ISSN 2347-3487

RENORMALIZATION ISSUES FOR A WHOLE ABELIAN MODEL

J. Chauca, R. Doria
Rio de Janeiro, Brazil; AprendaNet, Petrópolis, Brazil
AprendaNet, Petrópolis, Brazil

ABSTRACT

Considering that nature acts as a group, a whole abelian model is being developed. Classically, new aspects were observed as fields collective behavior and fields interacting among themselves and with mass through a global Lorentz force. This work analyzes some quantic aspects. Perturbation theory means that we know about 1-PI graphs. In a previous work, we have studied the quantum action principle, power-counting, primitively divergent graphs, Ward-Takahashi identities. This work concerns the study of counterterms and physical perturbation theory. It introduces a whole

renormalization programme which informations are obtained from the common gauge parameter which establishes the fields set. It derives relationships between renormalization constants and on perturbative persistence on one asslessness field in the *{A_I}* set. It also argues on finitude possibilities through a whole expansion for the graphs.

Indexing terms/Keywords

Whole abelian model; Ward-Takahashi identities; Renormalization.

Academic Discipline And Sub-Disciplines

Physics; Particle Physics

SUBJECT CLASSIFICATION

Gauge theory

TYPE (METHOD/APPROACH)

Survey

ISSN 2347-3487

1 Introduction

The actual physics belief is that physical processes should be understood through their parts [1]. The meaning of whole as guiding nature is not considered as a pattern for doing physics [2]. Our viewpoint is that nature should be taken not only as build up by elementary particles but also as acting as a group. Complexity works as example of such systemic behavior [3]. It supports us on searching for such whole gauge approach.

Our motivation is to introduce the whole concept through gauge theory [4]. Take its mechanism for providing physics with antireductionistic equations. Similarly to relativity, which do not isolate space and time but connect them as space-time, this whole approach considers that instead two fields $A_{\mu 1}$ and $A_{\mu 2}$ be interacting independently, they should be interconnected through a set $\{A_{\mu 1}, A_{\mu 2}\}$.

Under this consideration, we are studying a so-called non-linear abelian gauge model [5], [6]. It considers the presence of N-potential fields $A_{\mu I}$ transforming under a common abelian group as $A_{\mu I}' = A_{\mu I} + \partial_\mu P_I(\alpha)$ where $P_I(\alpha)$ means a generic polynomial expansion on gauge parameter. It yields a field set $\{A_{\mu I}\}$ transforming under a systemic symmetry based on a common gauge parameter $\alpha(x)$. Notice then, that through fields reparametrizations one can also study such systemic symmetry under different basis as the so-called constructor basis $\{D_\mu, X_\mu^i\}$ where i varies from 2 to N, and physical basis $\{G_{\mu I}\}$ [7], [8].

In this work, we are going to study the corresponding whole abelian gauge properties by taking $P_I'(\alpha) = 1$. It yields the following parametrization basis for analyzing such whole abelian symmetry:

$$\{A_{\mu I}\}: \quad A_{\mu I} \to A_{\mu I}' = A_{\mu I} + \partial_\mu \alpha(x), \tag{1}$$

$$\{D_\mu, X_\mu^i\}: \quad D_\mu \to D_\mu' = D_\mu + \partial_\mu \alpha(x),$$

$$X_\mu^i \to X_\mu^i{}' = X_\mu^i, \tag{2}$$

$$\{G_{\mu I}\}: \quad G_{\mu I} \to G_{\mu I}' = G_{\mu I} + \Omega_{I1}^{-1} \partial_\mu \alpha(x), \tag{3}$$

with

$$D_\mu = \sum_I A_{\mu I}, \quad X_{\mu(N-1)} = A_{\mu 1} - A_{\mu N}, \tag{4}$$

$$G_{\mu I} = \Omega_{I1}^{-1} D_\mu + \Omega_{1i}^{-1} X_\mu^i, \tag{5}$$

where Ω matriz diagonalize the transverse sector [9]. It depends just on Lagrangian coefficients.

