ISSN 2347-3487

Non-abelian whole gauge symmetry

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Abstract

The wholeness principle is analysed for non-abelian gauge symmetry. This principle states that nature acts through grouping. It says that physical laws should be derived from fields associations. At this work, we consider on the possibility of introducting a non-abelian fields set { $A_{\mu I}^{a}$ } under a common gauge parameter.

A Yang-Mills extension is studied. Taking the $SU(N)$ symmetry group with different potential fields rotating under a same group, new fields strengths are developed. They express covariant entities which are granular, collective, correlated, and not necessarily Lie algebra valued. They yield new scalars and a Lagrangian beyond Yang-Mills is obtained. Classical equations are derived and $(2N+7)$ equations are developed.

A further step is on how such non-abelian whole symmetry is implemented at $SU(N)$ gauge group. For this, it is studied on the algebra closure and Jacobi identities, Bianchi identities, Noether theorem, gauge fixing, BRST symmetry, conservation laws, covariance, charges algebra. As result, one notices that it is installed at $SU(N)$ symmetry independently on the number of involved fields. Given this consistency, Yang-Mills should not more be considered as the unique Lagrangian performed from $SU(N)$.

Introducting the BRST symmetry an invariant $\mathcal{L}_{eff}$ is stablished. The BRST charge associated to the N-potential fields system is calculated and its nilpotency property obtained. Others conservations laws involving ghost scale, global charges are evalued showing that this whole symmetry extension preserve the original Yang-Mills algebra. Also the ghost number is conserved. These results imply that Yang-Mills should be understood as a pattern and not as a specific Lagrangian.

Concluding, an extended Lagrangian can be constructed. It is possible to implement a non-abelian whole gauge symmetry based on a fields set { $A_{\mu I}^{a}$ }. Its physical feature is a systemic interpretation for the physical processes. Understand complexity from whole gauge symmetry.

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Keywords

Beyond Yang-Mills; Systemic Gauge Symmetry; Non-Abelian Whole Gauge Model.

1 Introduction

Gauge symmetry is guiding physics [1]. The physical laws search is being determined by symmetry groups. They carry the lemma where the numbers of gauge fields should be given by the number of generators of a given group. Under this principle Yang-Mills theories have been developed [2, 3]. Nevertheless it is possible to move beyond to this situation by including an undefined number of potential fields rotating under a same symmetry group [4]. Different origins based on Kaluza-Klein [5], supersymmetry [6], fibre bundle [?], σ-model [7] have already been studied to consider an initial set of fields transforming under a common gauge group as

$$A_{\mu I}' = UA_{\mu I}U^{-1} + \frac{i}{g_I}\partial_\mu U.U^{-1}, \qquad (1)$$

where $I = 1,...,\mathrm{N}$ and $U = e^{i\alpha_a t_a}$. The matrices t_a are the group generators of $SU(N)$. This matrices satisfy the Lie algebra for $SU(N)$. The index a is an internal indice and run according to the group's choice.

Eq. (1) indicates the existence of a non-abelian gauge symmetry involving different potential fields. It introduces the meaning of wholeness through gauge symmetry. Consequently emerges a new concept for physical laws be understood. Its whole symmetry deploys an ab initio for describing a systemic nature. For this, it constructs a fields association under a same gauge parameter. Considering that such fields satisfy the Borscherâ€™s theorem [8], one can redefine them. To get a better transparency on symmetry, one should write the model in terms of the $\{D_\mu, X_{\mu i}\}$ fields basis, where $D_\mu \equiv D_\mu^a t^a$ is defined as

$$D_\mu = \sum_I A_{\mu I}, \qquad (2)$$

with

$$D_\mu \to D_\mu' = UD_\mu U^{-1} + \frac{i}{g}\partial_\mu U.U^{-1}, \qquad (3)$$

and where $X_{\mu i} \equiv X_{\mu i}^a t^a$ are potential fields

$$X_{\mu 1} = A_{\mu 1} - A_{\mu 2}, A_{\mu(\mathrm{N}-1)} = A_{\mu 1} - A_{\mu\mathrm{N}}. \qquad (4)$$

with

$$X_{\mu i} \to X_{\mu i}' = UX_{\mu i}U^{-1}, \qquad (5)$$

where $i = 2,...,\mathrm{N}$. Thus the field D_μ works as the usual gauge field and the fields $X_{\mu i}$ as a kind of vctor-matter fields transforming in the adjoint representation. Geometrically, the potential fields $X_{\mu i}$ can be originated from the torsion tensor of the higher-dimensional manifold that spontaneously compactify to $M^4 \times B^4$,where B^4 is the Minkowski space-time and B^k some k-dimensional internal space. Thus the origin of the potential fields can be treated back to the vielbein, spin-connection and potential fields of higher-dimensional gravity-matter coupled theory spontaneously compactified for an internal space with torsion [5].

Nevertheless by definition, the physical fields are that ones which physical masses are the poles of two-point Green functions. For this, one has to diagonalize the transverse sector by introducting a matrix Ω [9]. The $\{D_\mu, X_{\mu i}\}$ basis is not the physical basis. It yields an operation guaranteed by the Borscher's theorem saying that physics must be nondependent under fields reparametrizations [8]. Thus, the physical basis $\{G_{\mu I}\}$ is obtained rotating as

$$D_\mu = \Omega_{1I}G_\mu^I, X_{\mu i} = \Omega_{iI}G_\mu^I \qquad (6)$$

and so, given the Ω matrix invertible condition

$$\Omega_{IK}\Omega_{KJ}^{-1} = \delta_{IJ} \qquad (7)$$

one gets

$$G_{\mu I} \rightarrow G_{\mu I}' = UG_{\mu I}U^{-1} + \frac{i}{g_I}\partial_\mu U.U^{-1}, \quad (8)$$

where $g_I = \frac{g}{\Omega_{I1}^{-1}}$. It is understood the notation $G_{\mu I} \equiv G_{\mu I}^a t^a$. The presence of different coupling constants means on the possibility for coupling with different currents.

The outline of the paper is organized as follows. The methodology is first to expose the new aspect originated from $SU(N)$ symmetry, and then, understand how through the gauge parameter and group generators the non-abelian symmetry is implemented. So at section 2, the non-abelian fields set symmetry { $A_{\mu I}^a$ } is proposed through an extended Lagrangian with respect to Yang-Mills. In section 3, from internal mechanisms one studies that this extended model is consistent with the symmetry skeleton which antecedes the Lagrangian. In the next two sections, Bianchi and Noether identities are derived. At section 6, the Lagrangian is divided in different pieces according to the scalars produced from generators decomposition. In section 7 one extracts on the classical equations showing about granular and collective space-time evolutions, covariance and with relationships beyond Lie algebra. Conserved currents are explored at section 8. The energy-momentum tensor is expressed at section 9. On symmetries as BRST, ghost scale and global gauge transformations, corresponding charges algebras and ghost number conservation are left for section 10. The corresponding Slavnov-Taylor Identity is written at section 11. Concluding remarks are posted at section 12, saying on the possibility of a systemic physics be described based on the whole symmetry principle.

2 Non-abelian whole Lagrangian

A non-abelian gauge symmetry association is defined through equations (1), (3), (5), (8). They are showing that the $SU(N)$ symmetry can be worked out through different fields basis as { $A_{\mu I}$ }, $\{D_\mu, X_{\mu i}\}$, $\{G_{\mu I}\}$. However, $\{D_\mu, X_{\mu i}\}$ is called the constructor basis due to the fact that, under this field-referential, the gauge invariance origin for the Lagrangian terms become more immediate. This is because the field D_μ works as the usual gauge field and the fields $X_{\mu i}$ transform covariantly.

The candidate for non-abelian whole Lagrangean will contain granular and collective contributions coming from antisymmetric, symmetric and semi-topological sectors [10]. Working out the Lagrangian in constructor basis, one gets

$$L_{GI}(D_\mu, X_\mu^i) = tr\left[(Z_{\mu\nu} + z_{\mu\nu})^2\right] + \eta tr\left[(Z_{\mu\nu} + z_{\mu\nu})(\tilde{Z}^{\mu\nu} + \tilde{z}^{\mu\nu})\right] - \frac{1}{2}m_{ij}X_\mu^i X^{\mu j}, \quad (9)$$

where $Z_{\mu\nu}$ is the most general covariant field strength with granular dependence on fields, and $z_{\mu\nu}$ is associated to collective fields. $\tilde{Z}^{\mu\nu}$ means $\varepsilon_{\mu\nu\rho\sigma}Z^{\rho\sigma}$.

Decomposing on antisymmetric and symmetric sectors

$$Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)} \quad (10)$$

where

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X_{[\mu\nu]}^i, \quad (11)$$

with the following granular field strength

$$D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu + ig[D_\mu, D_\nu], \quad (12)$$

and

$$X_{[\mu\nu]}^i = \partial_\mu X_\nu^i - \partial_\nu X_\mu^i + ig\left([D_\mu, X_\nu^i] - [D_\nu, X_\mu^i]\right). \quad (13)$$

For the symmetric sector,

$$Z_{(\mu\nu)} = \beta_i X_{(\mu\nu)}^i + \rho_i g_{\mu\nu} X_\alpha^{\alpha i}, \quad (14)$$

with $g_{\mu\nu}$ the metric tensor of Minkowski space and

$$X^{i}_{(\mu\nu)} = \partial_{\mu} X^{i}_{\nu} + \partial_{\nu} X^{i}_{\mu} + ig\left([D_{\mu}, X^{i}_{\nu}] + [D_{\nu}, X^{i}_{\mu}]\right) \quad (15)$$

Similarly for the collective field strength,

$$z_{\mu\nu} = z_{[\mu\nu]} + z_{(\mu\nu)} \quad (16)$$

where

$$z_{[\mu\nu]} = \gamma_{[ij]} X^{i}_{\mu} X^{j}_{\nu} + a_{(ij)} [X^{i}_{\mu}, X^{j}_{\nu}] + b_{[ij]} \{X^{i}_{\mu}, X^{j}_{\nu}\}, \quad (17)$$

and

$$z_{(\mu\nu)} = a_{[ij]} [X^{i}_{\mu}, X^{j}_{\nu}] + b_{(ij)} \{X^{i}_{\mu}, X^{j}_{\nu}\} + u_{[ij]} g_{\mu\nu} [X^{i}_{\alpha}, X^{\alpha j}] + v_{(ij)} g_{\mu\nu} \{X^{i}_{\alpha}, X^{\alpha j}\}. \quad (18)$$

Notice that $Z_{\mu\nu}$ and $z_{\mu\nu}$ are not necessarity Lie algebra valued as it is $F_{\mu\nu}$ in the usual Yang-Mills theory. However in order to explore the abundance of gauge scalars that such extended model offers one should also consider all possible group-valued structures in the non-irreducible sector contribution.

Besides that, one can yet to express the gauge fixing term so that the Lagrangian Eq. (9) become $L_{GI} + L_{GF}$, with

$$L_{GF} = \frac{1}{\xi}\left[\partial_{\mu}(D^{\mu} + \sigma_{i} X^{\mu i})\right]^{2} \quad (19)$$

The transverse diagonalized gauge invariant Lagrangian, which means the physical Lagrangian, is given by

$$L_{GI}(G_{\mu I}) = tr\left[(Z_{\mu\nu} + z_{\mu\nu})^{2}\right] + \eta tr\left[(Z_{\mu\nu} + z_{\mu\nu})(\tilde{Z}^{\mu\nu} + \tilde{z}^{\mu\nu})\right] - \frac{1}{2} m^{2}_{IJ} G^{I}_{\mu} G^{\mu J} + \xi_{IJ} (\partial_{\mu} G^{\mu I})(\partial_{\nu} G^{\nu J}),$$

(20)

where the corresponding field strengths one written in terms of physical fields. Rewriting Eq. (11),

$$Z_{[\mu\nu]} = a_{I} (\partial_{\mu} G^{I}_{\nu} - \partial_{\nu} G^{I}_{\mu}) - iga_{(IJ)} [G^{I}_{\mu}, G^{J}_{\nu}], \quad (21)$$

with

$$a_{I} = d\Omega_{1I} + \alpha_{i} \Omega^{i}_{I}, \; a_{(IJ)} = a_{I} \Omega_{1J} + \alpha_{i} \Omega_{1I} \Omega^{i}_{J}. \quad (22)$$

For Eq. (17),

$$z_{[\mu\nu]} = \gamma_{[IJ]} G^{I}_{\mu} G^{J}_{\nu} + b_{(IJ)} [G^{I}_{\mu}, G^{J}_{\nu}] + c_{[IJ]} \{G^{I}_{\mu}, G^{J}_{\nu}\} \quad (23)$$

with

$$\gamma_{[IJ]} = \gamma_{[ij]} \Omega^{i}_{I} \Omega^{j}_{J}$$

$$b_{(IJ)} = a_{(ij)} \Omega^{i}_{I} \Omega^{j}_{J}$$

$$c_{[IJ]} = b_{[ij]} \Omega^{i}_{I} \Omega^{j}_{J} \quad (24)$$

For Eq. (14),

$$Z_{(\mu\nu)} = \beta_{I} G^{I}_{(\mu\nu)} + \rho_{I} g_{\mu\nu} G^{\alpha I}_{\alpha}, \quad (25)$$

with

$$G^{I}_{(\mu\nu)} = \partial_{\mu} G^{I}_{\nu} + \partial_{\nu} G^{I}_{\mu} + ig_{J}\left([G^{I}_{\mu}, G^{J}_{\nu}] - [G^{I}_{\nu}, G^{J}_{\mu}]\right), \quad (26)$$

and

$$g_I = g\Omega_{1I}, \beta_I = \beta_i\Omega^i_I, \rho_I = \rho_i\Omega^i_I. \quad (27)$$

For Eq. (18),

$$z_{(\mu\nu)} = b_{[IJ]}[G^I_\mu, G^J_\nu] + c_{(IJ)}\{G^I_\mu, G^J_\nu\} + u_{[IJ]}g_{\mu\nu}[G^I_\alpha, G^{\alpha J}] + v_{(IJ)}g_{\mu\nu}\{G^I_\alpha, G^{\alpha J}\}. \quad (28)$$

with

$$b_{[IJ]} = b_{[ij]}\Omega^i_I\Omega^j_J, c_{(IJ)} = c_{(ij)}\Omega^i_I\Omega^j_J$$

$$u_{[IJ]} = u_{[ij]}\Omega^i_I\Omega^j_J, v_{(IJ)} = v_{(ij)}\Omega^i_I\Omega^j_J \quad (29)$$

Notice that eqs. (21), (23), (26), (28) transform covariantly.

3 Symmetry skeleton

We consider five preliminary types of fundamental mechanism analysis on the conditions for including more fields. They are based on counting the number of degrees of freedom, geometry, supersymmetry, symmetry and dynamics. This section intends to explore the most subtle, the fourth hability. It is based on the following instructions: algebra closure, BRST algorithm, Bianchi identities, local Noether theorem, covariance. The function of these symmetry topics will be to study whether the $SU(N)$ gauge group accomodates the presence of N potential fields rotating under the same group parameters. These arguments purely based on symmetry will work as basis for including an extended Lagrangian to SU(N) Symmetry Group. Gauge fields, being Lie algebra-valued carry group properties. Therefore, a first call of command from symmetry is to verify whether the set of gauge transformations implemented by such general gauge theory is able to build up one algebra. Considering the physical sector, one gets the field transformations

$$\delta G^a_{\mu I} = [\mathrm{D}_{\mu I}\alpha(x)]^a \quad (30)$$

where

$$[\mathrm{D}_{\mu I}\alpha(x)]^a = \Omega^{-1}_{I1}\partial_\mu\alpha^a(x) + [G_{\mu I}(x), \alpha(x)]^a \quad (31)$$

and taking two successive gauge transformations, one gets that the algebra of infinitesimal transformations closes:

$$[\delta(\alpha 1), \delta(\alpha 2)]G^a_{\mu I} = g f^a_{bc}\mathrm{D}_{\mu I}(\alpha^b_2\alpha^c_1) \quad (32)$$

The Jacobi identity of the Lie algebra imposes a next relationship. It is necessary to show that these infinitesimal transformations generate the whole invariance groups. Verify that the Jacobi identity acting on field $G_{\mu I}$ is satisfied. From Eq. (32), it yields

$$\{[\delta(\alpha 1), [\delta(\alpha 2), \delta(\alpha 3)]] + cycl.perm.\}G^a_{\mu I} = 0 \quad (33)$$

Eq. (32) and Eq. (33) apply to any tensorial combination. Concluding this first consistency test, one can state that the local properties for the N-potential fields of the classical transformations are summarized by Eq. (30), Eq. (32) and Eq. (33).

The next text includes quantum aspects. It is the BRST algorithm. BRST transformations [11] have been considered a very useful technique to probe the internal structure of a gauge theory. By taking supplementary fields with unphysical statistics it was noticed, initially, as a method to originate the Ward identities and also to compensate the effects due the quantum propagation of zero modes which are contained in a potential field. However it was later understood that the BRST framework also reveals more intrinsic aspects of the theory. Besides solving the gauge dependence of the gauge-fixing term, it brings a perspective where it anticipates the notion of Lagrangian. This means that BRST signature appears at the level of first principle for detecting a full Lagrangian. In this way, as the ghosts and the auxiliary fields are unphysical quanta, one could say that the BRST method works like the X-ray technique for detecting a possible physical illness embedded in the body of the theory. For instance, by computing the cohomology of the BRST charge, one is able to infer about the stability and absence of anomalies in the theory [12].

Considering that the BRST and anti-BRST symmetries [13] penetrate in the symmetry instructions for organizing the most general gauge invariant Lagrangian, our proposal is to use it for testing how for gauge theories will be able to absorb the presence of more potential fields. We are going to follow the Baulieu & Thierry-Mieg prescription [14]. There the ghost technical device takes from the very beginning, for predicting the Lagrangian, a set of basic fields D_μ, c, $\bar{c}$, b ; and in our case it should be complemented by the presence of X^i_μ, $\mathrm{N}-1$ massive vector fields.

The inclusion of the auxiliary field b, interpreted as a Lagrangian multiplier for the gauge-fixing condition, promotes the BRST and anti-BRST as fundamental symmetries of gauge theories. The symmetry generators s and $\bar{s}$ of fields into ghosts become independent of the notion of Lagrangian in the sense that transformations do not depend any more on the gauge-fixing term of the Lagrangian. Writing in terms of the fields,

$$sD_\mu = \nabla_\mu c, \bar{s}D_\mu = \nabla_\mu \bar{c},$$

$$sX^i_\mu = [X^i_\mu, c], \bar{s}X^i_\mu = [X^i_\mu, \bar{c}],$$

$$sc = -\frac{1}{2}[c,c], \bar{s}c = -[\bar{c},c]-b,$$

$$s\bar{c} = b, \bar{s}\bar{c} = -\frac{1}{2}[\bar{c},\bar{c}],$$

$$sb = 0, \bar{s}b = -[\bar{c},b], \tag{34}$$

where

$$\nabla_\mu = \partial_\mu \text{-} + g[D_\mu, \text{-}], \tag{35}$$

$$\mathrm{D}_\mu = \partial_\mu \text{-} + g[D_\mu, \text{-}] + g_i[X^i_\mu, \text{-}], \tag{36}$$

and

$$s\bar{c} + \bar{s}c = [\bar{c}, c]. \tag{37}$$

Information contained in Eq. (37) can be checked by Eq. (34). They completely determine the properties of an extended gauge symmetry.

