\partial \vec{M}}{\partial t} + \vec{v} . \nabla \vec{M} = \vec{\omega} \times \vec{M} - \frac{1}{\tau} \left(\vec{M} - \vec{M_0} \right), \tag{6}$$

where

$\vec{v}$ is the translational velocity, $\vec{\omega}$ is the spin velocity, $\vec{H}$ is the applied magnetic field, $\vec{M}$ is the Magnetization, $\vec{I}$ is the moment of inertia density, ρ is the density, τ is the magnetization relaxation time, η is the coefficient of shear viscosity, ζ is the coefficient of vortex viscosity, $\eta^{'}$ is the shear coefficient of spin viscosity, χ_0 is a constant depending on temperature and composition of suspension called magnetic susceptibility, ε is the polyadic,| is the unit dyadic, p is the pressure, g is the gravitation,

$$C = \eta^{'}\left(\nabla\vec{\omega}+\nabla\vec{\omega}^{T}\right) \text{ is the couple stress tensor,} \tag{7}$$

$$\vec{T} = 2\zeta\left(\nabla\times\vec{v}-2\vec{\omega}\right) \text{ is the antisymmetric vector of Cauchy stress,} \tag{8}$$

$$\vec{L} = \mu_0\vec{M}\times\vec{H} \text{ is the body couple density,} \tag{9}$$

$$\vec{f} = \mu_0\left(\vec{M}.\nabla\right)\vec{H} \text{ is the magnetic force density,} \tag{10}$$

$$T = -pI + \eta\left(\nabla\vec{v}+\nabla\vec{v}^{T}\right) + \zeta\varepsilon.\left(\nabla\times\vec{v}-2\vec{\omega}\right) \textbf{ is the Cauchy stress tensor, (11)}$$

$$\vec{M}_0 = \chi_0\vec{H} \text{ is the equilibrium magnetization,} \tag{12}$$

$$\vec{B} = \mu_0\left(\vec{H}+\vec{M}\right). \tag{13}$$

3. MATHEMATICAL FORMULATION AND SOLUTION

Consider a ferro fluid flow between two parallel plates separated by a distance d as shown in Figure.1 the upper plate is moving with velocity v and the magnetic field H_x or the magnetic flux density B_x is applied along x direction [2,6].
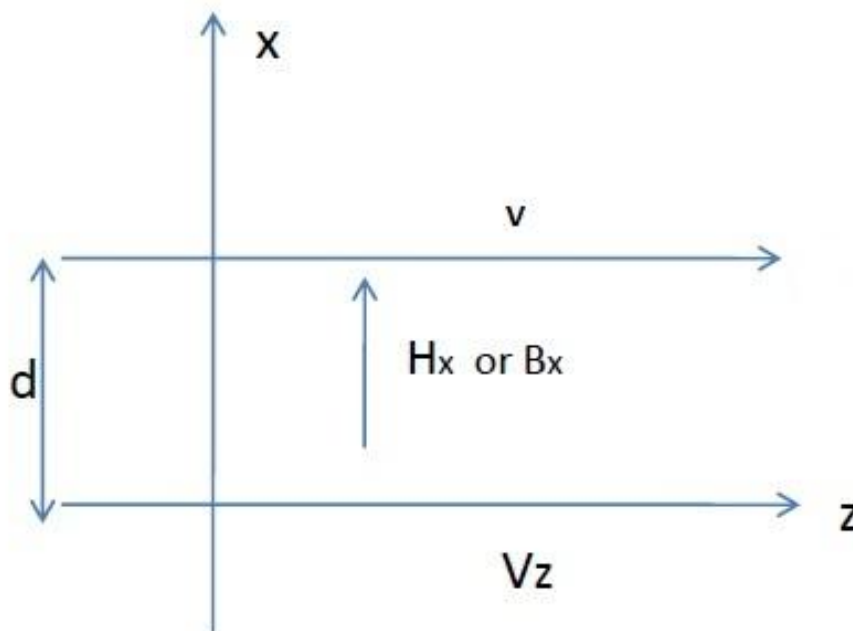

Figure 1: Geometry of Problem

The flow velocity $\vec{v}$ and the spin velocity $\vec{\omega}$ are considered as

$$\vec{v} = \left(0, 0, v_z(x)\right), \vec{\omega} = \left(0, \omega_y(x), 0\right). \tag{14}$$

Since the imposed magnetic field H_x or magnetic flux density B_x are spatially uniform with the y and z coordinate and are imposed on the system by external sources, all the field and flow variables are independent of y and z and can only vary with x.

$$\vec{H} = \left(H_x(x), 0, 0\right), \vec{B} = (B_x, 0, B_z(x)), \vec{M} = \left(M_x, 0, M_z\right), \vec{f} = \left(f_x, 0, 0\right), \vec{L} = (0, L_y, 0). \tag{15}$$