Being the whole model written from one gauge parameter, the parts are more well defined through the constructor basis $\{D_\mu, X_\mu^i\}$. There it departes a genuine gauge field D_μ and $(N-1)$ Proca fields X_μ^i. However, they do not correspond to the physical fields, which are that ones associated to the physical poles defined by two-point Green's functions. While gauge invariance is better defined at basis $\{D_\mu, X_\mu^i\}$, the parts (quanta) are through $\{G_{\mu I}\}$. The physical basis $\{G_{\mu I}\}$ is defined by Eq. (5), showing that every physical field is defined in terms of Ω matrix coefficients originated from the whole abelian Lagrangian.

2 Green's functions relationships

Our main goal here is to establish relationships among Green's functions with different external lines and, as consequence, relate renormalization constants of different vertices of theory. They arise as a direct consequence of the fact that the generating functional of Green's functions, $W[J, \xi, \bar{\xi}]$, is gauge invariant due to depend just on external sources.

So, although the quantized theory breaks gauge invariance through the gauge-fixing term and the coupling to external fields, it is preserved in terms of Green functions due to the Ward-Takahashi identities [10], [11], [12]. Considering the physical basis $\{G_{\mu I}\}$, one gets

$$\delta W = \mathrm{N}\int\prod_I \mathrm{D}G_{\mu I}\prod_j \mathrm{D}\psi_j \mathrm{D}\overline{\psi}_j e^{iS} i\delta S = 0,$$

for the following gauge transformations

$$\delta G_I = \Omega_{I1}^{-1}\partial_\mu \alpha, \quad \delta\psi_j = ig\alpha\psi_j, \quad \delta\overline{\psi}_j = -ig\alpha\overline{\psi}_j. \tag{6}$$

Notice that, there is just one gauge fixing-term which is associated to the existence of only one gauge parameter, but with the difference to the usual case that is involving a set of fields, $\mathrm{L}_{gf} = \frac{1}{\alpha}(\sigma_I \partial_\mu G^{\mu I})^2.$

Thus the non-gauge invariant term is written as

$$S_{ngi} = \int d^4x[\frac{-1}{2\alpha}(\sigma_I \partial_\mu G_I^\mu) + J_{\mu I}G^{\mu I} + \overline{\xi}_j \psi^j + \overline{\psi}_j \xi^j], \tag{7}$$

which yields the Ward-Takahashi identities for the generating functional of the proper vertices (1-PI Green's functions) in configuration space

$$\frac{i}{\alpha} a\sigma_I \mathrm{W}\partial_\mu G^{\mu I} - i\Omega_{I1}^{-1}\partial_\mu \frac{\delta\Gamma}{\delta G_{\mu I}}$$

$$+ g(\overline{\psi}_j \frac{\delta\Gamma}{\delta\overline{\psi}_j} - \psi_j \frac{\delta\Gamma}{\delta\psi_j}) = 0, \tag{8}$$

where $a \equiv \sigma^K \Omega_{K1}^{-1}$.

Eq. (8) allows to study aspects corresponding to the Green's functions involved on this whole abelian gauge model. It will play a crucial role in the renormalization of the model by restricting the number of independent UV divergences. For the two-point Green function corresponding to the vector bosons propagators, it yields

$$\Omega_{I1}^{-1}\partial_\mu \frac{\delta^2\Gamma}{\delta G_{\mu I}(x)\delta G_{\nu J}(y)} = \frac{a}{\alpha}\sigma_J \mathrm{W}\partial^\nu \delta(x-y). \tag{9}$$

Observe that here the Ward-Takahashi identity takes a matricial form involving N -equations. It says that it does not analysis one graph alone but a line of graphs together as $\sum_I {}^I gpg^J$.