The algebraic method will be scheduled in terms of Lorentz invariance, dimension analysis, ghost number, BRST and anti-BRST invariance, hermiticity and global invariance. It builds up the following expansion

$$\mathrm{L}^{ext}_Q = \mathrm{L}^{ext}_1(cl) + s\bar{K}^{ext}_3 + sK^{ext}_3 + s\bar{s}K^{ext}_2 \tag{38}$$

where L^{ext}_1, K^{ext}_2, $\bar{K}^{ext}_3$ and K^{ext}_3 are polynomial functions on all fields satisfying the above conditions. The operators s and $\bar{s}$ obey the following nilpotency relations

$$s^2 = s\bar{s} + \bar{s}s = \bar{s}^2 \tag{39}$$

which are equivalent to the closure of the classical algebra and to the Jacoby identity.

Now, the next step is to prove that this full extended BRST invariance Eq. (34) leads to the most general non-abelian gauge independent physics. From the fact that L_Q has dimension four and zero ghost number, one immediately extends the result K_2 for Yang-Mills theory by

$$K^{ext}_2 = K_2 + \alpha_{4i} D_\mu X^{\mu i} \tag{40}$$

where

$$K_2 = \alpha_3 D^2_\mu + \alpha_5 \bar{c}c \tag{41}$$

Notice that the term $\alpha_{4ij} X^i_\mu X^{j\mu}$ obeys all requisites to enter the K^{ext}_2 definition. However, they are s and $\bar{s}$ invariants giving a null contribution to L_Q. They are then irrelevant.

Looking for $\bar{K}^{ext}_3$, one explores a combination of four independent monomials, three present in the discussion of Yang-Mills theory

$$b\bar{c}, \tag{42}$$

$$b\bar{c}+\frac{1}{2}c[\bar{c},c]=-\bar{s}(c\bar{c}), \tag{43}$$

$$D^{\mu}\partial_{\mu}\bar{c}=\frac{1}{2}\bar{s}D_{\mu}^{2}, \tag{44}$$

and still fourth possibility in our case

$$\alpha_i X^{\mu i}\partial_{\mu}\bar{c}=\alpha_i\bar{s}(D_{\mu}X^{\mu i}) \tag{45}$$

However one can eliminate Eq. (43) - Eq. (45) from the game since they are of the type $\bar{s}K_2$. So the result here is the same as in ordinary Yang-Mills theories

$$K_2^{ext}=\bar{K}_3=\alpha_6 b\bar{c} \tag{46}$$

We should like to emphasize here that for most of the classical Lie groups, but not $SU(2)$, there exists an invariant symmetric tensor of $rank-3$, d_{abc}. In these cases one must still consider others candidates to $\bar{K}_3^{ext}$: $d_{abc}D_{\mu}^{a}D^{\mu b}\bar{c}^{c}$, and $\alpha_i d_{abc}D_{\mu}^{a}X^{\mu ib}\bar{c}^{c}$. However, according to the prescriptions, we discard them because they break the $\bar{s}$ -invariance of the Lagrangian. However a difference here is that these symmetric tensors can appear conveniently in the theory through the L_1 contribution.

In order to determine K_3^{ext}, one proceeds in the same way as for $\bar{K}_3^{ext}$. The result is again as in Yang-Mills theory

$$\bar{K}_3^{ext}=K_3=\alpha_6 b\bar{c}. \tag{47}$$

Observe without loss of generality that one can write

$$s\bar{K}_3=\bar{s}K_3=\alpha_6 b^2=\alpha_6 s(b\bar{c})=-\alpha_6\bar{s}(bc). \tag{48}$$

Finally we have to find L_1. From $\bar{K}_2^{ext}$ and K_3, Fadeev-Popov and the gauge-fixing terms are reproduced in the extended Yang-Mills approach. Nevertheless a main difference lies on the fact that L_1 is not only a function of the D_{μ} -gauge field but also a function of X_{μ}^{i}, vector fields. Then, as BRST invariance is equivalent to the classical gauge invariance, the most general possibility is given by

$$\mathsf{L}_1=tr\left[\alpha_1(Z_{\mu\nu}+z_{\mu\nu})(Z^{\mu\nu}+z^{\mu\nu})+\alpha_2\varepsilon_{\mu\nu\rho\sigma}(Z^{\mu\nu}+z^{\mu\nu})(Z^{\rho\sigma}+z^{\rho\sigma})-\frac{1}{2}m_{ij}^{2}X_{\mu}^{i}X^{\mu j}\right], \tag{49}$$

where $Z_{\mu\nu}$ and $z_{\mu\nu}$ are written in the previous section. One can indeed check, after some algebra, that really

$$\bar{s}\mathsf{L}_1=s\mathsf{L}_1=0 \tag{50}$$

which is the last requirement to establish our final Lagrangian. We have therefore shown that the most general non-abelian Lagrangian satisfying Baulieu and Thierry-Mieg programme is really the effective Lagrangian we have been using from the departure:

$$\mathsf{L}_Q=L_{GI}(D_{\mu},X_{\mu}^{i})+tr[s\bar{s}\left(\alpha_3 D_{\mu}^{2}+\alpha_{4i}D_{\mu}X^{\mu i}+\alpha_5\bar{c}c\right)+\alpha_6 b^2]. \tag{51}$$

The b field can be eliminated by using its equation of motion and in the limit of Landau gauge one gets the generating functional for Green functions:

$$Z[D_{\mu},X^{\mu i},c,\bar{c},J,J_i,\bar{\eta},\eta]=\int \mathsf{D}D_{\mu}\mathsf{D}X^{\mu i}\mathsf{D}c\mathsf{D}\bar{c}\,\delta[\partial^{\mu}(D_{\mu}+\sigma_i X_{\mu}^{i})]e^{iS_{eff}}, \tag{52}$$

where $J_i=J\sigma_i$, with J the external source associated to D_{μ} field and J_i to the X_{μ}^{i} vector field and

$$S_{eff}=\int d^4x\{L_{GI}(D_{\mu},X_{\mu}^{i})+tr[\partial_{\mu}\bar{c}\mathsf{D}^{\mu ext}c+J^{\mu}D_{\mu}+J_{\mu}^{i}X^{\mu i}+c\bar{\eta}+\bar{c}\eta]\}. \tag{53}$$

The third instruction dictated by the symmetry relies exclusively on algebraic identities, as for instance, the Bianchi identities. Mathematical considerations yield two relationships to be analyzed and explored by each particular theory. These are:

$$[A,[B,C]]+[C,[A,B]]+[B,[C,A]]=0, \tag{54}$$

$$+[C,\{A,B\}]+[B,\{C,A\}]=0, \tag{55}$$

Identities Eq. (54) and Eq. (55) will take different forms relative to the structure of the particular theory under consideration, as it becomes evident if we apply them to the cases of general relativity or Yang-Mills. Nevertheless, their implementation in physics is not immediate. In order to transform them into a type of constraint equation, they must first obey a kind of physical closure. This means that Eq. (54) and Eq. (55) must be consistent with dimensionality and covariance considerations. Thus a good candidate that gauge theories provide to surpass such a convenience is the covariant derivative. So, from Eq. (54), one gets the following identity

$$\mathrm{D}_{\rho}T_{\mu\nu}+\mathrm{D}_{\nu}T_{\rho\mu}+\mathrm{D}_{\mu}T_{\nu\rho}=0 \tag{56}$$

with

$$T_{\mu\nu}=[\mathrm{D}_{\mu},\mathrm{D}_{\nu}]. \tag{57}$$

From Eq. (55)

$$[\mathrm{D}_{\mu},\{\mathrm{D}_{\nu},\mathrm{D}_{\rho}\}]+[\mathrm{D}_{\rho},\{\mathrm{D}_{\mu},\mathrm{D}_{\nu}\}]+[\mathrm{D}_{\nu},\{\mathrm{D}_{\rho},\mathrm{D}_{\mu}\}]=0, \tag{58}$$

the operational identity will be

$$\mathrm{D}_{\mu}S_{\nu\rho}+\mathrm{D}_{\nu}S_{\rho\mu}+\mathrm{D}_{\rho}S_{\mu\nu}-6\partial_{\mu}\partial_{\nu}\partial_{\rho}=0 \tag{59}$$

with

$$S_{\mu\nu}=\{\mathrm{D}_{\mu},\mathrm{D}_{\nu}\}. \tag{60}$$

The significant physical question for the Bianchi identities of the extended theory concerns the possible covariant derivatives that can be built up. Since this model provides two basis $\{D_{\mu},X_{\mu i}\}$ and $\{G_{\mu I}\}$ one should take them both as a laboratory to grow the covariant derivatives. From the first set, one gets two types of covariant derivatives: $\nabla_{\mu}(D_{\mu})$ given by Eq. (35) and $\mathrm{D}_{\mu}(D_{\mu},X_{\mu i})$ through Eq. (36). Now, taking these covariant derivatives in Eq. (56) or Eq. (59), one gets different kinds of Bianchi identities. While the second Jacobi identity is more useful for effective theories, Eq. (54) serves our interest of exploring about the physical fields.

Thus taking the physical set, the corresponding covariant derivative is $\mathrm{D}_{\mu I}=\partial_{\mu}+g_{I}G_{\mu I}$. From eq (54) one gets the most general identity

$$[\mathrm{D}_{\mu I}(G_{I}),[\mathrm{D}_{\nu J}(G_{J}),\mathrm{D}_{\rho K}(G_{K})]]+cycl.perm.=0, \tag{61}$$

which contains the basic conditions for being proposed as a physical equation. It has the covariant property and correct dimensionality. Then, splitting up the corresponding field strength in symmetric and antisymmetric piece, one gets the following identity:

$$\partial_{\mu}G^{I}_{[\nu\gamma]}+\partial_{\nu}G^{I}_{[\gamma\mu]}+\partial_{\gamma}G^{I}_{[\mu\nu]}=0 \tag{62}$$

where

$$G^{I}_{[\mu\nu]}=[\mathrm{D}_{\mu}(G_{I}),\mathrm{D}_{\nu}(G_{I})]. \tag{63}$$

Eq. (62) means that this extended model contains N Bianchi identities, where each one is associated to a corresponding physical field. A similar result one gets from Eq. (59) for effective cases.

The attempt in this section is being to identify the existence of instructions in gauge theories for assuming a number of potential fields different from the number of group generators. So as a final aspect for analyzing a possible origin for this extended model is by means of invariance of the action. It leads to Euler-Lagrange equations which will be studied in the following sections and contributions from surface terms. The effort here will be just of introducing more fields at the minimal action principle. It gives,

$$\delta S=\int d^4x\left[\frac{\delta L}{\delta\Phi_I}-\partial_\mu\frac{\delta L}{\delta(\partial_\mu\Phi_I)}\right]\delta\Phi_I+\int d^4x\left[\frac{\delta L}{\delta(\partial_\mu\Phi_I)}\delta\partial_\mu\Phi_I+L\delta x^\mu\right] \tag{64}$$

where $\Phi_I\equiv\{D_\mu,X_{\mu i}\}$. Eq. (64) shows that while the conservation laws are to be manifested for all the system, the equation of motion appear individualized for each field, separately. Therefore an emphasis from this result is that the different identities which the Noether theorem and total angular momentum gives rise to are conservation laws for all system containing N fields.

The local Noether theorem for a non-abelian gauge involving N potential fields in the same group is understood by the three following equations:

$$\partial_\mu J_N^{\mu a}(D,X_i)=0, \tag{65}$$

$$\frac{i}{g}\frac{\delta L}{\delta D_\mu^a}=J_N^{\mu a}(D,X_i), \tag{66}$$

$$\frac{\delta L}{\delta(\partial_\mu D_\nu^a)}\partial^\mu\partial^\nu\alpha^a(x)=0, \tag{67}$$

where

$$J_N^{\mu a}(D,X_i)=\left[\frac{\delta L}{\delta(\partial_\mu D_\nu)},D_\nu\right]^a+\left[\frac{\delta L}{\delta(\partial_\mu X_\nu^{ia})},X_\nu^i\right]^a. \tag{68}$$

Thus, from the analysis of the global and local instructions given by Eq. (65) and Eq. (66), one gets that there is an explicit information on how symmetry moves a room to accommodate the X_μ^{ia} fields. It is calculated through Eq. (68). On the other hand, Eq. (67) only informs that $\frac{\delta L}{\delta(\partial_\mu D_\nu^a)}$ is totally antisymmetric. However, implicitly, from dimensional analysis and gauge invariance, it is also possible to guess that there can be X_μ^i fields. Eq. (67) contains indications for their presence through a coupling with the genuine gauge field D_μ. It can be made through mixed propagators and interacting terms. For instance, Eq. (11) plus Eq. (17) satisfy Eq. (67).

The inclusion of more potential fields should rather be characterized as an extension of the usual case. Therefore our preference in writing the Noether equations in terms of the set $\Phi_I\equiv\{D_\mu,X_{\mu i}\}$, where it is easy to get the boundary conditions by turning off the X_μ^i fields. From this basis, we will analyze three pieces of information from Noether theorem. First it is to reobtain the old result where symmetry current derived from inhomogeneous D_μ field will play a dual role. Its expression obtained from Noether theorem coincides with the relationship which will be performed for the corresponding D_μ-equation. Another consistency test is from Eq. (66), or taking its divergence. Then, the proposed Lagrangian must verify the equality between the left-hand side and right-hand side. The third information that Noether theorem provides should not be understood as a conservation law but as a constraint of the theory. Substituting the weaken condition Eq. (67) in Eq. (66) one gets

$$\partial_\mu T^{[\mu\nu]}=J^\nu, \tag{69}$$

where $T_{[\mu\nu]}$ is a skew-symmetric tensor depending on D_μ and the X_μ^i fields. J^ν in our case is essentially made of the "matter" of X_μ^i fields we have put in the game of the extended model. Thus the axiomatic approach to defining gauge theories as the theories where the equation

$$\partial_\mu F^{\mu\nu a}=J^{\nu a}, \tag{70}$$

should be obtained as a symmetry constraint is enlarged ($F^{\mu\nu a}$ is the QCD field strength). Eq. (69) reexamines this reflex between symmetry and Coulomb's law.

In the physical set the local Norther theorem is transposed as

$$\partial_\mu J_N^\mu(G) = 0, \tag{71}$$

$$\frac{i}{g_I}\partial_\mu \frac{\delta L}{\delta(\partial_\mu G_\mu^I)} = J_N^\mu, \tag{72}$$

$$\frac{i}{g_I}\frac{\delta L}{\delta(\partial_\mu G_\nu^I)}\partial^\mu\partial^\nu\alpha(x) = 0, \tag{73}$$

and

$$J_N^{\mu a}(G) = \left[\frac{\delta L}{\delta(\partial_\mu G_\nu^I)}, G_\nu^I\right]^a. \tag{74}$$

It then appears clearly that Coulomb's law does not contain a necessary compromise with a non-dynamical origin. It was just a coincidence for the case involving one field. Observe also that even its strongest condition, Eq. (73) does not require for the fields $G_{\mu I}$ being not dynamical.

Another conservation law concerns to the total angular momentum $J_{\mu\nu}^k = L_{\mu\nu}^k + S_{\mu\nu}^k$. It contains the orbital angular momentum plus the spin contribution. It gives

$$\partial_\mu J_{k\lambda}^\mu = 0, \tag{75}$$

where $L_{\mu\nu}^k = x_k\theta_\lambda^\mu - x_\lambda\theta_k^\mu$ and the spin-current $S_{\mu\nu}^k = \frac{\partial L}{\partial(\partial_\mu G_{\rho I})}(\Sigma_{k\lambda})^\sigma G_{\rho I}$.

To conclude this section, we would note that the so called four types of internal mechanisms work not only to detect the presence of N fields but also to isolate the identity carried by each of them. The first instruction shows, formally, the possibility of more than one field to be transformed under the same group parameter $\alpha^a(x)$; from the Baulieu & Thierry-Mieg procedure one gets a method to assume an extended Lagrangian; the existence of different equations associated to each field spots be developed through the Bianchi identities; the minimal action principle brings a conjunction between the whole system involving N fields and the individualization of each quanta through the variational principle. There the identity of each field is obtained through its correspondent covariant equation of motion, while the system identity is organized through conservation laws. This means that the conservation of energy-momentum, angular momentum and internal charges are instructions only for the system as a whole.