Using (15) in (3) and (4) we get

$$B_x = \text{constant},$$

$$H_y, H_z = \text{constant=0}. \tag{16}$$

Using (14) and (15) in (6), we obtain

$$M_x = \frac{\chi_0 H_x}{\left[\left(\omega_y \tau\right)^2 + 1\right]}, \tag{17}$$

$$M_z = \frac{-\chi_0 H_x \omega_y \tau}{\left[\left(\omega_y \tau\right)^2 + 1\right]},$$

or

$$M_x = \frac{\chi_0 \dfrac{B_x}{\mu_0}}{\left[\left(\omega_y \tau\right)^2 + 1 + \chi_0\right]}, \tag{18}$$

$$M_z = \frac{\chi_0 \dfrac{B_x}{\mu_0} \omega_y \tau}{\left[\left(\omega_y \tau\right)^2 + 1 + \chi_0\right]}.$$

Using (14) and (15) in (9) and (10) we get

$$f_x = \mu_0 M_x \frac{dH_x(x)}{dx},$$

or

$$\tag{19}$$

$$f_x = -\frac{d}{dx}\left(\frac{1}{2} \mu_0 M_x^2\right),$$

$$L_y = -\frac{\chi_0 \mu_0 \omega_y \tau H_x^2}{\left(\omega_y \tau\right)^2 + 1},$$

or

$$\tag{20}$$

$$L_y = -\frac{\chi_0 B_x^2 \omega_y \tau \left(1 + \left(\omega_y \tau\right)^2\right)}{\left(\omega_y \tau\right)^2 + 1 + \chi_0}.$$

Let us consider polar highly viscous fluid for which Reynold's number is negligible, therefore inertia is neglected. By neglecting spin viscosity, pressure gradient and gravity effects, and using equation (14) the flow and spin velocity equations (2) and (5) reduce to

$$\left(\zeta + \eta\right)\frac{d^2 v_z}{dx^2} + 2\zeta \frac{d\omega_y}{dx} = 0, \tag{21}$$

$$L_y - 2\zeta \left(\frac{dv_z}{dx} + 2\omega_y\right) = 0. \tag{22}$$

If we consider magnetic field or magnetic flux density are very small then we can neglect L_y also. Then the solution for flow and spin velocities are

$$v_z(x) = \frac{vx}{d}, \omega_y = \frac{v}{2d}, \tag{23}$$

with the boundary conditions $v_z(0) = 0$ and $v_z(d) = v$. \hfill (24)

Viscous-stress tensor for incompressible fluid is given by

$$T = -pI + \eta\left(\nabla\vec{v} + \nabla\vec{v}^T\right) + \zeta\varepsilon.\left(\nabla \times \vec{v} - 2\vec{\omega}\right). \tag{25}$$

Using equation(14) in equation (25) and neglecting pressure gradient, the shear stress at the lower plate is

$$T_{zx} = \left(\zeta + \eta\right)\frac{v}{d} + 2\zeta\omega_y. \tag{26}$$

Using equation equation (20) in equation (22) we obtain the third order and fifth order equations for the spin velocity ω_y as

$$r^3 - r^2 + \left[\frac{1}{4} + \frac{1 + P_H}{\left(\frac{v\tau}{d}\right)^2}\right]r - \frac{P_H}{2\left(\frac{v\tau}{d}\right)^2} = 0, \tag{27}$$

$$\left(r - \frac{1}{2}\right)^5 + \frac{1}{2}\left(r - \frac{1}{2}\right)^4 + \frac{\left[\left(1 + \chi_0\right)^2 P_B + 2\left(1 + \chi_0\right)\right]}{\left(\frac{v\tau}{d}\right)^2}\left(r - \frac{1}{2}\right)^3 +$$

$$\frac{\left(1 + \chi_0\right)}{\left(\frac{v\tau}{d}\right)^2}\left(r - \frac{1}{2}\right)^2 + \frac{\left(1 + \chi_0\right)^2}{\left(\frac{v\tau}{d}\right)^4}\left(r - \frac{1}{2}\right) + \frac{\left(1 + \chi_0\right)^2}{2\left(\frac{v\tau}{d}\right)^4} = 0, \tag{28}$$

where

$$r = \frac{\Delta\eta}{2\zeta} = \left(\frac{1}{2} + \frac{\omega_y d}{v}\right) \quad \text{is the non dimensional parameter, called as magnetoviscosity,} \quad (29)$$

$$\Delta\eta = \zeta\left(1 + \frac{2\omega_y d}{v}\right), \quad (30)$$

$$P_H = \frac{\mu_0 \chi_0 H_x^2 \tau}{4\zeta}, \quad (31)$$

$$P_B = \frac{\chi_0 B_x^2 \tau}{\mu_0 \left(1 + \chi_0\right)^2 4\zeta}. \quad (32)$$

Solving (27) and (28) for real roots, the graphs are drawn for the magnetoviscosity versus P_H and P_B which contain applied magnetic field and magnetic flux density respectively. These are depicted in figure 2 and figure 3 for different values of $\frac{v\tau}{d}$.

Now we study non polar fluids with constant pressure gradient, in which the body couple density is absent and the stress tensor is symmetric. For low Reynold's number flows, the inertia is negligible and neglecting the spin viscosity and gravity effects, the flow velocity equation reduces to

$$\eta\frac{d^2 v_z(x)}{dx^2} - \nabla p = 0, \quad (33)$$

with the boundary conditions $v_z(0) = 0$ and $v_z(d) = v$.