The question is if the longitudinal part of the two-point functions receive loop corrections? From [6], studying the Eq. (9), full propagators and by comparing with the free action, one gets that just for one field the propagator longitudinal part remains the same as that of the free propagator to all orders of perturbation theory. Physically, Eq. (9) says that given a set $\{G_{\mu I}\}$ the longitudinal mode of one of these fields does not exist because it decouples from all physical processes. However, differently from QED, it does not mark that the longitudinal photon must be that one to be decoupled. Its result is generic. It just points out that one longitudinal mode must be suppressed from the whole set. This means that this whole abelian model can accommodate a longitudinal photon.

For the three-point functions, there not exist fermions as external lines. It yields just the case with three bosons as external lines. In QED, this case is zero through Furry's theorem or by calculating explicitly [13]. However here these particles are different. From Eq. (9), one gets the following relationships

$$\Omega_{I1}^{-1}\partial^\mu \frac{\delta^3\Gamma}{\delta G_{\mu I}(x)\delta G_{\nu J}(y)\delta G_{\rho K}(z)} = 0. \tag{10}$$

From dimensional analysis, $\int d^4x \Gamma_{IJK}^{(3)\mu\nu\rho} G_\mu^I G_\nu^J G_\rho^K$ yields $[\Gamma^{(3)}] = [M]^1$, which means linearly divergent. However studying Eq. (10) in momentum space

$$\Omega_{I1}^{-1} k^\mu \Gamma_{\mu\nu\rho}^{(3)IJK}(G_I, G_J, G_K) = 0 \tag{11}$$

which implies

$$\Omega_{I1}^{-1}\Gamma_{\mu\nu\rho}^{(3)IJK} = (k^2\delta_\mu^\lambda - k_\mu k^\lambda)\Omega_{I1}^{-1}\Gamma_{\nu\rho}^{(3)\lambda,IJK} \quad (12)$$

that shows a finite three-point function.

Similarly for four point functions with four vector bosons as external legs

$$\Omega_{I1}^{-1}\partial^\mu \frac{\delta^4\Gamma}{\delta G_{\mu I}(x)\delta G_{\nu J}(y)\delta G_{\rho K}(z)\delta G_{\sigma L}(w)} = 0. \quad (13)$$

where $\Gamma^{(4)}$ is logarithmically divergent, but under the relationship

$$k^\mu[\Omega_{I1}^{(-1)}\Gamma_{\mu\nu\rho\sigma}^{(4)IJKL} = 0], \quad (14)$$

it propitiates finitude.

Applying the derivative $\frac{\delta}{\delta\overline{\psi}_p(y)}\frac{\delta}{\delta\psi_k(z)}$ with respect to the Ward-Takahashi identity, one derives the following relationship between the vertices and the fermionic propagator

$$\sum_I \Omega_{I1}^{-1}\partial_x^\mu\Gamma_\mu^{(3)}(x;y,z)$$

$$= g[\delta(x-y)-\delta(x-z)]\Gamma_{\overline{\psi}_p\psi_q}^{(2)}(z-y),$$

where

$$\Gamma_\mu^{(3)}(x;y,z) = \frac{\delta^3\Gamma}{\delta\overline{\psi}_p(y)\delta\psi_q(z)\delta G_I^\mu(x)},$$

$$\Gamma_{\overline{\psi}_p\psi_q}^{(2)}(z-y) = \frac{\delta^2\Gamma}{\delta\psi_q(z)\delta\overline{\psi}_p(y)}. \quad (15)$$

Eq. (15) says that the graphs

are related to all orders in perturbation theory.