Consequently the symmetry skeleton is able to support more "flash": the presence of more potential fields besides the usual gauge field. The principle that the number of potential fields must be equal to the number of group generators is enlarged. The $SU(N)$ group allows to introduce different fields rotating under the same symmetry and associated with different symmetry weights Ω_I^{-1}, and coupling constants g_I. However it is still necessary to ascertain a fifth consistency of the above skeleton for assuming more fields. It is to study on the covariance properties of the equations of motion. It will be considered in the subsequent sections

4 Bianchi identities

Considering the covariant derivatives Eq. (35), Eq. (36) and the collective expression $x_{\mu\nu}^{ij} = [X_\mu^i, X_\nu^j]$, one gets the following Bianchi identities:

$$\nabla_\mu D_{\nu\rho} + \nabla_\nu D_{\rho\mu} + \nabla_\rho D_{\mu\nu} = 0$$

$$D_\mu X_{[\nu\rho]}^i + D_\nu X_{[\rho\mu]}^i + D_\rho X_{[\mu\nu]}^i + ig([X_\mu^i, D_{\nu\rho}] + [X_\nu^i, D_{\rho\mu}] + [X_\rho^i, D_{\mu\nu}]) = 0$$

$$D_\mu X_{(\nu\rho)}^i + D_\nu X_{(\rho\mu)}^i - D_\rho X_{(\mu\nu)}^i + ig([X_\mu^i, D_{\nu\rho}] - [X_\nu^i, D_{\rho\mu}] + [X_\rho^i, D_{\mu\nu}]) = 0$$

$$\mathrm{D}_{\mu}x^{ij}_{\nu\rho}+\mathrm{D}_{\nu}x^{ij}_{\rho\mu}+\mathrm{D}_{\rho}x^{ij}_{\mu\nu}+[X^{j}_{\nu},X^{i}_{[\rho\mu]}]+[X^{j}_{\mu},X^{i}_{[\nu\rho]}]+[X^{j}_{\rho},X^{i}_{[\mu\nu]}]=0 \quad (76)$$

Then, defining $z'_{[\mu\nu]}=a_{(ij)}x^{ij}_{\mu\nu}$, and $z'_{(\mu\nu)}=a_{(ij)}x^{ij}_{\mu\nu}$, one also derives the following expressions

$$\mathrm{D}_{\mu}z'_{[\nu\rho]}+\mathrm{D}_{\nu}z'_{[\rho\mu]}+\mathrm{D}_{\rho}z'_{[\mu\nu]}+a_{(ij)}([X^{i}_{\nu},X^{j}_{[\rho\mu]}]+[X^{i}_{\mu},X^{j}_{[\nu\rho]}]+[X^{i}_{\rho},X^{j}_{[\mu\nu]}])=0$$

$$\mathrm{D}_{\mu}z'_{(\nu\rho)}+\mathrm{D}_{\nu}z'_{(\rho\mu)}+\mathrm{D}_{\rho}z'_{(\mu\nu)}-a_{[ij]}([X^{i}_{\mu},X^{j}_{(\nu\rho)}]+[X^{i}_{\nu},X^{j}_{(\rho\mu)}]+[X^{i}_{\rho},X^{j}_{(\mu\nu)}])=0$$

$$\mathrm{D}_{\mu}z'_{[\nu\rho]}-\mathrm{D}_{\nu}z'_{[\rho\mu]}+\mathrm{D}_{\rho}z'_{[\mu\nu]}-a_{(ij)}([X^{i}_{\mu},X^{j}_{(\nu\rho)}]-[X^{i}_{\nu},X^{j}_{(\rho\mu)}]-[X^{i}_{\rho},X^{j}_{(\mu\nu)})=0$$

$$\mathrm{D}_{\mu}z'_{(\nu\rho)}-\mathrm{D}_{\nu}z'_{(\rho\mu)}+\mathrm{D}_{\rho}z'_{(\mu\nu)}+a_{[ij]}([X^{i}_{\mu},X^{j}_{[\nu\rho]}]-[X^{i}_{\nu},X^{j}_{[\rho\mu]}]-[X^{i}_{\rho},X^{j}_{[\mu\nu]})=0 \quad (77)$$

5 Noether identities

The local Noether theorem provides three relationships

$$\partial_{\mu}\begin{Bmatrix}2\lambda_{1}[Z^{[\mu\nu]},dD_{\nu}+\alpha_{i}X^{i}_{\nu}]+\lambda_{3}d[z^{[\mu\nu]},D_{\nu}]+\lambda_{3}[z^{[\mu\nu]},dD_{\nu}+\alpha_{i}X^{i}_{\nu}]\\+\beta_{i}[Z^{(\mu\nu)}+z^{(\mu\nu)},(2\xi_{1}+\xi_{3})X^{i}_{\nu}]+\rho_{i}g^{\mu\nu}[Z^{\alpha)}_{(\alpha}+z^{\alpha)}_{(\alpha},X^{i}_{\nu}]\end{Bmatrix}^{a}\alpha_{a}=0, \quad (78)$$

$$\begin{Bmatrix}-2\frac{N}{g}d(\lambda_{1}Z^{[\mu\nu]}+\lambda_{3}z^{[\mu\nu]})+[2\lambda_{1}Z^{[\mu\nu]}+\lambda_{3}dz^{[\mu\nu]},dD_{\nu}+\alpha_{i}X^{i}_{\nu}]\\+2\xi_{1}\left(\beta_{i}[Z^{(\mu\nu)},X^{i}_{\nu}]+\rho_{i}g^{\mu\nu}[Z^{\alpha)}_{(\alpha},X^{i}_{\nu}]\right)+\xi_{3}\left(\beta_{i}[z^{(\mu\nu)},X^{i}_{\nu}]+\rho_{i}g^{\mu\nu}[z^{\alpha)}_{(\alpha},X^{i}_{\nu}]\right)\end{Bmatrix}^{a}\partial_{\mu}\alpha_{a}=0,$$
(79)

$$\left[4d\lambda_{1}Z^{[\mu\nu]}+2d\lambda_{3}z^{[\mu\nu]}\right]^{a}\partial_{\mu}\partial_{\nu}\alpha_{a}=0. \quad (80)$$

6 Lagrangian scalars

The potential fields Lagrangian plays with different quanta. From group theory arguments one knows that a quadrivector carries information about different spin states. Neverthless as gauge invariance acts differently one the vector and scalar sectors, one expects that it will work as a source for rendering explicit a different dynamics for each one of those parts. So we should now split the Lagrangean in antisymmetric and simmetric parts rewrite Eq. (9) as

$$L(D_{\mu},X_{\mu i})=tr\left[\lambda_{1}Z_{[\mu\nu]}Z^{[\mu\nu]}+\lambda_{2}z_{[\mu\nu]}z^{[\mu\nu]}+\lambda_{3}Z_{[\mu\nu]}z^{[\mu\nu]}\right]$$
$$+tr\left[\xi_{1}Z_{(\mu\nu)}Z^{(\mu\nu)}+\xi_{2}z_{(\mu\nu)}z^{(\mu\nu)}+\xi_{3}Z_{(\mu\nu)}z^{(\mu\nu)}\right] \quad (81)$$

A new aspect in this whole gauge model is that fields strength are not just Lie algebra valued. They can be decomposed through groups terms t_a, $t_a t_b$, $[t_a,t_b]$, $\{t_a,t_b\}$, and one gets an expansion where each term transforms covariantly. It yields a Lagrangian whole expansion which englobes the usual Yang-Mills sector and the whole extension. Defining the field strength

$$F_{\mu\nu}\equiv Z_{[\mu\nu]}+z_{[\mu\nu]} \quad (82)$$

one gets

$$F_{\mu\nu}=A_{\mu\nu}\mathrm{I}+B^{a}_{\mu\nu}t^{a}+C^{ab}_{\mu\nu}t^{a}t^{b} \quad (83)$$

where

$$A_{[\mu\nu]}=\frac{1}{N}b_{[ij]}X^{i}_{\mu a}X^{aj}_{\nu},$$

$$B^{a}_{[\mu\nu]}=dD^{a}_{\mu\nu}+\alpha_{i}X^{ia}_{[\mu\nu]}+c^{abc}_{(ij)}X^{ic}_{\mu}X^{jb}_{\nu},$$

$$c^{ab}_{\mu\nu}=\frac{1}{2}\gamma_{ij}(X^{ia}_{\mu}X^{jb}_{\nu}-X^{ia}_{\nu}X^{jb}_{\mu}), \quad (84)$$

with

$$C^{abc}_{[ij]} = -ia_{(ij)} f^{abc} + b_{[ij]} d^{abc}. \qquad (85)$$

Similarly from,

$$S_{\mu\nu} \equiv Z_{(\mu\nu)} + z_{(\mu\nu)} \qquad (86)$$

one expands the symmetric field strength

$$\begin{aligned} S_{\mu\nu} &= (\beta_i X^{ia}_{(\mu\nu)} + \rho_i g_{\mu\nu} X^{\alpha ia}_{\alpha}) t^a \\ &+ \gamma_{(ij)} X^{ia}_{(\mu} X^{jb}_{\nu)} t^a t^b + \left(a_{[ij]} X^{ia}_{\mu} X^{jb}_{\nu} + u_{[ij]} g_{\mu\nu} X^{ia}_{\alpha} X^{\alpha jb}\right)[t^a, t^b] \\ &+ \left(b_{(ij)} X^{ia}_{\mu} X^{jb}_{\nu} + v_{(ij)} g_{\mu\nu} X^{ia}_{\alpha} X^{\alpha jb}\right)\{t^a, t^b\}. \end{aligned} \qquad (87)$$

Splitting

$$L = L_A + L_S, \qquad (88)$$

considering the antisymmetric sector,

$$L_A = trF_{\mu\nu} F^{\mu\nu}, \qquad (89)$$

and performing calculations, one obtains L_A being build up by 5 scalar meshes

$$\begin{aligned} L_A &= NA^2_{\mu\nu} + 2NA_{\mu\nu} C^{\mu\nu a}_a + B^a_{\mu\nu} B^{\mu\nu}_a + iNf^{abc} B^a_{\mu\nu} C^{\mu\nu bc} \\ &+ N\left(C^a_{\mu\nu a} C^b_{\mu\nu b} + C^{ab}_{\mu\nu} C^{\mu\nu ab} + \frac{1}{4} d_{3abcd} C^{ab}_{\mu\nu} C^{\mu\nu cd} \right), \end{aligned} \qquad (90)$$

with

$$d_{3abcd} = d_{abf} d_{cdf} - d_{acf} d_{bdf} + d_{adf} d_{bcf}. \qquad (91)$$

Similarly for the symmetric sector, one obtains a 8 meshes decomposition. Given

$$L_S = S_{\mu\nu} S^{\mu\nu}, \qquad (92)$$

one gets,

$$L_S = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8, \qquad (93)$$

where

$$L_1 = NA^2_{\mu\nu},$$

$$L_2 = iNf^{abc} A^a_{\mu\nu} B^{\mu\nu bc},$$

$$L_3 = 2iNf^{abc} A^a_{\mu\nu} C^{\mu\nu bc},$$

$$L_4 = B^{aa}_{\mu\nu} B^{\mu\nu bb} + B^{ab}_{\mu\nu} B^{\mu\nu ba} + \frac{N}{4} d_{3abcd} B^{ab}_{\mu\nu} B^{\mu\nu cd},$$

$$L_5 = C^{ab}_{\mu\nu}\left[2(B^{\mu\nu ba} - B^{\mu\nu ab}) + \frac{N}{2}(d_{3abcd} - d_{3abdc}) B^{\mu\nu cd} \right],$$

$$L_6 = 4B^{aa}_{\mu\nu}D^{\mu\nu}_{bb} + 2(B^{ab}_{\mu\nu} + B^{ba}_{\mu\nu})D^{\mu\nu}_{ba} + \frac{N}{2}B^{ab}_{\mu\nu}(d_{3abcd} + d_{3abdc})D^{\mu\nu}_{dc},$$

$$L_7 = \left[2(C^{ab}_{\mu\nu} - C^{ba}_{\mu\nu}) + \frac{N}{4}(d_{3abcd} - d_{3abdc} - d_{3bacd} + d_{3badc})C^{ab}_{\mu\nu}\right]C^{\mu\nu}_{cd},$$

$$L_8 = 4D^{aa}_{\mu\nu}D^{\mu\nu}_{bb} + \left[2D^{dc}_{\mu\nu} + \frac{N}{4}(d_{3abcd} + d_{3abdc} + d_{3bacd} + d_{3badc})D^{ab}_{\mu\nu}\right]D^{\mu\nu}_{cd}, \quad (94)$$

with

$$A^a_{\mu\nu} = \beta_i X^{ia}_{(\mu\nu)} + \rho_i g_{\mu\nu} X^{\alpha ia}_{\alpha},$$

$$B^{ab}_{\mu\nu} = \gamma_{(ij)} X^{ia}_{\mu} X^{bj}_{\nu},$$

$$C^{ab}_{\mu\nu} = a_{[ij]} X^{ia}_{\mu} X^{jb}_{\nu} + u_{[ij]} g_{\mu\nu} X^{ia}_{\alpha} X^{\alpha jb},$$

$$D^{ab}_{\mu\nu} = b_{(ij)} X^{ia}_{\mu} X^{jb}_{\nu} + v_{(ij)} g_{\mu\nu} X^{ia}_{\alpha} X^{\alpha jb}, \quad (95)$$

where f^{abc} are the anti-symmetric structure constants and d^{abc} are the components of the completely symmetric invariant $rank-3$ tensor of the group.

7 Field Equations

The on-shell informations also will be depending on this generators expansions. It gives for D^a_μ field,

$$\lambda_1\left(4d\partial_\nu Z^{[\mu\nu]}t_a + 4i\frac{g}{N}(dD^b_\nu + \alpha_i X^{ib}_\nu)Z^{[\mu\nu]}[t_a,t_b]\right) +$$

$$+\lambda_3\left(2d\partial_\nu z^{[\mu\nu]}t_a + 2i\frac{g}{N}(dD^b_\nu - \alpha_i X^{ib}_\nu)z^{[\mu\nu]}[t_a,t_b]\right) +$$

$$+\xi_1\left(4i\frac{g}{N}\beta_i X^{ib}_\nu Z^{(\mu\nu)} + 4i\frac{g}{N}\rho_i X^{\mu ib} Z^{\nu)}_{(\mu}\right)[t_a,t_b] +$$

$$-\xi_3 2i\frac{g}{N}\left(\beta_i X^{ib}_\nu z^{(\mu\nu)} + \rho_i X^{\mu ib} g_{(\rho\sigma)} g^{(\rho\sigma)}\right)[t_a,t_b] = 0 \quad (96)$$

and for X^i_μ field

$$\lambda_1\left(4\alpha_i\partial_\nu Z^{[\mu\nu]}t_a + 4i\frac{g}{N}\alpha_i D^b_\nu Z^{[\mu\nu]}[t_a,t_b]\right) +$$

$$\lambda_2\left(4a_{(ij)} X^{ib}_\nu z^{[\mu\nu]}[t_a,t_b] + (4b_{[ij]} + 2\gamma_{[ij]})X^{ib}_\nu z^{[\mu\nu]}\{t_a,t_b\}\right) +$$

$$\lambda_3\left(2\alpha_i\partial_\nu z^{[\mu\nu]}t_a - 4i\frac{g}{N}\alpha_i D^b_\nu z^{[\mu\nu]}[t_a,t_b] + (2b_{[ij]} + \gamma_{[ij]})X^{ib}_\nu Z^{[\mu\nu]}\{t_a,t_b\}\right) +$$

$$\xi_1\left(-4\beta_i\partial_\nu Z^{(\mu\nu)}t_a - 4\rho_i\partial^\mu Z^{\nu)}_{(\nu}t_a - 4i\frac{g}{N}(\beta_i D^b_\nu Z^{(\mu\nu)} + \rho_i D^{\mu b} Z^{\nu)}_{(\nu})[t_a,t_b]\right) +$$

$$\xi_2\left(4(a_{[ij]} X^{ib}_\nu z^{(\mu\nu)} + u_{[ij]} X^{\mu ib} z^{\nu)}_{(\nu})[t_a,t_b] + 4b_{(ij)}\{X^i_\nu, z^{(\mu\nu)}\}t_a + 4v_{(ij)}\{X^{\mu j}, z^{\nu)}_{(\nu}\}t_a\right) +$$

$$\xi_3\left(\begin{array}{c} 2\left(\beta_i\partial_\nu z^{(\mu\nu)}-\rho_i\partial^\mu g_{(\rho\sigma)}z^{(\rho\sigma)}\right)t_a+2\left(b_{(ij)}X_\nu^{ib}Z^{[\mu\nu]}+v_{(ij)}X^{\mu ib}g_{(\rho\sigma)}Z^{(\rho\sigma)}\right)\{t_a,t_b\}+\\ \left(2i\frac{g}{N}(\beta_iD_\nu^b z^{(\mu\nu)}+\rho_iD^{\mu b}g_{(\rho\sigma)}z^{(\rho\sigma)})+2a_{[ij]}X_\nu^{ib}Z^{(\mu\nu)}+2u_{[ij]}X^{\mu ib}g_{(\rho\sigma)}Z^{(\rho\sigma)}\right)[t_a,t_b]\end{array}\right)=0 \quad (97)$$

Taking the trace in the above equation, one gets

$$\begin{aligned}&2\lambda_1(d\partial_\nu Z^{[\mu\nu]a}-gf_{abc}(dD_\nu^b+\alpha_iX_\nu{}^{ib})Z^{[\mu\nu]c})+\\&-2\xi_1 gf_{abc}(\beta_iX_\nu{}^{ib}Z^{(\mu\nu)c}+\rho_iX^{\mu ib}Z_{(\nu}{}^{\nu)c})+\\&+\lambda_3(d\partial_\nu z^{[\mu\nu]a}-gf_{abc}(dD_\nu^b+\alpha_iX_\nu{}^{ib})z^{[\mu\nu]c})+\\&-\xi_3 gf_{abc}(\beta_iX_\nu{}^{ib}z^{(\mu\nu)c}+\rho_iX^{\mu ib}z_{(\nu}{}^{\nu)c})=0\end{aligned} \quad (98)$$

Multiplying the equation of motion by t_k and taking again the corresponding trace, we have

$$\begin{aligned}&\lambda_1(d(d_{aek}-if_{aek})\partial_\nu Z^{[\mu\nu]e}+\\&+g(dD_\nu^{\nu b}+\alpha_iX_\nu{}^{ib})(if_{abc}f_{cek}-f_{abc}d_{cek})Z^{[\mu\nu]e})+\\&+\xi_1(g\rho_i(if_{abc}f_{cek}-f_{abc}d_{cek})X^{\mu ib}Z_{(\nu}{}^{\nu)e}+\\&+g\beta_i(if_{abc}f_{cek}-f_{abc}d_{cek})X_\nu{}^{ib}Z^{(\mu\nu)e})+\\&+\frac{\lambda_3}{2}(d(d_{aek}-if_{aek})\partial_\nu z^{[\mu\nu]a}+\\&+g(dD_\nu^b+\alpha_iX_\nu{}^{ib})(if_{abc}f_{cek}-f_{abc}d_{cek})z^{[\mu\nu]e})+\\&+\frac{\xi_3}{2}(g\rho_i(if_{abc}f_{cek}-f_{abc}d_{cek})X^{\mu ib}z_{(\nu}{}^{\nu)e}+\\&+g\beta_i(if_{abc}f_{cek}-f_{abc}d_{cek})X_\nu{}^{ib}z^{(\mu\nu)e})=0\end{aligned} \quad (99)$$