The solution of (33) is

$$v_z(x) = \left[\frac{v}{d} - \frac{\nabla p}{\eta}\frac{d}{2}\right]x + \frac{\nabla p}{\eta}\frac{x^2}{2}. \quad (34)$$

Consider the dimensionless pressure gradient as $a = -\dfrac{d^2\nabla p}{2\eta v}$. $\quad (35)$

The velocity distribution for various values of a is shown in the figure 4. The shearing stress for the given flow is

$$T_{zx} = \eta\frac{dv_z(x)}{dx} = \frac{\eta v}{d}\left[1 + a\left[1 - \frac{2x}{d}\right]\right]. \quad (36)$$

4. RESULTS AND DISCUSSIONS

We have studied velocity profile of both polar and non polar ferrofluid and observed that the velocity profile of non polar fluids is almost similar to non ferrofluids. For polar ferrofluid the solution for magnetoviscosity versus P_H is shown in the figure 2 and magnetoviscosity versus P_B is shown in the figure 3. For non polar ferrofluid the velocity distribution for various values of a is shown in the figure 4. In figure 2 and 3 we observe that magnetoviscosity increases as velocity increases i.e., for $\frac{v\tau}{d} = 10, 12, 15,$ but due to upper plate movement it decreases after half way and again it increases. Hence this upper half region can be treated as boundary layer region also. Where as for small velocity, magnetoviscosity shows gradual increase in graph.
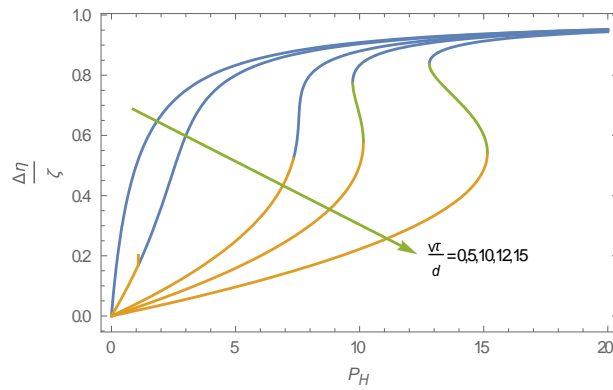
5. GRAPHS

Figure 2: Magnetoviscosity versus P_H for different values of $\dfrac{v\tau}{d}$
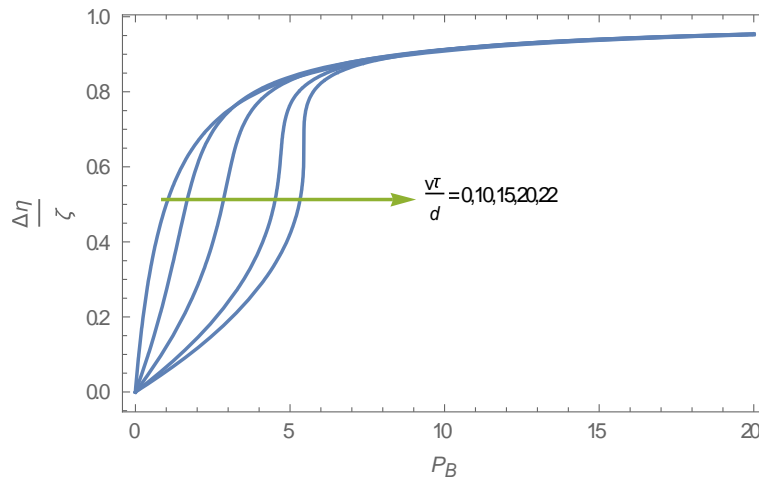

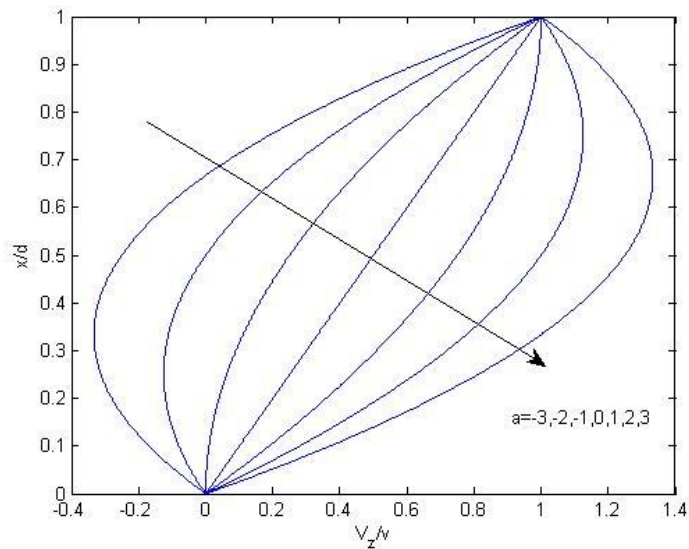**Figure 3: Magnetoviscosity versus P_H for different values of $\dfrac{v\tau}{d}$**

Figure 4: Velocity distribution for various values of pressure gradient

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