Similarly one obtains the relationship between two vector bosons - two fermions and one vector boson - two fermions graphs.

$$\Omega_{I1}^{-1}\partial_x^\mu\Gamma^{(4)}(x;w,y,z) =$$

$$= ig[\delta(x-z)-\delta(x-y)]\Gamma^{(3)}(w,y,z)$$

where

$$\Gamma^{(4)}(x;w,y,z) = \frac{\delta^4\Gamma}{\delta G_{\mu I}(x)\delta G_{\nu J}(w)\delta\overline{\psi}_p(y)\delta\psi_q(z)},$$

and

$$\Gamma^{(3)}(w,y,z) = \frac{\delta^3\Gamma}{\delta G_{\nu J}(w)\delta\overline{\psi}_p(y)\delta\psi_q(z)}. \quad (16)$$

Similarly Eq. (16) relates to all orders

ISSN 2347-3487

3 On the counterterms

In order to make the divergent diagrams finite, we must find a way to lower the associated degrees of divergence. This can be achieved by making subtractions in the Feynman integrand. In a renormalizable theory, we should be able to understand these subtractions as due to the inclusion of counterterms.

Thus, considering that physical fields are the ones which diagonalize the transverse sector, one obtains the following whole bare abelian Lagrangian [5], [6]

$$L^B = L_0^B + L_I^B + L_F^B,$$

where

$$L_0^B = -\frac{1}{2}G_{\mu I}\mathrm{W}P^{T\mu\nu}G_\nu^I - \frac{1}{2}K_{G_I}G_{\mu I}\mathrm{W}P^{T\mu\nu}G_\nu^I$$

$$-\frac{1}{2}l_{IJ}G_\mu^I\mathrm{W}P^{L\mu\nu}G_\nu^J - \frac{1}{2}K_{l_{IJ}}G_{\mu I}\mathrm{W}P^{L\mu\nu}G_\nu^I$$

$$+m_I^2G_{\mu I}G^{\mu I} + K_{m_I}m_I^2G_{\mu I}G^{\mu I},$$

$$L_I^B = a_{IJK}(\partial_\mu G_\nu^I)(G^{\mu J}G^{\nu K})$$

$$+K_{a_{IJ}}(\partial_\mu G_\nu^I)(G^{\mu J}G^{\nu K})$$

$$+b_{IJK}(\partial_\mu G^{\mu I})(G_\nu^J G^{\nu K})$$

$$+K_{b_{IJK}}(\partial_\mu G^{\mu I})(G_\nu^J G^{\nu K})$$

$$+a_{IJKL}G_\mu^I G_\nu^J G^{\mu K}G^{\nu L}$$

$$+K_{a_{IJKL}}G_\mu^I G_\nu^J G^{\mu K}G^{\nu L},$$

$$L_F^B = \overline{\psi}_p(F_{kq}\gamma\cdot\partial + g_{Ipq}G_\mu^I)\psi_q$$

$$+K_{\psi_p}K_\psi K_{\psi_p}\overline{\psi}_pF_{\mu p}\gamma\cdot\partial\psi_p$$

$$+K_{g_{Ipq}}\overline{\psi}_pG_\mu^I\psi_q \qquad (17)$$

which coefficients K_x are determined iteratively.

It yields the following renormalization constants

$$G_{\mu I}^B = Z_{G_I}^{1/2}G_{\mu I}, \quad e_{IJ}^B = Z_{e_{IJ}}Z_I^{-1/2}Z_J^{-1/2}e_{IJ},$$

$$m_I^B = Z_{m_I}Z_I^{-1}m_I,$$

$$a_{IJK}^B = Za_{IJK}Z_I^{-1/2}Z_J^{-1/2}Z_K^{-1/2}a_{IJK}^B,$$

$$a_{IJKL}^B = Za_{IJKL}Z_I^{-1/2}Z_J^{-1/2}Z_K^{-1/2}Z_L^{-1/2}a_{IJKL},$$

$$\psi_p^B = Z_{\psi_p}^{1/2}\psi_p, \quad g_{Ipq}^B = Z_1^{Ipq}Z_{\psi_p}^{-1/2}Z_{\psi_p}^{-1/2}Z_{G_I}^{-1/2}g_{Ipq}. \qquad (18)$$

The new aspect to be considered here is the presence of mixed propagators and self-interacting vector fields at three level. Notice, then, that the counterterms derived from Eqs. (17)-(18) and required to cancel divergences of each order of perturbation theory have the same form as the terms in the original Lagrangian density. So the resulting perturbation theory is renormalizable due to finite counterterms can be selected to cancel all divergences order-by-order [14].