The corresponding equations of motion at physical basis are

$$\begin{aligned}&\lambda_1\left(-4a_I\partial_\nu Z^{[\mu\nu]}t_a+4iga_{IJ}G_\nu^{Jb}Z^{[\mu\nu]}[t_a,t_b]\right)+\\&\lambda_2\left(4a_{(IJ)}G_\nu^{Jb}z^{[\mu\nu]}[t_a,t_b]+(4b_{[IJ]}+2\gamma_{[IJ]})G_\nu^{Jb}z^{[\mu\nu]}\{t_a,t_b\}\right)+\\&\lambda_3\left(2a_I\partial_\nu z^{[\mu\nu]}t_a-2a_{(IJ)}G_\nu^{Jb}(gz^{[\mu\nu]}+Z^{[\mu\nu]})[t_a,t_b]+2(b_{[IJ]}+\gamma_{[IJ]})G_\nu^{Jb}Z^{[\mu\nu]}\{t_a,t_b\}\right)+\\&-4\xi_1\left(\beta_I\partial_\nu Z^{(\mu\nu)}+\rho_Ig_{(\rho\sigma)}\partial^\mu Z^{(\rho\sigma)}\right)t_a+\end{aligned}$$

$$\begin{aligned}&4\xi_2\left((a_{[IJ]}G_\nu^{Jb}z^{(\mu\nu)}+u_{[IJ]}G_\nu^{Jb}g_{(\rho\sigma)}z^{(\rho\sigma)})[t_a,t_b]+(b_{(IJ)}G_\nu^{Jb}z^{(\mu\nu)}+v_{(IJ)}G_\nu^{Jb}g_{(\rho\sigma)}z^{(\rho\sigma)})\{t_a,t_b\}\right)+\\&2\xi_3\left(\begin{array}{c}-\left(\beta_I\partial_\nu z^{(\mu\nu)}+\rho_I\partial^\mu g_{(\rho\sigma)}z^{(\rho\sigma)}\right)t_a+\left(b_{[IJ]}G_\nu^{Jb}Z^{[\mu\nu]}+u_{[IJ]}G_\mu^{Jb}g_{(\rho\sigma)}z^{(\rho\sigma)}\right)[t_a,t_b]\\+\left(b_{(IJ)}G_\nu^{Jb}Z^{[\mu\nu]}+2v_{(IJ)}G_\mu^{Jb}g_{(\rho\sigma)}z^{(\rho\sigma)}\right)\{t_a,t_b\}\end{array}\right)=0\end{aligned} \quad (100)$$

In order to understand more specifically the model one should also express the equations of motion through Eq. (88) sectors. Considering first the D_μ-field equation of motion, one gets

$$\mathrm{D}_\mu^{ab} W^{[\mu\nu]b} = j_{S,D}^{\nu a} \tag{101}$$

where the L_A contribution is

$$W_{[\mu\nu]}^a = 4B_{[\mu\nu]}^a + 2i\gamma_{ij} f^{abc} X_{[\mu}^{bi} X_{\nu]}^{cj}, \tag{102}$$

where the covariant derivative is defined at Eq. (3.3) and the L_S contribution is

$$j_{S,D}^{\nu a} = j_1^{\nu a} + j_2^{\nu a} + j_3^{\nu a} \tag{103}$$

with

$$\begin{aligned} j_1^{\nu a} &= 4gf^{abc}\left[\beta_p\left(\beta_j X^{(\nu\mu)jb} + \rho_i g^{\mu\nu} X_\alpha^{\alpha jb}\right) X_\mu^{pc} + \rho_p\left(\beta_j + 4\rho_j\right) X_\alpha^{\alpha jb} X^{\nu pc}\right], \\ j_2^{\nu a} &= 2igN\gamma_{ij} f^{bmn} f^{bac}\left[\beta_p X^{im(\nu} X^{\mu)jn} X_\mu^{pc} + \rho_p g_{\alpha\beta} X^{im(\alpha} X^{\beta)jn} X^{\nu pc}\right], \\ j_3^{\nu a} &= 4igNf^{bmn} f^{bac}\left[(\beta_p + \rho_p) a_{[ij]} + (\beta_p + 4\rho_p) u_{[ij]}\right] X_\alpha^{im} X^{\alpha jn} X^{\nu pc}. \end{aligned} \tag{104}$$

Given that Eq. (7.7) depends only on $X\mu^{ie}$ fields, one gets that through this model $\{D_\mu^a, X_\mu^{ie}\}$ the X_μ^{ie} fields work as source to D_μ^a fields.

Eq. (101) contains three features. First, it is covariant which proves that the introduction of this extended symmetry is consistent. Notice that it not only show on covariance but also on the presence of a conserved current when Eq. (7.6) is not considered

$$j^{\mu a} = -\partial_\nu B^{\nu\mu a} = df^{abc}(dD_\nu^b + \alpha_i X_\nu^{bi}) W^{[\nu\mu]c} \tag{105}$$

The charge associated to this current as the same symmetry boundary condition as in the usual QCD [15].

Second, deriving the Noether theorem expression Eq. (68), one gets

$$J_N^{\mu a} = Nd\left[W^{[\nu\mu]}, dD_\nu + \alpha_i X_\nu^i\right] \tag{106}$$

which is exactly the D_μ-field equation of motion without the right hand side $j_{S,D}^{\nu a}$.

Third, due to the PoincarÃ© lemma, one derives the expression

$$\mathrm{D}_\mu^{ab} j_{S,D}^{\mu a} = 0, \tag{107}$$

showing that $j_{S,D}^{\mu}$ is conserved covariantly. Its relationship with Noether current is

$$j_N^{\mu a} = j_{S,D}^{\mu a} - f^{abc}(dD_\nu^b + \alpha_i X_\nu^{bi}) W^{[\nu\mu]} c. \tag{108}$$

Considering for $X_{\mu i}$ fields, we get the following covariant equations of motion

$$\alpha_i N \mathrm{D}_{ab}^{\mu} W_{[\mu\nu]}^b - m_{ij}^2 X_{\nu a}^j = J_{\nu a,i}^T, \tag{109}$$

where

$$J_{\nu a,i}^T = J_{\nu a,i}^A + J_{\nu a,i}^S, \tag{110}$$

which corresponding expressions are in Appendix B.

Considering that the main proposal at this section is to show that the introduction of a fields set in the $SU(N)$ gauge symmetry preserves covariance it will be not necessary to calculate the physical fields equations of motions.

Given that the minimal action expressions between two generic fields reference system $\{\varphi\}$ and $\{\Phi\}$ is given by $\frac{\delta S[\varphi]}{\delta\varphi} = \frac{\delta S'[\Phi]}{\delta\Phi} = 0$, one gets that its corresponding equations can be related to the $\{G_I\}$-basis through the transformations

$$\Omega_{I1}^{-1}\left(\frac{\partial L}{\partial G_{\nu I}} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu G_{\nu I})}\right) = 0$$

$$\Omega_{Ii}^{-1}\left(\frac{\partial L}{\partial G_{\nu I}} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu G_{\nu I})}\right) = 0 \qquad (111)$$

So eqs. (111) generate that the explicit covariance obtained through eqs. (7.4) and (7.11) are preserved at $\{G_I\}$ physical basis.

8 Directive and Circumstantial Symmetries

The whole physics introduces the meaning of an integral organization including two kinds of symmetries. They are the directive and the circumstantial symmetry. Their qualitative difference is that while the director symmetry appears as a natural instruction from the gauge parameter, the circumstantial symmetry will be depending on relationships between the so-called free coefficients studied at Apendice B.

From these two types of symmetries one derives currents conservations. Associated to the gauge parameter one gets the Slavnov-Taylor identity (off-shell) and the Noether identity (on-shell) which yield one conserved current with N-fields contributions

$$\int d^4x[\delta D_\mu^a \frac{\delta S}{\delta D_\mu^a} + \delta X_\mu^{ia} \frac{\delta S}{\delta X_\mu^{ia}}] = 0 \qquad (112)$$

which produces a directive conserved current

$$\partial_\mu J^\mu_{directive} = 0. \qquad (113)$$

Rewriting Eq. (7.1), one gets

$$\lambda_1 d\partial_\nu Z^{\mu\nu a} + \frac{1}{2} g^{\mu\nu}(\partial\cdot D^a + \sigma_i \partial\cdot X^{ia}) = J^{\mu a}(D) \qquad (114)$$

where $J^{\mu a}(D)$ current is explicitly derived at Appendix C. Considering that Eq. (8.3) coincides with Noether identity, $J^{\mu a}(D)$ conservation is a directive. It takes obligatory one degree of freedom from D_μ^a field.

Similarly for X_μ^{ia} fields, one gets

$$\lambda_1 \alpha_i \partial_\nu Z^{[\mu\nu]} + \frac{1}{2} m_{ij}^2 X^{\mu ja} = J^{\mu ia}(X) \qquad (115)$$

where $J^{\mu ia}(X)$ current is written at Appendix C. Consequently the classical decoupling of the longitudinal sector $\partial_\mu X^{\mu ia} = 0$ will depend on circumstances between the free coefficients.

9 Energy Momentum Tensor

Given the expression

$$\theta_{\mu\nu} = T_{\mu\nu} + \frac{1}{2}\partial^\rho(S_{\rho\mu\nu} + S_{\mu\nu\rho} - S_{\nu\rho\mu})$$

where

$$T_{\mu\nu} = \frac{\partial L}{\partial(\partial^\mu D_{\alpha a})}\partial_\nu D_{\alpha a} + \frac{\partial L}{\partial(\partial^\mu X^i_{\alpha a})}\partial_\nu X^i_{\alpha a} - \eta_{\mu\nu} L$$

$$S_{\rho\mu\nu} = \frac{\partial L}{\partial(\partial^{\rho} D_{\alpha a})} \Sigma_{\mu\nu}^{\alpha\beta} D_{\beta a} + \frac{\partial L}{\partial(\partial^{\rho} X_{\alpha a}^{i})} \Sigma_{\mu\nu}^{\alpha\beta} X_{\beta a}^{i} \quad (116)$$

with

$$\Sigma_{\mu\nu}^{\alpha\beta} = \delta_{\mu}^{\alpha}\delta_{\nu}^{\beta} - \delta_{\nu}^{\alpha}\delta_{\mu}^{\beta} \quad (117)$$

one gets

$$\theta_{\mu\nu} = \theta_{\mu\nu}^{A} + \theta_{\mu\nu}^{S} \quad (118)$$

with

$$\theta_{\mu\nu}^{A} = 4Z_{[\mu\rho]a} Z_{[\nu}^{\ \rho]a} + 4z_{[\mu\rho]a} z_{[\nu}^{\ \rho]a} + 2z_{[\mu\rho]a} Z_{[\nu}^{\ \rho]a} + 2Z_{[\mu\rho]a} z_{[\nu}^{\ \rho]a} - \eta_{\mu\nu} L^{A} \quad (119)$$

and

$$\begin{aligned}
\theta_{\mu\nu}^{S} &= 4\beta_i Z_{(\mu\rho)a} X_{(\nu}^{\ \rho)ia} + 4\rho_i Z_{(\rho\ a}^{\ \rho)} X_{(\mu\nu)}^{\ ia} \\
&+ 2\beta_i z_{(\mu\rho)a} X_{(\nu}^{\ \rho)ia} + 2\rho_i z_{(\rho\ a}^{\ \rho)} X_{(\mu\nu)}^{\ ia} \\
&+ 4z_{(\mu\rho)a} a_{[ij]} [X_{\nu}^{\ i}, X^{\rho j}]^{a} + 4z_{(\rho\ a}^{\ \rho)} u_{[ij]} [X_{\mu}^{\ i}, X_{\nu}^{\ j}]^{a} \\
&+ 4z_{(\mu\rho)a} b_{(ij)} \{X_{\nu}^{\ i}, X^{\rho ja}\} + 4z_{(\rho\ a}^{\ \rho)} v_{(ij)} \{X_{\mu}^{\ i}, X_{\nu}^{\ ja}\} \\
&+ 2Z_{(\mu\rho)a} a_{[ij]} [X_{\nu}^{\ i}, X^{\rho j}]^{a} + 2Z_{(\rho\ a}^{\ \rho)} u_{[ij]} [X_{\mu}^{\ i}, X_{\nu}^{\ j}]^{a} \\
&+ 2Z_{(\mu\rho)a} b_{(ij)} \{X_{\nu}^{\ i}, X^{\rho ja}\} + 2Z_{(\rho\ a}^{\ \rho)} v_{(ij)} \{X_{\mu}^{\ i}, X_{\nu}^{\ ja}\} \\
&- 4\beta_i \partial^{\rho} (Z_{(\mu\rho)a} X_{\nu}^{\ ia}) + 4\beta_i \partial^{\rho} (Z_{(\mu\nu)a} X_{\rho}^{\ ia}) \\
&- 4\rho_i \partial_{\mu} (Z_{(\rho\ a}^{\ \rho)} X_{\nu}^{\ ia}) - 4\beta_i \partial^{\rho} (Z_{(\nu\rho)a} X_{\mu}^{\ ia}) \\
&- 4\rho_i \partial_{\nu} (Z_{(\rho\ a}^{\ \rho)} X_{\mu}^{\ ia}) + 2\beta_i \partial^{\rho} (z_{(\mu\nu)a} X_{\rho}^{\ ia}) \\
&- 2\beta_i \partial^{\rho} (z_{(\mu\rho)a} X_{\nu}^{\ ia}) - 2\beta_i \partial^{\rho} (z_{(\nu\rho)a} X_{\mu}^{\ ia}) \\
&- 2\rho_i \partial_{\mu} (z_{(\rho\ a}^{\ \rho)} X_{\nu}^{\ ia}) + 4\rho_i \partial^{\rho} (Z_{(\alpha\ a}^{\ \alpha)} X_{\rho}^{\ ia}) \delta_{\mu}^{\nu} \\
&- 2\rho_i \partial_{\nu} (z_{(\rho\ a}^{\ \rho)} X_{\mu}^{\ ia}) + 2\rho_i \partial^{\rho} (z_{(\alpha\ a}^{\ \alpha)} X_{\rho}^{\ ia}) \delta_{\mu}^{\nu} - \eta_{\mu\nu} L^{S}
\end{aligned} \quad (120)$$

Eq. (9.3) provides the conservation law

$$\partial_{\mu}\theta^{\mu\nu} = 0. \quad (121)$$

10 Charges Algebra

Although the gauge fixing term breaks the gauge it is possible to show that there is a symmetry that is preserved in the Lagrangian which is the BRST symmetry. Considering the group parameter as $\alpha^{a} = gc^{a}\delta\lambda$ where α^{a} is a bose quantity, c^{a} a fermi quantity and $\delta\lambda$ some anticommuting global quantity, we will derive the BRST invariance. For convenience it will be studied at constructor basis.

Considering the general Lagrangian

$$L_{eff} = L_{GI} + L_{matter} + L_{GF} + L_{FP} \quad (122)$$

where the corresponding terms L_{GI} is defined at Eq. (9), the covariants derivatives with to this extended model are

$$\nabla_\mu = \partial_\mu \text{-} + g[D_\mu, \text{-}], \quad (123)$$

$$\mathsf{D}_\mu = \partial_\mu \text{-} + g[D_\mu, \text{-}] + g_i[X^i_\mu, \text{-}], \quad (124)$$

which means $\mathsf{D}^{ab}_\mu = \delta^{ab}\partial_\mu + gf^{abc}D^b_\mu + g_i f^{abc}X^{bi}_\mu$, and with a matter term as

$$\mathsf{L}_{matter} = \overline{\psi}^a \mathsf{D}^{ac}\psi^c. \quad (125)$$

Further, the gauge fixing term at Eq. (19) can be rewritten in terms of the scalar auxiliary field b^a as

$$\mathsf{L}_{GF} = -\frac{1}{\xi}\partial^\mu b^a (D^a_\mu + \sigma_i X^{ai}_\mu) + \frac{\xi}{2} b_a b^a \quad (126)$$

and the Faddeev Popov term is

$$\mathsf{L}_{FP} = -i(\partial^\mu \overline{c}^a)\mathsf{D}^{ab}_\mu c^b, \quad (127)$$

where $\overline{c}^a$ and c^b are the ghost fields.

Considering the infinitesimal BRST transformation

$$\begin{aligned} \delta D^a_\mu &= -\nabla^{ab}_\mu c^b \delta\lambda, \\ \delta X^{ai}_\mu &= gf^{abc} c^b X^{ci}_\mu \delta\lambda, \\ \delta\psi &= igc^a t^a \psi\delta\lambda, \\ \delta c^a &= -\frac{g}{2} f^{abc} c^b c^c \delta\lambda, \\ \delta\overline{c}^a &= ib^a \delta\lambda, \\ \delta b^a &= 0, \end{aligned} \quad (128)$$

one gets,

$$\delta(\mathsf{D}^{ab}_\mu c^b) = 0, \ \delta(c \times c)^a = 0, \quad (129)$$

which yields,

$$\delta\mathsf{L}_{eff} = 0 \quad (130)$$

and also that BRST transformations are idempotent

$$\delta^2 D^a_\mu = \delta^2 X^{ai}_\mu = \delta^2 c^a = \delta^2 \overline{c}^a = \delta^2 b^a = 0. \quad (131)$$

Similarly at $\{G_{\mu I}\}$ basis,

$$\delta G^a_{\mu I} = \mathsf{D}^{ab}_{\mu I} c^b \delta\lambda = \delta\lambda(sG^a_{\mu I}), \ \delta^2 G^a_{\mu I} = 0. \quad (132)$$

The fundamental object in a gauge theory is not the Lagrangian but the functional generator ot the Green's functions. It is given by

$$Z[J] = N' \int \mathcal{D}D_\mu \mathcal{D}X^i_\mu \mathcal{D}c\mathcal{D}\bar{c}.e^{i\int d^4x\left[\mathcal{L}_{eff} + J^a_\mu D^{\mu a} + j^{ai}_\mu X^{\mu ia}\right]}. \quad (133)$$

So we have to show that $Z[0]$ is invariant under BRST transformation. Considering that the part involving $\mathcal{L}_{eff}$ was already proved, we have now to demonstrate on the invariance of the measure $\mathcal{D}D_\mu \mathcal{D}X^i_\mu \mathcal{D}c\mathcal{D}\bar{c}$. Calculating the functional Jacobian of the BRST transformation,

$$J = \frac{\delta\left(D^{a'}_\mu(x), X^{ia'}_\mu(x), c^{a'}(x), \bar{c}^{a'}(x)\right)}{\delta\left(D^b_\nu(y), X^{ib}_\nu(y), c^b(y), \bar{c}^b(y)\right)}, \quad (134)$$

one gets that $detJ = 1$, which means that it is a constant that does not depend on fields and that can be absorved by a functional constant. In fact by introducing the fields set $\{D_\mu, X_{\mu i}\}$ the measure $\mathcal{D}D_\mu \mathcal{D}X^i_\mu \mathcal{D}c\mathcal{D}\bar{c}$ is preserved.