Substituting

$\Gamma_{\mu}^{(3)}(G_I,\psi_p,\overline{\psi}_q) = Z_{G_I}^{1/2} Z_{\psi_p}^{1/2} Z_{\overline{\psi}_q}^{1/2}\ \Gamma_{\mu}^{(3)Bare}(G_I^B,\psi_p^B,\overline{\psi}_q^B)$ in (15) and considering that the Ward-Takahashi identity for the Green's functions must be the same, one derives working with regularized theory,

$$Z_1^{Ipq} = Z_{\psi_p}^{1/2} Z_{\psi_q}^{1/2}, \tag{19}$$

Z_1^{Ipq} being given by $Z_1 = 1 + K_1^{Ipq}$, where K_1 is the vertex counterterm $K_1^{Ipq}\mu^{\varepsilon/2} g_{Ipq}\overline{\psi}_p\gamma^{\mu}\psi_q G_{\mu I}$, one gets that $K_1 = K_{\psi_p}^{1/2} K_{\psi_q}^{1/2}$. This means that we need not to compute the vertex correction in order to avoid its counterterm: this vertex is renormalized through the wave-function renormalization of the fermion fields.

Given that such abelian set $\{G_{\mu I}\}$ just extends QED, one notices like consistency the fact that its results, as Eq. (19), are able to reproduce the standard results [15], [16]. To introduce more fields in a same group does not bring a new qualitative aspect for the Ward-Takahashi identity. It just add new terms.

Thus we encounter four classes of divergent diagrams which renormalization is possible. They are the vacuum polarization diagrams, the fermion self-energy diagram, the boson-vertex and fermion-vertex connections diagrams. The new aspect is that such diagrams are constituted by a set of graphs.

4 Whole expansion for the graphs

A consequence from this whole gauge approach is the appearance of whole graphs. They are bringing a new physical reality to be understood. While the usual Feynman graphs associated to every field work as constituents, the systemic mechanism add them, and, creates as physical the so-called whole graphs.

In the resulting perturbation theory infinities appear as before, but now, there are additional pieces involving the counterterms. Differently from the usual case, this whole abelian model brings an abundance of graphs. For exemplifying on such whole graph, we will take the case $N = 2$ for 2 fermions - 1 boson graph. It yields at one loop an expansion in four graphs, with a whole graph defined as

So an interesting consequence from this series of graphs is that there is the possibility to control the infinities of the theory. The relationship between the 1-PI Green's functions and renormalization constants is not more one-to-one. While computing radiative corrections in a last paper [6], we encountered 1PI graphs with ultraviolet divergences. We sketch here that these divergences can yield finite expressions, as a whole. It says that masses and charges renormalizations will be depending on model whole aspects. Consequently, similarly to Supersymmetry, it is possible for theory be finite by infinities calculations [17].

5 Conclusion

Our purpose is to interpret the meaning of whole through gauge theory. There is a whole for defining the parts. For this, we have considered the gauge transformation for a fields set $\{A_{\mu I}\}$, and so, through Eqs. (1-3) one defines a whole-U(1) transformation. Then, at this conclusion, we should reflect how this whole abelian symmetry being analyzed, acts in terms of parts and whole. How this whole renormalization works.

Under a systemic approach the connection between parts and whole is expected. Eq. (5) already signs such correlation. Looking this initial definition, one notices that it is written in such way that the model incorporates the parts (physical fields) in terms of whole (Lagrangian coefficients). It derives a classical field theory where the fields do not appear defined isolatedely.