Given that the model contains the BRST symmetry the Noether theorem leads to the conserved current J^{BRST}_μ.

$$J^{BRST}_\mu = \delta D^{\nu a}\frac{\delta\mathcal{L}_{eff}}{\delta\partial^\mu D^{\nu a}} + \delta X^{\nu ai}\frac{\delta\mathcal{L}_{eff}}{\delta\partial^\mu X^{\nu ai}} + \delta\psi\frac{\delta\mathcal{L}_{eff}}{\delta\partial^\mu\psi} + \delta c^a\frac{\delta\mathcal{L}_{eff}}{\delta\partial^\mu c^a} + \delta\bar{c}^a\frac{\delta\mathcal{L}_{eff}}{\delta\partial^\mu\bar{c}^a}. \quad (135)$$

For simplicity, we are going to separete in antisymmetric and symmetric parts

$$J^{BRST}_\mu = J^{BRST}_{\mu A} + J^{BRST}_{\mu S} \quad (136)$$

where

$$J^{BRST}_{\mu A} = b^a \mathcal{D}^{ac}_\mu b^c - c^a\partial_\mu b^a - 4g(\partial^\nu c^a)D^a_{\nu\mu} - 4gf^{abc}(gD^{\nu b} + g_i X^{\nu bi})c^c D^a_{\nu\mu}$$
$$-4gc^a\left[f^{abc}(gD^{\nu b} + g_i X^{\nu bi})\right]D^c_{\nu\mu} + igc^a(\partial_\mu\bar{c}\times c)^a - i\frac{g}{2}(c\times c)^a\partial_\mu c^a \quad (137)$$

which gives the following expression for the BRST charge

$$Q^{BRST}_A = \int d^3x\left(b^a\mathcal{D}^{ac}_0 c^c - \partial_0 b^a c^a + i\frac{1}{2}g(\partial_0\bar{c})^a(c\times c)^a\right). \quad (138)$$

Considering that

$$2i\bar{\lambda}(Q^{BRST})^2 = i\bar{\lambda}\{Q^{BRST}, Q^{BRST}\} = [i\bar{\lambda}Q^{BRST}, Q^{BRST}] \equiv \delta Q^{BRST}, \quad (139)$$

and the relationships

$$[Q^{BRST}_A, D^a_\mu] = -i(\nabla_\mu c^a),$$
$$[Q^{BRST}_A, X^{ai}_\mu] =$$
$$[Q^{BRST}_A, b^a] = 0,$$
$$[Q^{BRST}_A, \psi] = gc^a t^a\psi,$$
$$\{Q^{BRST}_A, c^a\} = \frac{ig}{2}(c\times c)^a,$$
$$\{Q^{BRST}_A, \bar{c}^a\} = b^a. \quad (140)$$

One gets

$$\delta Q^{BRST}_A = \int d^3x\left(\frac{g}{2}(\partial_0 b)^a(c\times c)^a - \frac{g}{2}(\partial_0 b)^a(c\times c)^a\right)\delta\lambda = 0. \quad (141)$$

Including from Eq. (14) the symmetric sector $\mathsf{L}_S(D, X_i)$, one derives

$$J^{BRST}_{\mu S} = -4g_i f^{abc} c^b X^{\nu ci} X^{ai}_{(\mu\nu)}, \tag{142}$$

with

$$Q^{BRST}_S = \int d^3x \left(-4g_i f^{abc} c^b X^{\nu ci} X^{ai}_{(0\nu)}\right). \tag{143}$$

Given that $\delta_{BRST}(f^{abc} c^b X^{\nu ci}) = 0$, one gets

$$\delta Q^{BRST}_S = -4g_i \int d^3x \left(f^{abc} c^b X^{\nu ci} \delta X^{ai}_{(0\nu)}\right) = 0. \tag{144}$$

At this way we show that similarly to QCD

$$\delta Q^{BRST} = 0, \, and \, \delta^2 Q^{BRST} = 0. \tag{145}$$

Another conservation law is with respect to the scale global symmetry for ghosts

$$c^a \to c^{a'} = e^{\theta} c^a ; \bar{c}^a \to \bar{c}^{a'} = e^{-\theta} \bar{c}^a . \tag{146}$$

Consequently, one gets a Noether conserved current

$$J^c_\mu = i\left[\bar{c}^a \mathrm{D}^{ac}_\mu c^c) - (\partial^\mu \bar{c}^a) c^a\right], \tag{147}$$

and a conserved hermitean charge

$$Q_c = i\int d^3x \left(\bar{c}^a \overset{\leftrightarrow}{\partial^0} c^a + \bar{c}^a \left[g f^{abc} D^b_0 c^c + g_i f^{abc} X^{bi}_0 c^c \right]\right), \tag{148}$$

working as the “ghost-scale” generators of the fields operators transformations

$$[Q_c, D^a_\mu] = [Q_c, X^{ai}_\mu] = [Q_c, \psi^i] = 0,$$

$$[Q_c, c^a] = -ic^a,$$

$$[Q_c, \bar{c}^a] = i\bar{c}^a. \tag{149}$$

next symmetry to be studied corresponds to the global gauge transformation. The corresponding infinitesimal transformations are

$$\delta D^a_\mu = f^{abc} \varepsilon^b D^c_\mu,$$

$$\delta X^{ai}_\mu = f^{abc} \varepsilon^b X^{ci}_\mu,$$

$$\delta c^a = f^{abc} \varepsilon^b c^c,$$

$$\delta \bar{c}^a = f^{abc} \varepsilon^b \bar{c}^c,$$

$$\delta b^a = f^{abc} \varepsilon^b b^c, \tag{150}$$

where $\varepsilon^b = -g\alpha^b$. Considering L_{GI}, L_{matter}, L_{GF} and L_{FP} under Eq. (150), one gets that they are separatedely invariants. Thus one derives the following Noether conserved current

$$J^{Ga}_\mu = -4g(D^\nu \times D_{\nu\mu})^a + j^a_\mu - \left[(D_\mu + \sigma_i X^i_\mu) \times b\right]^a$$

$$-i(\bar{c} \times \mathrm{D}_\mu c)^a + i(\partial_\mu \bar{c} \times c)^a - 4g_i (X^\nu_i \times D_{\nu\mu})^a \tag{151}$$

where j^a_μ is the matter current. It gives

$$Q_G^a = \int d^3x\{-4g(D^\nu \times D_{\nu 0})^a + j_0^a + \left[(D_0 + \sigma_i X_0^i)\times b\right]^a + \tag{152}$$

$$-i(\bar{c}\times D_0 c)^a + i(\partial_o \bar{c}\times c)^a - 4g_i(X_i^\nu \times D_{\nu 0})^a \tag{153}$$

with

$$[Q_G^a, D_\mu^b] = if^{abc}D_\mu^c,$$

$$[Q_G^a, X_\mu^{ai}] = if^{abc}X_\mu^{ci},$$

$$[Q_G^a, b^b] = if^{abc}b^c,$$

$$[Q_G^a, \psi^i] = -t^a\psi^i,$$

$$[Q_G^a, c^b] = if^{abc}c^c,$$

$$[Q_G^a, \bar{c}^b] = if^{abc}\bar{c}^c. \tag{154}$$

Concluding, we obtain that the charges algebra is the same as in QCD:

$$[Q_{BRST}, Q_{BRST}] = 0,$$

$$[iQ_c, Q_{BRST}] = Q_{BRST},$$

$$[Q_c, Q_c] = 0,$$

$$[Q_G, Q_c] = 0,$$

$$[Q_G, Q_{BRST}] = 0,$$

$$[Q_G, Q_G] = if^{abc}Q_G. \tag{155}$$

Eq. (155) is showing that the charges algebra depends only on the symmetry involved. It does not depend on the number of potential fields being considered at the fields set.

Finally, in order to close this section we are going to calculate the ghost number operator. It is defined as

$$N_\mu^a = \{Q_{BRST}, D_\mu \bar{c}^a\}. \tag{156}$$

Calculating Eq. (156), one gets

$$N_\mu^a = 4\partial^\nu D_{\nu\mu}^a - J_\mu^{Ga}, \tag{157}$$

which gives

$$\partial_\mu N^{\mu a} = 0. \tag{158}$$

11 Slavnov-Taylor identity

Another ingredient on this non-abelian extension is to consider the Slavnov-Taylor identity. Now we perform those BRST transformation on generator functional to obtain the Slavnov-Taylor identities for the extended symmetry $SU(N)$. It is convenient to define the generator functional in terms of sources for fermions and bosons

$$Z(J, s, \bar{\sigma}, \sigma, \bar{\zeta}, \zeta; u, w, v, \bar{\theta}, \theta) = \int DDDX_i D\bar{c}DcD\bar{\chi}D\chi \exp(i\int d^4x\, L_{total}), \tag{159}$$

where the total Lagrangian in terms of fields and sources is

$$L_{total} = L_{eff} + J_\mu^a D^{\mu a} + s_\mu^a X_i^{\mu a} + \bar{\sigma}^a c^a + \bar{c}^a \sigma^a + \bar{\chi}\zeta + \bar{\zeta}\chi$$

$$+u_\mu^a \frac{1}{g_1} D^{\mu ab}(D)c^b - w_\mu^a f^{abc} c^b X_i^{\mu c} + v^a(-\frac{1}{2} f^{abc} c^b c^c) + \bar{\theta}(it^a c^a \chi) + (it^a c^a \bar{\chi})\theta, \quad (160)$$

in which J^μ and s^μ are sources of D_μ and $X_{\mu i}$, respectively, $(\bar{\sigma}^a, \sigma^a)$ anticommuting sources associated to Faddeev-Popov fields, and $(\bar{\zeta}, \zeta)$ are associated to the fermions $(\bar{\chi}, \chi)$, respectively. The three last terms of Eq. (160) have been introduced of a way that the total Lagrangian remains invariant by BRST transformations in accord with the nilpotent relations. The others sources $(u_\mu^a, w_\mu^a, v^a, \bar{\theta}, \theta)$ are anticommuting too. Now the invariance of the generator functional under BRST symmetry implies that

$$\int \mathrm{D}D\mathrm{D}X_i \mathrm{D}\bar{c}\mathrm{D}c\mathrm{D}\bar{\chi}\mathrm{D}\chi \exp^{iS} \int d^4x (J^{\mu a}\delta D_\mu^a + s^{\mu a}\delta X_{\mu i}^a + \bar{\sigma}^a \delta c^a + \delta \bar{c}^a \sigma^a + \bar{\zeta}\delta\chi + \delta\bar{\chi}\zeta) = 0,$$
(161)

and by substituting the BRST transformations, one gets

$$\int \mathrm{D}D\mathrm{D}X_i \mathrm{D}\bar{c}\mathrm{D}c\mathrm{D}\bar{\chi}\mathrm{D}\chi \exp^{iS} \int d^4x \{ -\frac{1}{g_1}[D_\mu^{ab}(D)c^b]\lambda J^{\mu a} + s^{\mu a} f^{abc} c^b \lambda X_{\mu i}^c$$

$$+\bar{\sigma}^a(-\frac{1}{2} f^{abc} c^b c^c)\lambda - \frac{1}{g_1 \xi}(\partial_\mu D^{\mu a})\lambda\sigma^a + \bar{\zeta}(iT^a c^a \lambda\chi) + (-iT^a c^a \lambda\bar{\chi})\zeta \} = 0 \quad (162)$$

in which it is easy to show that the Jacobian of those transformations is unity. Those expression is written in terms of derivatives of the generator functional in relation to sources

$$\int d^4x [J^{\mu a}\frac{\delta Z}{\delta u^{\mu a}} + s^{\mu a}\frac{\delta Z}{\delta w^{\mu a}} + \bar{\sigma}^a \frac{\delta Z}{\delta v^a} - \frac{1}{g_1 \xi}\partial_\mu(\frac{\delta Z}{\delta J^{\mu a}})\sigma^a + \bar{\zeta}\frac{\delta Z}{\delta \bar{\theta}} + \frac{\delta Z}{\delta \theta}\zeta] = 0. \quad (163)$$

Putting $Z = e^{iW}$, a same equation holds for W

$$\int d^4x [J^{\mu a}\frac{\delta W}{\delta u^{\mu a}} + s^{\mu a}\frac{\delta W}{\delta w^{\mu a}} + \bar{\sigma}^a \frac{\delta W}{\delta v^a} - \frac{1}{g_1 \xi}\partial_\mu(\frac{\delta W}{\delta J^{\mu a}})\sigma^a + \bar{\zeta}\frac{\delta W}{\delta \bar{\theta}} + \frac{\delta W}{\delta \theta}\zeta] = 0. \quad (164)$$

We convert this differential equation into an expression in terms of the one particle irreducible 1PI generating functional Γ, then we use the Legendre transformation by using W and Γ as function of the sources $(u, w, v, \bar{\theta}, \theta)$ too, that leads us to relations

$$J^{\mu a} = -\frac{\delta\Gamma}{\delta D_\mu^a}, \quad s^{\mu a} = -\frac{\delta\Gamma}{\delta X_\mu i^a}, \quad \bar{\sigma}^a = -\frac{\delta\Gamma}{\delta c^a}, \quad \sigma^a = -\frac{\delta\Gamma}{\delta \bar{c}^a}, \quad \bar{\zeta} = -\frac{\delta\Gamma}{\delta\chi},$$

$$\zeta = -\frac{\delta\Gamma}{\delta\bar{\chi}}, \quad \frac{\delta W}{\delta J^{\mu a}} = D_\mu^a, \quad \frac{\delta W}{\delta s^{\mu a}} = X_{\mu i}^a, \quad \frac{\delta W}{\delta u^{\mu a}} = \frac{\delta\Gamma}{\delta u^{\mu a}}, \quad \frac{\delta W}{\delta w^{\mu a}} = \frac{\delta\Gamma}{\delta w^{\mu a}},$$

$$\frac{\delta W}{\delta v^a} = \frac{\delta\Gamma}{\delta v^a}, \quad \frac{\delta W}{\delta\theta} = \frac{\delta\Gamma}{\delta\theta}, \quad \frac{\delta W}{\delta\bar{\theta}} = \frac{\delta\Gamma}{\delta\bar{\theta}}. \quad (165)$$

Hence the expression (11.6) becomes

$$\int d^4x [\frac{\delta\Gamma}{\delta D_\mu^a}\frac{\delta\Gamma}{\delta u^{\mu a}} + \frac{\delta\Gamma}{\delta X_{\mu i}^a}\frac{\delta\Gamma}{\delta w^{\mu a}} + \frac{\delta\Gamma}{\delta c^a}\frac{\delta\Gamma}{\delta v^a} - \frac{1}{g_1 \xi}(\partial_\mu D^{\mu a})\frac{\delta\Gamma}{\delta\bar{c}^a} + \frac{\delta\Gamma}{\delta\chi}\frac{\delta\Gamma}{\delta\bar{\theta}} + \frac{\delta\Gamma}{\delta\theta}\frac{\delta\Gamma}{\delta\bar{\chi}}] = 0. \quad (166)$$

For simplify the form of this expression, the generator functional Z has the following dependence in terms of $\bar{c}^a$ and σ^a

$$Z = \int \mathrm{D}D\mathrm{D}X_i \mathrm{D}\bar{c}\mathrm{D}c\mathrm{D}\bar{\chi}\mathrm{D}\chi \exp\{i\int d^4x[-\bar{c}^a \partial_\mu(D^{\mu ab}c^b) + \bar{c}^a \sigma^a + \cdots]\}, \quad (167)$$

that give us

$$\frac{\delta\Gamma}{\delta\bar{c}^{a}} = -g_{1}\partial_{\mu}(\frac{\delta\Gamma}{\delta u_{\mu}^{a}}), \tag{168}$$

and finally we get the functional differential equation

$$\int d^{4}x[\frac{\delta\Gamma'}{\delta D_{\mu}^{a}}\frac{\delta\Gamma'}{\delta u^{\mu a}} + \frac{\delta\Gamma'}{\delta X_{\mu i}^{a}}\frac{\delta\Gamma'}{\delta w^{\mu a}} + \frac{\delta\Gamma'}{\delta c^{a}}\frac{\delta\Gamma'}{\delta v^{a}} + \frac{\delta\Gamma'}{\delta\chi}\frac{\delta\Gamma'}{\delta\bar{\theta}} + \frac{\delta\Gamma'}{\delta\theta}\frac{\delta\Gamma'}{\delta\bar{\chi}}] = 0, \tag{169}$$

where

$$\Gamma' = \Gamma - \frac{1}{2\xi}\int d^{4}x(\partial_{\mu}D^{\mu a})^{2}. \tag{170}$$

The equation (169) is Slavnov-Taylor identity for extended $SU(N)$. It will give us the important relations between Green functions of the massless, massive gluons and Faddeev-Popov ghosts that imply into the renormalizability of the model. The question of full renormalizability will not demonstrated here cause it is necessary a detailed analysis on Slavnov-Taylor identities and redefinitions of the parameters into the Lagrangian. It will be dedicated in a next paper.

12 Conclusion

The effort in this work is to implement the whole gauge principle at non-abelian level. Gauge symmetry depends on two variables which are the gauge parameters and group generators. They define Lie algebra valued fields transforming under gauge symmetry. The purpose is to show that these two variables, α_a and t_a, also work to accommodate the gauge symmetry for a fields set transformation as Eq. (1). Consider on the possibility of an antireductionist physics where N -non-abelian fields act together. Given the $SU(N)$ symmetry drive new association features which go further than Yang-Mills understanding. This means to preserve the symmetry pattern and introduce a new Lagrangian.

Eight aspects attached to group generators and gauge parameters were analysed in order to express the consistency of introduction of this extended gauge model. From group generators: algebra closure and Jacobi identities, Bianchi identities; from gauge parameters: Noether theorem, gauge fixing, BRST symmetry, global transformations (BRST, ghost scale, gauge global); charges algebra, covariant equations of motion plus PoincarÃ© lemma from both symmetry variables. And so, they are showing that $SU(N)$ gauge group acts as an operator where it does not matter the number of fields involved on its transformations. Consequently, given a certain $SU(N)$ gauge group it is possible to derive a Lagrangian where the number of potential fields is not necessarily equal to the number of group generators as ruled by Yang-Mills theory.

Eq. (1) introduces that symmetry should be treated as an environment. A fields association physics appears. In a further work we will analyse on more details other classical aspects, renormalizability, unitarity. For instance, study on its consequences on the Slavnov-Taylor identity. And so, understand on possibilities for a systemic physical process be described through this non-abelian whole gauge principle. Complexity should be an achievement related through a gauge totality principle.

13 Group relationships

Gauge theory considers fields as Lie algebra valued. So one should express $A_{\mu I} \equiv A_{\mu I}^{a}t^{a}$, $D_{\mu} \equiv D_{\mu}^{a}t^{a}$, $X_{\mu i} \equiv X_{\mu i}^{a}t^{a}$, $G_{\mu I} \equiv G_{\mu I}^{a}t^{a}$ under the corresponding group generators properties

$$[t_{a}, t_{b}] = if_{abc}t_{c}, \tag{171}$$

$$\{t_{a}, t_{b}\} = \frac{1}{N}\delta_{ab} + d_{abc}t_{c}, \tag{172}$$

where Eq. (172) does not belong to the algebra. And with the following traces properties

$$tr(t_{a}) = 0$$

$$tr(t_{a}t_{b}) = N\delta_{ab}$$

$$tr(t_a t_b t_c) = i\frac{N}{2} f_{abc}$$

$$tr(t_a t_b t_c t_d) = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} +$$

$$+\frac{N}{4}\left[d_{abf}d_{cdf} - d_{acf}d_{bdf} + d_{adf}d_{bcf}\right] \quad (173)$$

14 Covariant $X_{\mu i}$-equations of motion

Eq. (7.11) corresponds to (N-1) covariant equations of motions related to $X_{\mu i}$-fields. At this Appendix it will be expressed the correspondent currents

$$J^{T,i}_{\nu a} = J^{A,i}_{\nu a} + J^{S,i}_{\nu a}, \quad (174)$$

where $J^{A,i}_{\nu a}$ is derived from the Lagrangian antisymmetric sector, eq (89). It gives

$$J^{A,i}_{\nu a} = Nj^{i}_{\nu a} + 2\gamma_{pq}\left[2\delta_{ae}\gamma^{[ij]}X^{pc}_{[\nu}X^{qc}_{\alpha]} + \gamma^{ij}X^{p}_{a[\nu}X^{eq}_{\alpha]} + \gamma^{ji}X^{pe}_{[\alpha}X^{q}_{\nu]e} + \frac{N}{2}d_{3aecd}\gamma^{[ij]}X^{pc}_{[\nu}X^{qd}_{\alpha]}\right]X^{\alpha e}_{j}$$

$$+b_{[ij]}d_{abc}W^{b}_{[\nu\mu]}X^{\mu jc} + b_{[pq]}4Nd_{abc}\alpha_i\partial^{\mu}(X^{pc}_{[\mu}X^{qb}_{\nu]}). \quad (175)$$

where

$$j^{i}_{\nu a} = 2if_{abc}\gamma_{(jk)}[D^{c}_{\nu\mu} + \alpha_i X^{ic}_{\nu\mu} + \alpha_i\alpha_j f_{cmn}X^{im}_{[\nu}X^{jn}_{\mu]}]X^{\mu jb} \quad (176)$$

Considering $J^{S,i}_{\nu a}$ from Eq. (92), one gets

$$J^{S,i}_{\nu a} = -\nabla^{\mu}_{ab}T^{i}_{(\mu\nu)b} + J^{i}_{\nu a}, \quad (177)$$

where

$$\nabla^{\mu}_{ab} = \partial^{\mu}\delta_{ab} + gf_{abc}D^{\mu}_{c}, \quad (178)$$

and $T^{i}_{(\mu\nu)}$ and $J^{i}_{\nu a}$ are calculed from the 8 meshes that build up L_S according to Eq. (94). It gives

$$T^{ia}_{(\mu\nu)} = \sum_{k=1}^{8} T^{ia,k}_{(\mu\nu)}, \quad (179)$$

$$J^{i}_{\nu} = \sum_{k=1}^{8} J^{i,k}_{\nu}, \quad (180)$$

where only the first three meshes in Eq. (94) contributes to $T^{ia,}_{\mu\nu}$. They are

$$T^{ia,1}_{(\mu\nu)} = 4N\left[\beta_i\beta_j X^{ja}_{(\mu\nu)} + (\rho_i\beta_j + 4\rho_i\rho_j + \beta_i\rho_j)g_{\mu\nu}X^{ja\alpha}_{\alpha}\right], \quad (181)$$

$$T^{ia,2}_{(\mu\nu)} = 2iNf^{amn}\gamma_{pq}\left[\beta^i X^{pm}_{(\mu}X^{qn}_{\nu)} + \rho^i g_{\mu\nu}X^{pm}_{\alpha}X^{\alpha qn}\right], \quad (182)$$

$$T^{ia,3}_{(\mu\nu)} = 4iNf^{amn}\left[\begin{array}{l}\beta^i(a_{[pq]}X^{pm}_{(\mu}X^{qn}_{\nu)} + u_{[pq]}g_{\mu\nu}X^{pm}_{\alpha}X^{\alpha qn} + \\ +\rho^i(a_{[pq]} + 4u_{[pq]})g_{\mu\nu}X^{pm}_{\alpha}X^{\alpha qn}\end{array}\right]. \tag{183}$$

Considering $J^i_{\nu a}$, one gets

$$J^{i,2}_{\nu a} = 2f_{abc}\gamma^{[iq]}\left[\beta_j X^{jc}_{\nu\mu}X^{\mu qb} + \rho_j X^{qb}_{\nu}X^{jc\alpha}_{\alpha}\right], \tag{184}$$

$$J^{i,3}_{\nu a} = f_{abc}\left[a_{iq}(\beta_j X^{jb}_{(\nu\mu)} + \rho_i g_{\mu\nu}X^{jb\alpha}_{\alpha})X^{\mu qc} + u_{[iq]}(\beta_j + 4\rho_j)X^{ib\alpha}_{\alpha}X^{qc}_{\nu}\right] \tag{185}$$

From mesh 5 to 8 there is no more propagating terms. Eq. (177) will receive just sources. Thus from

$$\frac{\delta(L_5 + ... + L_8)}{\delta X^{ia}_{\nu}} = \sum_{k=5}^{8} J^{\nu i,a}, \tag{186}$$

we get calculations which will not contribute to the scope of this work. For instance, mesh (5) gives

$$J^{i,5}_{\nu a} = \gamma_{pq}\left[\begin{array}{l}4\gamma^{(il)}X^{pn}_{\nu}X^{qn}_{\mu}X^{\mu}_{la} + 2(\gamma^{il}X^{pn(\nu}X^{\mu)qa} + \gamma^{li}X^{pa(\mu}X^{\nu)qn})X_{\mu nl} + \\ \frac{N}{4}(\gamma_{il}(d_{3aLmn} + d_{3mnaL}) + \gamma_{li}(d_{3Lamn} + d_{3mnLa}))X^{pm(\nu}X^{\mu)qn}X^{lL}_{\mu}\end{array}\right]. \tag{187}$$

Consequently, the above Eq. (187) shows how others sources from Eq. (186) will not interfere on the covariant property of $X_{\mu i}$ equations of motions due to the fact that they depend only on $X_{\mu i}$ -fields whose transform covariantly.

15 Conserved currents

Classically, in order to avoid undesired degrees of freedom we should relate them to conserved currents. For this every field in this whole model must be associated to a corresponding conserved current. Noether and Slavnov-Taylor identities already inform on the existence of only one natural conservation law. In this apendice one explores the conserved currents through the circumstantial symmetry.

Considering Eq. (7.1)-(8.3), one gets the following D_{μ} -current expression:

$$J^{\mu}_{a}(D) = \sum_{i=1}^{4} J^{\mu}_{a(i)}(D)$$

where

$$J^{\mu}_{a(1)}(D) = 4\lambda_1(ig(dD^b_{\nu} + \alpha_i X^{ib}_{\nu})Z^{[\mu\nu]}[t_a, t_b])$$

$$J^{\mu}_{a(2)}(D) = 4\xi_1 ig(\beta_i X^{ib}_{\nu}Z^{(\mu\nu)} + \rho_i X^{\mu ib}Z_{(\nu}{}^{\nu)})[t_a, t_b]$$

$$J^{\mu}_{a(3)}(D) = 2\lambda_3(ig(dD^b_{\nu} + \alpha_i X^{ib}_{\nu})z^{[\mu\nu]}[t_a, t_b])$$

$$J^{\mu}_{a(4)}(D) = 2\xi_3 ig(\beta_i X^{ib}_{\nu}z^{(\mu\nu)} + \rho_i X^{\mu ib}z_{(\nu}{}^{\nu)})[t_a, t_b] \tag{188}$$

Expanding

$$J^{\mu}_{a(1)}(D) = 4ig\lambda_1\{c^2 D^b_{\nu}(\partial^{\mu}D^{\nu} - \partial^{\nu}D^{\mu} + ig[D^{\mu}, D^{\nu}]) +$$

$$+ d\alpha_j D^b_{\nu}(\partial^{\mu}X^{\nu j} - \partial^{\nu}X^{\mu j} + ig([D^{\mu}, X^{\nu j}] - [D^{\nu}, X^{\mu j}])) +$$

$$+ d\alpha_i X^{ib}_{\nu}(\partial^{\mu}D^{\nu} - \partial^{\nu}D^{\mu} + ig[D^{\mu}, D^{\nu}]) +$$

$$+ \alpha_i\alpha_j X^{ib}_{\nu}(\partial^{\mu}X^{\nu j} - \partial^{\nu}X^{\mu j} + ig([D^{\mu}, X^{\nu j}] - [D^{\nu}, X^{\mu j}]))\}[t_a, t_b]$$

$$J^{\mu}_{a(2)}(D) = 4\xi_1 ig\{\beta_i\beta_j X^{ib}_{\nu}(\partial^{\mu}X^{\nu j} + \partial^{\nu}X^{\mu j} + ig([D^{\nu}, X^{\nu j}] + [D^{\nu}, X^{\mu j}])) +$$

$$+\beta_i\rho_j g^{\mu\nu}X^{ib}_{\nu}(\partial_{\alpha}X^{\alpha j} + \partial_{\alpha}X^{\alpha j} + 2ig)[D_{\alpha}, X^{\alpha j}] +$$

$$+\rho_i\beta_j X^{\mu ib}(\partial_{\nu}X^{\nu j} + \partial_{\nu}X^{\nu j} + 2ig[D_{\nu}, X^{\nu j}]) +$$

$$+\rho_i\rho_j g^{\nu}_{\nu}X^{\mu ib}(\partial_{\alpha}X^{\alpha j} + \partial_{\alpha}X^{\alpha j} + 2ig[D_{\alpha}, X^{\alpha j}])\}[t_a, t_b]$$

$$J^{\mu}_{a(3)}(D) = 2ig\lambda_3\{dD^{b}_{\nu}(a_{(ij)}[X^{\mu i}, X^{\nu j}] + b_{[ij]}\{X^{\mu i}, X^{\nu j}\} + \gamma_{[ij]}X^{\mu i}X^{\nu j})$$

$$+\alpha_k X^{kb}_{\nu}(a_{(ij)}[X^{\mu i}, X^{\nu j}] + b_{[ij]}\{X^{\mu i}, X^{\nu j}\} + \gamma_{[ij]}X^{\mu i}X^{\nu j})\}[t_a, t_b]$$

$$J^{\mu}_{a(4)}(D) = 2\xi_3 ig\{\beta_k X^{kb}_{\nu}(a_{[ij]}[X^{\mu i}, X^{\nu j}] + u_{[ij]}g^{\mu\nu}[X^{i}_{\alpha}, X^{\alpha j}]$$

$$+b_{(ij)}\{X^{\mu i}, X^{\nu j}\} + v_{(ij)}g^{\mu\nu}\{X^{i}_{\alpha}, X^{\alpha j}\}) +$$

$$+\rho_k X^{\mu kb}(a_{[ij]}[X^{i}_{\nu}, X^{\nu j}] + u_{[ij]}g^{\nu}_{\nu}[X^{i}_{\alpha}, X^{\alpha j}]$$

$$+b_{(ij)}\{X^{i}_{\nu}, X^{\nu j}\} + v_{(ij)}g^{\nu}_{\nu}\{X^{i}_{\alpha}, X^{\alpha j}\})\}[t_a, t_b] \tag{189}$$

it yields,

$$J^{\mu}_{a}(D) = 4ig\lambda_1(d^2D^{b}_{\nu} + d\alpha_i X^{ib}_{\nu}(\partial^{\mu}D^{\nu} - \partial^{\nu}D^{\mu}) +$$

$$+(d\alpha_j D^{b}_{\nu} + \alpha_i\alpha_j X^{ib}_{\nu})(\partial^{\mu}X^{\nu j} - \partial^{\nu}X^{\mu j}))[t_a, t_b] +$$

$$+4ig\xi_1(\beta_i\beta_j X^{ib}_{\nu}(\partial^{\mu}X^{\nu j} + \partial^{\nu}X^{\mu j}) + 2\rho_i\beta_j X^{\mu ib}\partial_{\nu}X^{\nu j} +$$

$$+2(\beta_i\rho_j + \rho_i\rho_j g^{\nu}_{\nu})X^{\mu ib}\partial_{\alpha}X^{\alpha j})[t_a, t_b]$$

$$-4g^2\lambda_1((d^2D^{b}_{\nu} + d\alpha_i X^{ib}_{\nu})[D^{\mu}, D^{\nu}] +$$

$$+(d\alpha_j D^{b}_{\nu} + \alpha_i\alpha_j X^{ib}_{\nu})([D^{\mu}, X^{\nu j}] - [D^{\nu}, X^{\mu j}]))[t_a, t_b]$$

$$-4g^2\xi_1(\beta_i\beta_j X^{ib}_{\nu}([D^{\mu}, X^{\nu j}] + [D^{\nu}, X^{\mu j}]) +$$

$$+2(\beta_i\rho_j + \rho_i\beta_j + \rho_i\rho_j g^{\nu}_{\nu})X^{\mu ib}[D_{\alpha}, X^{\alpha j}])[t_a, t_b]$$

$$+2ig\lambda_3(dD^{b}_{\nu} + \alpha_k X^{kb}_{\nu})(a_{(ij)}[X^{\mu i}, X^{\nu j}] + b_{[ij]}\{X^{\mu i}, X^{\nu j}\} +$$

$$+\gamma_{[ij]}X^{\mu i}X^{\nu j})[t_a, t_b]$$

$$+2ig\xi_3\{\beta_k X^{kb}_{\nu}(a_{[ij]}[X^{\mu i}, X^{\nu j}] + u_{(ij)}g^{\mu\nu}[X^{i}_{\alpha}, X^{\alpha j}] +$$

$$+b_{(ij)}\{X^{\mu i}, X^{\nu j}\} + v_{(ij)}g^{\nu}_{\nu}\{X^{i}_{\alpha}, X^{\alpha j}\}) +$$

$$+\rho_k X^{\mu kb}(a_{[ij]}[X^{i}_{\nu}, X^{\nu j}] + u_{[ij]}g^{\nu}_{\nu}[X^{i}_{\alpha}, X^{\alpha j}] +$$

$$+b_{(ij)}\{X^{i}_{\nu}, X^{\nu j}\} + v_{(ij)}g^{\nu}_{\nu}\{X^{i}_{\alpha}, X^{\alpha j}\})\}[t_a, t_b] \tag{190}$$

Eq. (C.3) can be rewritten as the following expansion

$$J_a^{\mu}(D) = \partial DD + \partial DX + \partial XX + DDD + DDX + DXX + XXX$$

where

$$\partial DD = D_\nu^b \partial^\mu D_\nu a_1 + D_\nu^b \partial^\nu D^\mu a_2 + D^{\mu b} \partial_\alpha D^\alpha a_3$$

$$a_1 = 4\lambda_1 igd^2 [t_a, t_b]$$

$$a_2 = -4\lambda_1 igd^2 [t_a, t_b]$$

$$a_3 = 0$$

$$\partial DX = D_\nu^b \partial^\mu X^{\nu i} b_1 + D_\nu^b \partial^\nu X^{\mu i} b_2 + D^{\mu b} \partial_\alpha X^{\alpha i} b_3$$

$$+ X_\nu^{ib} \partial^\mu D^\nu b_4 + X_\nu^{ib} \partial^\nu D^\mu b_5 + X^{\mu ib} \partial_\alpha D^\alpha b_6$$

$$b_1 = 4\lambda_1 igd\alpha_i [t_a, t_b]$$

$$b_2 = -4\lambda_1 igd\alpha_i [t_a, t_b]$$

$$b_3 = 0$$

$$b_4 = 4\lambda_1 igd\alpha_i [t_a, t_b]$$

$$b_5 = -4\lambda_1 igd\alpha_i [t_a, t_b]$$

$$b_6 = 0$$

$$\partial XX = X_\nu^{ib} \partial^\mu X^{\nu j} c_1 + X_\nu^{ib} \partial^\nu X^{\mu j} c_2 + X^{\mu ib} \partial_\alpha X^{\alpha j} c_3$$

$$c_1 = 4\lambda_1 ig\alpha_i\alpha_j [t_a, t_b] + 4\xi_1 ig\beta_i\beta_j [t_a, t_b]$$

$$c_2 = -4\lambda_1 ig\alpha_i\alpha_j [t_a, t_b] - 4\xi_1 ig\beta_i\beta_j [t_a, t_b]$$

$$c_3 = 8\xi_1 ig(\rho_i\beta_j + \beta_i\rho_j + \rho_i\rho_j g_\nu^\nu)[t_a, t_b]$$

$$DDD = D_\nu^b D^{\mu e} D^{\nu f} (-4\lambda_1 g^2 d^2)[t_e, t_f][t_a, t_b]$$

$$DDX = (X_\nu^{ib} D^{\mu e} D^{\nu f} + D_\nu^b D^{\mu e} X^{\nu i f} - D_\nu^b D^{\nu e} X^{\mu i f}).$$

$$.(-4\lambda_1 g^2 d\alpha_i)[t_e, t_f][t_a, t_b]$$

$$DXX = X_\nu^{ib} D^{\mu e} X^{\nu j f} (-4\lambda_1 g^2 \alpha_i\alpha_j - 4\xi_1 g^2 \beta_i\beta_j)[t_e, t_f][t_a, t_b] +$$

$$+ X_\nu^{ib} D^{\nu e} X^{\mu j f} (4\lambda_1 g^2 \alpha_i\alpha_j - 4\xi_1 g^2 \beta_i\beta_j)[t_e, t_f][t_a, t_b] +$$

$$+X^{\mu ib}D_{\alpha}^{e}(-8\xi_1 g^2)(\beta_i\rho_j+\rho_i\beta_j+\rho_i\rho_j g_\nu^\nu)[t_e,t_f][t_a,t_b]+$$

$$+D_\nu^{\ b}X^{\mu ie}X^{\nu jf}(2\lambda_2 igd)(a_{(ij)}[t_e,t_f]+b_{[ij]}\{t_e,t_f\}+\gamma_{[ij]}t_e t_f)[t_a,t_b]$$

$$XXX=X_\nu^{\ kb}X^{\mu ie}X^{\nu jf}(2\lambda_2 ig\alpha_k[t_e,t_f]+b_{[ij]}\{t_e,t_f\}+\gamma_{[ij]}t_e t_f)[t_a,t_b]+$$

$$+X_\nu^{\ kb}X^{\mu ie}X^{\nu jf}(\beta_k a_{[ij]}[t_e,t_f]+\beta_k b_{(ij)}\{t_e,t_f\})(2\xi_3 ig)[t_a,t_b]+$$

$$+X^{\mu kb}X_\alpha^{\ ie}X^{\alpha jf}((\beta_k u_{[ij]}+\rho_k a_{[ij]}+\rho_k u_{[ij]}g_\nu^\nu)[t_e,t_f]+$$

$$+(\beta_k v_{(ij)}+\rho_k b_{(ij)}+\rho_k v_{(ij)}g_\nu^\nu)\{t_e,t_f\})(2\xi_3 ig)[t_a,t_b] \quad (191)$$

Notice that eqs. (3.24) and (3.27) coincide.