Considering quantum aspects, studying on partition functions, one notices through Eq. (8) that instead just one equation, it appears N - Ward-Takahashi identities. Based in only one gauge parameter, the whole gauge symmetry stipulates N -coupled equations for studying the correspondent 1-PI graphs associated to the model. Physically, the information comes from the set $\{G_{\mu I}\}$, and so, our physical interpretation for Feynman diagrams must be in terms of a whole set of equations. Consequently, the parts (every physical field $G_{\mu I}$) renormalization must be derived in terms of the whole (fields set) renormalization.

Thus, the model provides a whole renormalization where the rule is given by the fields set. Although calculations are performed from every field contribution, they do not work isolatedely. It develops a situation where the renormalization program does not consider more one graph isolatedely. It appears an abundance of graphs (whole) for defining individual characteristics as renormalization constants (field, coupling constants) and finitudes.

Consequently, in this whole renormalization procedure introduced by Eqs. (1-5) one observes that, there is a whole divergence more crucial than the parts divergence. The renormalization constants calculations, the relationships between

1-PI graphs with different number of external legs, the finitude and other aspects will depend on the graphs whole expansion. Whole graphs is that will be measured.

Concluding, it is possible to introduce mass in gauge theory without requiring the Higgs mechanism. Based on whole symmetry it is possible to develop a massive and renormalizable gauge model.

Appendix A

Persistence of perturbative masslessness

Considering that at QED case, one can proof that order by order in perturbation theory there is no mechanism for generating a photon pole at $p^2 = 0$ in $\Pi(p)$, one should make a similar study for this $\{A_{\mu I}\}$ extended case. For this, it is better to consider the Ward-Takahashi identities written at constructor basis $\{D, X_i\}$. Considering the gauge fixing term $\mathcal{L}_{gf} = \frac{1}{2\alpha}[\partial\cdot(D+\bar{\sigma}_i X^i)]^2$ and coupling to external sources as $J\cdot D + j_i X^i$, it gives the following identities

$$\frac{i}{\alpha}\mathrm{W}\partial^\mu\left(\frac{\delta Z}{\delta J^\mu(x)}+\bar{\sigma}_i\frac{\delta Z}{\delta j^{\mu i}(x)}\right)$$

$$-\partial^\mu J_\mu + g\bar{\xi}\frac{\delta Z}{\delta\bar{\xi}} - g\xi\frac{\delta Z}{\delta\xi} = 0,$$

and

$$\frac{i}{\alpha}\mathrm{W}\partial^\mu\left(\frac{\partial W}{\delta J^\mu(x)}+\bar{\sigma}_i\frac{\delta W}{\delta j^{\mu i}(x)}\right)$$

$$-(\partial^\mu J_\mu)W + g\bar{\xi}\frac{\partial W}{\partial\bar{\xi}} - g\xi\frac{\partial W}{\partial\bar{\xi}} = 0,$$

and

$$\frac{i}{\alpha}\mathrm{W}\partial^\mu(D_\mu+\bar{\sigma}_i X^{\mu i}) + \partial^\mu\frac{\delta\Gamma}{\delta D^\mu}$$

$$-g\psi\frac{\delta\Gamma}{\delta\psi} + g\bar{\psi}\frac{\delta\Gamma}{\delta\psi} = 0, \tag{20}$$

which yields,

$$\partial_\mu\Gamma^{\mu\nu}_{DD}(x-y) = \frac{1}{\alpha}\mathrm{W}_x\partial^\nu\delta^4(x-y),$$

$$\Gamma^{\mu\nu}_{DD} = \frac{\delta^2\Gamma}{\delta D_\mu\delta D_\nu}, \tag{21}$$

$$\partial_\mu\Gamma^{\mu\nu}_{DX_i}(x-y) = \frac{\bar{\sigma}_i}{\alpha}\mathrm{W}_x\partial^\nu\delta^4(x-y),$$

$$\Gamma^{\mu\nu}_{DX_i} = \frac{\delta^2\Gamma}{\delta D_\mu\delta DX^i_\nu}, \tag{22}$$