Similarly for X_{ia}^{μ} fields equations of motion, eqs. (7.2) and (8.4), are the following J_{ia}^{μ} currents expression:

$$J_{ia}^{\mu}(X)=\sum_{k=1}^{6}J_{ia(k)}^{\mu}$$

where

$$J_{ia(1)}^{\mu}(X)=4ig\lambda_1\alpha_i(D_\nu^b Z^{[\mu\nu]}[t_a,t_b])$$

$$J_{ia(2)}^{\mu}(X)=-4\xi_1(\beta_i\partial_\nu Z^{(\mu\nu)}t_a+\rho_i\partial^\mu Z_{(\nu}^{\ \nu)}t_a+$$

$$+ig(\beta_i D_\nu^b Z^{(\mu\nu)}+\rho_i D^{\mu b}Z_{(\nu}^{\ \nu)})[t_a,t_b])$$

$$J_{ia(3)}^{\mu}(X)=2\lambda_2(2a_{(ij)}X_\nu^{\ jb}z^{[\mu\nu]}[t_a,t_b]+(2b_{[ij]}+\gamma_{[ij]})X_\nu^{\ jb}z^{[\mu\nu]}\{t_a,t_b\})$$

$$J_{ia(4)}^{\mu}(X)=4\xi_2\{(a_{[ij]}X_\nu^{\ jb}z^{(\mu\nu)}+u_{[ij]}X^{\mu jb}z_{(\nu}^{\ \nu)})[t_a,t_b]+$$

$$+(b_{(ij)}X_\nu^{\ jb}z^{(\mu\nu)}+v_{(ij)}X^{\mu jb}z_{(\nu}^{\ \nu)})\{t_a,t_b\}\}$$

$$J_{ia(5)}^{\mu}(X)=\lambda_3(2ig\alpha_i D_\nu^b z^{[\mu\nu]}[t_a,t_b]+2a_{(ij)}X_\nu^{\ jb}Z^{[\mu\nu]}[t_a,t_b]+$$

$$+(2b_{[ij]}+\gamma_{[ij]})X_\nu^{\ jb}Z^{[\mu\nu]}\{t_a,t_b\})$$

$$J_{ia(6)}^{\mu}(X)=2\xi_3\{-\beta_i\partial_\nu z^{(\mu\nu)}t_a-\rho_i\partial^\mu z_{(\nu}^{\ \nu)}t_a+$$

$$-ig(\beta_i D_\nu^b z^{(\mu\nu)}+\rho_i D^{\mu b}z_{(\nu}^{\ \nu)})[t_a,t_b]+$$

$$+(a_{[ij]}X_\nu^{\ jb}Z^{(\mu\nu)}+u_{[ij]}X^{\mu jb}Z_{(\nu}^{\ \nu)})[t_a,t_b]+$$

$$+(b_{(ij)}X_\nu^{\ jb}Z^{(\mu\nu)}+v_{(ij)}X^{\mu jb}Z_{(\nu}^{\ \nu)})\{t_a,t_b\}\} \quad (192)$$

Expanding

$$J^{\mu}_{ia(1)}(X)=4ig\lambda_1(d\alpha_i D^b_\nu(\partial^\mu D^\nu-\partial^\nu D^\mu+ig[D^\mu,D^\nu])+$$
$$+\alpha_i\alpha_j D^b_\nu(\partial^\mu X^{\nu j}-\partial^\nu X^{\mu j}+ig([D^\mu,X^{\nu j}]-[D^\nu,X^{\mu j}])))[t_a,t_b]$$

$$J^{\mu}_{ia(2)}(X)=-4\xi_1\{(\beta_i\beta_j\partial_\nu(\partial^\mu X^{\nu j}+\partial^\nu X^{\mu j}+ig([D,X^{\nu j}]+[D^\nu,X^{\mu j}]))+$$
$$+2\beta_i\rho_j g^{\mu\nu}\partial_\nu(\partial_\alpha X^{\alpha j}+ig[D_\alpha,X^{\alpha j}])+$$
$$+2\beta_j\rho_i\partial^\mu(\partial_\nu X^{\nu j}+ig[D_\nu,X^{\nu j}])+$$
$$+2\rho_i\rho_j g^\nu_\nu\partial^\mu(\partial_\alpha X^{\alpha j}+ig[D_\alpha,X^{\alpha j}]))t_a+$$
$$+ig(\beta_i\beta_j D^b_\nu(\partial^\mu X^{\nu j}+\partial^\nu X^{\mu j}+ig[D^\mu,X^{\nu j}]+[D^\nu,X^{\mu j}])+$$
$$+2\beta_i\rho_j g^{\mu\nu}D^b_\nu(\partial_\alpha X^{\alpha j}+ig[D_\alpha,X^{\alpha j}])+$$
$$+2\beta_j\rho_i D^{\mu b}(\partial_\nu X^{\nu j}+ig[D_\nu,X^{\nu j}])+$$
$$+2\rho_i\rho_j g^\nu_\nu D^{\mu b}(\partial_\alpha X^{\alpha j}+ig[D_\alpha,X^{\alpha j}]))[t_a,t_b]\}$$

$$J^{\mu}_{ia(3)}(X)=2\lambda_2(2a_{(ij)}X_\nu{}^{jb}(a_{(kl)}[X^{\mu k},X^{\nu l}]+b_{[kl]}\{X^{\mu k},X^{\nu l}\}+\gamma_{[kl]}X^{\mu k}X^{\nu l})[t_a,t_b]+$$
$$+(2b_{[ij]}+\gamma_{[ij]})X_\nu{}^{jb}(a_{(kl)}[X^{\mu k},X^{\nu l}]+b_{[kl]}\{X^{\mu k},X^{\nu l}+\gamma_{[kl]}X^{\mu k}X^{\nu l})\{t_a,t_b\})$$

$$J^{\mu}_{ia(4)}(X)=4\xi_2\{(a_{[ij]}X_\nu{}^{jb}(a_{[kl]}[X^{\mu k},X^{\nu l}]+u_{[kl]}g^{\mu\nu}[X_\alpha{}^k,X^{\alpha l}]+$$
$$+b_{(kl)}\{X^{\mu k},X^{\nu l}\}+v_{(kl)}g^{\mu\nu}\{X_\alpha{}^k,X^{\alpha l}\})+$$
$$+u_{[ij]}X^{\mu jb}(a_{[kl]}[X_\nu{}^k,X^{\nu l}]+u_{[kl]}g^\nu_\nu[X_\alpha{}^k,X^{\alpha l}]+$$
$$+b_{(kl)}\{X_\nu{}^k,X^{\nu l}\}+v_{(kl)}g^\nu_\nu\{X_\alpha{}^k,X^{\alpha l}\})[t_a,t_b]+$$
$$+(b_{(ij)}X_\nu{}^{jb}(a_{[kl]}[X^{\mu k},X^{\nu l}]+u_{[kl]}g^{\mu\nu}[X_\alpha{}^k,X^{\alpha l}]+$$
$$+b_{(kl)}\{X^{\mu k},X^{\nu l}]+u_{(kl)}g^{\mu\nu}[X_\alpha{}^k,X^{\alpha l}])+$$
$$+v_{(ij)}X^{\mu jb}(a_{[kl]}[X_\nu{}^k,X^{\nu l}+u_{[kl]}g^\nu_\nu[X_\alpha{}^k,X^{\alpha l}]+$$
$$+b_{(kl)}\{X_\nu{}^k,X^{\nu l}\}+v_{(kl)}g^\nu_\nu\{X_\alpha{}^k,X^{\alpha l}\}))\{t_a,t_b\}\}$$

$$J^{\mu}_{ia(5)}(X)=\lambda_3\{2ig\alpha_i D^b_\nu(a_{(jk)}[X^{\mu j},X^{\nu k}]+b_{[jk]}\{X^{\mu j},X^{\nu k}\}+\gamma_{[jk]}X^{\mu j}X^{\nu k})[t_a,t_b]+$$
$$+2da_{(ij)}X_\nu{}^{jb}(\partial^\mu D^\nu-\partial^\nu D^\mu+ig[D^\mu,D^\nu])[t_a,t_b]+$$
$$+2\alpha_k a_{(ij)}X_\nu{}^{jb}(\partial^\mu X^{\nu k}-\partial^\nu X^{\mu k}+ig([D^\mu,X^{\nu k}]-[D^\nu,X^{\mu k}]))[t_a,t_b]+$$

$$+d(2b_{[ij]}+\gamma_{[ij]})X_{\nu}^{\ jb}(\partial^{\mu}D^{\nu}-\partial^{\nu}D^{\mu}+ig[D^{\mu},D^{\nu}])\{t_a,t_b\}+$$

$$+\alpha_k(2b_{[ij]}+\gamma_{[ij]})X_{\nu}^{\ jb}(\partial^{\mu}X^{\nu k}-\partial^{\nu}X^{\mu k}+ig([D^{\mu},X^{\nu k}]-[D^{\nu},X^{\mu k}]))\{t_a,t_b\}\}$$

$$J^{\mu}_{ia(6)}(X)=2\xi_3\{-\beta_i\partial_{\nu}(a_{[jk]}[X^{\mu j},X^{\nu k}]+u_{[jk]}g^{\mu\nu}[X_{\alpha}^{\ j},X^{\alpha k}]+$$

$$+b_{(jk)}\{X^{\mu j},X^{\nu k}\}+v_{(jk)}g^{\mu\nu}\{X_{\alpha}^{\ j},X^{\alpha k}\})t_a+$$

$$-\rho_i\partial^{\mu}(a_{[jk]}[X_{\nu}^{\ j},X^{\nu k}]+u_{[jk]}g^{\nu}_{\nu}[X_{\alpha}^{\ j},X^{\alpha k}]+$$

$$+b_{(jk)}\{X_{\nu}^{\ j},X^{\nu k}\}+v_{(jk)}g^{\nu}_{\nu}\{X_{\alpha}^{\ j},X^{\alpha k}\})t_a+$$

$$-ig\beta_i D^b_{\nu}(a_{[jk]}[X^{\mu j},X^{\nu k}]+u_{[jk]}g^{\mu\nu}[X_{\alpha}^{\ j},X^{\alpha k}]+$$

$$+b_{(jk)}\{X^{\mu j},X^{\nu k}\}+v_{(jk)}g^{\mu\nu}\{X_{\alpha}^{\ j},X^{\alpha k}\})[t_a,t_b]+$$

$$-ig\rho_i D^{\mu b}(a_{[jk]}[X_{\nu}^{\ j},X^{\nu k}]+u_{[jk]}g^{\nu}_{\nu}[X_{\alpha}^{\ j},X^{\alpha k}]+$$

$$+b_{(jk)}\{X_{\nu}^{\ j},X^{\nu k}\}+v_{(jk)}g^{\nu}_{\nu}\{X_{\alpha}^{\ j},X^{\alpha k}\})[t_a,t_b]+$$

$$+\beta_k a_{[ij]}X_{\nu}^{\ jb}(\partial^{\mu}X^{\nu k}+\partial^{\nu}X^{\mu k}+ig([D^{\mu},X^{\nu k}]+[D^{\nu},X^{\mu k}]))[t_a,t_b]+$$

$$+2\rho_k a_{[ij]}g^{\mu\nu}X_{\nu}^{\ jb}(\partial_{\alpha}X^{\alpha k}+ig[D_{\alpha},X^{\alpha k}])[t_a,t_b]+$$

$$+2\beta_k u_{[ij]}X^{\mu jb}(\partial_{\nu}X^{\nu k}+ig[D_{\nu},X^{\nu k}])[t_a,t_b]+$$

$$+2\rho_k u_{[ij]}g^{\nu}_{\nu}X^{\mu jb}(\partial_{\alpha}X^{\alpha k}+ig[D_{\alpha},X^{\alpha k}])[t_a,t_b]+$$

$$+\beta_k b_{(ij)}X_{\nu}^{\ jb}(\partial^{\mu}X^{\nu k}+\partial^{\nu}X^{\mu k}+ig([D^{\mu},X^{\nu k}]+[D^{\nu},X^{\mu k}]))\{t_a,t_b\}+$$

$$+2\rho_k b_{(ij)}g^{\mu\nu}X_{\nu}^{\ jb}(\partial_{\alpha}X^{\alpha k}+ig[D_{\alpha},X^{\alpha k}])\{t_a,t_b\}+$$

$$+2\beta_k v_{(ij)}X^{\mu jb}(\partial_{\nu}X^{\nu k}+ig[D_{\nu},X^{\nu k}])\{t_a,t_b\}+$$

$$+2\rho_k v_{(ij)}g^{\nu}_{\nu}X^{\mu jb}(\partial_{\alpha}X^{\alpha k}+ig[D_{\alpha},X^{\alpha k}])\{t_a,t_b\}\} \tag{193}$$

which yields

$$J^{\mu}_{ia}(X)=-4\xi_1(\beta_i\beta_j\partial_{\nu}(\partial^{\mu}X^{\nu j}+\partial^{\nu}X^{\mu j})+2(\beta_i\rho_j+\beta_j\rho_i+\rho_i\rho_j g^{\nu}_{\nu})\partial^{\mu}\partial_{\alpha}X^{\alpha j})t_a+$$

$$+4ig\lambda_1(d\alpha_i D^b_{\nu}(\partial^{\mu}D^{\nu}-\partial^{\nu}D^{\mu})+\alpha_i\alpha_j D^b_{\nu}(\partial^{\mu}X^{\nu j}-\partial^{\nu}X^{\mu j}))[t_a,t_b]+$$

$$-4ig\xi_1(\beta_i\beta_j D^b_{\nu}(\partial^{\mu}X^{\nu j}+\partial^{\nu}X^{\mu j})+2(\beta_i\rho_j+\beta_j\rho_i+\rho_i\rho_j g^{\nu}_{\nu})D^{\mu b}\partial_{\alpha}X^{\alpha j})[t_a,t_b]+$$

$$+\lambda_3\{X_{\nu}^{\ jb}(\partial^{\mu}D^{\nu}-\partial^{\nu}D^{\mu})(2da_{(ij)}[t_a,t_b]+d(2b_{[ij]}+\gamma_{[ij]})\{t_a,t_b\})+$$

$$+X_{\nu}^{\ jb}(\partial^{\mu}X^{\nu k}-\partial^{\nu}X^{\mu k})(2\alpha_k a_{(ij)}[t_a,t_b]+\alpha_k(2b_{[ij]}+\gamma_{[ij]})\{t_a,t_b\})\}+$$

$$+2\xi_3\{X_{\nu}^{\ jb}(\partial^{\mu}X^{\nu k}+\partial^{\nu}X^{\mu k})(\beta_k a_{[ij]}[t_a,t_b]+\beta_k b_{(ij)}\{t_a,t_b\})+$$

$$+2X^{\mu jb}\partial_{\alpha}X^{\alpha k}((\rho_k a_{[ij]}+\beta_k u_{[ij]}+\rho_k u_{[ij]}g^{\nu}_{\nu})[t_a,t_b]+$$

ISSN 2347-3487

$$+(\rho_k b_{(ij)} + \beta_k v_{(ij)} + \rho_k v_{(ij)} g_\nu^\nu)\{t_a, t_b\})\} +$$

$$-4ig\xi_1(\beta_i\beta_j\partial_\nu([D^\mu, X^{\nu j}]) + 2(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)\partial^\mu[D_\alpha, X^{\alpha j}])t_a +$$

$$-2\xi_3\{\beta_i\partial_\nu(a_{[jk]}[X^{\mu j}, X^{\nu k}] + b_{(jk)}\{X^{\mu j}, X^{\nu k}\}) +$$

$$+(\beta_i u_{[jk]} + \rho_i(a_{(jk)} + u_{[jk]} g_\nu^\nu))\partial^\mu[X_\alpha^{\ j}, X^{\alpha k}] +$$

$$+(\beta_i v_{(jk)} + \rho_i(b_{(jk)} + v_{(jk)} g_\nu^\nu))\partial^\mu\{X_\alpha^{\ j}, X^{\alpha k}\}\}t_a +$$

$$-4g^2\lambda_1(d\alpha_i D_\nu^b[D^\mu, D^\nu] + \alpha_i\alpha_j D_\nu^b([D^\mu, X^{\nu j}] - [D^\nu, X^{\mu j}]))[t_a, t_b] +$$

$$+4g^2\xi_1(\beta_i\beta_j D_\nu^b([D^\mu, X^{\nu j}] + [D^\nu, X^{\mu j}]) +$$

$$+2(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)D^{\mu b}[D_\alpha, X^{\alpha j}])[t_a, t_b] +$$

$$+ig\lambda_3\{X_\nu^{\ jb}[D^\mu, D^\nu](2da_{(ij)}[t_a, t_b] + d(2b_{[ij]} + \gamma_{[ij]})\{t_a, t_b\}) +$$

$$+2\alpha_i D_\nu^b(a_{(jk)}[X, X] + b_{[jk]}\{X, X\} + \gamma_{[jk]} X^{\mu j} X^{\nu k})[t_a, t_b] +$$

$$+X_\nu^{\ jb}([D^\mu, X^{\nu k}] - [D^\nu, X^{\mu k}])(2\alpha_k a_{(ij)}[t_a, t_b] + \alpha_k(2b_{[ij]} + \gamma_{[ij]}))\{t_a, t_b\}\} +$$

$$-2ig\xi_3\{\beta_i D_\nu^b(a_{[jk]}[X^{\mu j}, X^{\nu k}] + b_{(jk)}\{X^{\mu j}, X^{\nu k}\}) +$$

$$+(\beta_i u_{[jk]} + \rho_i(a_{[jk]} + u_{[jk]} g_\nu^\nu))D^{\mu b}[X_\alpha^{\ j}, X^{\alpha k}] +$$

$$+(\beta_i v_{(jk)} + \rho_i(b_{(jk)} + v_{(jk)} g_\nu^\nu))D^{\mu b}\{X_\alpha^{\ j}, X^{\alpha k}\}\}[t_a, t_b] +$$

$$+2ig\xi_3\{X_\nu^{\ jb}([D^\mu, X^{\nu k}] + [D^\nu, X^{\mu k}])(\beta_k a_{[ij]}[t_a, t_b] + \beta_k b_{(ij)}\{t_a, t_b\})$$

$$+2X^{\mu jb}[D_\alpha, X^{\alpha k}]((\rho_k a_{[ij]} + \beta_k u_{[ij]} + \rho_k u_{[ij]} g_\nu^\nu)[t_a, t_b] +$$

$$+(\rho_k b_{(ij)} + \beta_k v_{(ij)} + \rho_k v_{(ij)} g_\nu^\nu)\{t_a, t_b\})\} +$$

$$+2\lambda_2 X_\nu^{\ jb}(a_{(kl)}[X^{\mu k}, X^{\nu l}] + b_{[kl]}\{X^{\mu k}, X^{\nu l}\})(2a_{(ij)}[t_a, t_b] + (2b_{[ij]} + \gamma_{[ij]})\{t_a, t_b\}) +$$

$$+4\xi_2\{X_\nu^{\ jb}(a_{[kl]}[X^{\mu k}, X^{\nu l}] + b_{(kl)}\{X^{\mu k}, X^{\nu l}])(a_{[ij]}[t_a, t_b] + b_{(ij)}\{t_a, t_b\}) +$$

$$+X^{\mu jb}([X_\alpha^{\ k}, X^{\alpha l}](u_{[kl]}(a_{[ij]}[t_a, t_b] + b_{(ij)}\{t_a, t_b\}) +$$

$$+(a_{[kl]} + u_{[kl]} g_\nu^\nu)(u_{[ij]}[t_a, t_b] + v_{(ij)}\{t_a, t_b\})) +$$

$$+\{X_\alpha^{\ k}, X^{\alpha l}\}(v_{(kl)}(a_{[ij]}[t_a, t_b] + b_{(ij)}\{t_a, t_b\}) +$$

$$+(b_{(kl)} + v_{(kl)} g_\nu^\nu)(u_{[ij]}[t_a, t_b] + v_{(ij)}\{t_a, t_b\}))))\} \tag{194}$$

Eq. (C.6) can be rewritten expanding as

$$J_{ia}^{\ \mu}(X) = \partial\partial X + \partial DD + \partial DX + \partial XX + DDD + DDX + DXX + XXX$$

$$\partial\partial X = \partial_\nu\partial^\mu X^{\nu j} a_1 + \partial_\nu\partial^\nu X^{\mu j} a_2 + \partial^\mu\partial_\alpha X^{\alpha j} a_3$$

$$a_1 = -4\xi_1\beta_i\beta_j t_a$$

$$a_2 = -4\xi_1\beta_i\beta_j t_a$$

$$a_3 = -8\xi_1(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)t_a$$

$$\partial DD = D_\nu^{\ b}\partial^\mu D^\nu b_1 + D_\nu^{\ b}\partial^\nu D^\mu b_2 + D^{\mu b}\partial_\alpha D^\alpha b_3$$

$$b1 = 4\lambda_1 igd\alpha_i[t_a, t_b]$$

$$b2 = -4\lambda_1 igd\alpha_i[t_a, t_b]$$

$$\partial DX = D_\nu^{\ b}\partial^\mu X^{\nu j}c_1 + D_\nu^{\ b}\partial^\nu X^{\mu j}c_2 + D^{\mu b}\partial_\alpha X^{\alpha j}c_3$$

$$+ X_\nu^{\ jb}\partial^\mu D^\nu c_4 + X_\nu^{\ jb}\partial^\nu D^\mu c_5 + X^{\nu\ jb}_{\ \nu}\partial_\alpha D^\alpha c_6$$

$$+ D^{\mu e}\partial_\nu X^{\nu jf}(-4\xi_1 ig\beta_i\beta_j)[t_e, t_f]t_a +$$

$$+ X^{\nu jf}\partial_\nu D^{\mu e}(-4\xi_1 ig\beta_i\beta_j)[t_e, t_f]t_a +$$

$$+ D^{\nu e}\partial_\nu X^{\mu jf}(-4\xi_1 ig\beta_i\beta_j)[t_e, t_f]t_a +$$

$$+ X^{\mu jf}\partial_\nu D^{\nu e}(-4\xi_1 ig\beta_i\beta_j)[t_e, t_f]t_a +$$

$$+ D_\alpha^{\ e}\partial^\mu X^{\alpha jf}(-8\xi_1 ig)(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)[t_e, t_f]t_a$$

$$+ X^{\alpha jf}\partial^\mu D_\alpha^{\ e}(-8\xi_1 ig)(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)[t_e, t_f]t_a$$

$$c_1 = 4\lambda_1 ig\alpha_i\alpha_j[t_a, t_b] - 4\xi_1 ig\beta_i\beta_j[t_a, t_b]$$

$$c_2 = -4\lambda_1 ig\alpha_i\alpha_j[t_a, t_b] - 4\xi_1 ig\beta_i\beta_j[t_a, t_b]$$

$$c_3 = -8\xi_1 ig(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g_\nu^\nu)[t_a, t_b]$$

$$c_4 = \lambda_3(2da_{(ij)}[t_a, t_b] + d(2b_{[ij]} + \gamma_{[ij]})\{t_a, t_b\})$$

$$c_5 = -\lambda_3(2da_{(ij)}[t_a, t_b] + d(2b_{[ij]} + \gamma_{[ij]})\{t_a, t_b\})$$

$$\partial XX = X_\nu^{\ jb}\partial^\mu X^{\nu k}d_1 + X_\nu^{\ jb}\partial^\nu X^{\mu k}d_2 + X^{\mu jb}\partial_\alpha X^{\alpha k}d_3 +$$

$$+ X^{\mu je}\partial_\nu X^{\nu kf}(-2\xi_3\beta_i(a_{[jk]}[t_e, t_f] + b_{(ij)}\{t_e, t_f\})t_a) +$$

$$+ X^{\nu kf}\partial_\nu X^{\mu je}(-2\xi_3\beta_i(a_{[jk]}[t_e, t_f] + b_{(ij)}\{t_e, t_f\})t_a) +$$

$$+ (X_\alpha^{\ je}\partial^\mu X^{\alpha kf} + X^{\alpha kf}\partial^\mu X_\alpha^{\ je}).(-2\xi_3(\beta_i u_{[jk]} + \rho_i(a_{[jk]} +$$

$$+ u_{[jk]}g_\nu^\nu))[t_e, t_f]t_a - 2\xi_3(\beta_i v_{(jk)} + \rho_i(b_{(jk)} + v_{(jk)}g_\nu^\nu))\{t_e, t_f\}t_a)$$

$$d_1 = \lambda_3(2\alpha_k a_{(ij)}[t_a,t_b] + \alpha_k(2b_{[ij]} + \gamma_{[ij]})\{t_a,t_b\}) +$$
$$+2\xi_3(\beta_k a_{[ij]}[t_a,t_b] + \beta_k b_{(ij)}\{t_a,t_b\})$$
$$d_2 = -\lambda_3(2\alpha_k a_{(ij)}[t_a,t_b] + \alpha_k(2b_{[ij]} + \gamma_{[ij]})\{t_a,t_b\}) +$$
$$+2\xi_3(\beta_k a_{[ij]}[t_a,t_b] + \beta_k b_{(ij)}\{t_a,t_b\})$$
$$d_3 = 4\xi_3((\rho_k a_{[ij]} + \beta_k u_{[ij]} + \rho_k u_{[ij]} g^\nu_\nu)[t_a,t_b] +$$
$$+(\rho_k b_{(ij)} + \beta_k u_{[ij]} + \rho_k v_{(ij)} g^\nu_\nu)\{t_a,t_b\})$$

$$DDD = D_\nu D^{\mu e} D^{\nu f}(-4\lambda_1 g^2 d\alpha_i)[t_e,t_f][t_a,t_b]$$

$$DDX = D_\nu^{\ b} D^{\mu e} X^{\nu jf}(-4\lambda_1 g^2 \alpha_i\alpha_j + 4\xi_1 g^2 \beta_i\beta_j)[t_e,t_f][t_a,t_b] +$$
$$+D_\nu^{\ b} D^{\mu e} X^{\nu jf}(-4\lambda_1 g^2 \alpha_i\alpha_j + 4\xi_1 g^2 \beta_i\beta_j)[t_e,t_f][t_a,t_b] +$$
$$+D^{\mu b} D_\alpha^{\ e} X^{\alpha jf}(8\xi_1 g^2(\beta_i\rho_j + \beta_j\rho_i + \rho_i\rho_j g^\nu_\nu))[t_e,t_f][t_a,t_b]$$
$$+X_\nu^{\ jb} D^{\mu e} D^{\nu f}(\lambda_3 ig).(2da_{(ij)}[t_e,t_f][t_a,t_b] +$$
$$+d(2b_{[ij]} + \gamma_{[ij]})[t_e,t_f][t_a,t_b])$$

$$DXX = D_\nu^{\ b} X^{\mu je} X^{\nu kf}(2\lambda_3 ig\alpha_i(a_{(jk)}[t_e,t_f] + b_{[jk]}\{t_e,t_f\} + \gamma_{[jk]}t_e t_f) +$$
$$-2\xi_3 ig\beta_i(a_{[jk]}[t_e,t_f] + b_{(jk)}\{t_e,t_f\}))[t_a,t_b] +$$
$$+D^{\mu b} X_\alpha^{\ je} X^{\alpha kf}(-2\xi_3 ig(\beta_i u_{[jk]} + \rho_i(a_{[jk]} + u_{[jk]} g^\nu_\nu))[t_e,t_f] +$$
$$-2\xi_3 ig(\beta_i v_{(jk)} + \rho_i(b_{(jk)} + v_{(jk)} g^\nu_\nu))\{t_e,t_f\})[t_a,t_b] +$$
$$+X_\nu^{\ jb} D^{\mu e} X^{\nu kf}(\lambda_3 ig\alpha_k(2a_{(ij)}[t_e,t_f][t_a,t_b] + (2b_{[ij]} + \gamma_{[ij]})[t_e,t_f]\{t_a,t_b\}) +$$
$$+2\xi_2 ig\beta_k(a_{[ij]}[t_e,t_f][t_a,t_b] + b_{(ij)}[t_e,t_f]\{t_a,t_b\})) +$$
$$+X_\nu^{\ jb} D^{\nu e} X^{\mu kf}(-\lambda_3 ig\alpha_k(2a_{(ij)}[t_e,t_f][t_a,t_b] + (2b_{[ij]} + \gamma_{[ij]})[t_e,t_f]\{t_a,t_b\}) +$$
$$+2\xi_2 ig\beta_k(a_{[ij]}[t_e,t_f][t_a,t_b] + b_{(ij)}[t_e,t_f]\{t_a,t_b\})) +$$
$$+X^{\mu jb} D_\alpha^{\ e} X^{\alpha kf}(4\xi_3 ig(\rho_k a_{[ij]} + \beta_k u_{[ij]} + \rho_k u_{[ij]} g^\nu_\nu)[t_e,t_f][t_a,t_b] +$$
$$+4\xi_3 ig(\rho_k b_{(ij)} + \beta_k v_{(ij)} + \rho_k v_{(ij)} g^\nu_\nu)[t_e,t_f]\{t_a,t_b\})$$

$$XXX = X_\nu^{\ jb} X^{\mu ke} X^{\nu kl}(2\lambda_2(a_{(kl)}[t_e,t_f] + b_{[kl]}\{t_e,t_f\} + \gamma_{[kl]}t_e t_f).$$
$$.(2a_{(ij)}[t_a,t_b] + (2b_{[ij]} + \gamma_{[ij]})\{t_a,t_b\}) +$$

$$+4\xi_2(a_{(kl)}[t_e,t_f]+b_{(kl)}\{t_e,t_f\}).(a_{[ij]}[t_a,t_b]+b_{(ij)}\{t_a,t_b\}))+$$

$$+X^{\mu jb}X_{\alpha}^{\ ke}X^{\alpha lf}(4\xi_2)(u_{[kl]}(a_{[ij]}[t_e,t_f][t_a,t_b]+b_{(ij)}[t_e,t_f]\{t_a,t_b\})+$$

$$+(a_{[kl]}+u_{[kl]}g_\nu^{\nu})(u_{[ij]}[t_e,t_f]\{t_a,t_b\}+v_{(ij)}[t_e,t_f]\{t_a,t_b\})+$$

$$+v_{(ij)}(a_{[ij]}\{t_e,t_f\}[t_a,t_b]+b_{(ij)}\{t_e,t_f\}\{t_a,t_b\})+$$

$$+(b_{(kl)}+v_{(kl)}g_\nu^{\nu})(u_{[ij]}\{t_e,t_f\}\{t_a,t_b\}+v_{(ij)}\{t_e,t_f\}\{t_a,t_b\})) \quad (195)$$

Thus depending on free coefficients expressions one can decouple the longitudinal sector. Given the model symmetry circumstance, one gets $(N-1)$ conserved currents

$$\partial_\mu J_{ia}^{\mu}(X)=\frac{1}{2}m_{ij}^2\partial_\mu X^{\mu j}=0 \quad (196)$$

16 Volume of circumstances

The volume of circumstances measure the number of invariant terms in the Lagrangian. It is an interesting property that fields association physics can offer. It relates the free coefficients associated to scalar terms as d^2, $d\alpha_i$, $\alpha_i\alpha_j$ and so on. Physically these free coefficients can take any value without violating gauge symmetry.

As an example, we are going to the case $L_A=trZ_{\mu\nu}Z^{\mu\nu}$, which yields th following volume of circumstance

$$\frac{5}{4}N^4-\frac{9}{2}N^3-\frac{33}{4}N^2-7N+3, \quad (197)$$

given by the structure

$$D_{\mu\nu a}D^{\mu\nu a}:1$$

$$D_{\mu\nu a}X^{[\mu\nu]ia}:(N-1)$$

$$X_{[\mu\nu]a}^{i}X^{[\mu\nu]ak}:(N-1)^2$$

$$D_{\mu\nu}^{a}X^{\mu bi}X^{\nu cj}:(N-1)^2$$

$$X_{[\mu\nu]}^{ak}X^{\mu bi}X^{\nu cj}:(N-1)^3$$

$$X_{\mu a}^{\ i}X_{\nu}^{aj}X_{b}^{\mu k}X^{\nu bl}:\left[\frac{(N-1)(N-2)}{2}\right]^2$$

$$X_{\mu}^{bi}X_{\nu}^{cj}X^{\mu sk}X^{\nu tl}:(N-1)^4 \quad (198)$$

It is still possible to rewrite some of these structures in more elementary terms

$$D_{\mu\nu}^{a}X^{\mu bi}X^{\nu cj}:$$

$$f_{abc}D_{\mu\nu}^{a}X^{\mu bi}X^{\nu cj}:\frac{N(N-1)}{2}$$

$$d_{abc}D_{\mu\nu}^{a}X^{\mu bi}X^{\nu cj}:\frac{(N-1)(N-2)}{2} \quad (199)$$

$$X_{[\mu\nu]}^{ak}X^{\mu bi}X^{\nu cj}:$$

$$f_{abc}X^{ak}_{[\mu\nu]}X^{\mu bi}X^{\nu cj}:\frac{N(N-1)^2}{2}$$

$$d_{abc}X^{ak}_{[\mu\nu]}X^{\mu bi}X^{\nu cj}:\frac{(N-1)^2(N-2)}{2} \tag{200}$$

$$X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:$$

$$f_{abc}f^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:\left[\frac{N(N-1)}{2}\right]^2$$

$$f_{abc}d^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:\frac{N(N-1)^2(N-2)}{4}$$

$$d_{abc}f^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:\frac{N(N-1)^2(N-2)}{4}$$

$$d_{abc}d^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:\left[\frac{(N-1)(N-2)}{2}\right]^2 \tag{201}$$

Similarly one gets for $L_S = tr(Z_{(\mu\nu)}Z^{(\mu\nu)}$ the following volume of circumstances

$$(5N^2-6N+4)(N-1)^2, \tag{202}$$

given by the structure

$$X^{i}_{(\mu\nu)a}X^{(\mu\nu)ak}:(N-1)^2$$

$$X^{\mu i}_{\mu a}X^{\nu ak}_{\nu}:2(N-1)^2$$

$$X^{ai}_{(\mu\nu)}X^{\mu bk}X^{\nu cl}:(N-1)^3$$

$$X^{\mu ai}_{\mu}X^{bk}_{\nu}X^{\nu cl}:3(N-1)^3$$

$$X^{i}_{\mu a}X^{aj}_{\nu}X^{\mu k}_{b}X^{\nu bl}:\left[\frac{N(N-1)}{2}\right]^2$$

$$X^{i}_{\mu a}X^{\mu aj}X^{k}_{\nu b}X^{\nu bl}:3\left[\frac{N(N-1)}{2}\right]^2$$

$$X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl}:(N-1)^4$$

$$X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu bl}:(N-1)^2(N-2)(3N-2) \tag{203}$$

It is still possible to rewrite some of these structures in more elementary terms

$$X^{ai}_{(\mu\nu)}X^{\mu bk}X^{\nu cl}:$$

$$f_{abc}X^{ai}_{(\mu\nu)}X^{\mu bk}X^{\nu cl}:\frac{(N-1)^2(N-2)}{2}$$

$$d_{abc}X^{ai}_{(\mu\nu)}X^{\mu bk}X^{\nu cl} : \frac{N(N-1)^2}{2} \quad (204)$$

$$X^{\mu ai}_{\mu}X^{bk}_{\nu}X^{\nu cl} :$$

$$f_{abc}X^{\mu ai}_{\mu}X^{bk}_{\nu}X^{\nu cl} : 3\frac{(N-1)^2(N-2)}{2}$$

$$d_{abc}X^{\mu ai}_{\mu}X^{bk}_{\nu}X^{\nu cl} : 3\frac{N(N-1)^2}{2} \quad (205)$$

$$X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl} :$$

$$f_{abc}f^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl} : \left[\frac{(N-1)(N-2)}{2}\right]^2$$

$$f_{abc}d^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl} : \frac{N(N-1)^2(N-2)}{4}$$

$$d_{abc}f^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl} : \frac{N(N-1)^2(N-2)}{4}$$

$$d_{abc}d^{a}_{st}X^{bi}_{\mu}X^{cj}_{\nu}X^{\mu sk}X^{\nu tl} : \left[\frac{N(N-1)}{2}\right]^2 \quad (206)$$

$$X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu tbl} :$$

$$f_{abc}f^{a}_{st}X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu tbl} : \frac{(N-1)^2(N-2)(3N-4)}{4}$$

$$f_{abc}d^{a}_{st}X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu tbl} : \frac{3N(N-1)^2(N-2)}{4}$$

$$d_{abc}f^{a}_{st}X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu tbl} : \frac{(N-1)^2(N-2)(3N-4)}{4}$$

$$d_{abc}d^{a}_{st}X^{bi}_{\mu}X^{\mu cj}X^{sk}_{\nu}X^{\nu tbl} : \frac{3N(N-1)^2(N-2)}{4} \quad (207)$$

With this, we have, in general, that the total volume of circumstance of the Lagrangian is

$$L : \frac{25}{4}N^4 - \frac{41}{2}N^3 + \frac{117}{4}N^2 - 21N + 7, \quad (208)$$

which outshines Yang-Mills ($N = 1$), $L : 1$ free coeficient.

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