Given that

$$\Gamma_{Free}^{(2)} = \frac{1}{2}\int d^4x (D, X^i)_\mu [AWP_T^{\mu\nu} +$$

$$+(A+B)WP_L^{\mu\nu} + M^2\eta^{\mu\nu}](D, X^i)_\nu, \tag{23}$$

where from Eq. (4) one derives $L_m = m_{ij}^2 X_\mu^i X^{\mu j}$, that results $M^2 = \begin{pmatrix} 0 & 0 \\ 0 & m_{ij}^2 \end{pmatrix}$ with $m_{DD}^2 = 0 = m_{DX_i}^2$. Thus, considering that the effect of the radioactive correction means to add a selfinteracting term $\Pi^{\mu\nu}(p)$, one gets

$$\Gamma_{DD}^{\mu\nu}(p,-p) = (a_1 + m_{DD}^2)\eta^{\mu\nu} +$$

$$+(-a_1 - \frac{1}{2\alpha})p^\mu p^\nu - i\hbar\Pi_{DD}^{\mu\nu}(p), \tag{24}$$

$$\Gamma_{DX_i}^{\mu\nu}(p,-p) = (c_i + m_{DX_i}^2)\eta^{\mu\nu} +$$

$$+(-c_i - \frac{\bar{\sigma}_i}{2\alpha})p^\mu p^\nu - i\hbar\Pi_{DX_i}^{\mu\nu}(p), \tag{25}$$

where a_1, c_i are coefficients coming from A matrix.

Substituting (24), (25) respectively in (21), (22), one derives

$$i\hbar p_\mu \Pi_{DD}^{\mu\nu} = m_{DD}^2 p^\nu = 0, \tag{26}$$

$$i\hbar p_\mu \Pi_{DX_i}^{\mu\nu} = m_{DX_i}^2 p^\nu = 0. \tag{27}$$

Eqs. (26) and (27) are the necessary ingredient for a massless D_μ, implying $\Pi_{DD}^{\mu\nu} = (g^{\mu\nu} - p^\mu p^\nu)\Pi_{DD}(p)$ and hence forbidding terms like $m^2\eta_{\mu\nu}$ that would give rise to a mass. Similarly to $\Pi_{DX_i}^{\mu\nu}$.

References

[1] M. Gell-Mann. The Quark and the Jaguar, W. H. Freeman and Company, New York, 1994.

[2] G. t'Hooft (Editor). 50 years of Yang-Mills theory, World Scientific, Singapore, 2005.

[3] K. Mainzer. Thinking in Complexity, Springer-Verlag, Berlin, 2007.

[4] R. Doria. Whole Gauge Theory, XXX Encontro Nacional de Fí-sica de Partículas e Campos, 2009.

[5] R. Doria, JAP, Vol 7, Number 3 (2015) 1840-1856.

[6] J. Chauca, R. Doria, J. L. Valle. Rev. Mex. Fis. Vol 58, Nº 2 (2012)152-159.

[7] H. J. Borchers. Comm. Math. Phys. **1** (1965) 281.

[8] R. M. Doria, J. A. Helayë-Neto. Acta Physica Hungarica Vol 73 Numbers 2-4 (1993) 243.

[9] J. Domingos, R. M. Doria, R. Portugal. Acta Physica Hungarica Vol 73 Numbers 2-4 (1993) 205.

[10] J. C. Ward. Phys. Rev. **78** (1950) 182.

[11] J. C. Taylor. Nucl. Phys. **B 33** (1971) 436.

[12] A. A. Slavnov. Theor. Math. Phys. **10** (1972) 99.

[13] W. H. Furry. Phys. Rev. **51** (1937) 127.

[14] S. Coleman. Aspects of Symmetry, Cambridge University Press, 1985

[15] T-Y Wu, W-Y Pauchy Hwang. Relativistic Quantum Mechanics and Quantum Fields, World Scientific, 1991.

[16] S. J. Chang. Introduction to Quantum Field Theory, World Scientific, 1991.

[17] S. Weinberg. The Quantum Theory of Fields II, Cambridge University Press, 1996.